

# Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.5-x<sup>m</sup>-trig-a+b-log-c-x<sup>n</sup>-<sup>p</sup>

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3.224	$\int \frac{\cot^4(a+b \log(cx^n))}{x} dx$	696
3.225	$\int \frac{\cot^5(a+b \log(cx^n))}{x} dx$	700
3.226	$\int (ex)^m \cot(d(a + b \log(cx^n))) dx$	706
3.227	$\int (ex)^m \cot^2(d(a + b \log(cx^n))) dx$	708
3.228	$\int (ex)^m \cot^3(d(a + b \log(cx^n))) dx$	711

3.229	$\int \cot^p (d(a + b \log(cx^n))) dx$	716
3.230	$\int (ex)^m \cot^p (d(a + b \log(cx^n))) dx$	718
3.231	$\int \frac{\cot^{\frac{5}{2}}(a+b \log(cx^n))}{x} dx$	720
3.232	$\int \frac{\cot^{\frac{3}{2}}(a+b \log(cx^n))}{x} dx$	724
3.233	$\int \frac{\sqrt{\cot(a+b \log(cx^n))}}{x} dx$	728
3.234	$\int \frac{1}{x\sqrt{\cot(a+b \log(cx^n))}} dx$	732
3.235	$\int \frac{1}{x \cot^{\frac{3}{2}}(a+b \log(cx^n))} dx$	736
3.236	$\int \frac{1}{x \cot^{\frac{5}{2}}(a+b \log(cx^n))} dx$	740
3.237	$\int x^2 \sec(a + b \log(cx^n)) dx$	744
3.238	$\int x \sec(a + b \log(cx^n)) dx$	747
3.239	$\int \sec(a + b \log(cx^n)) dx$	750
3.240	$\int \frac{\sec(a+b \log(cx^n))}{x} dx$	753
3.241	$\int \frac{\sec(a+b \log(cx^n))}{x^2} dx$	755
3.242	$\int \frac{\sec(a+b \log(cx^n))}{x^3} dx$	758
3.243	$\int x^2 \sec^2(a + b \log(cx^n)) dx$	761
3.244	$\int x \sec^2(a + b \log(cx^n)) dx$	764
3.245	$\int \sec^2(a + b \log(cx^n)) dx$	767
3.246	$\int \frac{\sec^2(a+b \log(cx^n))}{x} dx$	770
3.247	$\int \frac{\sec^2(a+b \log(cx^n))}{x^2} dx$	773
3.248	$\int \frac{\sec^2(a+b \log(cx^n))}{x^3} dx$	776
3.249	$\int x \sec^3(a + b \log(cx^n)) dx$	779
3.250	$\int \sec^3(a + b \log(cx^n)) dx$	782
3.251	$\int \frac{\sec^3(a+b \log(cx^n))}{x} dx$	785
3.252	$\int \frac{\sec^3(a+b \log(cx^n))}{x^2} dx$	788
3.253	$\int \frac{\sec^3(a+b \log(cx^n))}{x^3} dx$	791
3.254	$\int x \sec^4(a + b \log(cx^n)) dx$	794
3.255	$\int \sec^4(a + b \log(cx^n)) dx$	799
3.256	$\int \frac{\sec^4(a+b \log(cx^n))}{x} dx$	804
3.257	$\int \frac{\sec^4(a+b \log(cx^n))}{x^2} dx$	807
3.258	$\int \frac{\sec^4(a+b \log(cx^n))}{x^3} dx$	812
3.259	$\int \left( - (1 + b^2 n^2) \sec(a + b \log(cx^n)) + 2b^2 n^2 \sec^3(a + b \log(cx^n)) \right) dx$	817
3.260	$\int x^m \sec^3 \left( a + 2 \log \left( cx^{\frac{1}{2} \sqrt{-(1+m)^2}} \right) \right) dx$	821
3.261	$\int x \sec^3 \left( a + 2 \log \left( cx^i \right) \right) dx$	825
3.262	$\int \sec^3 \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) dx$	828
3.263	$\int \sec^3 \left( a + 2 \log \left( cx^{-\frac{i}{2}} \right) \right) dx$	831
3.264	$\int \sec^p \left( a + \frac{i \log(cx^n)}{n(-2+p)} \right) dx$	834
3.265	$\int \sec^p \left( a - \frac{i \log(cx^n)}{n(-2+p)} \right) dx$	837
3.266	$\int \sqrt{\sec(a + b \log(cx^n))} dx$	840
3.267	$\int \frac{\sqrt{\sec(a+b \log(cx^n))}}{x} dx$	843
3.268	$\int \sec^{\frac{3}{2}}(a + b \log(cx^n)) dx$	846
3.269	$\int \frac{\sec^{\frac{3}{2}}(a+b \log(cx^n))}{x} dx$	849
3.270	$\int \sec^{\frac{5}{2}}(a + b \log(cx^n)) dx$	852

3.271	$\int \frac{\sec^{\frac{5}{2}}(a+b \log (c x^n))}{x} d x$	855
3.272	$\int \frac{1}{\sqrt{\sec (a+b \log (c x^n))}} d x$	858
3.273	$\int \frac{1}{x \sqrt{\sec (a+b \log (c x^n))}} d x$	861
3.274	$\int \frac{1}{\sec^{\frac{3}{2}}(a+b \log (c x^n))} d x$	864
3.275	$\int \frac{1}{x \sec^{\frac{3}{2}}(a+b \log (c x^n))} d x$	867
3.276	$\int \frac{1}{\sec^{\frac{5}{2}}(a+b \log (c x^n))} d x$	870
3.277	$\int \frac{1}{x \sec^{\frac{5}{2}}(a+b \log (c x^n))} d x$	873
3.278	$\int x^m \sec^3(a+b \log (c x^n)) d x$	876
3.279	$\int x^m \sec^2(a+b \log (c x^n)) d x$	879
3.280	$\int x^m \sec(a+b \log (c x^n)) d x$	882
3.281	$\int x^m \sec^{\frac{5}{2}}(a+b \log (c x^n)) d x$	885
3.282	$\int x^m \sec^{\frac{3}{2}}(a+b \log (c x^n)) d x$	888
3.283	$\int x^m \sqrt{\sec (a+b \log (c x^n))} d x$	891
3.284	$\int \frac{x^m}{\sqrt{\sec (a+b \log (c x^n))}} d x$	894
3.285	$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b \log (c x^n))} d x$	897
3.286	$\int (e x)^m \sec^p(d(a+b \log (c x^n))) d x$	900
3.287	$\int x \sec^p(a+b \log (c x^n)) d x$	903
3.288	$\int \sec^p(a+b \log (c x^n)) d x$	906
3.289	$\int x^2 \csc (a+b \log (c x^n)) d x$	909
3.290	$\int x \csc (a+b \log (c x^n)) d x$	912
3.291	$\int \csc (a+b \log (c x^n)) d x$	915
3.292	$\int \frac{\csc (a+b \log (c x^n))}{x} d x$	918
3.293	$\int \frac{\csc (a+b \log (c x^n))}{x^2} d x$	921
3.294	$\int \frac{\csc (a+b \log (c x^n))}{x^3} d x$	924
3.295	$\int \csc^2(a+b \log (c x^n)) d x$	927
3.296	$\int \frac{\csc^2(a+b \log (c x^n))}{x} d x$	930
3.297	$\int \csc^3(a+b \log (c x^n)) d x$	933
3.298	$\int \frac{\csc^3(a+b \log (c x^n))}{x} d x$	938
3.299	$\int \csc^4(a+b \log (c x^n)) d x$	942
3.300	$\int \frac{\csc^4(a+b \log (c x^n))}{x} d x$	948
3.301	$\int \left(-\left(1+b^2 n^2\right) \csc (a+b \log (c x^n))+2 b^2 n^2 \csc ^3(a+b \log (c x^n))\right) d x$	951
3.302	$\int x^m \csc ^3\left(a+2 \log \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right)\right) d x$	955
3.303	$\int x \csc ^3\left(a+2 \log \left(c x^i\right)\right) d x$	959
3.304	$\int \csc ^3\left(a+2 \log \left(c x^{\frac{i}{2}}\right)\right) d x$	962
3.305	$\int \csc ^3\left(a+2 \log \left(c x^{-\frac{i}{2}}\right)\right) d x$	965
3.306	$\int \csc ^p\left(a+\frac{i \log (c x^n)}{n(-2+p)}\right) d x$	968
3.307	$\int \csc ^p\left(a-\frac{i \log (c x^n)}{n(-2+p)}\right) d x$	971
3.308	$\int \sqrt{\csc (a+b \log (c x^n))} d x$	974
3.309	$\int \frac{\sqrt{\csc (a+b \log (c x^n))}}{x} d x$	977
3.310	$\int \csc ^{\frac{3}{2}}(a+b \log (c x^n)) d x$	980
3.311	$\int \frac{\csc ^{\frac{3}{2}}(a+b \log (c x^n))}{x} d x$	983
3.312	$\int \csc ^{\frac{5}{2}}(a+b \log (c x^n)) d x$	986



3.313	$\int \frac{\csc^{\frac{5}{2}}(a+b \log(cx^n))}{x} dx$	989
3.314	$\int \frac{1}{\sqrt{\csc(a+b \log(cx^n))}} dx$	992
3.315	$\int \frac{1}{x \sqrt{\csc(a+b \log(cx^n))}} dx$	995
3.316	$\int \frac{1}{\csc^{\frac{3}{2}}(a+b \log(cx^n))} dx$	998
3.317	$\int \frac{1}{x \csc^{\frac{3}{2}}(a+b \log(cx^n))} dx$	1001
3.318	$\int \frac{1}{\csc^{\frac{5}{2}}(a+b \log(cx^n))} dx$	1004
3.319	$\int \frac{1}{x \csc^{\frac{5}{2}}(a+b \log(cx^n))} dx$	1007
3.320	$\int (ex)^m \csc^3(d(a+b \log(cx^n))) dx$	1010
3.321	$\int (ex)^m \csc^2(d(a+b \log(cx^n))) dx$	1016
3.322	$\int (ex)^m \csc(d(a+b \log(cx^n))) dx$	1019
3.323	$\int x^m \csc^{\frac{5}{2}}(a+b \log(cx^n)) dx$	1022
3.324	$\int x^m \csc^{\frac{3}{2}}(a+b \log(cx^n)) dx$	1025
3.325	$\int x^m \sqrt{\csc(a+b \log(cx^n))} dx$	1028
3.326	$\int \frac{x^m}{\sqrt{\csc(a+b \log(cx^n))}} dx$	1031
3.327	$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b \log(cx^n))} dx$	1034
3.328	$\int (ex)^m \csc^p(d(a+b \log(cx^n))) dx$	1037
3.329	$\int x \csc^p(a+b \log(cx^n)) dx$	1040
3.330	$\int \csc^p(a+b \log(cx^n)) dx$	1043

#### 4 Listing of Grading functions

1047



# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 330 ]. This is test number [ 139 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 76.97 ( 254 )	% 23.03 ( 76 )
Mathematica	% 92.42 ( 305 )	% 7.58 ( 25 )
Maple	% 30.3 ( 100 )	% 69.7 ( 230 )
Maxima	% 42.12 ( 139 )	% 57.88 ( 191 )
Fricas	% 36.06 ( 119 )	% 63.94 ( 211 )
Sympy	% 16.67 ( 55 )	% 83.33 ( 275 )
Giac	% 23.03 ( 76 )	% 76.97 ( 254 )

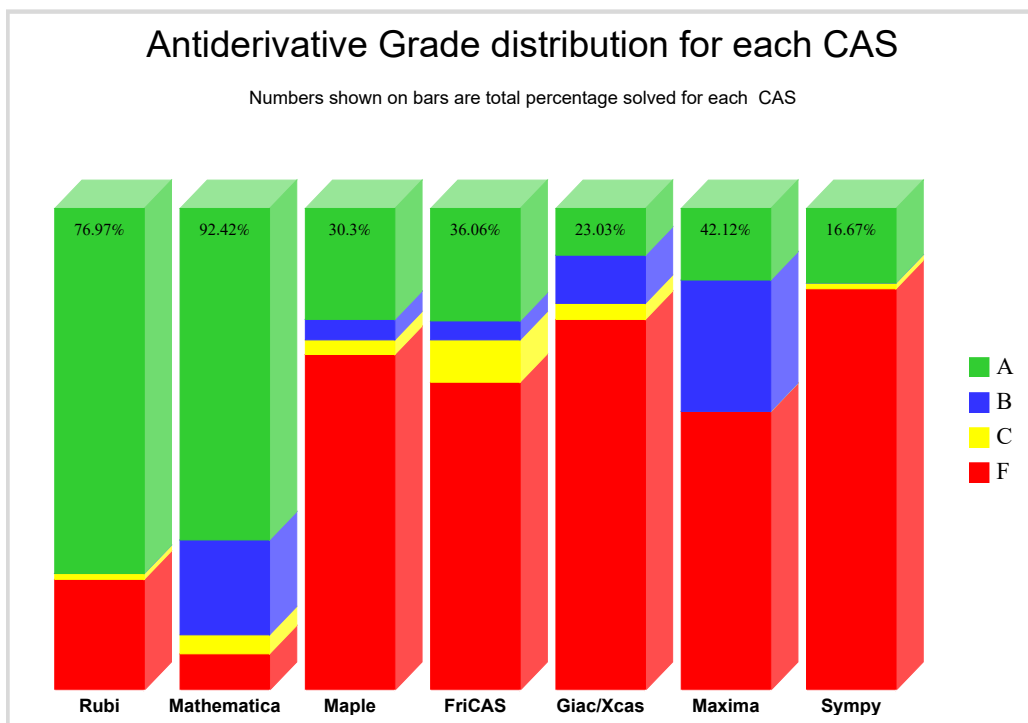
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

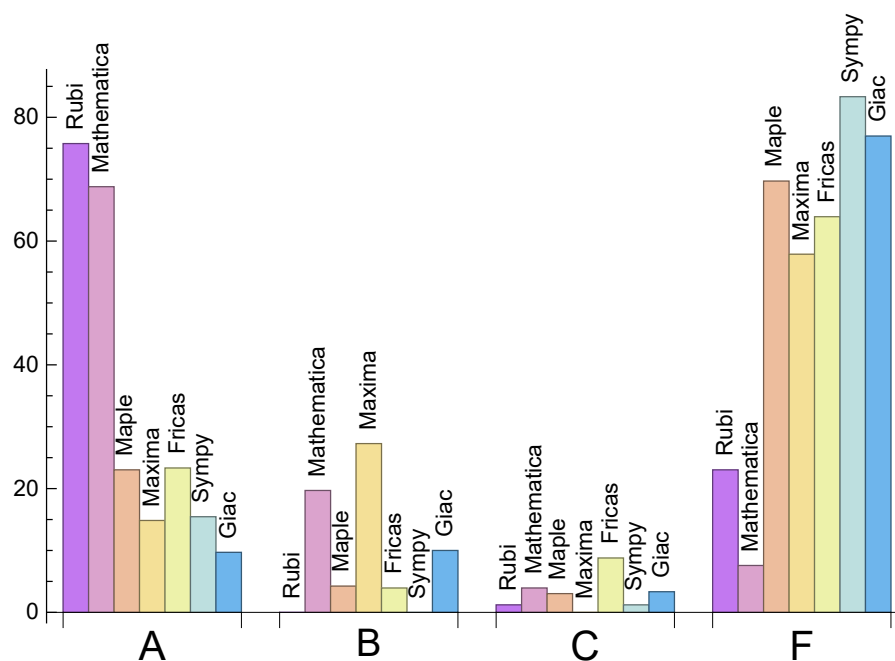
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	75.76	0.	1.21	23.03
Mathematica	68.79	19.7	3.94	7.58
Maple	23.03	4.24	3.03	69.7
Maxima	14.85	27.27	0.	57.88
Fricas	23.33	3.94	8.79	63.94
Sympy	15.45	0.	1.21	83.33
Giac	9.7	10.	3.33	76.97

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	101.85	1.02	98.	1.
Mathematica	2.03	150.93	1.56	122.	1.08
Maple	0.43	114.12	2.04	75.5	1.32
Maxima	1.26	671.3	9.3	261.	4.4
Fricas	0.49	224.48	2.99	188.	2.29
Sympy	17.07	90.	1.75	51.	1.11
Giac	2.1	604.83	7.79	112.	2.31

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {259, 260, 262, 301, 302, 304}

**Mathematica** {53, 54, 56, 57, 58, 59, 61, 62, 63, 65, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 110, 112, 114, 116, 118, 120, 127, 128, 129, 130, 131, 132, 133, 134, 153, 155, 156, 157, 177, 178, 204, 206, 207, 208, 228, 229, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 266, 268, 270, 272, 274, 276, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 295, 297, 299, 306, 307, 308, 310, 312, 314, 316, 318, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1

```

For Sympy, called directly from Python, the following code is used

```

try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1

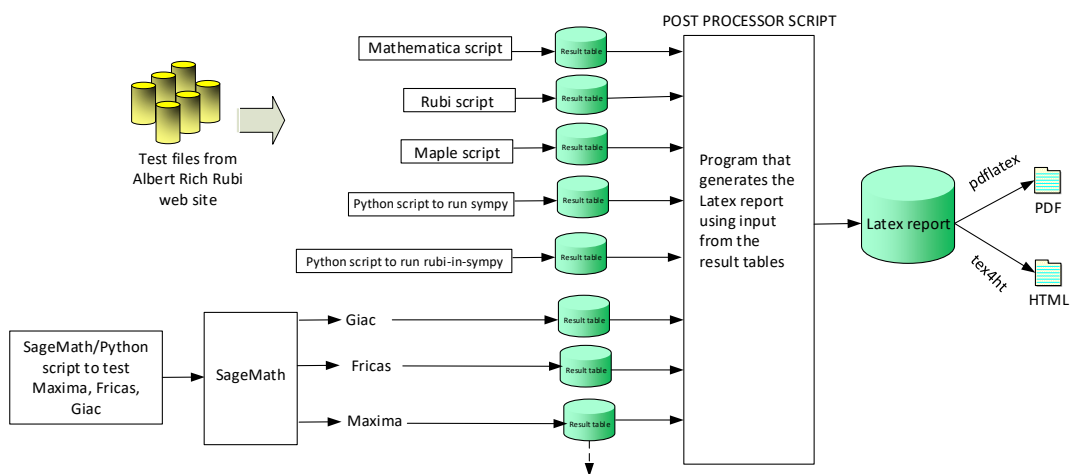
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.





**One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)**

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

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June 22, 2018



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 139, 147, 162, 169, 172, 173, 174, 180, 181, 182, 183, 184, 185, 190, 198, 213, 220, 223, 224, 225, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

B grade: { }

C grade: { 259, 260, 301, 302 }

F grade: { 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230 }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 37, 40, 44, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 111, 112, 113, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 129, 131, 132, 133, 134, 136, 138, 139, 140, 142, 144, 146, 147, 148, 150, 152, 154, 155, 156, 157, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 179, 181, 183, 187, 189, 190, 191, 193, 195, 197, 199, 201, 203, 205, 206, 207, 208, 213, 216, 217, 218, 219, 221, 222, 223, 225, 226, 228, 230, 232, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 256, 259, 260, 264, 265, 266, 267, 269, 270, 271, 273, 274, 275, 277, 278, 280, 281, 283, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 300, 301, 302, 306, 307, 308, 309, 311, 312, 313, 315, 316, 317, 319, 322, 323, 325, 327, 328, 329, 330 }

B grade: { 75, 77, 89, 110, 114, 118, 128, 130, 135, 137, 141, 143, 145, 149, 151, 153, 158, 159, 160, 161, 163, 164, 176, 178, 186, 188, 192, 194, 196, 200, 202, 204, 209, 210, 211, 212, 214, 215, 227, 229, 254, 255, 257, 258, 261, 262, 263, 268, 272, 276, 279, 282, 284, 292, 299, 303, 304, 305, 310, 314, 318, 320, 321, 324, 326 }

C grade: { 72, 125, 180, 182, 184, 185, 198, 220, 224, 231, 233, 235, 236 }

F grade: { 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 41, 42, 43, 45, 46, 47, 49, 51, 104, 105, 106, 107, 109 }

### 2.1.3 Maple

A grade: { 4, 10, 16, 22, 30, 37, 44, 55, 60, 64, 66, 68, 89, 94, 99, 102, 119, 135, 137, 139, 141, 143, 145, 147, 149, 162, 169, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 213, 223, 224, 225, 231, 232, 233, 234, 235, 236, 240, 246, 251, 256, 269, 292, 296, 298, 300, 309, 311, 313, 315, 317, 319 }

B grade: { 25, 48, 50, 52, 111, 113, 115, 121, 220, 267, 271, 273, 275, 277 }

C grade: { 103, 117, 259, 261, 262, 263, 301, 303, 304, 305 }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 23, 24, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49, 51, 53, 54, 56, 57, 58, 59, 61, 62, 63, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 100, 101, 104, 105, 106, 107, 108, 109, 110, 112, 114, 116, 118, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 138, 140, 142, 144, 146, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 260, 264, 265, 266, 268, 270, 272, 274, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 302, 306, 307, 308, 310, 312, 314, 316, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

### 2.1.4 Maxima

A grade: { 4, 10, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 89, 94, 102, 103, 104, 105, 106, 107, 108, 109, 139, 147, 162, 190, 198, 213, 240, 292 }

B grade: { 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 69, 72, 73, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 122, 123, 124, 125, 126, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 148, 169, 172, 173, 174, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 199, 220, 223, 224, 225, 246, 256, 259, 260, 261, 262, 263, 296, 298, 300, 301, 302, 303, 304, 305 }

C grade: { }

F grade: { 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 37, 44, 69, 70, 71, 72, 73, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 122, 123, 124, 125, 126, 139, 162, 172, 190, 213, 223, 246, 251, 256, 259, 261, 262, 263, 264, 296, 300, 301, 303, 304, 305, 306 }

B grade: { 147, 169, 173, 174, 198, 220, 224, 225, 240, 265, 292, 298, 307 }

C grade: { 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49, 51, 104, 105, 106, 107, 108, 109, 260, 302 }

F grade: { 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

## 2.1.6 Sympy

A grade: { 4, 5, 6, 10, 12, 16, 22, 25, 30, 37, 44, 89, 90, 94, 99, 102, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 162, 172, 173, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 213, 224, 240, 292 }

B grade: { }

C grade: { 31, 32, 38, 39 }

F grade: { 1, 2, 3, 7, 8, 9, 11, 13, 14, 15, 17, 18, 19, 20, 21, 23, 24, 26, 27, 28, 29, 33, 34, 35, 36, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 95, 96, 97, 98, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 139, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 190, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

## 2.1.7 Giac

A grade: { 25, 27, 28, 29, 30, 34, 35, 36, 37, 44, 48, 50, 103, 105, 107, 135, 136, 137, 138, 139, 140, 141, 142, 148, 186, 187, 188, 193, 195, 199, 262, 304 }

B grade: { 1, 2, 3, 7, 8, 9, 73, 86, 87, 88, 91, 92, 93, 126, 143, 144, 145, 146, 147, 149, 169, 173, 189, 190, 191, 192, 194, 196, 197, 198, 200, 263, 305 }

C grade: { 26, 33, 40, 47, 49, 51, 104, 106, 108, 260, 302 }

F grade: { 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 38, 39, 41, 42, 43, 45, 46, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 94, 95, 96, 97, 98, 99, 100, 101, 102, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	44	0	296	130	0	1246
normalized size	1	1.	0.77	0.	5.19	2.28	0.	21.86
time (sec)	N/A	0.017	0.075	0.039	1.201	0.484	0.	1.315

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	44	0	296	130	0	1246
normalized size	1	1.	0.77	0.	5.19	2.28	0.	21.86
time (sec)	N/A	0.013	0.059	0.033	1.235	0.483	0.	1.262

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	40	0	278	122	0	1191
normalized size	1	1.	0.77	0.	5.35	2.35	0.	22.9
time (sec)	N/A	0.011	0.051	0.033	1.2	0.488	0.	1.192

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	38	20	26	53	39	0
normalized size	1	1.	2.	1.05	1.37	2.79	2.05	0.
time (sec)	N/A	0.015	0.027	0.017	1.139	0.495	3.066	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	40	0	282	122	286	0
normalized size	1	1.	0.7	0.	4.95	2.14	5.02	0.
time (sec)	N/A	0.018	0.058	0.036	1.238	0.494	32.247	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	44	0	292	127	352	0
normalized size	1	1.	0.77	0.	5.12	2.23	6.18	0.
time (sec)	N/A	0.015	0.061	0.032	1.213	0.489	59.515	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	61	0	406	215	0	1125
normalized size	1	1.	0.63	0.	4.19	2.22	0.	11.6
time (sec)	N/A	0.031	0.143	0.078	1.299	0.493	0.	1.492

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	0	381	209	0	1107
normalized size	1	1.	0.58	0.	3.89	2.13	0.	11.3
time (sec)	N/A	0.022	0.11	0.069	1.161	0.49	0.	1.447

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	0	378	198	0	1061
normalized size	1	1.	0.64	0.	4.3	2.25	0.	12.06
time (sec)	N/A	0.019	0.09	0.068	1.252	0.496	0.	1.388

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	74	119	56	0
normalized size	1	1.	0.92	1.33	1.9	3.05	1.44	0.
time (sec)	N/A	0.03	0.071	0.022	1.147	0.491	23.001	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	57	0	382	193	0	0
normalized size	1	1.	0.6	0.	4.02	2.03	0.	0.
time (sec)	N/A	0.026	0.098	0.059	1.133	0.496	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	58	0	378	198	643	0
normalized size	1	1.	0.59	0.	3.86	2.02	6.56	0.
time (sec)	N/A	0.026	0.101	0.058	1.151	0.499	82.082	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	122	0	1361	343	0	0
normalized size	1	1.	0.76	0.	8.51	2.14	0.	0.
time (sec)	N/A	0.055	0.507	0.065	1.23	0.51	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	125	0	1372	350	0	0
normalized size	1	1.	0.79	0.	8.68	2.22	0.	0.
time (sec)	N/A	0.045	0.476	0.063	1.43	0.52	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	121	0	1337	329	0	0
normalized size	1	1.	0.81	0.	8.97	2.21	0.	0.
time (sec)	N/A	0.037	0.483	0.069	1.245	0.504	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	45	35	315	109	83	0
normalized size	1	1.	1.05	0.81	7.33	2.53	1.93	0.
time (sec)	N/A	0.032	0.059	0.025	1.103	0.492	55.585	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	125	0	1343	321	0	0
normalized size	1	1.	0.79	0.	8.5	2.03	0.	0.
time (sec)	N/A	0.047	0.335	0.063	1.286	0.51	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	125	0	1359	333	0	0
normalized size	1	1.	0.79	0.	8.6	2.11	0.	0.
time (sec)	N/A	0.048	0.385	0.063	1.24	0.51	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	171	0	1494	454	0	0
normalized size	1	1.	0.85	0.	7.4	2.25	0.	0.
time (sec)	N/A	0.078	0.496	0.098	1.269	0.518	0.	0.



Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	169	0	1465	443	0	0
normalized size	1	1.	0.8	0.	6.98	2.11	0.	0.
time (sec)	N/A	0.061	0.436	0.079	1.411	0.518	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	168	0	1455	429	0	0
normalized size	1	1.	0.88	0.	7.62	2.25	0.	0.
time (sec)	N/A	0.051	0.401	0.083	1.325	0.521	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	84	126	177	110	0
normalized size	1	1.	0.7	1.15	1.73	2.42	1.51	0.
time (sec)	N/A	0.049	0.091	0.027	1.142	0.501	169.546	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	170	0	1465	420	0	0
normalized size	1	1.	0.84	0.	7.25	2.08	0.	0.
time (sec)	N/A	0.066	0.5	0.075	1.371	0.523	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	169	0	1461	423	0	0
normalized size	1	1.	0.8	0.	6.96	2.01	0.	0.
time (sec)	N/A	0.063	0.457	0.069	1.321	0.53	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	76	36	92	56	47
normalized size	1	1.	0.74	1.95	0.92	2.36	1.44	1.21
time (sec)	N/A	0.014	0.016	0.024	1.117	0.483	2.02	1.143

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	0	0	111	159	0	367
normalized size	1	1.	0.	0.	0.83	1.2	0.	2.76
time (sec)	N/A	0.277	0.253	0.059	1.199	0.497	0.	1.813

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	42	108	0	1
normalized size	1	1.	0.	0.	0.48	1.23	0.	0.01
time (sec)	N/A	0.099	0.146	0.037	1.095	0.468	0.	1.464

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	42	107	0	1
normalized size	1	1.	0.	0.	0.48	1.22	0.	0.01
time (sec)	N/A	0.052	0.116	0.035	1.212	0.471	0.	1.495

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	0	0	39	101	0	1
normalized size	1	1.	0.	0.	0.48	1.23	0.	0.01
time (sec)	N/A	0.052	0.086	0.034	1.17	0.466	0.	1.277

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	20	5	8
normalized size	1	1.	1.	1.2	1.4	4.	1.	1.6
time (sec)	N/A	0.004	0.001	0.012	1.064	0.431	0.279	1.14

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	C	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	45	107	214	0
normalized size	1	1.	0.	0.	0.52	1.24	2.49	0.
time (sec)	N/A	0.061	0.072	0.038	1.131	0.472	14.281	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	C	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	47	112	240	0
normalized size	1	1.	0.	0.	0.53	1.27	2.73	0.
time (sec)	N/A	0.053	0.09	0.035	1.242	0.472	71.619	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	234	316	0	672
normalized size	1	1.	0.	0.	2.	2.7	0.	5.74
time (sec)	N/A	0.159	0.356	0.086	1.373	0.496	0.	2.53

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F(-1)	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	63	157	0	1
normalized size	1	1.	0.	0.	0.83	2.07	0.	0.01
time (sec)	N/A	0.076	0.251	0.08	1.273	0.471	0.	4.354

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	63	146	0	1
normalized size	1	1.	0.	0.	0.83	1.92	0.	0.01
time (sec)	N/A	0.058	0.152	0.076	1.116	0.475	0.	1.824

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	0	0	55	144	0	1
normalized size	1	1.	0.	0.	0.81	2.12	0.	0.01
time (sec)	N/A	0.055	0.101	0.079	1.165	0.469	0.	1.585

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	32	7	11
normalized size	1	1.	1.	1.14	1.29	4.57	1.	1.57
time (sec)	N/A	0.006	0.001	0.012	1.093	0.437	0.339	1.107

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	C	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	65	150	240	0
normalized size	1	1.	0.	0.	0.88	2.03	3.24	0.
time (sec)	N/A	0.068	0.149	0.065	1.164	0.471	75.767	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	C	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	73	151	464	0
normalized size	1	1.	0.	0.	0.96	1.99	6.11	0.
time (sec)	N/A	0.062	0.129	0.061	1.101	0.47	55.452	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	169	0	263	387	0	2525
normalized size	1	1.	0.75	0.	1.16	1.71	0.	11.17
time (sec)	N/A	0.079	1.188	0.084	1.298	0.503	0.	3.681

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	0	0	122	207	0	0
normalized size	1	1.	0.	0.	0.71	1.2	0.	0.
time (sec)	N/A	0.161	0.216	0.072	1.199	0.47	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	0	0	151	250	0	0
normalized size	1	1.	0.	0.	0.85	1.4	0.	0.
time (sec)	N/A	0.111	0.246	0.07	1.152	0.48	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F(-1)	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	0	0	143	238	0	0
normalized size	1	1.	0.	0.	0.85	1.42	0.	0.
time (sec)	N/A	0.105	0.133	0.069	1.159	0.475	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	42	7	11
normalized size	1	1.	1.	1.14	1.29	6.	1.	1.57
time (sec)	N/A	0.005	0.001	0.013	0.994	0.432	0.389	1.107

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	0	0	165	244	0	0
normalized size	1	1.	0.	0.	0.94	1.39	0.	0.
time (sec)	N/A	0.132	0.17	0.069	1.15	0.479	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	0	0	173	255	0	0
normalized size	1	1.	0.	0.	0.97	1.43	0.	0.
time (sec)	N/A	0.114	0.205	0.072	1.143	0.48	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	0	0	65	149	0	255
normalized size	1	1.	0.	0.	0.58	1.33	0.	2.28
time (sec)	N/A	0.194	0.23	0.064	1.062	0.486	0.	1.469

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	44	106	42	0	0	39
normalized size	1	1.	0.85	2.04	0.81	0.	0.	0.75
time (sec)	N/A	0.035	0.059	0.039	1.009	0.	0.	1.185

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	181	252	0	473
normalized size	1	1.	0.	0.	1.71	2.38	0.	4.46
time (sec)	N/A	0.145	0.317	0.061	1.062	0.487	0.	1.895

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	173	65	0	0	43
normalized size	1	1.	1.13	3.26	1.23	0.	0.	0.81
time (sec)	N/A	0.045	0.1	0.069	1.022	0.	0.	1.269

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	218	218	0	0	278	350	0	1751
normalized size	1	1.	0.	0.	1.28	1.61	0.	8.03
time (sec)	N/A	0.305	0.429	0.06	1.082	0.503	0.	2.456

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	103	284	101	0	0	0
normalized size	1	1.	1.05	2.9	1.03	0.	0.	0.
time (sec)	N/A	0.061	0.121	0.081	1.086	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	94	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	1.394	0.349	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	96	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	1.329	0.169	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	32	129	0	0	0	0
normalized size	1	1.	1.1	4.45	0.	0.	0.	0.
time (sec)	N/A	0.027	0.083	0.938	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	99	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	1.402	0.168	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	95	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	1.364	0.169	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	159	0	0	0	0	0
normalized size	1	1.	1.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	1.777	0.167	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	161	0	0	0	0	0
normalized size	1	1.	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	1.798	0.166	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	58	131	0	0	0	0
normalized size	1	1.	0.85	1.93	0.	0.	0.	0.
time (sec)	N/A	0.042	0.142	1.328	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	172	0	0	0	0	0
normalized size	1	1.	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	1.158	0.166	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	168	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	1.199	0.16	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	96	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.382	0.168	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	32	102	0	0	0	0
normalized size	1	1.	1.1	3.52	0.	0.	0.	0.
time (sec)	N/A	0.027	0.091	0.692	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	96	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.918	0.168	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	190	0	0	0	0
normalized size	1	1.	0.89	2.97	0.	0.	0.	0.
time (sec)	N/A	0.042	0.184	1.102	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	125	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	1.507	0.167	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	131	0	0	0	0
normalized size	1	1.	0.9	1.93	0.	0.	0.	0.
time (sec)	N/A	0.042	0.214	1.235	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	81	0	543	146	0	0
normalized size	1	1.	1.65	0.	11.08	2.98	0.	0.
time (sec)	N/A	0.039	0.132	0.415	2.442	0.459	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	341	0	0	1083	0	0
normalized size	1	1.	1.01	0.	0.	3.21	0.	0.
time (sec)	N/A	0.17	1.909	0.115	0.	0.59	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	326	0	0	695	0	0
normalized size	1	1.	1.27	0.	0.	2.71	0.	0.
time (sec)	N/A	0.118	1.208	0.092	0.	0.549	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	102	0	3444	401	0	0
normalized size	1	1.	0.66	0.	22.36	2.6	0.	0.
time (sec)	N/A	0.055	0.287	0.089	1.537	0.511	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	63	0	1705	238	0	7776
normalized size	1	1.	0.68	0.	18.53	2.59	0.	84.52
time (sec)	N/A	0.025	0.135	0.05	1.264	0.496	0.	1.751

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	145	235	0	0	0	0	0
normalized size	1	0.97	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	1.973	0.431	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	145	488	0	0	0	0	0
normalized size	1	0.97	3.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	5.461	0.269	0.	0.	0.	0.



Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	131	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.517	0.263	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	145	544	0	0	0	0	0
normalized size	1	0.97	3.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	5.175	0.261	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	145	214	0	0	0	0	0
normalized size	1	0.97	1.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	2.412	0.261	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	122	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.932	0.283	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	100	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.647	0.146	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	98	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.59	0.117	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	98	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.538	0.125	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.15	0.13	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	102	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.594	0.135	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	100	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.628	0.117	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	43	0	294	128	0	1246
normalized size	1	1.	0.77	0.	5.25	2.29	0.	22.25
time (sec)	N/A	0.018	0.072	0.039	1.054	0.496	0.	1.304

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	43	0	294	128	0	1235
normalized size	1	1.	0.77	0.	5.25	2.29	0.	22.05
time (sec)	N/A	0.012	0.064	0.036	1.053	0.485	0.	1.277

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	39	0	277	120	0	1185
normalized size	1	1.	0.76	0.	5.43	2.35	0.	23.24
time (sec)	N/A	0.009	0.047	0.045	1.063	0.484	0.	1.17

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	24	51	37	0
normalized size	1	1.	2.06	1.06	1.33	2.83	2.06	0.
time (sec)	N/A	0.015	0.027	0.021	0.989	0.483	1.394	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	41	0	281	120	287	0
normalized size	1	1.	0.73	0.	5.02	2.14	5.12	0.
time (sec)	N/A	0.015	0.057	0.036	1.098	0.487	25.175	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	61	0	406	205	0	1125
normalized size	1	1.	0.63	0.	4.19	2.11	0.	11.6
time (sec)	N/A	0.03	0.144	0.082	1.111	0.491	0.	1.488

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	54	0	381	200	0	1107
normalized size	1	1.	0.55	0.	3.89	2.04	0.	11.3
time (sec)	N/A	0.023	0.097	0.066	1.113	0.493	0.	1.526

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	54	0	378	189	0	1061
normalized size	1	1.	0.61	0.	4.3	2.15	0.	12.06
time (sec)	N/A	0.016	0.08	0.069	1.117	0.49	0.	1.378

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	72	119	56	0
normalized size	1	1.	0.92	1.33	1.85	3.05	1.44	0.
time (sec)	N/A	0.029	0.067	0.027	1.107	0.495	10.133	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	57	0	385	188	0	0
normalized size	1	1.	0.6	0.	4.05	1.98	0.	0.
time (sec)	N/A	0.027	0.117	0.057	1.373	0.504	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	120	0	1359	321	0	0
normalized size	1	1.	0.75	0.	8.49	2.01	0.	0.
time (sec)	N/A	0.051	0.525	0.073	1.266	0.511	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	123	0	1370	327	0	0
normalized size	1	1.	0.78	0.	8.67	2.07	0.	0.
time (sec)	N/A	0.045	0.487	0.065	1.23	0.508	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	117	0	1335	308	0	0
normalized size	1	1.	0.79	0.	8.96	2.07	0.	0.
time (sec)	N/A	0.036	0.414	0.095	1.227	0.501	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	313	109	82	0
normalized size	1	1.	1.	0.83	7.45	2.6	1.95	0.
time (sec)	N/A	0.033	0.056	0.033	1.118	0.489	27.563	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	122	0	1342	304	0	0
normalized size	1	1.	0.77	0.	8.49	1.92	0.	0.
time (sec)	N/A	0.048	0.452	0.066	1.235	0.509	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	167	0	1455	381	0	0
normalized size	1	1.	0.87	0.	7.62	1.99	0.	0.
time (sec)	N/A	0.045	0.435	0.09	1.273	0.515	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	84	126	177	110	0
normalized size	1	1.	0.7	1.15	1.73	2.42	1.51	0.
time (sec)	N/A	0.044	0.101	0.027	1.144	0.5	121.549	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	34	27	85	0	34
normalized size	1	1.	0.76	1.17	0.93	2.93	0.	1.17
time (sec)	N/A	0.014	0.012	0.037	0.968	0.47	0.	1.144

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	111	150	0	360
normalized size	1	1.	0.	0.	1.1	1.49	0.	3.56
time (sec)	N/A	0.146	0.253	0.054	1.239	0.487	0.	1.854

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	0	0	39	96	0	1
normalized size	1	1.	0.	0.	0.63	1.55	0.	0.02
time (sec)	N/A	0.045	0.086	0.117	1.117	0.472	0.	1.284

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	232	315	0	672
normalized size	1	1.	0.	0.	1.98	2.69	0.	5.74
time (sec)	N/A	0.117	0.34	0.082	1.269	0.5	0.	2.713

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	0	0	55	143	0	1
normalized size	1	1.	0.	0.	0.81	2.1	0.	0.01
time (sec)	N/A	0.056	0.101	0.079	1.291	0.472	0.	1.619

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	158	0	263	379	0	2525
normalized size	1	1.	0.7	0.	1.16	1.68	0.	11.17
time (sec)	N/A	0.082	1.352	0.094	1.329	0.5	0.	4.34

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	C	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	143	227	0	0
normalized size	1	1.	0.	0.	1.12	1.77	0.	0.
time (sec)	N/A	0.096	0.139	0.083	1.144	0.478	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	377	0	0	0	0	0
normalized size	1	1.	3.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	3.427	0.205	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	181	0	0	0	0
normalized size	1	1.	1.	7.54	0.	0.	0.	0.
time (sec)	N/A	0.027	0.093	2.044	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	163	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	1.668	0.162	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	54	247	0	0	0	0
normalized size	1	1.	0.86	3.92	0.	0.	0.	0.
time (sec)	N/A	0.043	0.113	2.182	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	696	0	0	0	0	0
normalized size	1	1.	6.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	7.171	0.151	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	280	0	0	0	0
normalized size	1	1.	0.92	4.44	0.	0.	0.	0.
time (sec)	N/A	0.042	0.133	2.823	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	99	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.38	0.16	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	0	0	0	0
normalized size	1	1.	1.	1.08	0.	0.	0.	0.
time (sec)	N/A	0.028	0.085	0.037	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	431	0	0	0	0	0
normalized size	1	1.	3.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	3.613	0.153	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	139	0	0	0	0
normalized size	1	1.	0.92	2.36	0.	0.	0.	0.
time (sec)	N/A	0.041	0.153	2.377	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	147	0	0	0	0	0
normalized size	1	1.	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	1.102	0.163	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	54	291	0	0	0	0
normalized size	1	1.	0.86	4.62	0.	0.	0.	0.
time (sec)	N/A	0.044	0.146	2.319	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	82	0	252	103	0	0
normalized size	1	1.	1.71	0.	5.25	2.15	0.	0.
time (sec)	N/A	0.036	0.115	0.343	1.805	0.457	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	260	312	0	4775	667	0	0
normalized size	1	0.98	1.17	0.	17.95	2.51	0.	0.
time (sec)	N/A	0.125	4.035	0.096	2.065	0.562	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	292	0	3175	467	0	0
normalized size	1	1.	1.45	0.	15.8	2.32	0.	0.
time (sec)	N/A	0.078	1.882	0.082	1.596	0.532	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	91	0	872	275	0	0
normalized size	1	1.	0.76	0.	7.27	2.29	0.	0.
time (sec)	N/A	0.032	0.32	0.07	1.259	0.501	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	53	0	423	158	0	6969
normalized size	1	1.	0.76	0.	6.04	2.26	0.	99.56
time (sec)	N/A	0.016	0.13	0.042	1.177	0.484	0.	1.692

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	204	0	0	0	0	0
normalized size	1	0.97	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	2.009	0.174	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	126	436	0	0	0	0	0
normalized size	1	0.98	3.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	5.238	0.2	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.584	0.178	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	487	0	0	0	0	0
normalized size	1	0.97	3.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	5.059	0.188	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	205	0	0	0	0	0
normalized size	1	0.97	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	2.21	0.168	0.	0.	0.	0.



Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	123	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.985	0.185	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	102	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.633	0.127	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	102	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.557	0.113	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	47	0	132	49	122	0	37	46
normalized size	1	0.	2.81	1.04	2.6	0.	0.79	0.98
time (sec)	N/A	0.03	0.032	0.07	1.058	0.	0.683	1.172

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	43	0	66	0	204	0	44	35
normalized size	1	0.	1.53	0.	4.74	0.	1.02	0.81
time (sec)	N/A	0.023	0.018	0.068	1.633	0.	0.697	1.17

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	33	0	114	37	99	0	26	34
normalized size	1	0.	3.45	1.12	3.	0.	0.79	1.03
time (sec)	N/A	0.017	0.022	0.062	1.058	0.	0.518	1.193

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	42	0	165	0	27	41
normalized size	1	0.	1.56	0.	6.11	0.	1.	1.52
time (sec)	N/A	0.007	0.007	0.058	1.708	0.	0.502	1.145

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	14	59	0	14
normalized size	1	1.	1.	1.21	1.	4.21	0.	1.
time (sec)	N/A	0.013	0.021	0.013	0.998	0.485	0.	1.141

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	29	0	44	0	171	0	27	38
normalized size	1	0.	1.52	0.	5.9	0.	0.93	1.31
time (sec)	N/A	0.028	0.024	0.059	1.633	0.	0.457	1.176

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	35	0	132	43	130	0	39	45
normalized size	1	0.	3.77	1.23	3.71	0.	1.11	1.29
time (sec)	N/A	0.027	0.032	0.057	1.088	0.	0.696	1.137

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	70	0	212	0	53	38
normalized size	1	0.	1.56	0.	4.71	0.	1.18	0.84
time (sec)	N/A	0.027	0.025	0.058	1.675	0.	1.257	1.142

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	63	0	155	62	312	0	54	352
normalized size	1	0.	2.46	0.98	4.95	0.	0.86	5.59
time (sec)	N/A	0.07	0.167	0.078	1.159	0.	0.672	1.251

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	62	0	100	0	363	0	66	190
normalized size	1	0.	1.61	0.	5.85	0.	1.06	3.06
time (sec)	N/A	0.05	0.12	0.078	1.645	0.	0.655	1.243

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	51	0	135	50	261	0	42	298
normalized size	1	0.	2.65	0.98	5.12	0.	0.82	5.84
time (sec)	N/A	0.032	0.114	0.072	1.027	0.	0.742	1.286

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	46	0	70	0	305	0	51	154
normalized size	1	0.	1.52	0.	6.63	0.	1.11	3.35
time (sec)	N/A	0.01	0.089	0.061	1.599	0.	0.61	1.187

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	28	23	23	97	22	130
normalized size	1	1.	1.56	1.28	1.28	5.39	1.22	7.22
time (sec)	N/A	0.025	0.039	0.014	1.509	0.463	0.53	1.163

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	60	0	72	0	312	0	51	99
normalized size	1	0.	1.2	0.	5.2	0.	0.85	1.65
time (sec)	N/A	0.049	0.109	0.05	1.619	0.	0.673	1.217

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	F(-2)	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	55	0	150	61	0	0	60	240
normalized size	1	0.	2.73	1.11	0.	0.	1.09	4.36
time (sec)	N/A	0.053	0.17	0.065	0.	0.	1.046	1.216

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	71	0	124	0	0	0	0	0
normalized size	1	0.	1.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.185	0.201	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	77	0	172	0	0	0	0	0
normalized size	1	0.	2.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.414	0.054	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	184	0	255	0	0	0	0	0
normalized size	1	0.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.93	0.067	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-1)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	142	0	330	0	0	0	0	0
normalized size	1	0.	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.665	0.374	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	162	0	157	0	0	0	0	0
normalized size	1	0.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.648	0.411	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	120	0	240	0	0	0	0	0
normalized size	1	0.	2.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.491	0.339	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F(-1)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	120	0	240	0	0	0	0	0
normalized size	1	0.	2.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.495	0.322	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F(-1)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	120	0	240	0	0	0	0	0
normalized size	1	0.	2.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.489	0.318	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	71	0	146	0	0	0	0	0
normalized size	1	0.	2.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	6.508	1.566	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	75	0	155	0	0	0	0	0
normalized size	1	0.	2.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	6.065	1.357	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	69	0	146	0	0	0	0	0
normalized size	1	0.	2.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	6.198	1.217	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	67	0	151	0	0	0	0	0
normalized size	1	0.	2.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	10.97	1.083	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	30	32	97	44	0
normalized size	1	1.	0.96	1.15	1.23	3.73	1.69	0.
time (sec)	N/A	0.018	0.047	0.014	0.991	0.493	10.905	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	71	0	153	0	0	0	0	0
normalized size	1	0.	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	4.034	1.364	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	69	0	147	0	0	0	0	0
normalized size	1	0.	2.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	3.703	1.454	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	159	0	179	0	0	0	0	0
normalized size	1	0.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	6.586	1.464	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	163	0	189	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	6.292	1.36	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	159	0	179	0	0	0	0	0
normalized size	1	0.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	6.549	1.244	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	154	0	185	0	0	0	0	0
normalized size	1	0.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	11.231	1.099	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	51	50	432	242	0	135
normalized size	1	1.	1.76	1.72	14.9	8.34	0.	4.66
time (sec)	N/A	0.029	0.082	0.019	1.332	0.479	0.	2.286

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	157	0	184	0	0	0	0	0
normalized size	1	0.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	4.315	1.37	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	156	0	179	0	0	0	0	0
normalized size	1	0.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	4.195	1.565	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	38	47	1677	209	70	0
normalized size	1	1.	0.88	1.09	39.	4.86	1.63	0.
time (sec)	N/A	0.034	0.156	0.018	1.142	0.508	18.474	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	62	61	2931	416	66	1430
normalized size	1	1.	1.38	1.36	65.13	9.24	1.47	31.78
time (sec)	N/A	0.038	0.101	0.018	1.357	0.486	32.256	5.261

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	55	68	6029	387	0	0
normalized size	1	1.	0.82	1.01	89.99	5.78	0.	0.
time (sec)	N/A	0.044	0.163	0.017	1.53	0.508	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	101	0	186	0	0	0	0	0
normalized size	1	0.	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	14.507	1.721	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	196	0	550	0	0	0	0	0
normalized size	1	0.	2.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	17.453	1.703	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	351	0	642	0	0	0	0	0
normalized size	1	0.	1.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	17.782	1.796	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-1)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	190	0	458	0	0	0	0	0
normalized size	1	0.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.414	0.177	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	210	0	205	0	0	0	0	0
normalized size	1	0.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	1.139	0.189	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	50	161	0	0	0	0
normalized size	1	1.	0.25	0.8	0.	0.	0.	0.
time (sec)	N/A	0.139	0.259	0.051	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	175	161	0	0	0	0
normalized size	1	1.	0.88	0.81	0.	0.	0.	0.
time (sec)	N/A	0.128	0.243	0.031	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	48	140	0	0	0	0
normalized size	1	1.	0.27	0.8	0.	0.	0.	0.
time (sec)	N/A	0.12	0.097	0.03	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	142	140	0	0	0	0
normalized size	1	1.	0.81	0.8	0.	0.	0.	0.
time (sec)	N/A	0.121	0.135	0.031	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	46	161	0	0	0	0
normalized size	1	1.	0.23	0.81	0.	0.	0.	0.
time (sec)	N/A	0.134	0.118	0.037	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	48	161	0	0	0	0
normalized size	1	1.	0.24	0.8	0.	0.	0.	0.
time (sec)	N/A	0.128	0.209	0.038	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	49	0	137	65	184	0	39	68
normalized size	1	0.	2.8	1.33	3.76	0.	0.8	1.39
time (sec)	N/A	0.026	0.034	0.079	1.086	0.	0.607	1.552

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	43	0	66	62	176	0	46	63
normalized size	1	0.	1.53	1.44	4.09	0.	1.07	1.47
time (sec)	N/A	0.022	0.018	0.089	1.187	0.	0.498	1.465



Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	35	0	118	53	154	0	27	55
normalized size	1	0.	3.37	1.51	4.4	0.	0.77	1.57
time (sec)	N/A	0.015	0.023	0.069	1.099	0.	0.505	1.379

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	42	44	132	0	29	51
normalized size	1	0.	1.56	1.63	4.89	0.	1.07	1.89
time (sec)	N/A	0.007	0.008	0.071	1.057	0.	0.495	1.342

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	25	17	14	61	0	78
normalized size	1	1.	1.79	1.21	1.	4.36	0.	5.57
time (sec)	N/A	0.013	0.025	0.017	1.122	0.483	0.	1.23

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	29	0	44	47	139	0	29	54
normalized size	1	0.	1.52	1.62	4.79	0.	1.	1.86
time (sec)	N/A	0.025	0.023	0.06	1.048	0.	0.474	1.264

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	36	0	136	59	188	0	39	66
normalized size	1	0.	3.78	1.64	5.22	0.	1.08	1.83
time (sec)	N/A	0.024	0.03	0.066	1.389	0.	0.697	1.318

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	70	59	192	0	54	66
normalized size	1	0.	1.56	1.31	4.27	0.	1.2	1.47
time (sec)	N/A	0.025	0.022	0.061	1.142	0.	0.626	1.377

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	67	0	162	77	489	0	54	188
normalized size	1	0.	2.42	1.15	7.3	0.	0.81	2.81
time (sec)	N/A	0.065	0.175	0.09	1.197	0.	0.75	1.433

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	64	0	100	75	475	0	60	112
normalized size	1	0.	1.56	1.17	7.42	0.	0.94	1.75
time (sec)	N/A	0.051	0.127	0.086	1.235	0.	0.949	1.315

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	55	0	142	65	400	0	42	159
normalized size	1	0.	2.58	1.18	7.27	0.	0.76	2.89
time (sec)	N/A	0.036	0.124	0.083	1.088	0.	0.603	1.319

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	48	0	70	56	375	0	42	107
normalized size	1	0.	1.46	1.17	7.81	0.	0.88	2.23
time (sec)	N/A	0.011	0.085	0.073	1.243	0.	0.48	1.338

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	34	27	26	97	20	43
normalized size	1	1.	1.89	1.5	1.44	5.39	1.11	2.39
time (sec)	N/A	0.024	0.05	0.018	1.725	0.476	0.539	1.318

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	B	F	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	64	0	72	62	385	0	44	117
normalized size	1	0.	1.12	0.97	6.02	0.	0.69	1.83
time (sec)	N/A	0.048	0.121	0.069	1.154	0.	1.009	1.256

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	A	F(-2)	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	57	0	153	76	0	0	60	257
normalized size	1	0.	2.68	1.33	0.	0.	1.05	4.51
time (sec)	N/A	0.051	0.221	0.074	0.	0.	1.178	1.344

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	70	0	103	0	0	0	0	0
normalized size	1	0.	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.241	0.158	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	77	0	169	0	0	0	0	0
normalized size	1	0.	2.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.43	0.06	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	169	0	250	0	0	0	0	0
normalized size	1	0.	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.951	0.072	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	142	0	330	0	0	0	0	0
normalized size	1	0.	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.594	0.372	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	162	0	157	0	0	0	0	0
normalized size	1	0.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.638	0.409	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	120	0	238	0	0	0	0	0
normalized size	1	0.	1.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.479	0.322	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	120	0	238	0	0	0	0	0
normalized size	1	0.	1.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	0.465	0.357	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	120	0	238	0	0	0	0	0
normalized size	1	0.	1.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.465	0.336	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	70	0	220	0	0	0	0	0
normalized size	1	0.	3.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	5.4	1.831	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	74	0	229	0	0	0	0	0
normalized size	1	0.	3.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	5.741	1.595	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	68	0	219	0	0	0	0	0
normalized size	1	0.	3.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	5.719	1.45	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	66	0	141	0	0	0	0	0
normalized size	1	0.	2.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	10.645	1.309	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	40	30	32	97	46	0
normalized size	1	1.	1.6	1.2	1.28	3.88	1.84	0.
time (sec)	N/A	0.018	0.059	0.019	0.971	0.503	5.006	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	70	0	217	0	0	0	0	0
normalized size	1	0.	3.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	4.675	1.589	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	68	0	211	0	0	0	0	0
normalized size	1	0.	3.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	4.313	1.779	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	158	0	175	0	0	0	0	0
normalized size	1	0.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	4.991	1.815	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	162	0	185	0	0	0	0	0
normalized size	1	0.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	5.329	1.632	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	158	0	175	0	0	0	0	0
normalized size	1	0.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	5.13	1.48	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	153	0	178	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	9.637	1.293	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	51	63	435	216	0	0
normalized size	1	1.	1.7	2.1	14.5	7.2	0.	0.
time (sec)	N/A	0.03	0.11	0.022	1.23	0.491	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	156	0	181	0	0	0	0	0
normalized size	1	0.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	4.361	1.628	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F(-1)	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	155	0	175	0	0	0	0	0
normalized size	1	0.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	4.028	1.863	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	52	47	2313	212	0	0
normalized size	1	1.	1.18	1.07	52.57	4.82	0.	0.
time (sec)	N/A	0.037	0.224	0.021	1.304	0.498	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	46	69	2932	385	66	0
normalized size	1	1.	1.05	1.57	66.64	8.75	1.5	0.
time (sec)	N/A	0.039	0.116	0.021	1.412	0.49	15.288	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	68	8097	387	0	0
normalized size	1	1.	1.05	1.03	122.68	5.86	0.	0.
time (sec)	N/A	0.046	0.231	0.025	1.888	0.501	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	100	0	182	0	0	0	0	0
normalized size	1	0.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	13.969	1.946	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	195	0	547	0	0	0	0	0
normalized size	1	0.	2.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	16.599	1.967	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F(-2)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	350	0	639	0	0	0	0	0
normalized size	1	0.	1.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	17.039	2.033	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F(-2)
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	190	0	458	0	0	0	0	0
normalized size	1	0.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	1.303	0.233	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F(-2)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	210	0	205	0	0	0	0	0
normalized size	1	0.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	1.127	0.213	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	50	161	0	0	0	0
normalized size	1	1.	0.25	0.8	0.	0.	0.	0.
time (sec)	N/A	0.139	0.268	0.062	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	175	161	0	0	0	0
normalized size	1	1.	0.88	0.81	0.	0.	0.	0.
time (sec)	N/A	0.131	0.291	0.034	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	48	140	0	0	0	0
normalized size	1	1.	0.27	0.8	0.	0.	0.	0.
time (sec)	N/A	0.121	0.098	0.032	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	142	140	0	0	0	0
normalized size	1	1.	0.81	0.8	0.	0.	0.	0.
time (sec)	N/A	0.128	0.156	0.035	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	46	161	0	0	0	0
normalized size	1	1.	0.23	0.81	0.	0.	0.	0.
time (sec)	N/A	0.132	0.145	0.038	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	48	161	0	0	0	0
normalized size	1	1.	0.24	0.8	0.	0.	0.	0.
time (sec)	N/A	0.134	0.216	0.038	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.168	0.275	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.136	0.228	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.112	0.208	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	42	130	51	0
normalized size	1	1.	1.	1.68	2.21	6.84	2.68	0.
time (sec)	N/A	0.016	0.04	0.022	1.074	0.499	4.007	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.142	0.262	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.145	0.313	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	160	0	0	0	0	0
normalized size	1	1.	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	5.531	1.485	0.	0.	0.	0.



Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	149	0	0	0	0	0
normalized size	1	1.	1.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	5.346	1.363	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	147	0	0	0	0	0
normalized size	1	1.	1.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	6.407	1.22	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	223	93	0	0
normalized size	1	1.	1.	1.06	12.39	5.17	0.	0.
time (sec)	N/A	0.028	0.091	0.033	1.167	0.467	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	160	0	0	0	0	0
normalized size	1	1.	1.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	3.881	1.473	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	150	0	0	0	0	0
normalized size	1	1.	1.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	3.718	1.615	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	118	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	4.746	2.13	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	120	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	4.418	1.74	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	64	0	311	0	0
normalized size	1	1.	1.	1.16	0.	5.65	0.	0.
time (sec)	N/A	0.038	0.073	0.039	0.	0.508	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	123	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	4.629	2.144	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	119	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	4.656	2.151	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	204	0	0	0	0	0
normalized size	1	1.	2.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	12.915	1.415	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	213	0	0	0	0	0
normalized size	1	1.	2.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	10.798	1.325	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	37	1786	157	0	0
normalized size	1	1.	0.86	0.88	42.52	3.74	0.	0.
time (sec)	N/A	0.034	0.116	0.043	1.16	0.472	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	215	0	0	0	0	0
normalized size	1	1.	2.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	9.452	1.551	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	203	0	0	0	0	0
normalized size	1	1.	2.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	9.379	1.699	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	C	B	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	41	175	29	537	2290	146	0	0
normalized size	1	4.27	0.71	13.1	55.85	3.56	0.	0.
time (sec)	N/A	0.133	0.655	0.48	2.875	0.474	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	B	C	F(-1)	C
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	110	146	198	0	1318	231	0	1126
normalized size	1	1.33	1.8	0.	11.98	2.1	0.	10.24
time (sec)	N/A	0.217	2.111	0.228	1.426	0.477	0.	15.441

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	127	215	189	127	0	0
normalized size	1	1.	2.82	4.78	4.2	2.82	0.	0.
time (sec)	N/A	0.043	0.151	0.199	1.12	0.449	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	58	48	137	214	208	151	0	100
normalized size	1	0.83	2.36	3.69	3.59	2.6	0.	1.72
time (sec)	N/A	0.035	0.114	0.204	1.245	0.449	0.	4.758

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	139	238	224	158	0	112
normalized size	1	1.	2.9	4.96	4.67	3.29	0.	2.33
time (sec)	N/A	0.041	0.139	0.204	1.233	0.45	0.	4.556

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	67	0	0	340	0	0
normalized size	1	1.	0.71	0.	0.	3.58	0.	0.
time (sec)	N/A	0.091	0.858	0.372	0.	0.506	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	0	0	346	0	0
normalized size	1	1.	0.89	0.	0.	4.94	0.	0.
time (sec)	N/A	0.075	0.792	0.322	0.	0.503	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	99	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.482	0.357	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	181	0	0	0	0
normalized size	1	1.	1.	3.35	0.	0.	0.	0.
time (sec)	N/A	0.043	0.116	1.984	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	415	0	0	0	0	0
normalized size	1	1.	3.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	5.862	0.273	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	139	0	0	0	0
normalized size	1	1.	0.76	1.56	0.	0.	0.	0.
time (sec)	N/A	0.061	0.138	2.643	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	1.351	0.275	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	69	291	0	0	0	0
normalized size	1	1.	0.74	3.13	0.	0.	0.	0.
time (sec)	N/A	0.061	0.156	2.697	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	380	0	0	0	0	0
normalized size	1	1.	3.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	4.315	0.283	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	181	0	0	0	0
normalized size	1	1.	1.	3.35	0.	0.	0.	0.
time (sec)	N/A	0.044	0.105	1.849	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	168	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	1.595	0.283	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	72	247	0	0	0	0
normalized size	1	1.	0.77	2.66	0.	0.	0.	0.
time (sec)	N/A	0.06	0.133	2.337	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	867	0	0	0	0	0
normalized size	1	1.	7.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	8.653	0.289	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	83	280	0	0	0	0
normalized size	1	1.	0.89	3.01	0.	0.	0.	0.
time (sec)	N/A	0.061	0.175	2.312	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	134	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	5.658	2.368	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	482	0	0	0	0	0
normalized size	1	1.	4.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	17.541	1.779	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	99	94	0	0	0	0	0
normalized size	1	0.96	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.212	0.349	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	182	0	0	0	0	0
normalized size	1	0.97	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	2.137	0.287	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	470	0	0	0	0	0
normalized size	1	0.97	3.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	9.581	0.295	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.784	0.306	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	126	437	0	0	0	0	0
normalized size	1	0.98	3.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	6.987	0.268	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	202	0	0	0	0	0
normalized size	1	0.97	1.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	2.516	0.288	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	133	169	0	0	0	0	0
normalized size	1	0.96	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	1.559	0.27	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	142	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.963	0.212	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	142	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.781	0.178	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	1.498	0.384	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	78	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	1.473	0.31	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	80	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	1.271	0.259	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	54	33	43	147	49	0
normalized size	1	1.	2.7	1.65	2.15	7.35	2.45	0.
time (sec)	N/A	0.016	0.057	0.027	0.996	0.497	2.881	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	1.076	0.377	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	1.092	0.454	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	146	0	0	0	0	0
normalized size	1	1.	1.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	5.209	1.417	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	227	95	0	0
normalized size	1	1.	1.	1.05	11.95	5.	0.	0.
time (sec)	N/A	0.028	0.096	0.034	1.209	0.467	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	117	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	5.616	2.277	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	107	66	2927	352	0	0
normalized size	1	1.	1.95	1.2	53.22	6.4	0.	0.
time (sec)	N/A	0.04	0.089	0.046	1.292	0.503	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	782	0	0	0	0	0
normalized size	1	1.	9.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	13.475	1.441	0.	0.	0.	0.



Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	56	36	1798	211	0	0
normalized size	1	1.	1.3	0.84	41.81	4.91	0.	0.
time (sec)	N/A	0.034	0.088	0.049	1.181	0.473	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	C	B	A	F(-1)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	42	172	30	535	2296	154	0	0
normalized size	1	4.1	0.71	12.74	54.67	3.67	0.	0.
time (sec)	N/A	0.127	0.418	0.479	2.035	0.474	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	B	C	F(-1)	C
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	110	142	79	0	1315	236	0	1133
normalized size	1	1.29	0.72	0.	11.95	2.15	0.	10.3
time (sec)	N/A	0.184	2.08	0.181	1.446	0.489	0.	14.201

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	127	215	192	131	0	0
normalized size	1	1.	2.59	4.39	3.92	2.67	0.	0.
time (sec)	N/A	0.042	0.179	0.126	1.124	0.442	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	58	51	137	215	215	155	0	100
normalized size	1	0.88	2.36	3.71	3.71	2.67	0.	1.72
time (sec)	N/A	0.035	0.147	0.136	1.182	0.447	0.	4.537

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	137	239	224	161	0	112
normalized size	1	1.	2.69	4.69	4.39	3.16	0.	2.2
time (sec)	N/A	0.04	0.147	0.144	1.218	0.45	0.	4.751

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	155	0	0	343	0	0
normalized size	1	1.	1.61	0.	0.	3.57	0.	0.
time (sec)	N/A	0.088	2.069	0.278	0.	0.51	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	128	0	0	350	0	0
normalized size	1	1.	1.8	0.	0.	4.93	0.	0.
time (sec)	N/A	0.076	3.005	0.278	0.	0.499	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	115	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.645	0.364	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	102	0	0	0	0
normalized size	1	1.	0.98	1.73	0.	0.	0.	0.
time (sec)	N/A	0.041	0.117	0.799	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	411	0	0	0	0	0
normalized size	1	1.	3.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	6.058	0.295	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	72	190	0	0	0	0
normalized size	1	1.	0.77	2.02	0.	0.	0.	0.
time (sec)	N/A	0.055	0.152	1.276	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	174	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	1.824	0.296	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	73	131	0	0	0	0
normalized size	1	1.	0.74	1.34	0.	0.	0.	0.
time (sec)	N/A	0.058	0.186	1.118	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	377	0	0	0	0	0
normalized size	1	1.	3.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	4.037	0.319	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	129	0	0	0	0
normalized size	1	1.	0.98	2.19	0.	0.	0.	0.
time (sec)	N/A	0.04	0.108	1.305	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	186	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	2.311	0.299	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	76	131	0	0	0	0
normalized size	1	1.	0.78	1.34	0.	0.	0.	0.
time (sec)	N/A	0.059	0.166	1.423	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	876	0	0	0	0	0
normalized size	1	1.	7.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	8.759	0.3	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	88	205	0	0	0	0
normalized size	1	1.	0.9	2.09	0.	0.	0.	0.
time (sec)	N/A	0.058	0.22	1.183	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	367	0	0	0	0	0
normalized size	1	1.	3.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	2.285	5.298	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	534	0	0	0	0	0
normalized size	1	1.	4.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	6.543	2.25	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	118	181	0	0	0	0	0
normalized size	1	0.96	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.426	0.94	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	165	0	0	0	0	0
normalized size	1	0.97	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	2.955	0.389	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	466	0	0	0	0	0
normalized size	1	0.97	3.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	9.531	0.298	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.969	0.289	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	126	441	0	0	0	0	0
normalized size	1	0.98	3.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	7.348	0.279	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	126	218	0	0	0	0	0
normalized size	1	0.97	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	2.407	0.307	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	133	169	0	0	0	0	0
normalized size	1	0.96	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	1.693	0.283	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	142	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	1.118	0.232	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	142	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.87	0.208	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [180] had the largest ratio of [ 0.4737 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	15	0.067
2	A	1	1	1.	13	0.077
3	A	1	1	1.	11	0.091
4	A	2	1	1.	15	0.067
5	A	1	1	1.	15	0.067
6	A	1	1	1.	15	0.067
7	A	2	2	1.	17	0.118
8	A	2	2	1.	15	0.133
9	A	2	2	1.	13	0.154
10	A	3	2	1.	17	0.118
11	A	2	2	1.	17	0.118
12	A	2	2	1.	17	0.118
13	A	2	2	1.	17	0.118
14	A	2	2	1.	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	2	2	1.	13	0.154
16	A	3	1	1.	17	0.059
17	A	2	2	1.	17	0.118
18	A	2	2	1.	17	0.118
19	A	3	2	1.	17	0.118
20	A	3	2	1.	15	0.133
21	A	3	2	1.	13	0.154
22	A	4	2	1.	17	0.118
23	A	3	2	1.	17	0.118
24	A	3	2	1.	17	0.118
25	A	2	1	1.	7	0.143
26	A	3	2	1.	28	0.071
27	A	3	2	1.	24	0.083
28	A	3	2	1.	22	0.091
29	A	3	2	1.	19	0.105
30	A	2	2	1.	6	0.333
31	A	3	2	1.	23	0.087
32	A	3	2	1.	24	0.083
33	A	3	2	1.	33	0.061
34	A	3	2	1.	28	0.071
35	A	3	2	1.	23	0.087
36	A	3	2	1.	24	0.083
37	A	2	2	1.	8	0.25
38	A	3	2	1.	28	0.071
39	A	3	2	1.	25	0.08
40	A	2	2	1.	33	0.061
41	A	3	2	1.	25	0.08
42	A	3	2	1.	26	0.077
43	A	3	2	1.	24	0.083
44	A	2	2	1.	8	0.25
45	A	3	2	1.	28	0.071
46	A	3	2	1.	28	0.071
47	A	3	2	1.	28	0.071
48	A	3	2	1.	15	0.133
49	A	3	2	1.	30	0.067
50	A	3	2	1.	17	0.118
51	A	3	2	1.	30	0.067
52	A	3	2	1.	17	0.118
53	A	3	3	1.	17	0.176
54	A	3	3	1.	15	0.2
55	A	2	1	1.	19	0.053
56	A	3	3	1.	19	0.158
57	A	3	3	1.	19	0.158
58	A	3	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	3	3	1.	15	0.2
60	A	3	2	1.	19	0.105
61	A	3	3	1.	19	0.158
62	A	3	3	1.	19	0.158
63	A	3	3	1.	15	0.2
64	A	2	1	1.	19	0.053
65	A	3	3	1.	15	0.2
66	A	3	2	1.	19	0.105
67	A	3	3	1.	15	0.2
68	A	3	2	1.	19	0.105
69	A	3	3	1.	15	0.2
70	A	3	2	1.	21	0.095
71	A	2	2	1.	21	0.095
72	A	2	2	1.	21	0.095
73	A	1	1	1.	19	0.053
74	A	3	3	0.97	23	0.13
75	A	3	3	0.97	23	0.13
76	A	3	3	1.	23	0.13
77	A	3	3	0.97	23	0.13
78	A	3	3	0.97	23	0.13
79	A	3	3	1.	21	0.143
80	A	3	3	1.	17	0.176
81	A	3	3	1.	15	0.2
82	A	3	3	1.	13	0.231
83	A	2	1	1.	17	0.059
84	A	3	3	1.	17	0.176
85	A	3	3	1.	17	0.176
86	A	1	1	1.	15	0.067
87	A	1	1	1.	13	0.077
88	A	1	1	1.	11	0.091
89	A	2	1	1.	15	0.067
90	A	1	1	1.	15	0.067
91	A	2	2	1.	17	0.118
92	A	2	2	1.	15	0.133
93	A	2	2	1.	13	0.154
94	A	3	2	1.	17	0.118
95	A	2	2	1.	17	0.118
96	A	2	2	1.	17	0.118
97	A	2	2	1.	15	0.133
98	A	2	2	1.	13	0.154
99	A	3	1	1.	17	0.059
100	A	2	2	1.	17	0.118
101	A	3	2	1.	13	0.154
102	A	4	2	1.	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
103	A	2	1	1.	7	0.143
104	A	3	2	1.	28	0.071
105	A	3	2	1.	19	0.105
106	A	3	2	1.	33	0.061
107	A	3	2	1.	24	0.083
108	A	2	2	1.	33	0.061
109	A	3	2	1.	24	0.083
110	A	3	3	1.	15	0.2
111	A	2	1	1.	19	0.053
112	A	3	3	1.	15	0.2
113	A	3	2	1.	19	0.105
114	A	3	3	1.	15	0.2
115	A	3	2	1.	19	0.105
116	A	3	3	1.	15	0.2
117	A	2	1	1.	19	0.053
118	A	3	3	1.	15	0.2
119	A	3	2	1.	19	0.105
120	A	3	3	1.	15	0.2
121	A	3	2	1.	19	0.105
122	A	3	3	1.	15	0.2
123	A	3	2	0.98	17	0.118
124	A	2	2	1.	17	0.118
125	A	2	2	1.	17	0.118
126	A	1	1	1.	15	0.067
127	A	3	3	0.97	19	0.158
128	A	3	3	0.98	19	0.158
129	A	3	3	1.	19	0.158
130	A	3	3	0.97	19	0.158
131	A	3	3	0.97	19	0.158
132	A	3	3	1.	21	0.143
133	A	3	3	1.	15	0.2
134	A	3	3	1.	13	0.231
135	F	0	0	N/A	0	N/A
136	F	0	0	N/A	0	N/A
137	F	0	0	N/A	0	N/A
138	F	0	0	N/A	0	N/A
139	A	2	1	1.	13	0.077
140	F	0	0	N/A	0	N/A
141	F	0	0	N/A	0	N/A
142	F	0	0	N/A	0	N/A
143	F	0	0	N/A	0	N/A
144	F	0	0	N/A	0	N/A
145	F	0	0	N/A	0	N/A
146	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	3	2	1.	15	0.133
148	F	0	0	N/A	0	N/A
149	F	0	0	N/A	0	N/A
150	F	0	0	N/A	0	N/A
151	F	0	0	N/A	0	N/A
152	F	0	0	N/A	0	N/A
153	F	0	0	N/A	0	N/A
154	F	0	0	N/A	0	N/A
155	F	0	0	N/A	0	N/A
156	F	0	0	N/A	0	N/A
157	F	0	0	N/A	0	N/A
158	F	0	0	N/A	0	N/A
159	F	0	0	N/A	0	N/A
160	F	0	0	N/A	0	N/A
161	F	0	0	N/A	0	N/A
162	A	2	1	1.	17	0.059
163	F	0	0	N/A	0	N/A
164	F	0	0	N/A	0	N/A
165	F	0	0	N/A	0	N/A
166	F	0	0	N/A	0	N/A
167	F	0	0	N/A	0	N/A
168	F	0	0	N/A	0	N/A
169	A	3	2	1.	19	0.105
170	F	0	0	N/A	0	N/A
171	F	0	0	N/A	0	N/A
172	A	3	2	1.	17	0.118
173	A	4	2	1.	17	0.118
174	A	4	2	1.	17	0.118
175	F	0	0	N/A	0	N/A
176	F	0	0	N/A	0	N/A
177	F	0	0	N/A	0	N/A
178	F	0	0	N/A	0	N/A
179	F	0	0	N/A	0	N/A
180	A	13	9	1.	19	0.474
181	A	13	9	1.	19	0.474
182	A	12	8	1.	19	0.421
183	A	12	8	1.	19	0.421
184	A	13	9	1.	19	0.474
185	A	13	9	1.	19	0.474
186	F	0	0	N/A	0	N/A
187	F	0	0	N/A	0	N/A
188	F	0	0	N/A	0	N/A
189	F	0	0	N/A	0	N/A
190	A	2	1	1.	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
191	F	0	0	N/A	0	N/A
192	F	0	0	N/A	0	N/A
193	F	0	0	N/A	0	N/A
194	F	0	0	N/A	0	N/A
195	F	0	0	N/A	0	N/A
196	F	0	0	N/A	0	N/A
197	F	0	0	N/A	0	N/A
198	A	3	2	1.	15	0.133
199	F	0	0	N/A	0	N/A
200	F	0	0	N/A	0	N/A
201	F	0	0	N/A	0	N/A
202	F	0	0	N/A	0	N/A
203	F	0	0	N/A	0	N/A
204	F	0	0	N/A	0	N/A
205	F	0	0	N/A	0	N/A
206	F	0	0	N/A	0	N/A
207	F	0	0	N/A	0	N/A
208	F	0	0	N/A	0	N/A
209	F	0	0	N/A	0	N/A
210	F	0	0	N/A	0	N/A
211	F	0	0	N/A	0	N/A
212	F	0	0	N/A	0	N/A
213	A	2	1	1.	17	0.059
214	F	0	0	N/A	0	N/A
215	F	0	0	N/A	0	N/A
216	F	0	0	N/A	0	N/A
217	F	0	0	N/A	0	N/A
218	F	0	0	N/A	0	N/A
219	F	0	0	N/A	0	N/A
220	A	3	2	1.	19	0.105
221	F	0	0	N/A	0	N/A
222	F	0	0	N/A	0	N/A
223	A	3	2	1.	17	0.118
224	A	4	2	1.	17	0.118
225	A	4	2	1.	17	0.118
226	F	0	0	N/A	0	N/A
227	F	0	0	N/A	0	N/A
228	F	0	0	N/A	0	N/A
229	F	0	0	N/A	0	N/A
230	F	0	0	N/A	0	N/A
231	A	13	9	1.	19	0.474
232	A	13	9	1.	19	0.474
233	A	12	8	1.	19	0.421
234	A	12	8	1.	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	13	9	1.	19	0.474
236	A	13	9	1.	19	0.474
237	A	3	3	1.	15	0.2
238	A	3	3	1.	13	0.231
239	A	3	3	1.	11	0.273
240	A	2	1	1.	15	0.067
241	A	3	3	1.	15	0.2
242	A	3	3	1.	15	0.2
243	A	3	3	1.	17	0.176
244	A	3	3	1.	15	0.2
245	A	3	3	1.	13	0.231
246	A	3	2	1.	17	0.118
247	A	3	3	1.	17	0.176
248	A	3	3	1.	17	0.176
249	A	3	3	1.	15	0.2
250	A	3	3	1.	13	0.231
251	A	3	2	1.	17	0.118
252	A	3	3	1.	17	0.176
253	A	3	3	1.	17	0.176
254	A	3	3	1.	15	0.2
255	A	3	3	1.	13	0.231
256	A	3	1	1.	17	0.059
257	A	3	3	1.	17	0.176
258	A	3	3	1.	17	0.176
259	C	7	3	4.27	44	0.068
260	C	3	3	1.33	31	0.097
261	A	3	3	1.	17	0.176
262	A	3	3	0.83	17	0.176
263	A	3	3	1.	17	0.176
264	A	3	3	1.	23	0.13
265	A	3	3	1.	23	0.13
266	A	3	3	1.	15	0.2
267	A	3	2	1.	19	0.105
268	A	3	3	1.	15	0.2
269	A	4	3	1.	19	0.158
270	A	3	3	1.	15	0.2
271	A	4	3	1.	19	0.158
272	A	3	3	1.	15	0.2
273	A	3	2	1.	19	0.105
274	A	3	3	1.	15	0.2
275	A	4	3	1.	19	0.158
276	A	3	3	1.	15	0.2
277	A	4	3	1.	19	0.158
278	A	3	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
279	A	3	3	1.	17	0.176
280	A	3	3	0.96	15	0.2
281	A	3	3	0.97	19	0.158
282	A	3	3	0.97	19	0.158
283	A	3	3	1.	19	0.158
284	A	3	3	0.98	19	0.158
285	A	3	3	0.97	19	0.158
286	A	3	3	0.96	21	0.143
287	A	3	3	1.	15	0.2
288	A	3	3	1.	13	0.231
289	A	3	3	1.	15	0.2
290	A	3	3	1.	13	0.231
291	A	3	3	1.	11	0.273
292	A	2	1	1.	15	0.067
293	A	3	3	1.	15	0.2
294	A	3	3	1.	15	0.2
295	A	3	3	1.	13	0.231
296	A	3	2	1.	17	0.118
297	A	3	3	1.	13	0.231
298	A	3	2	1.	17	0.118
299	A	3	3	1.	13	0.231
300	A	3	1	1.	17	0.059
301	C	7	3	4.1	44	0.068
302	C	3	3	1.29	31	0.097
303	A	3	3	1.	17	0.176
304	A	3	3	0.88	17	0.176
305	A	3	3	1.	17	0.176
306	A	3	3	1.	23	0.13
307	A	3	3	1.	23	0.13
308	A	3	3	1.	15	0.2
309	A	3	2	1.	19	0.105
310	A	3	3	1.	15	0.2
311	A	4	3	1.	19	0.158
312	A	3	3	1.	15	0.2
313	A	4	3	1.	19	0.158
314	A	3	3	1.	15	0.2
315	A	3	2	1.	19	0.105
316	A	3	3	1.	15	0.2
317	A	4	3	1.	19	0.158
318	A	3	3	1.	15	0.2
319	A	4	3	1.	19	0.158
320	A	3	3	1.	21	0.143
321	A	3	3	1.	21	0.143
322	A	3	3	0.96	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	3	3	0.97	19	0.158
324	A	3	3	0.97	19	0.158
325	A	3	3	1.	19	0.158
326	A	3	3	0.98	19	0.158
327	A	3	3	0.97	19	0.158
328	A	3	3	0.96	21	0.143
329	A	3	3	1.	15	0.2
330	A	3	3	1.	13	0.231



# Chapter 3

## Listing of integrals

### 3.1 $\int x^2 \sin(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=57

$$\frac{3x^3 \sin(a + b \log(cx^n))}{b^2n^2 + 9} - \frac{bnx^3 \cos(a + b \log(cx^n))}{b^2n^2 + 9}$$

[Out]  $-\frac{(b^n x^3 \cos[a + b \log[c x^n]])}{(9 + b^2 n^2)} + \frac{(3 x^3 \sin[a + b \log[c x^n]])}{(9 + b^2 n^2)}$

---

**Rubi [A]** time = 0.0171281, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {4485}

$$\frac{3x^3 \sin(a + b \log(cx^n))}{b^2n^2 + 9} - \frac{bnx^3 \cos(a + b \log(cx^n))}{b^2n^2 + 9}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sin[a + b\*Log[c\*x^n]],x]

[Out]  $-\frac{(b^n x^3 \cos[a + b \log[c x^n]])}{(9 + b^2 n^2)} + \frac{(3 x^3 \sin[a + b \log[c x^n]])}{(9 + b^2 n^2)}$

#### Rule 4485

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\int x^2 \sin(a + b \log(cx^n)) dx = -\frac{bnx^3 \cos(a + b \log(cx^n))}{9 + b^2n^2} + \frac{3x^3 \sin(a + b \log(cx^n))}{9 + b^2n^2}$$

**Mathematica [A]** time = 0.0751232, size = 44, normalized size = 0.77

$$\frac{x^3 (bn \cos(a + b \log(cx^n)) - 3 \sin(a + b \log(cx^n)))}{b^2n^2 + 9}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sin[a + b\*Log[c\*x^n]],x]

[Out] -((x^3\*(b\*n\*Cos[a + b\*Log[c\*x^n]] - 3\*Sin[a + b\*Log[c\*x^n]]))/(9 + b^2\*n^2))

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int x^2 \sin(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sin(a+b\*ln(c\*x^n)),x)

[Out] int(x^2\*sin(a+b\*ln(c\*x^n)),x)

**Maxima [B]** time = 1.20056, size = 296, normalized size = 5.19

$$\frac{((b \cos(2 b \log(c)) \cos(b \log(c)) + b \sin(2 b \log(c)) \sin(b \log(c)) + b \cos(b \log(c)))n - 3 \cos(b \log(c)) \sin(2 b \log(c)))}{(b^2 \cos(b \log(c))^2 + b^2 \sin(b \log(c))^2) n^2 + 9 \cos(b \log(c))^2 + 9 \sin(b \log(c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] -1/2\*(((b\*cos(2\*b\*log(c))\*cos(b\*log(c)) + b\*sin(2\*b\*log(c))\*sin(b\*log(c)) + b\*cos(b\*log(c)))\*n - 3\*cos(b\*log(c))\*sin(2\*b\*log(c)) + 3\*cos(2\*b\*log(c))\*sin(b\*log(c)) - 3\*sin(b\*log(c)))\*x^3\*cos(b\*log(x^n) + a) - ((b\*cos(b\*log(c))\*sin(2\*b\*log(c)) - b\*cos(2\*b\*log(c))\*sin(b\*log(c)) + b\*sin(b\*log(c)))\*n + 3\*cos(2\*b\*log(c))\*cos(b\*log(c)) + 3\*sin(2\*b\*log(c))\*sin(b\*log(c)) + 3\*cos(b\*log(c)))\*x^3\*sin(b\*log(x^n) + a))/((b^2\*cos(b\*log(c))^2 + b^2\*sin(b\*log(c))^2)\*n^2 + 9\*cos(b\*log(c))^2 + 9\*sin(b\*log(c))^2)

**Fricas [A]** time = 0.484201, size = 130, normalized size = 2.28

$$\frac{bnx^3 \cos(bn \log(x) + b \log(c) + a) - 3x^3 \sin(bn \log(x) + b \log(c) + a)}{b^2n^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] -(b\*n\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a) - 3\*x^3\*sin(b\*n\*log(x) + b\*log(c) + a))/(b^2\*n^2 + 9)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.31496, size = 1246, normalized size = 21.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] -1/2*(b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 - b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 - 4*b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - 4*b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a)^2 - b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 + 6*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 6*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 6*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + 6*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b) + b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b) - 6*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - 6*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - 6*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a) - 6*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a))/(b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b^2*n^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + b^2*n^2*tan(1/2*a)^2 + b^2*n^2 + 9*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + 9*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + 9*tan(1/2*a)^2 + 9)
```

### 3.2 $\int x \sin(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=57

$$\frac{2x^2 \sin(a + b \log(cx^n))}{b^2 n^2 + 4} - \frac{bnx^2 \cos(a + b \log(cx^n))}{b^2 n^2 + 4}$$

[Out]  $-\left(\frac{b n x^2 \cos[a + b \log[c x^n]]}{4 + b^2 n^2}\right) + \left(\frac{2 x^2 \sin[a + b \log[c x^n]]}{4 + b^2 n^2}\right)$

**Rubi [A]** time = 0.0126992, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {4485}

$$\frac{2x^2 \sin(a + b \log(cx^n))}{b^2 n^2 + 4} - \frac{bnx^2 \cos(a + b \log(cx^n))}{b^2 n^2 + 4}$$

Antiderivative was successfully verified.

[In] Int[x\*Sin[a + b\*Log[c\*x^n]],x]

[Out]  $-\left(\frac{b n x^2 \cos[a + b \log[c x^n]]}{4 + b^2 n^2}\right) + \left(\frac{2 x^2 \sin[a + b \log[c x^n]]}{4 + b^2 n^2}\right)$

#### Rule 4485

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] := Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\int x \sin(a + b \log(cx^n)) dx = -\frac{bnx^2 \cos(a + b \log(cx^n))}{4 + b^2 n^2} + \frac{2x^2 \sin(a + b \log(cx^n))}{4 + b^2 n^2}$$

**Mathematica [A]** time = 0.0587051, size = 44, normalized size = 0.77

$$-\frac{x^2 (bn \cos(a + b \log(cx^n)) - 2 \sin(a + b \log(cx^n)))}{b^2 n^2 + 4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sin[a + b\*Log[c\*x^n]],x]

[Out]  $-\left(\frac{x^2 (b n \cos[a + b \log[c x^n]] - 2 \sin[a + b \log[c x^n]])}{4 + b^2 n^2}\right)$

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int x \sin(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+b*ln(c*x^n)),x)
```

```
[Out] int(x*sin(a+b*ln(c*x^n)),x)
```

**Maxima [B]** time = 1.23481, size = 296, normalized size = 5.19

$$\frac{((b \cos(2b \log(c)) \cos(b \log(c)) + b \sin(2b \log(c)) \sin(b \log(c)) + b \cos(b \log(c)))n - 2 \cos(b \log(c)) \sin(2b \log(c)))}{...}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) +
b*cos(b*log(c)))*n - 2*cos(b*log(c))*sin(2*b*log(c)) + 2*cos(2*b*log(c))*s
in(b*log(c)) - 2*sin(b*log(c)))*x^2*cos(b*log(x^n) + a) - ((b*cos(b*log(c))
*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n + 2
*cos(2*b*log(c))*cos(b*log(c)) + 2*sin(2*b*log(c))*sin(b*log(c)) + 2*cos(b*
log(c))*x^2*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))
^2)*n^2 + 4*cos(b*log(c))^2 + 4*sin(b*log(c))^2)
```

**Fricas [A]** time = 0.482559, size = 130, normalized size = 2.28

$$\frac{bnx^2 \cos(bn \log(x) + b \log(c) + a) - 2x^2 \sin(bn \log(x) + b \log(c) + a)}{b^2n^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] -(b*n*x^2*cos(b*n*log(x) + b*log(c) + a) - 2*x^2*sin(b*n*log(x) + b*log(c)
+ a))/(b^2*n^2 + 4)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n)),x)
```

```
[Out] Exception raised: TypeError
```

**Giac [B]** time = 1.26222, size = 1246, normalized size = 21.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*(b*n*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 + b*n*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 - b*n*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 - b*n*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 - 4*b*n*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))*\tan(1/2*a) - 4*b*n*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))*\tan(1/2*a) - b*n*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & \tan(1/2*a)^2 - b*n*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*a)^2 + 4*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a) + 4*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))*\tan(1/2*a) + 4*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))*\tan(1/2*a)^2 + 4*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))*\tan(1/2*a)^2 + b*n*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & + b*n*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & - 4*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) - 4*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) - 4*x^2*e^{(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)} \\ & \tan(1/2*a) - 4*x^2*e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))*\tan(1/2*a) - 4*x^2 \\ & e^{(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))*\tan(1/2*a)^2 + b^2*n^2*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 + \\ & b^2*n^2*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + b^2*n^2*\tan(1/2*a)^2 + b^2*n^2 + 4*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 + 4*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + 4*\tan(1/2*a)^2 + 4 \end{aligned}$$

### 3.3 $\int \sin(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=52

$$\frac{x \sin(a + b \log(cx^n))}{b^2 n^2 + 1} - \frac{bnx \cos(a + b \log(cx^n))}{b^2 n^2 + 1}$$

[Out]  $-\frac{(b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]])}{(1 + b^2*n^2)} + \frac{(x*\text{Sin}[a + b*\text{Log}[c*x^n]])}{(1 + b^2*n^2)}$

**Rubi [A]** time = 0.0112486, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {4475}

$$\frac{x \sin(a + b \log(cx^n))}{b^2 n^2 + 1} - \frac{bnx \cos(a + b \log(cx^n))}{b^2 n^2 + 1}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]], x]

[Out]  $-\frac{(b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]])}{(1 + b^2*n^2)} + \frac{(x*\text{Sin}[a + b*\text{Log}[c*x^n]])}{(1 + b^2*n^2)}$

**Rule 4475**

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.), x\_Symbol] := Simp[(x\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] - Simp[(b\*d\*n\*x\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2\*d^2\*n^2 + 1, 0]

**Rubi steps**

$$\int \sin(a + b \log(cx^n)) dx = -\frac{bnx \cos(a + b \log(cx^n))}{1 + b^2 n^2} + \frac{x \sin(a + b \log(cx^n))}{1 + b^2 n^2}$$

**Mathematica [A]** time = 0.0509788, size = 40, normalized size = 0.77

$$\frac{x(\sin(a + b \log(cx^n)) - bn \cos(a + b \log(cx^n)))}{b^2 n^2 + 1}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]], x]

[Out]  $(x*(-(b*n*\text{Cos}[a + b*\text{Log}[c*x^n]]) + \text{Sin}[a + b*\text{Log}[c*x^n]]))/(1 + b^2*n^2)$

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int \sin(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a+b*ln(c*x^n)),x)`

[Out] `int(sin(a+b*ln(c*x^n)),x)`

**Maxima [B]** time = 1.19993, size = 278, normalized size = 5.35

$$\frac{((b \cos(2b \log(c)) \cos(b \log(c)) + b \sin(2b \log(c)) \sin(b \log(c)) + b \cos(b \log(c)))n - \cos(b \log(c)) \sin(2b \log(c)) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out] 
$$-1/2 * (((b * \cos(2 * b * \log(c)) * \cos(b * \log(c)) + b * \sin(2 * b * \log(c)) * \sin(b * \log(c)) + b * \cos(b * \log(c))) * n - \cos(b * \log(c)) * \sin(2 * b * \log(c)) + \cos(2 * b * \log(c)) * \sin(b * \log(c)) - \sin(b * \log(c))) * x * \cos(b * \log(x^n) + a) - ((b * \cos(b * \log(c)) * \sin(2 * b * \log(c)) - b * \cos(2 * b * \log(c)) * \sin(b * \log(c)) + b * \sin(b * \log(c))) * n + \cos(2 * b * \log(c)) * \cos(b * \log(c)) + \sin(2 * b * \log(c)) * \sin(b * \log(c)) + \cos(b * \log(c))) * x * \sin(b * \log(x^n) + a)) / ((b^2 * \cos(b * \log(c))^2 + b^2 * \sin(b * \log(c))^2) * n^2 + \cos(b * \log(c))^2 + \sin(b * \log(c))^2)$$

**Fricas [A]** time = 0.488431, size = 122, normalized size = 2.35

$$\frac{bnx \cos(bn \log(x) + b \log(c) + a) - x \sin(bn \log(x) + b \log(c) + a)}{b^2 n^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out] 
$$-(b * n * x * \cos(b * n * \log(x) + b * \log(c) + a) - x * \sin(b * n * \log(x) + b * \log(c) + a)) / (b^2 * n^2 + 1)$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*ln(c*x**n)),x)`

[Out] Exception raised: TypeError

**Giac [B]** time = 1.19184, size = 1191, normalized size = 22.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*(b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 + b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} \\ & + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 \\ & - b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 \\ & - b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 \\ & - 4*b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) * \tan(1/2*a) \\ & - 4*b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) * \tan(1/2*a) \\ & - b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} * \tan(1/2*a)^2 - b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*a)^2 \\ & + 2*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a) \\ & + 2*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a) \\ & + 2*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) * \tan(1/2*a)^2 \\ & + 2*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) * \tan(1/2*a)^2 \\ & + b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} + b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & - 2*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) - 2*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) \\ & - 2*x*e^{(1/2*\pi*b*n*\text{sgn}(x)} - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} * \tan(1/2*a) - 2*x*e^{(-1/2*\pi*b*n*\text{sgn}(x)} + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*a) \\ & / (b^2*n^2*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 + b^2*n^2*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 \\ & + b^2*n^2*\tan(1/2*a)^2 + b^2*n^2 + \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 + \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + \tan(1/2*a)^2 + 1) \end{aligned}$$

$$3.4 \quad \int \frac{\sin(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=19

$$-\frac{\cos(a+b \log(cx^n))}{bn}$$

[Out] -(Cos[a + b\*Log[c\*x^n]]/(b\*n))

**Rubi [A]** time = 0.0151146, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2638}

$$-\frac{\cos(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]/x,x]

[Out] -(Cos[a + b\*Log[c\*x^n]]/(b\*n))

**Rule 2638**

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\sin(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sin(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cos(a+b \log(cx^n))}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0266074, size = 38, normalized size = 2.

$$\frac{\sin(a) \sin(b \log(cx^n))}{bn} - \frac{\cos(a) \cos(b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]/x,x]

[Out] -((Cos[a]\*Cos[b\*Log[c\*x^n]])/(b\*n)) + (Sin[a]\*Sin[b\*Log[c\*x^n]])/(b\*n)

**Maple [A]** time = 0.017, size = 20, normalized size = 1.1

$$-\frac{\cos(a+b \ln(cx^n))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(sin(a+b*ln(c*x^n))/x,x)`

[Out] `-cos(a+b*ln(c*x^n))/b/n`

**Maxima [A]** time = 1.13855, size = 26, normalized size = 1.37

$$\frac{\cos(b \log(cx^n) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))/x,x, algorithm="maxima")`

[Out] `-cos(b*log(c*x^n) + a)/(b*n)`

**Fricas [A]** time = 0.494714, size = 53, normalized size = 2.79

$$\frac{\cos(bn \log(x) + b \log(c) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))/x,x, algorithm="fricas")`

[Out] `-cos(b*n*log(x) + b*log(c) + a)/(b*n)`

**Sympy [A]** time = 3.06634, size = 39, normalized size = 2.05

$$\begin{cases} \log(x) \sin(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \sin(a + b \log(c)) & \text{for } n = 0 \\ -\frac{\cos(a + bn \log(x) + b \log(c))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*ln(c*x**n))/x,x)`

[Out] `Piecewise((log(x)*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sin(a + b*log(c)), Eq(n, 0)), (-cos(a + b*n*log(x) + b*log(c))/(b*n), True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))/x,x, algorithm="giac")`

[Out] `integrate(sin(b*log(c*x^n) + a)/x, x)`

$$3.5 \quad \int \frac{\sin(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=57

$$-\frac{\sin(a+b \log(cx^n))}{x(b^2n^2+1)} - \frac{bn \cos(a+b \log(cx^n))}{x(b^2n^2+1)}$$

[Out]  $-\left(\frac{b^n \cos[a + b \log[cx^n]]}{(1 + b^2n^2)x}\right) - \frac{\sin[a + b \log[cx^n]]}{(1 + b^2n^2)x}$

**Rubi [A]** time = 0.0178563, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {4485}

$$-\frac{\sin(a+b \log(cx^n))}{x(b^2n^2+1)} - \frac{bn \cos(a+b \log(cx^n))}{x(b^2n^2+1)}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]/x^2,x]

[Out]  $-\left(\frac{b^n \cos[a + b \log[cx^n]]}{(1 + b^2n^2)x}\right) - \frac{\sin[a + b \log[cx^n]]}{(1 + b^2n^2)x}$

**Rule 4485**

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_ Symbol] := Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

**Rubi steps**

$$\int \frac{\sin(a+b \log(cx^n))}{x^2} dx = -\frac{bn \cos(a+b \log(cx^n))}{(1+b^2n^2)x} - \frac{\sin(a+b \log(cx^n))}{(1+b^2n^2)x}$$

**Mathematica [A]** time = 0.0582826, size = 40, normalized size = 0.7

$$-\frac{\sin(a+b \log(cx^n)) + bn \cos(a+b \log(cx^n))}{b^2n^2x + x}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]/x^2,x]

[Out]  $-\left(\frac{b^n \cos[a + b \log[cx^n]] + \sin[a + b \log[cx^n]]}{(x + b^2n^2x)}\right)$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\sin(a+b \ln(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))/x^2,x)

[Out] int(sin(a+b\*ln(c\*x^n))/x^2,x)

**Maxima [B]** time = 1.23783, size = 282, normalized size = 4.95

$$\frac{((b \cos(2 b \log(c)) \cos(b \log(c)) + b \sin(2 b \log(c)) \sin(b \log(c)) + b \cos(b \log(c)))n + \cos(b \log(c)) \sin(2 b \log(c)))}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))/x^2,x, algorithm="maxima")

[Out] 
$$-1/2 * ((b * \cos(2 * b * \log(c)) * \cos(b * \log(c)) + b * \sin(2 * b * \log(c)) * \sin(b * \log(c)) + b * \cos(b * \log(c))) * n + \cos(b * \log(c)) * \sin(2 * b * \log(c)) - \cos(2 * b * \log(c)) * \sin(b * \log(c)) + \sin(b * \log(c)) * \cos(b * \log(c))) * \cos(b * \log(x^n) + a) - ((b * \cos(b * \log(c)) * \sin(2 * b * \log(c)) - b * \cos(2 * b * \log(c)) * \sin(b * \log(c)) + b * \sin(b * \log(c))) * n - \cos(2 * b * \log(c)) * \cos(b * \log(c)) - \sin(2 * b * \log(c)) * \sin(b * \log(c)) - \cos(b * \log(c)) * \sin(b * \log(x^n) + a)) / (((b^2 * \cos(b * \log(c))^2 + b^2 * \sin(b * \log(c))^2) * n^2 + \cos(b * \log(c))^2 + \sin(b * \log(c))^2) * x)$$

**Fricas [A]** time = 0.493786, size = 122, normalized size = 2.14

$$\frac{bn \cos(bn \log(x) + b \log(c) + a) + \sin(bn \log(x) + b \log(c) + a)}{(b^2 n^2 + 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))/x^2,x, algorithm="fricas")

[Out] 
$$-(b * n * \cos(b * n * \log(x) + b * \log(c) + a) + \sin(b * n * \log(x) + b * \log(c) + a)) / ((b^2 * n^2 + 1) * x)$$

**Sympy [A]** time = 32.2466, size = 286, normalized size = 5.02

$$\left\{ \begin{array}{l} \frac{\log(x) \sin\left(-a + i \log(x) + \frac{i \log(c)}{n}\right)}{2x} - \frac{i \log(x) \cos\left(-a + i \log(x) + \frac{i \log(c)}{n}\right)}{2x} + \frac{\sin\left(-a + i \log(x) + \frac{i \log(c)}{n}\right)}{2x} - \frac{\log(c) \sin\left(-a + i \log(x) + \frac{i \log(c)}{n}\right)}{2nx} - \frac{i \log(c) \cos\left(-a + i \log(x) + \frac{i \log(c)}{n}\right)}{2nx} \\ \frac{\log(x) \sin\left(a + i \log(x) + \frac{i \log(c)}{n}\right)}{2x} + \frac{i \log(x) \cos\left(a + i \log(x) + \frac{i \log(c)}{n}\right)}{2x} - \frac{\sin\left(a + i \log(x) + \frac{i \log(c)}{n}\right)}{2x} + \frac{\log(c) \sin\left(a + i \log(x) + \frac{i \log(c)}{n}\right)}{2nx} + \frac{i \log(c) \cos\left(a + i \log(x) + \frac{i \log(c)}{n}\right)}{2nx} \\ \frac{bn \cos(a + bn \log(x) + b \log(c))}{b^2 n^2 x + x} - \frac{\sin(a + bn \log(x) + b \log(c))}{b^2 n^2 x + x} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*ln(c\*x\*\*n))/x\*\*2,x)

[Out] 
$$\text{Piecewise}\left(\left(-\log(x) * \sin(-a + I * \log(x) + I * \log(c) / n) / (2 * x) - I * \log(x) * \cos(-a + I * \log(x) + I * \log(c) / n) / (2 * x) + \sin(-a + I * \log(x) + I * \log(c) / n) / (2 * x) - \log(c) * \sin(-a + I * \log(x) + I * \log(c) / n) / (2 * n * x) - I * \log(c) * \cos(-a + I * \log(x) + I * \log(c) / n) / (2 * n * x), \text{Eq}(b, -I / n)\right), \left(\log(x) * \sin(a + I * \log(x) + I * \log(c) / n) / (2 * n * x) - I * \log(x) * \cos(a + I * \log(x) + I * \log(c) / n) / (2 * n * x) + \sin(a + I * \log(x) + I * \log(c) / n) / (2 * n * x) - \log(c) * \sin(a + I * \log(x) + I * \log(c) / n) / (2 * n * x) - I * \log(c) * \cos(a + I * \log(x) + I * \log(c) / n) / (2 * n * x), \text{Eq}(b, I / n)\right)\right)$$

```
(2*x) + I*log(x)*cos(a + I*log(x) + I*log(c)/n)/(2*x) - sin(a + I*log(x) +
I*log(c)/n)/(2*x) + log(c)*sin(a + I*log(x) + I*log(c)/n)/(2*n*x) + I*log(c)
)*cos(a + I*log(x) + I*log(c)/n)/(2*n*x), Eq(b, I/n)), (-b*n*cos(a + b*n*lo
g(x) + b*log(c))/(b**2*n**2*x + x) - sin(a + b*n*log(x) + b*log(c))/(b**2*n
**2*x + x), True))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)/x^2, x)
```

$$3.6 \quad \int \frac{\sin(a+b \log(cx^n))}{x^3} dx$$

**Optimal.** Leaf size=57

$$-\frac{2 \sin(a+b \log(cx^n))}{x^2(b^2n^2+4)} - \frac{bn \cos(a+b \log(cx^n))}{x^2(b^2n^2+4)}$$

[Out] -((b\*n\*Cos[a + b\*Log[c\*x^n]])/((4 + b^2\*n^2)\*x^2)) - (2\*Sin[a + b\*Log[c\*x^n]])/((4 + b^2\*n^2)\*x^2)

**Rubi [A]** time = 0.0153199, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {4485}

$$-\frac{2 \sin(a+b \log(cx^n))}{x^2(b^2n^2+4)} - \frac{bn \cos(a+b \log(cx^n))}{x^2(b^2n^2+4)}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]/x^3,x]

[Out] -((b\*n\*Cos[a + b\*Log[c\*x^n]])/((4 + b^2\*n^2)\*x^2)) - (2\*Sin[a + b\*Log[c\*x^n]])/((4 + b^2\*n^2)\*x^2)

**Rule 4485**

Int[((e\_.)\*(x\_)^(m\_.)\*Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)], x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

**Rubi steps**

$$\int \frac{\sin(a+b \log(cx^n))}{x^3} dx = -\frac{bn \cos(a+b \log(cx^n))}{(4+b^2n^2)x^2} - \frac{2 \sin(a+b \log(cx^n))}{(4+b^2n^2)x^2}$$

**Mathematica [A]** time = 0.0613524, size = 44, normalized size = 0.77

$$-\frac{2 \sin(a+b \log(cx^n)) + bn \cos(a+b \log(cx^n))}{x^2(b^2n^2+4)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]/x^3,x]

[Out] -((b\*n\*Cos[a + b\*Log[c\*x^n]] + 2\*Sin[a + b\*Log[c\*x^n]])/((4 + b^2\*n^2)\*x^2))

**Maple [F]** time = 0.032, size = 0, normalized size = 0.

$$\int \frac{\sin(a + b \ln(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))/x^3,x)

[Out] int(sin(a+b\*ln(c\*x^n))/x^3,x)

**Maxima [B]** time = 1.21295, size = 292, normalized size = 5.12

$$\frac{((b \cos(2b \log(c)) \cos(b \log(c)) + b \sin(2b \log(c)) \sin(b \log(c)) + b \cos(b \log(c)))n + 2 \cos(b \log(c)) \sin(2b \log(c)))}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))/x^3,x, algorithm="maxima")

[Out] 
$$-1/2 * (((b * \cos(2 * b * \log(c)) * \cos(b * \log(c)) + b * \sin(2 * b * \log(c)) * \sin(b * \log(c)) + b * \cos(b * \log(c))) * n + 2 * \cos(b * \log(c)) * \sin(2 * b * \log(c))) * \cos(b * \log(x^n) + a) - ((b * \cos(b * \log(c)) * \sin(2 * b * \log(c)) - b * \cos(2 * b * \log(c)) * \sin(b * \log(c)) + b * \sin(b * \log(c))) * n - 2 * \cos(2 * b * \log(c)) * \cos(b * \log(c)) - 2 * \sin(2 * b * \log(c)) * \sin(b * \log(c)) - 2 * \cos(b * \log(c))) * \sin(b * \log(x^n) + a)) / (((b^2 * \cos(b * \log(c))^2 + b^2 * \sin(b * \log(c))^2) * n^2 + 4 * \cos(b * \log(c))^2 + 4 * \sin(b * \log(c))^2) * x^2)$$

**Fricas [A]** time = 0.489054, size = 127, normalized size = 2.23

$$\frac{bn \cos(bn \log(x) + b \log(c) + a) + 2 \sin(bn \log(x) + b \log(c) + a)}{(b^2 n^2 + 4) x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))/x^3,x, algorithm="fricas")

[Out] 
$$-(b * n * \cos(b * n * \log(x) + b * \log(c) + a) + 2 * \sin(b * n * \log(x) + b * \log(c) + a)) / ((b^2 * n^2 + 4) * x^2)$$

**Sympy [A]** time = 59.5154, size = 352, normalized size = 6.18

$$\left\{ \begin{array}{l} \frac{\log(x) \sin\left(-a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{2x^2} - \frac{i \log(x) \cos\left(-a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{2x^2} + \frac{\sin\left(-a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{4x^2} - \frac{\log(c) \sin\left(-a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{2nx^2} - \frac{i \log(c) \cos\left(-a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{2nx^2} \\ \frac{\log(x) \sin\left(a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{2x^2} + \frac{i \log(x) \cos\left(a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{2x^2} - \frac{\sin\left(a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{4x^2} + \frac{\log(c) \sin\left(a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{2nx^2} + \frac{i \log(c) \cos\left(a + 2i \log(x) + \frac{2i \log(c)}{n}\right)}{2nx^2} \\ - \frac{bn \cos(a + bn \log(x) + b \log(c))}{b^2 n^2 x^2 + 4x^2} - \frac{2 \sin(a + bn \log(x) + b \log(c))}{b^2 n^2 x^2 + 4x^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*ln(c\*x\*\*n))/x\*\*3,x)

```
[Out] Piecewise((-log(x)*sin(-a + 2*I*log(x) + 2*I*log(c)/n)/(2*x**2) - I*log(x)*
cos(-a + 2*I*log(x) + 2*I*log(c)/n)/(2*x**2) + sin(-a + 2*I*log(x) + 2*I*log
(c)/n)/(4*x**2) - log(c)*sin(-a + 2*I*log(x) + 2*I*log(c)/n)/(2*n*x**2) -
I*log(c)*cos(-a + 2*I*log(x) + 2*I*log(c)/n)/(2*n*x**2), Eq(b, -2*I/n)), (l
og(x)*sin(a + 2*I*log(x) + 2*I*log(c)/n)/(2*x**2) + I*log(x)*cos(a + 2*I*log
(x) + 2*I*log(c)/n)/(2*x**2) - sin(a + 2*I*log(x) + 2*I*log(c)/n)/(4*x**2)
+ log(c)*sin(a + 2*I*log(x) + 2*I*log(c)/n)/(2*n*x**2) + I*log(c)*cos(a +
2*I*log(x) + 2*I*log(c)/n)/(2*n*x**2), Eq(b, 2*I/n)), (-b*n*cos(a + b*n*log
(x) + b*log(c))/(b**2*n**2*x**2 + 4*x**2) - 2*sin(a + b*n*log(x) + b*log(c)
)/(b**2*n**2*x**2 + 4*x**2), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)/x^3, x)
```

### 3.7 $\int x^2 \sin^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=97

$$\frac{3x^3 \sin^2(a + b \log(cx^n))}{4b^2n^2 + 9} - \frac{2bnx^3 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 9} + \frac{2b^2n^2x^3}{3(4b^2n^2 + 9)}$$

[Out]  $(2*b^2*n^2*x^3)/(3*(9 + 4*b^2*n^2)) - (2*b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(9 + 4*b^2*n^2) + (3*x^3*Sin[a + b*Log[c*x^n]]^2)/(9 + 4*b^2*n^2)$

**Rubi [A]** time = 0.031048, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 30}

$$\frac{3x^3 \sin^2(a + b \log(cx^n))}{4b^2n^2 + 9} - \frac{2bnx^3 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 9} + \frac{2b^2n^2x^3}{3(4b^2n^2 + 9)}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sin[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(2*b^2*n^2*x^3)/(3*(9 + 4*b^2*n^2)) - (2*b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(9 + 4*b^2*n^2) + (3*x^3*Sin[a + b*Log[c*x^n]]^2)/(9 + 4*b^2*n^2)$

#### Rule 4487

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 \sin^2(a + b \log(cx^n)) dx &= -\frac{2bnx^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{9 + 4b^2n^2} + \frac{3x^3 \sin^2(a + b \log(cx^n))}{9 + 4b^2n^2} + \frac{(2b^2n^2)}{9 + 4b^2n^2} \\ &= \frac{2b^2n^2x^3}{3(9 + 4b^2n^2)} - \frac{2bnx^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{9 + 4b^2n^2} + \frac{3x^3 \sin^2(a + b \log(cx^n))}{9 + 4b^2n^2} \end{aligned}$$

**Mathematica [A]** time = 0.142716, size = 61, normalized size = 0.63

$$\frac{x^3(-6bn \sin(2(a + b \log(cx^n))) - 9 \cos(2(a + b \log(cx^n))) + 4b^2n^2 + 9)}{6(4b^2n^2 + 9)}$$



Antiderivative was successfully verified.

[In] Integrate[x^2\*Sin[a + b\*Log[c\*x^n]]^2,x]

[Out] (x^3\*(9 + 4\*b^2\*n^2 - 9\*Cos[2\*(a + b\*Log[c\*x^n])] - 6\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])]))/(6\*(9 + 4\*b^2\*n^2))

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int x^2 (\sin(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sin(a+b\*ln(c\*x^n))^2,x)

[Out] int(x^2\*sin(a+b\*ln(c\*x^n))^2,x)

**Maxima [B]** time = 1.2991, size = 406, normalized size = 4.19

$$\frac{3(2(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(2b \log(c)) + b \sin(2b \log(c)))n + 3 \cos(4b \log(c)) \cos(2b \log(c)) - 3 \sin(4b \log(c)) \sin(2b \log(c)) + 3 \cos(2b \log(c)) \cos(4b \log(c)) - 3 \sin(2b \log(c)) \sin(4b \log(c)) + 3 \cos(2b \log(c)) \cos(2b \log(c)) + 3 \sin(4b \log(c)) \sin(2b \log(c)) + 3 \cos(2b \log(c)) \cos(4b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c)) + b \cos(2b \log(c)) \cos(4b \log(c)) - 3 \cos(2b \log(c)) \sin(4b \log(c)) + 3 \cos(4b \log(c)) \sin(2b \log(c)) - 3 \sin(2b \log(c)) \sin(4b \log(c)) + 3 \cos(4b \log(c)) \cos(2b \log(c)) - 3 \sin(2b \log(c)) \cos(4b \log(c)) + 3 \cos(2b \log(c)) \cos(2b \log(c)) + 3 \sin(4b \log(c)) \sin(2b \log(c)) + 3 \cos(2b \log(c)) \cos(4b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c)) + b \cos(2b \log(c)) \cos(4b \log(c)) - 3 \cos(2b \log(c)) \sin(4b \log(c)) + 3 \cos(4b \log(c)) \sin(2b \log(c)) - 3 \sin(2b \log(c)) \sin(4b \log(c)) + 3 \cos(2b \log(c)) \cos(4b \log(c)) - 2*(4*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + 9*\cos(2*b*\log(c))^2 + 9*\sin(2*b*\log(c))^2)*x^3)/(4*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + 9*\cos(2*b*\log(c))^2 + 9*\sin(2*b*\log(c))^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -1/12\*(3\*(2\*(b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)) + b\*sin(2\*b\*log(c)))\*n + 3\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + 3\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)) + 3\*cos(2\*b\*log(c))\*cos(4\*b\*log(c)))\*x^3\*cos(2\*b\*log(x^n) + 2\*a) + 3\*(2\*(b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)) + b\*cos(2\*b\*log(c)))\*n - 3\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) + 3\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)) - 3\*sin(2\*b\*log(c))\*sin(4\*b\*log(c)) + 3\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) - 3\*sin(2\*b\*log(c))\*cos(4\*b\*log(c)) + 3\*cos(2\*b\*log(c))\cos(2\*b\*log(c)) + 3\*sin(4\*b\*log(c))\sin(2\*b\*log(c)) + 3\*cos(2\*b\*log(c))\cos(4\*b\*log(c)) + b\*sin(4\*b\*log(c))\sin(2\*b\*log(c)) + b\*cos(2\*b\*log(c))\cos(4\*b\*log(c)) - 3\*cos(2\*b\*log(c))\sin(4\*b\*log(c)) + 3\*cos(4\*b\*log(c))\sin(2\*b\*log(c)) - 3\*sin(2\*b\*log(c))\sin(4\*b\*log(c)) + 3\*cos(4\*b\*log(c))\cos(2\*b\*log(c)) - 3\*sin(2\*b\*log(c))\cos(4\*b\*log(c)) + 3\*cos(2\*b\*log(c))\cos(2\*b\*log(c)) + 3\*sin(4\*b\*log(c))\sin(2\*b\*log(c)) + 3\*cos(2\*b\*log(c))\cos(4\*b\*log(c)) + b\*sin(4\*b\*log(c))\sin(2\*b\*log(c)) + b\*cos(2\*b\*log(c))\cos(4\*b\*log(c)) - 3\*cos(2\*b\*log(c))\sin(4\*b\*log(c)) + 3\*cos(4\*b\*log(c))\sin(2\*b\*log(c)) - 3\*sin(2\*b\*log(c))\sin(4\*b\*log(c)) + 3\*cos(2\*b\*log(c))\cos(4\*b\*log(c)) - 2\*(4\*(b^2\*\cos(2\*b\*\log(c))^2 + b^2\*\sin(2\*b\*\log(c))^2)\*n^2 + 9\*\cos(2\*b\*\log(c))^2 + 9\*\sin(2\*b\*\log(c))^2)\*x^3)/(4\*(b^2\*\cos(2\*b\*\log(c))^2 + b^2\*\sin(2\*b\*\log(c))^2)\*n^2 + 9\*\cos(2\*b\*\log(c))^2 + 9\*\sin(2\*b\*\log(c))^2)

**Fricas [A]** time = 0.492592, size = 215, normalized size = 2.22

$$\frac{6bnx^3 \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) + 9x^3 \cos(bn \log(x) + b \log(c) + a)^2 - (2b^2n^2 - 3(4b^2n^2 + 9))}{3(4b^2n^2 + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -1/3\*(6\*b\*n\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a)\*sin(b\*n\*log(x) + b\*log(c) + a) + 9\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a)^2 - (2\*b^2\*n^2 + 9)\*x^3)/(4\*b^2\*n^2 + 9)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*sin(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.49213, size = 1125, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$\frac{1}{6}x^3 + \frac{1}{4}(4b^2n^2x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4b^2n^2x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4b^2n^2x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4b^2n^2x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) - 3x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 3x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 4b^2n^2x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4b^2n^2x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4b^2n^2x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(a) - 4b^2n^2x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(a) + 3x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 3x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 12x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 12x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 3x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(a)^2 + 3x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(a)^2 - 3x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b) - 3x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)}}/(4b^2n^2tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4b^2n^2tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4b^2n^2tan(a)^2 + 4b^2n^2 + 9tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 9tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 9tan(a)^2 + 9)$$

### 3.8 $\int x \sin^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=98

$$\frac{x^2 \sin^2(a + b \log(cx^n))}{2(b^2 n^2 + 1)} - \frac{bnx^2 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{2(b^2 n^2 + 1)} + \frac{b^2 n^2 x^2}{4(b^2 n^2 + 1)}$$

[Out] (b^2\*n^2\*x^2)/(4\*(1 + b^2\*n^2)) - (b\*n\*x^2\*Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(2\*(1 + b^2\*n^2)) + (x^2\*Sin[a + b\*Log[c\*x^n]]^2)/(2\*(1 + b^2\*n^2))

**Rubi [A]** time = 0.0222139, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {4487, 30}

$$\frac{x^2 \sin^2(a + b \log(cx^n))}{2(b^2 n^2 + 1)} - \frac{bnx^2 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{2(b^2 n^2 + 1)} + \frac{b^2 n^2 x^2}{4(b^2 n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[x\*Sin[a + b\*Log[c\*x^n]]^2,x]

[Out] (b^2\*n^2\*x^2)/(4\*(1 + b^2\*n^2)) - (b\*n\*x^2\*Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(2\*(1 + b^2\*n^2)) + (x^2\*Sin[a + b\*Log[c\*x^n]]^2)/(2\*(1 + b^2\*n^2))

#### Rule 4487

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x \sin^2(a + b \log(cx^n)) dx &= -\frac{bnx^2 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{2(1 + b^2 n^2)} + \frac{x^2 \sin^2(a + b \log(cx^n))}{2(1 + b^2 n^2)} + \frac{(b^2 n^2) \int}{2(1 + b^2 n^2)} \\ &= \frac{b^2 n^2 x^2}{4(1 + b^2 n^2)} - \frac{bnx^2 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{2(1 + b^2 n^2)} + \frac{x^2 \sin^2(a + b \log(cx^n))}{2(1 + b^2 n^2)} \end{aligned}$$

**Mathematica [A]** time = 0.109502, size = 57, normalized size = 0.58

$$\frac{x^2 (-bn \sin(2(a + b \log(cx^n))) - \cos(2(a + b \log(cx^n))) + b^2 n^2 + 1)}{4b^2 n^2 + 4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sin[a + b\*Log[c\*x^n]]^2,x]

[Out] (x^2\*(1 + b^2\*n^2 - Cos[2\*(a + b\*Log[c\*x^n])] - b\*n\*Sin[2\*(a + b\*Log[c\*x^n]
)))/(4 + 4\*b^2\*n^2)

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int x (\sin(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+b\*ln(c\*x^n))^2,x)

[Out] int(x\*sin(a+b\*ln(c\*x^n))^2,x)

**Maxima [B]** time = 1.16108, size = 381, normalized size = 3.89

$$\frac{((b \cos(2 b \log(c)) \sin(4 b \log(c)) - b \cos(4 b \log(c)) \sin(2 b \log(c)) + b \sin(2 b \log(c)))n + \cos(4 b \log(c)) \cos(2 b \log(c)))x^2}{4(b^2 n^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -1/8\*(((b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)
)) + b\*sin(2\*b\*log(c)))\*n + cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c)
))\*sin(2\*b\*log(c)) + cos(2\*b\*log(c)))\*x^2\*cos(2\*b\*log(x^n) + 2\*a) + ((b\*cos
(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)) + b\*cos(2\*
b\*log(c)))\*n - cos(2\*b\*log(c))\*sin(4\*b\*log(c)) + cos(4\*b\*log(c))\*sin(2\*b\*lo
g(c)) - sin(2\*b\*log(c)))\*x^2\*sin(2\*b\*log(x^n) + 2\*a) - 2\*((b^2\*cos(2\*b\*log(
c))^2 + b^2\*sin(2\*b\*log(c))^2)\*n^2 + cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2
)\*x^2)/((b^2\*cos(2\*b\*log(c))^2 + b^2\*sin(2\*b\*log(c))^2)\*n^2 + cos(2\*b\*log(c)
)^2 + sin(2\*b\*log(c))^2)

**Fricas [A]** time = 0.489977, size = 209, normalized size = 2.13

$$\frac{2 b n x^2 \cos(b n \log(x) + b \log(c) + a) \sin(b n \log(x) + b \log(c) + a) + 2 x^2 \cos(b n \log(x) + b \log(c) + a)^2 - (b^2 n^2 + 2) x}{4(b^2 n^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -1/4\*(2\*b\*n\*x^2\*cos(b\*n\*log(x) + b\*log(c) + a)\*sin(b\*n\*log(x) + b\*log(c) +
a) + 2\*x^2\*cos(b\*n\*log(x) + b\*log(c) + a)^2 - (b^2\*n^2 + 2)\*x^2)/(b^2\*n^2 +
1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.44718, size = 1107, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$\frac{1}{4}x^2 + \frac{1}{8}(2b^2n^2x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 2b^2n^2x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 2b^2n^2x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 2b^2n^2x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - 2b^2n^2x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - 2b^2n^2x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - 2b^2n^2x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - 2b^2n^2x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 4x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 4x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(a)^2} + x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(a)^2} - x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b) - x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)})/(b^2n^2tan(b*n*log(abs(x)) + b*log(abs(c)))^2tan(a)^2 + b^2n^2tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + b^2n^2tan(a)^2 + b^2n^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2tan(a)^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + tan(a)^2 + 1)$$

### 3.9 $\int \sin^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=88

$$\frac{x \sin^2(a + b \log(cx^n))}{4b^2n^2 + 1} - \frac{2bnx \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 1} + \frac{2b^2n^2x}{4b^2n^2 + 1}$$

[Out]  $(2*b^2*n^2*x)/(1 + 4*b^2*n^2) - (2*b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(1 + 4*b^2*n^2) + (x*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/(1 + 4*b^2*n^2)$

**Rubi [A]** time = 0.0187392, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {4477, 8}

$$\frac{x \sin^2(a + b \log(cx^n))}{4b^2n^2 + 1} - \frac{2bnx \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 1} + \frac{2b^2n^2x}{4b^2n^2 + 1}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(2*b^2*n^2*x)/(1 + 4*b^2*n^2) - (2*b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(1 + 4*b^2*n^2) + (x*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/(1 + 4*b^2*n^2)$

#### Rule 4477

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Simp[x*Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1), x] + (Dist[(b^2*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 + 1), Int[Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x] - Simp[(b*d*n*p*x*Cos[d*(a + b*Log[c*x^n])] * Sin[d*(a + b*Log[c*x^n])]^(p - 1))/(b^2*d^2*n^2*p^2 + 1), x]) /; FreeQ[{a, b, c, d, n}, x] && I GtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

#### Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

#### Rubi steps

$$\begin{aligned} \int \sin^2(a + b \log(cx^n)) dx &= -\frac{2bnx \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 4b^2n^2} + \frac{x \sin^2(a + b \log(cx^n))}{1 + 4b^2n^2} + \frac{(2b^2n^2) \int 1 dx}{1 + 4b^2n^2} \\ &= \frac{2b^2n^2x}{1 + 4b^2n^2} - \frac{2bnx \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 4b^2n^2} + \frac{x \sin^2(a + b \log(cx^n))}{1 + 4b^2n^2} \end{aligned}$$

**Mathematica [A]** time = 0.0902609, size = 56, normalized size = 0.64

$$\frac{x(-2bn \sin(2(a + b \log(cx^n))) - \cos(2(a + b \log(cx^n))) + 4b^2n^2 + 1)}{8b^2n^2 + 2}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(x*(1 + 4*b^2*n^2 - \text{Cos}[2*(a + b*\text{Log}[c*x^n])]) - 2*b*n*\text{Sin}[2*(a + b*\text{Log}[c*x^n])]))/(2 + 8*b^2*n^2)$

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int (\sin(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a+b*ln(c*x^n))^2,x)`

[Out] `int(sin(a+b*ln(c*x^n))^2,x)`

**Maxima [B]** time = 1.25203, size = 378, normalized size = 4.3

$$(2(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(2b \log(c)) + b \sin(2b \log(c)))n + \cos(4b \log(c)) \cos(2b \log(c)))x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))^2,x, algorithm="maxima")`

[Out] 
$$\frac{-1/4*((2*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)) + b*\sin(2*b*\log(c)))n + \cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)) + \cos(2*b*\log(c)))x*\cos(2*b*\log(x^n) + 2*a) + (2*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)) + b*\cos(2*b*\log(c)))n - \cos(2*b*\log(c))*\sin(4*b*\log(c)) + \cos(4*b*\log(c))*\sin(2*b*\log(c)) - \sin(2*b*\log(c)))x*\sin(2*b*\log(x^n) + 2*a) - 2*(4*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + \cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*x)/(4*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + \cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)}$$

**Fricas [A]** time = 0.496488, size = 198, normalized size = 2.25

$$\frac{2bnx \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) + x \cos(bn \log(x) + b \log(c) + a)^2 - (2b^2n^2 + 1)}{4b^2n^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))^2,x, algorithm="fricas")`

[Out] 
$$-(2*b*n*x*\cos(b*n*\log(x) + b*\log(c) + a)*\sin(b*n*\log(x) + b*\log(c) + a) + x*\cos(b*n*\log(x) + b*\log(c) + a)^2 - (2*b^2*n^2 + 1)*x)/(4*b^2*n^2 + 1)$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**2,x)
```

```
[Out] Exception raised: TypeError
```

**Giac [B]** time = 1.38757, size = 1061, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*(4*b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*
n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x*e^(-pi*b*n*sgn(x) + pi*b*
n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b
*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) +
b*log(abs(c)))*tan(a)^2 + 4*b*n*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c)
+ pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x*e^(pi*b*n*sgn(x)
- pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a
)^2 - x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)
) + b*log(abs(c)))^2*tan(a)^2 - 4*b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sg
n(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b*n*x*e^(-pi*b*n*sgn(
x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*
b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a) - 4*b*n*x*e^(-
pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a) + x*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + x*e^(
-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(a
bs(c)))^2 + 4*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log
(abs(x)) + b*log(abs(c)))*tan(a) + 4*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sg
n(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + x*e^(pi*b*n*sgn(
x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a)^2 + x*e^(-pi*b*n*sgn(x) + pi*b*n -
pi*b*sgn(c) + pi*b)*tan(a)^2 - x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) -
pi*b) - x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b))/(4*b^2*n^2*tan
(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*tan(b*n*log(abs(x)
) + b*log(abs(c)))^2 + 4*b^2*n^2*tan(a)^2 + 4*b^2*n^2 + tan(b*n*log(abs(x))
+ b*log(abs(c)))^2*tan(a)^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + tan
(a)^2 + 1)
```



$$3.10 \quad \int \frac{\sin^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=39

$$\frac{\log(x)}{2} - \frac{\sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{2bn}$$

[Out] Log[x]/2 - (Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(2\*b\*n)

**Rubi [A]** time = 0.0304173, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2635, 8}

$$\frac{\log(x)}{2} - \frac{\sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^2/x,x]

[Out] Log[x]/2 - (Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(2\*b\*n)

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x] \*(b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^2(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sin^2(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int 1 dx, x, \log(cx^n)\right)}{2n} \\ &= \frac{\log(x)}{2} - \frac{\cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{2bn} \end{aligned}$$

**Mathematica [A]** time = 0.0714815, size = 36, normalized size = 0.92

$$-\frac{\sin(2(a+b \log(cx^n))) - 2(a+b \log(cx^n))}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^2/x,x]

[Out] -(-2\*(a + b\*Log[c\*x^n]) + Sin[2\*(a + b\*Log[c\*x^n]]))/(4\*b\*n)

---

**Maple [A]** time = 0.022, size = 52, normalized size = 1.3

$$-\frac{\cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{2bn} + \frac{\ln(cx^n)}{2n} + \frac{a}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^2/x,x)

[Out] -1/2\*cos(a+b\*ln(c\*x^n))\*sin(a+b\*ln(c\*x^n))/b/n+1/2/n\*ln(c\*x^n)+1/2/b/n\*a

---

**Maxima [A]** time = 1.14662, size = 74, normalized size = 1.9

$$\frac{2bn \log(x) - \cos(2b \log(x^n) + 2a) \sin(2b \log(c)) - \cos(2b \log(c)) \sin(2b \log(x^n) + 2a)}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^2/x,x, algorithm="maxima")

[Out] 1/4\*(2\*b\*n\*log(x) - cos(2\*b\*log(x^n) + 2\*a)\*sin(2\*b\*log(c)) - cos(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a))/(b\*n)

---

**Fricas [A]** time = 0.49112, size = 119, normalized size = 3.05

$$\frac{bn \log(x) - \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^2/x,x, algorithm="fricas")

[Out] 1/2\*(b\*n\*log(x) - cos(b\*n\*log(x) + b\*log(c) + a)\*sin(b\*n\*log(x) + b\*log(c) + a))/(b\*n)

---

**Sympy [A]** time = 23.0007, size = 56, normalized size = 1.44

$$-\frac{\begin{cases} \log(x) \cos(2a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cos(2a + 2b \log(c)) & \text{for } n = 0 \\ \frac{\sin(2a + 2bn \log(x) + 2b \log(c))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\log(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*ln(c\*x\*\*n))\*\*2/x,x)

[Out] -Piecewise((log(x)\*cos(2\*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)\*cos(2\*a + 2\*b\*log(c)), Eq(n, 0)), (sin(2\*a + 2\*b\*n\*log(x) + 2\*b\*log(c))/(2\*b\*n), True))/2 + log(x)/2

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^2/x, x)
```

### 3.11 $\int \frac{\sin^2(a+b \log(cx^n))}{x^2} dx$

**Optimal.** Leaf size=95

$$-\frac{\sin^2(a+b \log(cx^n))}{x(4b^2n^2+1)} - \frac{2bn \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x(4b^2n^2+1)} - \frac{2b^2n^2}{x(4b^2n^2+1)}$$

[Out]  $(-2*b^2*n^2)/((1+4*b^2*n^2)*x) - (2*b*n*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]])/((1+4*b^2*n^2)*x) - \text{Sin}[a+b*\text{Log}[c*x^n]]^2/((1+4*b^2*n^2)*x)$

**Rubi [A]** time = 0.0258384, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 30}

$$-\frac{\sin^2(a+b \log(cx^n))}{x(4b^2n^2+1)} - \frac{2bn \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x(4b^2n^2+1)} - \frac{2b^2n^2}{x(4b^2n^2+1)}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^2/x^2, x]

[Out]  $(-2*b^2*n^2)/((1+4*b^2*n^2)*x) - (2*b*n*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]])/((1+4*b^2*n^2)*x) - \text{Sin}[a+b*\text{Log}[c*x^n]]^2/((1+4*b^2*n^2)*x)$

#### Rule 4487

Int[((e.\_)\*(x.\_))^(m.\_)\*Sin[((a.\_) + Log[(c.\_)\*(x.\_)^(n.\_)]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m.\_), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^2(a+b \log(cx^n))}{x^2} dx &= -\frac{2bn \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{(1+4b^2n^2)x} - \frac{\sin^2(a+b \log(cx^n))}{(1+4b^2n^2)x} + \frac{(2b^2n^2) \int \frac{1}{x^2} dx}{1+4b^2n^2} \\ &= -\frac{2b^2n^2}{(1+4b^2n^2)x} - \frac{2bn \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{(1+4b^2n^2)x} - \frac{\sin^2(a+b \log(cx^n))}{(1+4b^2n^2)x} \end{aligned}$$

**Mathematica [A]** time = 0.0984482, size = 57, normalized size = 0.6

$$\frac{-2bn \sin(2(a+b \log(cx^n))) + \cos(2(a+b \log(cx^n))) - 4b^2n^2 - 1}{2(4b^2n^2x + x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^2/x^2,x]
```

```
[Out] (-1 - 4*b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[2*(a + b*Log[c*x^n]
)))/(2*(x + 4*b^2*n^2*x))
```

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^2/x^2,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^2/x^2,x)
```

**Maxima [B]** time = 1.13302, size = 382, normalized size = 4.02

$$\frac{8(b^2 \cos(2b \log(c))^2 + b^2 \sin(2b \log(c))^2)n^2 + 2 \cos(2b \log(c))^2 + (2(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)))n + \cos(4b \log(c)) \cos(2b \log(c)) - \sin(4b \log(c)) \sin(2b \log(c)) - \cos(2b \log(c)) \cos(2b \log(x^n) + 2a) + 2 \sin(2b \log(c))^2 + (2(b \cos(4b \log(c)) \cos(2b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c)) + b \cos(2b \log(c)))n + \cos(2b \log(c)) \sin(4b \log(c)) - \cos(4b \log(c)) \sin(2b \log(c)) + \sin(2b \log(c)) \sin(2b \log(x^n) + 2a))}{(4(b^2 \cos(2b \log(c))^2 + b^2 \sin(2b \log(c))^2)n^2 + \cos(2b \log(c))^2 + \sin(2b \log(c))^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*cos(2*b*log
(c))^2 + (2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*
log(c)) + b*sin(2*b*log(c)))n - cos(4*b*log(c))*cos(2*b*log(c)) - sin(4*b*
log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 2*sin(
2*b*log(c))^2 + (2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*s
in(2*b*log(c)) + b*cos(2*b*log(c)))n + cos(2*b*log(c))*sin(4*b*log(c)) - c
os(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a))/
((4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2
+ sin(2*b*log(c))^2)*x)
```

**Fricas [A]** time = 0.495563, size = 193, normalized size = 2.03

$$\frac{2b^2n^2 + 2bn \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) - \cos(bn \log(x) + b \log(c) + a)^2 + 1}{(4b^2n^2 + 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*n^2 + 2*b*n*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c)
) + a) - cos(b*n*log(x) + b*log(c) + a)^2 + 1)/((4*b^2*n^2 + 1)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*ln(c\*x\*\*n))\*\*2/x\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^2/x^2,x, algorithm="giac")

[Out] integrate(sin(b\*log(c\*x^n) + a)^2/x^2, x)

### 3.12 $\int \frac{\sin^2(a+b \log(cx^n))}{x^3} dx$

**Optimal.** Leaf size=98

$$\frac{\sin^2(a+b \log(cx^n))}{2x^2(b^2n^2+1)} - \frac{bn \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{2x^2(b^2n^2+1)} - \frac{b^2n^2}{4x^2(b^2n^2+1)}$$

[Out]  $-(b^2n^2)/(4*(1+b^2n^2)*x^2) - (b*n*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]])/(2*(1+b^2n^2)*x^2) - \text{Sin}[a+b*\text{Log}[c*x^n]]^2/(2*(1+b^2n^2)*x^2)$

**Rubi [A]** time = 0.0260508, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 30}

$$\frac{\sin^2(a+b \log(cx^n))}{2x^2(b^2n^2+1)} - \frac{bn \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{2x^2(b^2n^2+1)} - \frac{b^2n^2}{4x^2(b^2n^2+1)}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^2/x^3, x]

[Out]  $-(b^2n^2)/(4*(1+b^2n^2)*x^2) - (b*n*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]])/(2*(1+b^2n^2)*x^2) - \text{Sin}[a+b*\text{Log}[c*x^n]]^2/(2*(1+b^2n^2)*x^2)$

#### Rule 4487

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] := Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^2(a+b \log(cx^n))}{x^3} dx &= -\frac{bn \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{2(1+b^2n^2)x^2} - \frac{\sin^2(a+b \log(cx^n))}{2(1+b^2n^2)x^2} + \frac{(b^2n^2) \int \frac{1}{x^3} dx}{2(1+b^2n^2)} \\ &= -\frac{b^2n^2}{4(1+b^2n^2)x^2} - \frac{bn \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{2(1+b^2n^2)x^2} - \frac{\sin^2(a+b \log(cx^n))}{2(1+b^2n^2)x^2} \end{aligned}$$

**Mathematica [A]** time = 0.100985, size = 58, normalized size = 0.59

$$\frac{bn \sin(2(a+b \log(cx^n))) - \cos(2(a+b \log(cx^n))) + b^2n^2 + 1}{4x^2(b^2n^2+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^2/x^3,x]

[Out]  $-(1 + b^2 n^2 - \cos[2(a + b \log[cx^n])] + b n \sin[2(a + b \log[cx^n])]) / (4(1 + b^2 n^2)x^2)$

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^2/x^3,x)

[Out] int(sin(a+b\*ln(c\*x^n))^2/x^3,x)

**Maxima [B]** time = 1.15096, size = 378, normalized size = 3.86

$$2 \frac{(b^2 \cos(2b \log(c))^2 + b^2 \sin(2b \log(c))^2)n^2 + 2 \cos(2b \log(c))^2 + ((b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)))n + \cos(4b \log(c)) \sin(2b \log(c)) + b \sin(2b \log(c)))n - \cos(4b \log(c)) \cos(2b \log(c)) - \sin(4b \log(c)) \sin(2b \log(c)) - \cos(2b \log(c)) \cos(2b \log(x^n) + 2a) + 2 \sin(2b \log(c))^2 + ((b \cos(4b \log(c)) \cos(2b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c)) + b \cos(2b \log(c)))n + \cos(2b \log(c)) \sin(4b \log(c)) - \cos(4b \log(c)) \sin(2b \log(c)) + \sin(2b \log(c)) \sin(2b \log(x^n) + 2a)) / ((b^2 \cos(2b \log(c))^2 + b^2 \sin(2b \log(c))^2)n^2 + \cos(2b \log(c))^2 + \sin(2b \log(c))^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^2/x^3,x, algorithm="maxima")

[Out]  $-1/8*(2*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + 2*\cos(2*b*\log(c))^2 + ((b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)) + b*\sin(2*b*\log(c)))n - \cos(4*b*\log(c))*\cos(2*b*\log(c)) - \sin(4*b*\log(c))*\sin(2*b*\log(c)) - \cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 2*\sin(2*b*\log(c))^2 + ((b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)) + b*\cos(2*b*\log(c)))n + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)) + \sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a)) / ((b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + \cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*x^2)$

**Fricas [A]** time = 0.499001, size = 198, normalized size = 2.02

$$\frac{b^2 n^2 + 2bn \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) - 2 \cos(bn \log(x) + b \log(c) + a)^2 + 2}{4(b^2 n^2 + 1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^2/x^3,x, algorithm="fricas")

[Out]  $-1/4*(b^2*n^2 + 2*b*n*\cos(b*n*\log(x) + b*\log(c) + a)*\sin(b*n*\log(x) + b*\log(c) + a) - 2*\cos(b*n*\log(x) + b*\log(c) + a)^2 + 2) / ((b^2*n^2 + 1)*x^2)$



**Sympy [A]** time = 82.0819, size = 643, normalized size = 6.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*ln(c\*x\*\*n))\*\*2/x\*\*3,x)

[Out] Piecewise((log(x)\*sin(-a + I\*log(x) + I\*log(c)/n)\*\*2/(4\*x\*\*2) + I\*log(x)\*sin(-a + I\*log(x) + I\*log(c)/n)\*cos(-a + I\*log(x) + I\*log(c)/n)/(2\*x\*\*2) - log(x)\*cos(-a + I\*log(x) + I\*log(c)/n)\*\*2/(4\*x\*\*2) - 3\*sin(-a + I\*log(x) + I\*log(c)/n)\*\*2/(8\*x\*\*2) - cos(-a + I\*log(x) + I\*log(c)/n)\*\*2/(8\*x\*\*2) + log(c)\*sin(-a + I\*log(x) + I\*log(c)/n)\*\*2/(4\*n\*x\*\*2) + I\*log(c)\*sin(-a + I\*log(x) + I\*log(c)/n)\*cos(-a + I\*log(x) + I\*log(c)/n)/(2\*n\*x\*\*2) - log(c)\*cos(-a + I\*log(x) + I\*log(c)/n)\*\*2/(4\*n\*x\*\*2), Eq(b, -I/n)), (log(x)\*sin(a + I\*log(x) + I\*log(c)/n)\*\*2/(4\*x\*\*2) + I\*log(x)\*sin(a + I\*log(x) + I\*log(c)/n)\*cos(a + I\*log(x) + I\*log(c)/n)/(2\*x\*\*2) - log(x)\*cos(a + I\*log(x) + I\*log(c)/n)\*\*2/(4\*x\*\*2) - 3\*sin(a + I\*log(x) + I\*log(c)/n)\*\*2/(8\*x\*\*2) - cos(a + I\*log(x) + I\*log(c)/n)\*\*2/(8\*x\*\*2) + log(c)\*sin(a + I\*log(x) + I\*log(c)/n)\*\*2/(4\*n\*x\*\*2) + I\*log(c)\*sin(a + I\*log(x) + I\*log(c)/n)\*cos(a + I\*log(x) + I\*log(c)/n)/(2\*n\*x\*\*2) - log(c)\*cos(a + I\*log(x) + I\*log(c)/n)\*\*2/(4\*n\*x\*\*2), Eq(b, I/n)), (-b\*\*2\*n\*\*2\*sin(a + b\*n\*log(x) + b\*log(c))\*\*2/(4\*b\*\*2\*n\*\*2\*x\*\*2 + 4\*x\*\*2) - b\*\*2\*n\*\*2\*cos(a + b\*n\*log(x) + b\*log(c))\*\*2/(4\*b\*\*2\*n\*\*2\*x\*\*2 + 4\*x\*\*2) - 2\*b\*n\*sin(a + b\*n\*log(x) + b\*log(c))\*cos(a + b\*n\*log(x) + b\*log(c))/(4\*b\*\*2\*n\*\*2\*x\*\*2 + 4\*x\*\*2) - 2\*sin(a + b\*n\*log(x) + b\*log(c))\*\*2/(4\*b\*\*2\*n\*\*2\*x\*\*2 + 4\*x\*\*2), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^2/x^3,x, algorithm="giac")

[Out] integrate(sin(b\*log(c\*x^n) + a)^2/x^3, x)

### 3.13 $\int x^2 \sin^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=160

$$\frac{x^3 \sin^3(a + b \log(cx^n))}{3(b^2 n^2 + 1)} + \frac{2b^2 n^2 x^3 \sin(a + b \log(cx^n))}{b^4 n^4 + 10b^2 n^2 + 9} - \frac{2b^3 n^3 x^3 \cos(a + b \log(cx^n))}{3(b^4 n^4 + 10b^2 n^2 + 9)} - \frac{bnx^3 \sin^2(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{3(b^2 n^2 + 1)}$$

[Out]  $(-2*b^3*n^3*x^3*Cos[a + b*Log[c*x^n]])/(3*(9 + 10*b^2*n^2 + b^4*n^4)) + (2*b^2*n^2*x^3*Sin[a + b*Log[c*x^n]])/(9 + 10*b^2*n^2 + b^4*n^4) - (b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(3*(1 + b^2*n^2)) + (x^3*Sin[a + b*Log[c*x^n]]^3)/(3*(1 + b^2*n^2))$

**Rubi [A]** time = 0.0553367, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 4485}

$$\frac{x^3 \sin^3(a + b \log(cx^n))}{3(b^2 n^2 + 1)} + \frac{2b^2 n^2 x^3 \sin(a + b \log(cx^n))}{b^4 n^4 + 10b^2 n^2 + 9} - \frac{2b^3 n^3 x^3 \cos(a + b \log(cx^n))}{3(b^4 n^4 + 10b^2 n^2 + 9)} - \frac{bnx^3 \sin^2(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{3(b^2 n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sin[a + b\*Log[c\*x^n]]^3,x]

[Out]  $(-2*b^3*n^3*x^3*Cos[a + b*Log[c*x^n]])/(3*(9 + 10*b^2*n^2 + b^4*n^4)) + (2*b^2*n^2*x^3*Sin[a + b*Log[c*x^n]])/(9 + 10*b^2*n^2 + b^4*n^4) - (b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(3*(1 + b^2*n^2)) + (x^3*Sin[a + b*Log[c*x^n]]^3)/(3*(1 + b^2*n^2))$

#### Rule 4487

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 4485

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int x^2 \sin^3(a + b \log(cx^n)) dx &= -\frac{bnx^3 \cos(a + b \log(cx^n)) \sin^2(a + b \log(cx^n))}{3(1 + b^2 n^2)} + \frac{x^3 \sin^3(a + b \log(cx^n))}{3(1 + b^2 n^2)} + \frac{(2b^2 n^2) \int x \sin^3(a + b \log(cx^n)) dx}{3(1 + b^2 n^2)} \\ &= -\frac{2b^3 n^3 x^3 \cos(a + b \log(cx^n))}{3(9 + 10b^2 n^2 + b^4 n^4)} + \frac{2b^2 n^2 x^3 \sin(a + b \log(cx^n))}{9 + 10b^2 n^2 + b^4 n^4} - \frac{bnx^3 \cos(a + b \log(cx^n)) \sin^2(a + b \log(cx^n))}{3(1 + b^2 n^2)} \end{aligned}$$

**Mathematica [A]** time = 0.507012, size = 122, normalized size = 0.76

$$\frac{x^3 \left( -9bn \left( b^2n^2 + 1 \right) \cos \left( a + b \log \left( cx^n \right) \right) + bn \left( b^2n^2 + 9 \right) \cos \left( 3 \left( a + b \log \left( cx^n \right) \right) \right) - 2 \sin \left( a + b \log \left( cx^n \right) \right) \left( \left( b^2n^2 + 9 \right) \cos \left( a + b \log \left( cx^n \right) \right) + \left( b^2n^2 + 1 \right) \sin \left( a + b \log \left( cx^n \right) \right) \right) \right)}{12 \left( b^4n^4 + 10b^2n^2 + 9 \right)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sin[a + b\*Log[c\*x^n]]^3,x]

[Out] (x^3\*(-9\*b\*n\*(1 + b^2\*n^2)\*Cos[a + b\*Log[c\*x^n]] + b\*n\*(9 + b^2\*n^2)\*Cos[3\*(a + b\*Log[c\*x^n])] - 2\*(-9 - 13\*b^2\*n^2 + (9 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])])\*Sin[a + b\*Log[c\*x^n]])/(12\*(9 + 10\*b^2\*n^2 + b^4\*n^4))

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int x^2 (\sin(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sin(a+b\*ln(c\*x^n))^3,x)

[Out] int(x^2\*sin(a+b\*ln(c\*x^n))^3,x)

**Maxima [B]** time = 1.22961, size = 1361, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] 1/24\*(((b^3\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c)))\*n^3 - (b^2\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) - b^2\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) + b^2\*sin(3\*b\*log(c)))\*n^2 + 9\*(b\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(3\*b\*log(c)))\*n - 9\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) + 9\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) - 9\*sin(3\*b\*log(c)))\*x^3\*cos(3\*b\*log(x^n) + 3\*a) - 9\*((b^3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 - 3\*(b^2\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^2\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n - 3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) + 3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) - 3\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) + 3\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*x^3\*cos(b\*log(x^n) + a) - ((b^3\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c)))\*n^3 + (b^2\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c)))\*n^2 + 9\*(b\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c)))\*n + 9\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + 9\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + 9\*cos(3\*b\*log(c)))\*x^3\*sin(3\*b\*log(x^n) + 3\*a) + 9\*((b^3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^3\*cos(3\*b\*log(c))\*sin(2\*b\*log(c))

$$\begin{aligned} & c)) * n^3 + 3 * (b^2 * \cos(4 * b * \log(c)) * \cos(3 * b * \log(c)) + b^2 * \cos(3 * b * \log(c)) * \cos( \\ & (2 * b * \log(c)) + b^2 * \sin(4 * b * \log(c)) * \sin(3 * b * \log(c)) + b^2 * \sin(3 * b * \log(c)) * \sin( \\ & n(2 * b * \log(c))) * n^2 + (b * \cos(3 * b * \log(c)) * \sin(4 * b * \log(c)) - b * \cos(4 * b * \log(c)) \\ & * \sin(3 * b * \log(c)) + b * \cos(2 * b * \log(c)) * \sin(3 * b * \log(c)) - b * \cos(3 * b * \log(c)) * \sin( \\ & n(2 * b * \log(c))) * n + 3 * \cos(4 * b * \log(c)) * \cos(3 * b * \log(c)) + 3 * \cos(3 * b * \log(c)) * \cos( \\ & s(2 * b * \log(c)) + 3 * \sin(4 * b * \log(c)) * \sin(3 * b * \log(c)) + 3 * \sin(3 * b * \log(c)) * \sin(2 \\ & * b * \log(c))) * x^3 * \sin(b * \log(x^n) + a) / ((b^4 * \cos(3 * b * \log(c))^2 + b^4 * \sin(3 * b * \\ & \log(c))^2) * n^4 + 10 * (b^2 * \cos(3 * b * \log(c))^2 + b^2 * \sin(3 * b * \log(c))^2) * n^2 + 9 \\ & * \cos(3 * b * \log(c))^2 + 9 * \sin(3 * b * \log(c))^2) \end{aligned}$$

**Fricas [A]** time = 0.509526, size = 343, normalized size = 2.14

$$\frac{(b^3 n^3 + 9 b n) x^3 \cos(b n \log(x) + b \log(c) + a)^3 - 3(b^3 n^3 + 3 b n) x^3 \cos(b n \log(x) + b \log(c) + a) - ((b^2 n^2 + 9) x^3 \cos(b n \log(x) + b \log(c) + a)^2 - (7 b^2 n^2 + 9) x^3) \sin(b n \log(x) + b \log(c) + a)}{3(b^4 n^4 + 10 b^2 n^2 + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/3\*((b^3\*n^3 + 9\*b\*n)\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a)^3 - 3\*(b^3\*n^3 + 3\*b\*n)\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a) - ((b^2\*n^2 + 9)\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a)^2 - (7\*b^2\*n^2 + 9)\*x^3)\*sin(b\*n\*log(x) + b\*log(c) + a)) / (b^4\*n^4 + 10\*b^2\*n^2 + 9)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*sin(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] Timed out

### 3.14 $\int x \sin^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=158

$$\frac{2x^2 \sin^3(a + b \log(cx^n))}{9b^2n^2 + 4} + \frac{12b^2n^2x^2 \sin(a + b \log(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} - \frac{6b^3n^3x^2 \cos(a + b \log(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} - \frac{3bnx^2 \sin^2(a + b \log(cx^n))}{9b^2n^2}$$

[Out]  $(-6*b^3*n^3*x^2*\text{Cos}[a + b*\text{Log}[c*x^n]])/(16 + 40*b^2*n^2 + 9*b^4*n^4) + (12*b^2*n^2*x^2*\text{Sin}[a + b*\text{Log}[c*x^n]])/(16 + 40*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x^2*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/(4 + 9*b^2*n^2) + (2*x^2*\text{Sin}[a + b*\text{Log}[c*x^n]]^3)/(4 + 9*b^2*n^2)$

**Rubi [A]** time = 0.0448578, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {4487, 4485}

$$\frac{2x^2 \sin^3(a + b \log(cx^n))}{9b^2n^2 + 4} + \frac{12b^2n^2x^2 \sin(a + b \log(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} - \frac{6b^3n^3x^2 \cos(a + b \log(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} - \frac{3bnx^2 \sin^2(a + b \log(cx^n))}{9b^2n^2}$$

Antiderivative was successfully verified.

[In] Int[x\*Sin[a + b\*Log[c\*x^n]]^3,x]

[Out]  $(-6*b^3*n^3*x^2*\text{Cos}[a + b*\text{Log}[c*x^n]])/(16 + 40*b^2*n^2 + 9*b^4*n^4) + (12*b^2*n^2*x^2*\text{Sin}[a + b*\text{Log}[c*x^n]])/(16 + 40*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x^2*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/(4 + 9*b^2*n^2) + (2*x^2*\text{Sin}[a + b*\text{Log}[c*x^n]]^3)/(4 + 9*b^2*n^2)$

#### Rule 4487

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 4485

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)], x\_Symbol] := Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int x \sin^3(a + b \log(cx^n)) dx &= -\frac{3bnx^2 \cos(a + b \log(cx^n)) \sin^2(a + b \log(cx^n))}{4 + 9b^2n^2} + \frac{2x^2 \sin^3(a + b \log(cx^n))}{4 + 9b^2n^2} + \frac{(6b^2n^2x^2 \sin(a + b \log(cx^n)))}{16 + 40b^2n^2 + 9b^4n^4} \\ &= -\frac{6b^3n^3x^2 \cos(a + b \log(cx^n))}{16 + 40b^2n^2 + 9b^4n^4} + \frac{12b^2n^2x^2 \sin(a + b \log(cx^n))}{16 + 40b^2n^2 + 9b^4n^4} - \frac{3bnx^2 \cos(a + b \log(cx^n))}{4} \end{aligned}$$

**Mathematica [A]** time = 0.47611, size = 125, normalized size = 0.79

$$\frac{x^2 \left( -3bn \left( 9b^2n^2 + 4 \right) \cos \left( a + b \log \left( cx^n \right) \right) + 3bn \left( b^2n^2 + 4 \right) \cos \left( 3 \left( a + b \log \left( cx^n \right) \right) \right) - 4 \sin \left( a + b \log \left( cx^n \right) \right) \left( \left( b^2n^2 + 4 \right) \cos \left( a + b \log \left( cx^n \right) \right) \right) \right)}{4 \left( 9b^4n^4 + 40b^2n^2 + 16 \right)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sin[a + b\*Log[c\*x^n]]^3,x]

[Out] (x^2\*(-3\*b\*n\*(4 + 9\*b^2\*n^2)\*Cos[a + b\*Log[c\*x^n]] + 3\*b\*n\*(4 + b^2\*n^2)\*Cos[3\*(a + b\*Log[c\*x^n])] - 4\*(-4 - 13\*b^2\*n^2 + (4 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])])\*Sin[a + b\*Log[c\*x^n]])/(4\*(16 + 40\*b^2\*n^2 + 9\*b^4\*n^4))

**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int x (\sin(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+b\*ln(c\*x^n))^3,x)

[Out] int(x\*sin(a+b\*ln(c\*x^n))^3,x)

**Maxima [B]** time = 1.4297, size = 1372, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] 1/8\*((3\*(b^3\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c)))\*n^3 - 2\*(b^2\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c)))\*n^2 + 12\*(b\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(3\*b\*log(c)))\*n - 8\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) + 8\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) - 8\*sin(3\*b\*log(c)))\*x^2\*cos(3\*b\*log(x^n) + 3\*a) - 3\*(9\*(b^3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 - 18\*(b^2\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^2\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + 4\*(b\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n - 8\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) + 8\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) - 8\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) + 8\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*x^2\*cos(b\*log(x^n) + a) - (3\*(b^3\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c)))\*n^3 + 2\*(b^2\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c)))\*n^2 + 12\*(b\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c)))\*n + 8\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + 8\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + 8\*cos(3\*b\*log(c))\*x^2\*sin(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^3\*cos(3\*b\*log(c))

$$c)) \sin(2b \log(c)) n^3 + 18(b^2 \cos(4b \log(c)) \cos(3b \log(c)) + b^2 \cos(3b \log(c)) \cos(2b \log(c)) + b^2 \sin(4b \log(c)) \sin(3b \log(c)) + b^2 \sin(3b \log(c)) \sin(2b \log(c))) n^2 + 4(b \cos(3b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(3b \log(c)) + b \cos(2b \log(c)) \sin(3b \log(c)) - b \cos(3b \log(c)) \sin(2b \log(c))) n + 8 \cos(4b \log(c)) \cos(3b \log(c)) + 8 \cos(3b \log(c)) \cos(2b \log(c)) + 8 \sin(4b \log(c)) \sin(3b \log(c)) + 8 \sin(3b \log(c)) \sin(2b \log(c)) x^2 \sin(b \log(x^n) + a) / (9(b^4 \cos(3b \log(c))^2 + b^4 \sin(3b \log(c))^2) n^4 + 40(b^2 \cos(3b \log(c))^2 + b^2 \sin(3b \log(c))^2) n^2 + 16 \cos(3b \log(c))^2 + 16 \sin(3b \log(c))^2)$$

**Fricas [A]** time = 0.52004, size = 350, normalized size = 2.22

$$\frac{3(b^3 n^3 + 4bn)x^2 \cos(bn \log(x) + b \log(c) + a)^3 - 3(3b^3 n^3 + 4bn)x^2 \cos(bn \log(x) + b \log(c) + a) - 2((b^2 n^2 + 4))}{9b^4 n^4 + 40b^2 n^2 + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] (3\*(b^3\*n^3 + 4\*b\*n)\*x^2\*cos(b\*n\*log(x) + b\*log(c) + a)^3 - 3\*(3\*b^3\*n^3 + 4\*b\*n)\*x^2\*cos(b\*n\*log(x) + b\*log(c) + a) - 2\*((b^2\*n^2 + 4)\*x^2\*cos(b\*n\*log(x) + b\*log(c) + a)^2 - (7\*b^2\*n^2 + 4)\*x^2)\*sin(b\*n\*log(x) + b\*log(c) + a))/(9\*b^4\*n^4 + 40\*b^2\*n^2 + 16)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] Timed out

### 3.15 $\int \sin^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=149

$$\frac{x \sin^3(a + b \log(cx^n))}{9b^2n^2 + 1} + \frac{6b^2n^2x \sin(a + b \log(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} - \frac{6b^3n^3x \cos(a + b \log(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} - \frac{3bnx \sin^2(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{9b^2n^2 + 1}$$

[Out]  $(-6*b^3*n^3*x*Cos[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) + (6*b^2*n^2*x*Sin[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(1 + 9*b^2*n^2) + (x*Sin[a + b*Log[c*x^n]]^3)/(1 + 9*b^2*n^2)$

**Rubi [A]** time = 0.0365887, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {4477, 4475}

$$\frac{x \sin^3(a + b \log(cx^n))}{9b^2n^2 + 1} + \frac{6b^2n^2x \sin(a + b \log(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} - \frac{6b^3n^3x \cos(a + b \log(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} - \frac{3bnx \sin^2(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{9b^2n^2 + 1}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^3,x]

[Out]  $(-6*b^3*n^3*x*Cos[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) + (6*b^2*n^2*x*Sin[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(1 + 9*b^2*n^2) + (x*Sin[a + b*Log[c*x^n]]^3)/(1 + 9*b^2*n^2)$

#### Rule 4477

Int[Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] := Simp[(x\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*n^2\*p^2 + 1), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + 1), Int[Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*x\*Cos[d\*(a + b\*Log[c\*x^n]])\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*n^2\*p^2 + 1), x]) /; FreeQ[{a, b, c, d, n}, x] && I GtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + 1, 0]

#### Rule 4475

Int[Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] := Simp[(x\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] - Simp[(b\*d\*n\*x\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2\*d^2\*n^2 + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \sin^3(a + b \log(cx^n)) dx &= -\frac{3bnx \cos(a + b \log(cx^n)) \sin^2(a + b \log(cx^n))}{1 + 9b^2n^2} + \frac{x \sin^3(a + b \log(cx^n))}{1 + 9b^2n^2} + \frac{(6b^2n^2) \int \sin^2(a + b \log(cx^n)) dx}{1 + 9b^2n^2} \\ &= -\frac{6b^3n^3x \cos(a + b \log(cx^n))}{1 + 10b^2n^2 + 9b^4n^4} + \frac{6b^2n^2x \sin(a + b \log(cx^n))}{1 + 10b^2n^2 + 9b^4n^4} - \frac{3bnx \cos(a + b \log(cx^n)) \sin^2(a + b \log(cx^n))}{1 + 9b^2n^2} \end{aligned}$$

**Mathematica [A]** time = 0.482902, size = 121, normalized size = 0.81

$$\frac{x(3bn(9b^2n^2 + 1) \cos(a + b \log(cx^n)) - 3(b^3n^3 + bn) \cos(3(a + b \log(cx^n))) + 2 \sin(a + b \log(cx^n))((b^2n^2 + 1) \cos(a + b \log(cx^n)) - \sin(a + b \log(cx^n))))}{36b^4n^4 + 40b^2n^2 + 4}$$



Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*Log[c*x^n]]^3,x]
```

```
[Out] -((x*(3*b*n*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] - 3*(b*n + b^3*n^3)*Cos[3
*(a + b*Log[c*x^n]]) + 2*(-1 - 13*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[
c*x^n]]))*Sin[a + b*Log[c*x^n]]))/(4 + 40*b^2*n^2 + 36*b^4*n^4))
```

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int (\sin(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a+b*ln(c*x^n))^3,x)
```

```
[Out] int(sin(a+b*ln(c*x^n))^3,x)
```

**Maxima [B]** time = 1.24488, size = 1337, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*
log(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*cos(3*b*log(c))*sin(6*b*log(c)) -
b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 3*(b*cos(
6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b
*log(c))*n - cos(3*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(3*b*log
(c)) - sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) - 3*(9*(b^3*cos(4*b*log(c
))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4*b*log(
c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 - 9*(b^2*cos
(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c)) + b^2*co
s(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c)))*n^2 +
(b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log(c)) + b*
sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n - co
s(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)) - cos(2*b*l
og(c))*sin(3*b*log(c)) + cos(3*b*log(c))*sin(2*b*log(c)))*x*cos(b*log(x^n)
+ a) - (3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*
b*log(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*cos(6*b*log(c))*cos(3*b*log(c))
+ b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 3*(b*co
s(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(3
*b*log(c)))*n + cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*l
og(c)) + cos(3*b*log(c)))*x*sin(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(3*b*log
(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b*lo
g(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 + 9*(b^2*c
os(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(c)) + b^2*
sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log(c)))*n^2
+ (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)) +
b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n +
cos(4*b*log(c))*cos(3*b*log(c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b
```

```
*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(2*b*log(c)))*x*sin(b*log(x^n)
) + a))/(9*(b^4*cos(3*b*log(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 10*(b^2*cos
(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2)*n^2 + cos(3*b*log(c))^2 + sin(3*b*
log(c))^2)
```

**Fricas [A]** time = 0.504358, size = 329, normalized size = 2.21

$$\frac{3(b^3n^3 + bn)x \cos(bn \log(x) + b \log(c) + a)^3 - 3(3b^3n^3 + bn)x \cos(bn \log(x) + b \log(c) + a) - ((b^2n^2 + 1)x \cos(bn \log(x) + b \log(c) + a))^2}{9b^4n^4 + 10b^2n^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] (3*(b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 + b*n)
*x*cos(b*n*log(x) + b*log(c) + a) - ((b^2*n^2 + 1)*x*cos(b*n*log(x) + b*log
(c) + a)^2 - (7*b^2*n^2 + 1)*x)*sin(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4
+ 10*b^2*n^2 + 1)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.16 \quad \int \frac{\sin^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=43

$$\frac{\cos^3(a+b \log(cx^n))}{3bn} - \frac{\cos(a+b \log(cx^n))}{bn}$$

[Out]  $-(\text{Cos}[a + b*\text{Log}[c*x^n]]/(b*n)) + \text{Cos}[a + b*\text{Log}[c*x^n]]^3/(3*b*n)$

**Rubi [A]** time = 0.0317094, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2633}

$$\frac{\cos^3(a+b \log(cx^n))}{3bn} - \frac{\cos(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^3/x,x]

[Out]  $-(\text{Cos}[a + b*\text{Log}[c*x^n]]/(b*n)) + \text{Cos}[a + b*\text{Log}[c*x^n]]^3/(3*b*n)$

**Rule 2633**

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{\sin^3(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sin^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\text{Subst}\left(\int (1-x^2) dx, x, \cos(a+b \log(cx^n))\right)}{bn} \\ &= -\frac{\cos(a+b \log(cx^n))}{bn} + \frac{\cos^3(a+b \log(cx^n))}{3bn} \end{aligned}$$

**Mathematica [A]** time = 0.0591127, size = 45, normalized size = 1.05

$$\frac{\cos(3(a+b \log(cx^n)))}{12bn} - \frac{3 \cos(a+b \log(cx^n))}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^3/x,x]

[Out]  $(-3*\text{Cos}[a + b*\text{Log}[c*x^n]])/(4*b*n) + \text{Cos}[3*(a + b*\text{Log}[c*x^n])]/(12*b*n)$

**Maple [A]** time = 0.025, size = 35, normalized size = 0.8

$$\frac{(2 + (\sin(a + b \ln(cx^n)))^2) \cos(a + b \ln(cx^n))}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a+b*ln(c*x^n))^3/x,x)`

[Out] `-1/3/n/b*(2+sin(a+b*ln(c*x^n))^2)*cos(a+b*ln(c*x^n))`

**Maxima [B]** time = 1.10253, size = 315, normalized size = 7.33

$$\frac{(\cos(6b \log(c)) \cos(3b \log(c)) + \sin(6b \log(c)) \sin(3b \log(c)) + \cos(3b \log(c))) \cos(3b \log(x^n) + 3a) - 9(\cos(4b \log(c)) \cos(3b \log(c)) + \sin(4b \log(c)) \sin(3b \log(c)) + \cos(3b \log(c)) \cos(2b \log(c)) + \sin(4b \log(c)) \sin(3b \log(c)) + \sin(3b \log(c)) \sin(2b \log(c))) \cos(b \log(x^n) + a) - (\cos(3b \log(c)) \sin(6b \log(c)) - \cos(6b \log(c)) \sin(3b \log(c)) + \sin(3b \log(c)) \sin(3b \log(x^n) + 3a) + 9(\cos(3b \log(c)) \sin(4b \log(c)) - \cos(4b \log(c)) \sin(3b \log(c)) + \cos(2b \log(c)) \sin(3b \log(c)) - \cos(3b \log(c)) \sin(2b \log(c))) \sin(b \log(x^n) + a)}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="maxima")`

[Out] `1/24*((cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 9*(cos(4*b*log(c))*cos(3*b*log(c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(2*b*log(c)))*cos(b*log(x^n) + a) - (cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(3*b*log(x^n) + 3*a) + 9*(cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)) + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*sin(b*log(x^n) + a))/(b*n)`

**Fricas [A]** time = 0.491843, size = 109, normalized size = 2.53

$$\frac{\cos(bn \log(x) + b \log(c) + a)^3 - 3 \cos(bn \log(x) + b \log(c) + a)}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="fricas")`

[Out] `1/3*(cos(b*n*log(x) + b*log(c) + a)^3 - 3*cos(b*n*log(x) + b*log(c) + a))/(b*n)`

**Sympy [A]** time = 55.5855, size = 83, normalized size = 1.93

$$\begin{cases} \log(x) \sin^3(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \sin^3(a + b \log(c)) & \text{for } n = 0 \\ -\frac{\sin^2(a + bn \log(x) + b \log(c)) \cos(a + bn \log(x) + b \log(c))}{bn} - \frac{2 \cos^3(a + bn \log(x) + b \log(c))}{3bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*ln(c*x**n))**3/x,x)`

[Out] `Piecewise((log(x)*sin(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sin(a + b*log(c))**3, Eq(n, 0)), (-sin(a + b*n*log(x) + b*log(c))**2*cos(a + b*n*log(x) + b*log(c))/(b*n) - 2*cos(a + b*n*log(x) + b*log(c))**3/(3*b*n), True))`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^3/x, x)
```

$$3.17 \quad \int \frac{\sin^3(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=158

$$\frac{\sin^3(a+b \log(cx^n))}{x(9b^2n^2+1)} - \frac{6b^2n^2 \sin(a+b \log(cx^n))}{x(9b^4n^4+10b^2n^2+1)} - \frac{6b^3n^3 \cos(a+b \log(cx^n))}{x(9b^4n^4+10b^2n^2+1)} - \frac{3bn \sin^2(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x(9b^2n^2+1)}$$

[Out]  $(-6*b^3*n^3*Cos[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - (6*b^2*n^2*Sin[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - (3*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/((1 + 9*b^2*n^2)*x) - Sin[a + b*Log[c*x^n]]^3/((1 + 9*b^2*n^2)*x)$

**Rubi [A]** time = 0.0468833, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 4485}

$$\frac{\sin^3(a+b \log(cx^n))}{x(9b^2n^2+1)} - \frac{6b^2n^2 \sin(a+b \log(cx^n))}{x(9b^4n^4+10b^2n^2+1)} - \frac{6b^3n^3 \cos(a+b \log(cx^n))}{x(9b^4n^4+10b^2n^2+1)} - \frac{3bn \sin^2(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x(9b^2n^2+1)}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^3/x^2,x]

[Out]  $(-6*b^3*n^3*Cos[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - (6*b^2*n^2*Sin[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - (3*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/((1 + 9*b^2*n^2)*x) - Sin[a + b*Log[c*x^n]]^3/((1 + 9*b^2*n^2)*x)$

#### Rule 4487

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 4485

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^3(a+b \log(cx^n))}{x^2} dx &= -\frac{3bn \cos(a+b \log(cx^n)) \sin^2(a+b \log(cx^n))}{(1+9b^2n^2)x} - \frac{\sin^3(a+b \log(cx^n))}{(1+9b^2n^2)x} + \frac{(6b^2n^2) \int \frac{\sin(a+b \log(cx^n))}{x} dx}{1+9b^2n^2} \\ &= -\frac{6b^3n^3 \cos(a+b \log(cx^n))}{(1+10b^2n^2+9b^4n^4)x} - \frac{6b^2n^2 \sin(a+b \log(cx^n))}{(1+10b^2n^2+9b^4n^4)x} - \frac{3bn \cos(a+b \log(cx^n)) \sin^2(a+b \log(cx^n))}{(1+9b^2n^2)x} \end{aligned}$$

**Mathematica [A]** time = 0.334706, size = 125, normalized size = 0.79

$$\frac{-3bn(9b^2n^2 + 1)\cos(a + b\log(cx^n)) + 3(b^3n^3 + bn)\cos(3(a + b\log(cx^n))) + 2\sin(a + b\log(cx^n))((b^2n^2 + 1)\cos(n))}{4x(9b^4n^4 + 10b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^3/x^2,x]

[Out] (-3\*b\*n\*(1 + 9\*b^2\*n^2)\*Cos[a + b\*Log[c\*x^n]] + 3\*(b\*n + b^3\*n^3)\*Cos[3\*(a + b\*Log[c\*x^n])] + 2\*(-1 - 13\*b^2\*n^2 + (1 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])])\*Sin[a + b\*Log[c\*x^n]]/(4\*(1 + 10\*b^2\*n^2 + 9\*b^4\*n^4)\*x)

**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^3/x^2,x)

[Out] int(sin(a+b\*ln(c\*x^n))^3/x^2,x)

**Maxima [B]** time = 1.28572, size = 1343, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^3/x^2,x, algorithm="maxima")

[Out] 1/8\*((3\*(b^3\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c)))\*n^3 + (b^2\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c)))\*n^2 + 3\*(b\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(3\*b\*log(c))\*n + cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + sin(3\*b\*log(c))\*cos(3\*b\*log(x^n) + 3\*a) - 3\*(9\*(b^3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 9\*(b^2\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^2\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n + cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(b\*log(x^n) + a) - (3\*(b^3\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c)))\*n^3 - (b^2\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c)))\*n^2 + 3\*(b\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c))\*n - cos(6\*b\*log(c))\*cos(3\*b\*log(c)) - sin(6\*b\*log(c))\*sin(3\*b\*log(c)) - cos(3\*b\*log(c))\*sin(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*

$$\begin{aligned} & \sin(3*b*\log(c)) - b^3*\cos(3*b*\log(c))*\sin(2*b*\log(c)))*n^3 - 9*(b^2*\cos(4*b \\ & * \log(c))*\cos(3*b*\log(c)) + b^2*\cos(3*b*\log(c))*\cos(2*b*\log(c)) + b^2*\sin(4* \\ & b*\log(c))*\sin(3*b*\log(c)) + b^2*\sin(3*b*\log(c))*\sin(2*b*\log(c)))*n^2 + (b*c \\ & \cos(3*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(3*b*\log(c)) + b*\cos( \\ & 2*b*\log(c))*\sin(3*b*\log(c)) - b*\cos(3*b*\log(c))*\sin(2*b*\log(c)))*n - \cos(4* \\ & b*\log(c))*\cos(3*b*\log(c)) - \cos(3*b*\log(c))*\cos(2*b*\log(c)) - \sin(4*b*\log(c) \\ & ))*\sin(3*b*\log(c)) - \sin(3*b*\log(c))*\sin(2*b*\log(c))*\sin(b*\log(x^n) + a))/ \\ & ((9*(b^4*\cos(3*b*\log(c))^2 + b^4*\sin(3*b*\log(c))^2)*n^4 + 10*(b^2*\cos(3*b*\log(c))^2 + b^2*\sin(3*b*\log(c))^2)*n^2 + \cos(3*b*\log(c))^2 + \sin(3*b*\log(c))^2)*x) \end{aligned}$$

**Fricas [A]** time = 0.509677, size = 321, normalized size = 2.03

$$\frac{3(b^3n^3 + bn)\cos(bn\log(x) + b\log(c) + a)^3 - 3(3b^3n^3 + bn)\cos(bn\log(x) + b\log(c) + a) - (7b^2n^2 - (b^2n^2 + 1)\cos(bn\log(x) + b\log(c) + a)^2 + 1)\sin(bn\log(x) + b\log(c) + a)}{(9b^4n^4 + 10b^2n^2 + 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^3/x^2,x, algorithm="fricas")

[Out] (3\*(b^3\*n^3 + b\*n)\*cos(b\*n\*log(x) + b\*log(c) + a)^3 - 3\*(3\*b^3\*n^3 + b\*n)\*cos(b\*n\*log(x) + b\*log(c) + a) - (7\*b^2\*n^2 - (b^2\*n^2 + 1)\*cos(b\*n\*log(x) + b\*log(c) + a)^2 + 1)\*sin(b\*n\*log(x) + b\*log(c) + a))/((9\*b^4\*n^4 + 10\*b^2\*n^2 + 1)\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*ln(c\*x\*\*n))\*\*3/x\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^3/x^2,x, algorithm="giac")

[Out] integrate(sin(b\*log(c\*x^n) + a)^3/x^2, x)



### 3.18 $\int \frac{\sin^3(a+b \log(cx^n))}{x^3} dx$

**Optimal.** Leaf size=158

$$\frac{2 \sin^3(a+b \log(cx^n))}{x^2(9b^2n^2+4)} - \frac{12b^2n^2 \sin(a+b \log(cx^n))}{x^2(9b^4n^4+40b^2n^2+16)} - \frac{6b^3n^3 \cos(a+b \log(cx^n))}{x^2(9b^4n^4+40b^2n^2+16)} - \frac{3bn \sin^2(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x^2(9b^2n^2+4)}$$

[Out]  $(-6*b^3*n^3*\text{Cos}[a + b*\text{Log}[c*x^n]])/((16 + 40*b^2*n^2 + 9*b^4*n^4)*x^2) - (12*b^2*n^2*\text{Sin}[a + b*\text{Log}[c*x^n]])/((16 + 40*b^2*n^2 + 9*b^4*n^4)*x^2) - (3*b^3*n^3*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/((4 + 9*b^2*n^2)*x^2) - (2*\text{Sin}[a + b*\text{Log}[c*x^n]]^3)/((4 + 9*b^2*n^2)*x^2)$

**Rubi [A]** time = 0.0477091, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 4485}

$$\frac{2 \sin^3(a+b \log(cx^n))}{x^2(9b^2n^2+4)} - \frac{12b^2n^2 \sin(a+b \log(cx^n))}{x^2(9b^4n^4+40b^2n^2+16)} - \frac{6b^3n^3 \cos(a+b \log(cx^n))}{x^2(9b^4n^4+40b^2n^2+16)} - \frac{3bn \sin^2(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x^2(9b^2n^2+4)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[a + b*\text{Log}[c*x^n]]^3/x^3, x]$

[Out]  $(-6*b^3*n^3*\text{Cos}[a + b*\text{Log}[c*x^n]])/((16 + 40*b^2*n^2 + 9*b^4*n^4)*x^2) - (12*b^2*n^2*\text{Sin}[a + b*\text{Log}[c*x^n]])/((16 + 40*b^2*n^2 + 9*b^4*n^4)*x^2) - (3*b^3*n^3*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/((4 + 9*b^2*n^2)*x^2) - (2*\text{Sin}[a + b*\text{Log}[c*x^n]]^3)/((4 + 9*b^2*n^2)*x^2)$

#### Rule 4487

$\text{Int}[(e_{.}*(x_{.}))^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}]]*(b_{.})*(d_{.})]^{(p_{.})}, x\_Symbol] \rightarrow \text{Simp}[(m+1)*(e*x)^{(m+1)}*\text{Sin}[d*(a+b*\text{Log}[c*x^n])]^{(p)}/(b^2*d^2*e*n^2*p^2 + e*(m+1)^2), x] + (\text{Dist}[(b^2*d^2*n^2*p*(p-1))/(b^2*d^2*n^2*p^2 + (m+1)^2), \text{Int}[(e*x)^m*\text{Sin}[d*(a+b*\text{Log}[c*x^n])]^{(p-2)}, x], x] - \text{Simp}[(b*d*n*p*(e*x)^{(m+1)}*\text{Cos}[d*(a+b*\text{Log}[c*x^n])] *\text{Sin}[d*(a+b*\text{Log}[c*x^n])]^{(p-1)}/(b^2*d^2*e*n^2*p^2 + e*(m+1)^2), x]) /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{IGtQ}[p, 1] \&\& \text{NeQ}[b^2*d^2*n^2*p^2 + (m+1)^2, 0]$

#### Rule 4485

$\text{Int}[(e_{.}*(x_{.}))^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}]]*(b_{.})*(d_{.})], x\_Symbol] \rightarrow \text{Simp}[(m+1)*(e*x)^{(m+1)}*\text{Sin}[d*(a+b*\text{Log}[c*x^n])]/(b^2*d^2*e*n^2 + e*(m+1)^2), x] - \text{Simp}[(b*d*n*(e*x)^{(m+1)}*\text{Cos}[d*(a+b*\text{Log}[c*x^n])]/(b^2*d^2*e*n^2 + e*(m+1)^2), x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[b^2*d^2*n^2 + (m+1)^2, 0]$

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^3(a+b \log(cx^n))}{x^3} dx &= -\frac{3bn \cos(a+b \log(cx^n)) \sin^2(a+b \log(cx^n))}{(4+9b^2n^2)x^2} - \frac{2 \sin^3(a+b \log(cx^n))}{(4+9b^2n^2)x^2} + \frac{(6b^2n^2) \int \frac{\sin^2(a+b \log(cx^n))}{x^3} dx}{4+9b^2n^2} \\ &= -\frac{6b^3n^3 \cos(a+b \log(cx^n))}{(16+40b^2n^2+9b^4n^4)x^2} - \frac{12b^2n^2 \sin(a+b \log(cx^n))}{(16+40b^2n^2+9b^4n^4)x^2} - \frac{3bn \cos(a+b \log(cx^n)) \sin^2(a+b \log(cx^n))}{(4+9b^2n^2)x^2} \end{aligned}$$

**Mathematica [A]** time = 0.384536, size = 125, normalized size = 0.79

$$\frac{-3bn(b^2n^2 + 4)\cos(a + b\log(cx^n)) + 3bn(b^2n^2 + 4)\cos(3(a + b\log(cx^n))) + 4\sin(a + b\log(cx^n))((b^2n^2 + 4)\cos(a + b\log(cx^n)))}{4x^2(9b^4n^4 + 40b^2n^2 + 16)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^3/x^3,x]

[Out] (-3\*b\*n\*(4 + 9\*b^2\*n^2)\*Cos[a + b\*Log[c\*x^n]] + 3\*b\*n\*(4 + b^2\*n^2)\*Cos[3\*(a + b\*Log[c\*x^n])] + 4\*(-4 - 13\*b^2\*n^2 + (4 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])])\*Sin[a + b\*Log[c\*x^n]]/(4\*(16 + 40\*b^2\*n^2 + 9\*b^4\*n^4)\*x^2)

**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^3/x^3,x)

[Out] int(sin(a+b\*ln(c\*x^n))^3/x^3,x)

**Maxima [B]** time = 1.24026, size = 1359, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^3/x^3,x, algorithm="maxima")

[Out] 1/8\*((3\*(b^3\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c)))\*n^3 + 2\*(b^2\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c)))\*n^2 + 12\*(b\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(3\*b\*log(c)))\*n + 8\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - 8\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + 8\*sin(3\*b\*log(c))\*cos(3\*b\*log(x^n) + 3\*a) - 3\*(9\*(b^3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 18\*(b^2\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^2\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + 4\*(b\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n + 8\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - 8\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + 8\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - 8\*cos(3\*b\*log(c))\*sin(2\*b\*log(c))\*cos(b\*log(x^n) + a) - (3\*(b^3\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c)))\*n^3 - 2\*(b^2\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c)))\*n^2 + 12\*(b\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c)))\*n - 8\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) - 8\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) - 8\*cos(3\*b\*log(c))\*sin(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c))



### 3.19 $\int x^2 \sin^4(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=202

$$\frac{36b^2n^2x^3 \sin^2(a + b \log(cx^n))}{64b^4n^4 + 180b^2n^2 + 81} + \frac{3x^3 \sin^4(a + b \log(cx^n))}{16b^2n^2 + 9} - \frac{4bnx^3 \sin^3(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{16b^2n^2 + 9} - \frac{24b^3n^3x^3}{16b^2n^2 + 9}$$

[Out]  $(8*b^4*n^4*x^3)/(81 + 180*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(81 + 180*b^2*n^2 + 64*b^4*n^4) + (36*b^2*n^2*x^3*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/(81 + 180*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]]^3)/(9 + 16*b^2*n^2) + (3*x^3*\text{Sin}[a + b*\text{Log}[c*x^n]]^4)/(9 + 16*b^2*n^2)$

**Rubi [A]** time = 0.0783835, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 30}

$$\frac{36b^2n^2x^3 \sin^2(a + b \log(cx^n))}{64b^4n^4 + 180b^2n^2 + 81} + \frac{3x^3 \sin^4(a + b \log(cx^n))}{16b^2n^2 + 9} - \frac{4bnx^3 \sin^3(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{16b^2n^2 + 9} - \frac{24b^3n^3x^3}{16b^2n^2 + 9}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sin[a + b\*Log[c\*x^n]]^4,x]

[Out]  $(8*b^4*n^4*x^3)/(81 + 180*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(81 + 180*b^2*n^2 + 64*b^4*n^4) + (36*b^2*n^2*x^3*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/(81 + 180*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]]^3)/(9 + 16*b^2*n^2) + (3*x^3*\text{Sin}[a + b*\text{Log}[c*x^n]]^4)/(9 + 16*b^2*n^2)$

#### Rule 4487

Int[((e.\_)\*(x.\_))^(m.\_)\*Sin[((a.\_) + Log[(c.\_)\*(x.\_)^(n.\_)]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x.\_)^(m.\_), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 \sin^4(a + b \log(cx^n)) dx &= -\frac{4bnx^3 \cos(a + b \log(cx^n)) \sin^3(a + b \log(cx^n))}{9 + 16b^2n^2} + \frac{3x^3 \sin^4(a + b \log(cx^n))}{9 + 16b^2n^2} + \frac{(12b^2n^2x^3)}{9 + 16b^2n^2} \\ &= -\frac{24b^3n^3x^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{81 + 180b^2n^2 + 64b^4n^4} + \frac{36b^2n^2x^3 \sin^2(a + b \log(cx^n))}{81 + 180b^2n^2 + 64b^4n^4} - \frac{12b^2n^2x^3}{81 + 180b^2n^2 + 64b^4n^4} \\ &= \frac{8b^4n^4x^3}{81 + 180b^2n^2 + 64b^4n^4} - \frac{24b^3n^3x^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{81 + 180b^2n^2 + 64b^4n^4} + \frac{36b^2n^2x^3 \sin^2(a + b \log(cx^n))}{81 + 180b^2n^2 + 64b^4n^4} - \frac{12b^2n^2x^3}{81 + 180b^2n^2 + 64b^4n^4} \end{aligned}$$

**Mathematica [A]** time = 0.495872, size = 171, normalized size = 0.85

$$\frac{x^3 \left( -128b^3n^3 \sin(2(a + b \log(cx^n))) + 16b^3n^3 \sin(4(a + b \log(cx^n))) - 12(16b^2n^2 + 9) \cos(2(a + b \log(cx^n))) + 3 \right)}{8(64b^4n^4)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sin[a + b\*Log[c\*x^n]]^4,x]

[Out] (x^3\*(81 + 180\*b^2\*n^2 + 64\*b^4\*n^4 - 12\*(9 + 16\*b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])] + 3\*(9 + 4\*b^2\*n^2)\*Cos[4\*(a + b\*Log[c\*x^n])] - 72\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])] - 128\*b^3\*n^3\*Sin[2\*(a + b\*Log[c\*x^n])] + 36\*b\*n\*Sin[4\*(a + b\*Log[c\*x^n])] + 16\*b^3\*n^3\*Sin[4\*(a + b\*Log[c\*x^n])]))/(8\*(81 + 180\*b^2\*n^2 + 64\*b^4\*n^4))

**Maple [F]** time = 0.098, size = 0, normalized size = 0.

$$\int x^2 (\sin(a + b \ln(cx^n)))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sin(a+b\*ln(c\*x^n))^4,x)

[Out] int(x^2\*sin(a+b\*ln(c\*x^n))^4,x)

**Maxima [B]** time = 1.2689, size = 1494, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n))^4,x, algorithm="maxima")

[Out] 1/16\*((16\*(b^3\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^3\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*sin(4\*b\*log(c)))\*n^3 + 12\*(b^2\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c)))\*n^2 + 36\*(b\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*sin(4\*b\*log(c)))\*n + 27\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + 27\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + 27\*cos(4\*b\*log(c))\*x^3\*cos(4\*b\*log(x^n) + 4\*a) - 4\*(32\*(b^3\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 48\*(b^2\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + 18\*(b\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n + 27\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + 27\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + 27\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + 27\*sin(4\*b\*log(c))\*sin(2\*b\*log(c))\*x^3\*cos(2\*b\*log(x^n) + 2\*a) + (16\*(b^3\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^3\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*cos(4\*b\*log(c)))\*n^3 - 12\*(b^2\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^2\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*sin(4\*b\*log(c)))\*n^2 + 36\*(b\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*cos(4\*b\*log(c)))\*n - 27\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) + 27\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) - 27\*sin(4\*b\*log(c))

```

c))) * x^3 * sin(4*b*log(x^n) + 4*a) - 4*(32*(b^3*cos(6*b*log(c))*cos(4*b*log(c))
)) + b^3*cos(4*b*log(c))*cos(2*b*log(c)) + b^3*sin(6*b*log(c))*sin(4*b*log(c))
+ b^3*sin(4*b*log(c))*sin(2*b*log(c)))*n^3 - 48*(b^2*cos(4*b*log(c))*sin(6*b*log(c))
- b^2*cos(6*b*log(c))*sin(4*b*log(c)) + b^2*cos(2*b*log(c))*sin(4*b*log(c))
- b^2*cos(4*b*log(c))*sin(2*b*log(c)))*n^2 + 18*(b*cos(6*b*log(c))*cos(4*b*log(c))
+ b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c))
+ b*sin(4*b*log(c))*sin(2*b*log(c)))*n - 27*cos(4*b*log(c))*sin(6*b*log(c))
+ 27*cos(6*b*log(c))*sin(4*b*log(c)) - 27*cos(2*b*log(c))*sin(4*b*log(c))
+ 27*cos(4*b*log(c))*sin(2*b*log(c)))*x^3 * sin(2*b*log(x^n) + 2*a)
+ 2*(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 180*(b^2*cos(4*b*log(c))^2
+ b^2*sin(4*b*log(c))^2)*n^2 + 81*cos(4*b*log(c))^2 + 81*sin(4*b*log(c))^2)*x^3
/(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 180*(b^2*cos(4*b*log(c))^2
+ b^2*sin(4*b*log(c))^2)*n^2 + 81*cos(4*b*log(c))^2 + 81*sin(4*b*log(c))^2)

```

**Fricas [A]** time = 0.517694, size = 454, normalized size = 2.25

$$3(4b^2n^2 + 9)x^3 \cos(bn \log(x) + b \log(c) + a)^4 - 6(10b^2n^2 + 9)x^3 \cos(bn \log(x) + b \log(c) + a)^2 + (8b^4n^4 + 48b^2n^2 + 9)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] (3*(4*b^2*n^2 + 9)*x^3*cos(b*n*log(x) + b*log(c) + a)^4 - 6*(10*b^2*n^2 + 9)
)*x^3*cos(b*n*log(x) + b*log(c) + a)^2 + (8*b^4*n^4 + 48*b^2*n^2 + 27)*x^3
+ 4*((4*b^3*n^3 + 9*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*n^3
+ 9*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a
))/(64*b^4*n^4 + 180*b^2*n^2 + 81)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.20 $\int x \sin^4(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=210

$$\frac{3b^2n^2x^2 \sin^2(a + b \log(cx^n))}{2(4b^4n^4 + 5b^2n^2 + 1)} + \frac{x^2 \sin^4(a + b \log(cx^n))}{2(4b^2n^2 + 1)} - \frac{bnx^2 \sin^3(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 1} - \frac{3b^3n^3x^2 \sin^2(a + b \log(cx^n))}{2(4b^4n^4 + 5b^2n^2 + 1)}$$

[Out]  $(3*b^4*n^4*x^2)/(4*(1 + 5*b^2*n^2 + 4*b^4*n^4)) - (3*b^3*n^3*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*(1 + 5*b^2*n^2 + 4*b^4*n^4)) + (3*b^2*n^2*x^2*Sin[a + b*Log[c*x^n]]^2)/(2*(1 + 5*b^2*n^2 + 4*b^4*n^4)) - (b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(1 + 4*b^2*n^2) + (x^2*Sin[a + b*Log[c*x^n]]^4)/(2*(1 + 4*b^2*n^2))$

**Rubi [A]** time = 0.0609191, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {4487, 30}

$$\frac{3b^2n^2x^2 \sin^2(a + b \log(cx^n))}{2(4b^4n^4 + 5b^2n^2 + 1)} + \frac{x^2 \sin^4(a + b \log(cx^n))}{2(4b^2n^2 + 1)} - \frac{bnx^2 \sin^3(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 1} - \frac{3b^3n^3x^2 \sin^2(a + b \log(cx^n))}{2(4b^4n^4 + 5b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[x\*Sin[a + b\*Log[c\*x^n]]^4,x]

[Out]  $(3*b^4*n^4*x^2)/(4*(1 + 5*b^2*n^2 + 4*b^4*n^4)) - (3*b^3*n^3*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*(1 + 5*b^2*n^2 + 4*b^4*n^4)) + (3*b^2*n^2*x^2*Sin[a + b*Log[c*x^n]]^2)/(2*(1 + 5*b^2*n^2 + 4*b^4*n^4)) - (b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(1 + 4*b^2*n^2) + (x^2*Sin[a + b*Log[c*x^n]]^4)/(2*(1 + 4*b^2*n^2))$

#### Rule 4487

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x \sin^4(a + b \log(cx^n)) dx &= -\frac{bnx^2 \cos(a + b \log(cx^n)) \sin^3(a + b \log(cx^n))}{1 + 4b^2n^2} + \frac{x^2 \sin^4(a + b \log(cx^n))}{2(1 + 4b^2n^2)} + \frac{(3b^2n^2)}{2(1 + 4b^2n^2)} \\ &= -\frac{3b^3n^3x^2 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)} + \frac{3b^2n^2x^2 \sin^2(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)} - \frac{3b^3n^3x^2 \sin^2(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)} \\ &= \frac{3b^4n^4x^2}{4(1 + 5b^2n^2 + 4b^4n^4)} - \frac{3b^3n^3x^2 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)} + \frac{3b^2n^2x^2 \sin^2(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)} \end{aligned}$$

**Mathematica [A]** time = 0.435803, size = 169, normalized size = 0.8

$$\frac{x^2 \left( -16b^3n^3 \sin(2(a + b \log(cx^n))) + 2b^3n^3 \sin(4(a + b \log(cx^n))) - 4(4b^2n^2 + 1) \cos(2(a + b \log(cx^n))) + (b^2n^2 + 1) \right)}{16(4b^4n^4 + 5b^2n^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sin[a + b\*Log[c\*x^n]]^4,x]

[Out] (x^2\*(3 + 15\*b^2\*n^2 + 12\*b^4\*n^4 - 4\*(1 + 4\*b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])]) + (1 + b^2\*n^2)\*Cos[4\*(a + b\*Log[c\*x^n])] - 4\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])] - 16\*b^3\*n^3\*Sin[2\*(a + b\*Log[c\*x^n])] + 2\*b\*n\*Sin[4\*(a + b\*Log[c\*x^n])] + 2\*b^3\*n^3\*Sin[4\*(a + b\*Log[c\*x^n])])/(16\*(1 + 5\*b^2\*n^2 + 4\*b^4\*n^4))

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int x (\sin(a + b \ln(cx^n)))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+b\*ln(c\*x^n))^4,x)

[Out] int(x\*sin(a+b\*ln(c\*x^n))^4,x)

**Maxima [B]** time = 1.41093, size = 1465, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^4,x, algorithm="maxima")

[Out] 1/32\*((2\*(b^3\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^3\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*sin(4\*b\*log(c)))\*n^3 + (b^2\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c)))\*n^2 + 2\*(b\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*sin(4\*b\*log(c)))\*n + cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + cos(4\*b\*log(c)))\*x^2\*cos(4\*b\*log(x^n) + 4\*a) - 4\*(4\*(b^3\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 4\*(b^2\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n + cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*x^2\*cos(2\*b\*log(x^n) + 2\*a) + (2\*(b^3\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^3\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*cos(4\*b\*log(c)))\*n^3 - (b^2\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^2\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*sin(4\*b\*log(c)))\*n^2 + 2\*(b\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*cos(4\*b\*log(c)))\*n - cos(4\*b\*log(c))\*sin(8\*b\*log(c)) + cos(8\*b\*log(c))\*sin(4\*b\*log(c)) - sin(4\*b\*log(c)))\*x^2\*sin(4\*b\*log(x^n) + 4\*a) - 4\*(4\*(b^3\*



$$\begin{aligned} & \cos(6b \log(c)) \cos(4b \log(c)) + b^3 \cos(4b \log(c)) \cos(2b \log(c)) + b^3 \\ & \sin(6b \log(c)) \sin(4b \log(c)) + b^3 \sin(4b \log(c)) \sin(2b \log(c)) \cdot n^3 \\ & - 4(b^2 \cos(4b \log(c)) \sin(6b \log(c)) - b^2 \cos(6b \log(c)) \sin(4b \log(c)) \\ & + b^2 \cos(2b \log(c)) \sin(4b \log(c)) - b^2 \cos(4b \log(c)) \sin(2b \log(c))) \cdot n^2 \\ & + (b \cos(6b \log(c)) \cos(4b \log(c)) + b \cos(4b \log(c)) \cos(2b \log(c)) \\ & + b \sin(6b \log(c)) \sin(4b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c))) \cdot n \\ & - \cos(4b \log(c)) \sin(6b \log(c)) + \cos(6b \log(c)) \sin(4b \log(c)) \\ & - \cos(2b \log(c)) \sin(4b \log(c)) + \cos(4b \log(c)) \sin(2b \log(c)) \cdot x^2 \\ & \sin(2b \log(x^n) + 2a) + 6(4(b^4 \cos(4b \log(c))^2 + b^4 \sin(4b \log(c))^2) \cdot n^4 \\ & + 5(b^2 \cos(4b \log(c))^2 + b^2 \sin(4b \log(c))^2) \cdot n^2 + \cos(4b \log(c))^2 \\ & + \sin(4b \log(c))^2) \cdot x^2) / (4(b^4 \cos(4b \log(c))^2 + b^4 \sin(4b \log(c))^2) \cdot n^4 \\ & + 5(b^2 \cos(4b \log(c))^2 + b^2 \sin(4b \log(c))^2) \cdot n^2 + \cos(4b \log(c))^2 \\ & + \sin(4b \log(c))^2) \end{aligned}$$

**Fricas [A]** time = 0.518291, size = 443, normalized size = 2.11

$$2(b^2 n^2 + 1)x^2 \cos(bn \log(x) + b \log(c) + a)^4 - 2(5b^2 n^2 + 2)x^2 \cos(bn \log(x) + b \log(c) + a)^2 + (3b^4 n^4 + 8b^2 n^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^4,x, algorithm="fricas")

[Out]  $\frac{1}{4}(2(b^2 n^2 + 1)x^2 \cos(bn \log(x) + b \log(c) + a)^4 - 2(5b^2 n^2 + 2)x^2 \cos(bn \log(x) + b \log(c) + a)^2 + (3b^4 n^4 + 8b^2 n^2 + 2)x^2 + 2(2(b^3 n^3 + b^n)x^2 \cos(bn \log(x) + b \log(c) + a)^3 - (5b^3 n^3 + 2b^n)x^2 \cos(bn \log(x) + b \log(c) + a)) \sin(bn \log(x) + b \log(c) + a)) / (4b^4 n^4 + 5b^2 n^2 + 1)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*ln(c\*x\*\*n))\*\*4,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^4,x, algorithm="giac")

[Out] Timed out

### 3.21 $\int \sin^4(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=191

$$\frac{12b^2n^2x \sin^2(a + b \log(cx^n))}{64b^4n^4 + 20b^2n^2 + 1} + \frac{x \sin^4(a + b \log(cx^n))}{16b^2n^2 + 1} - \frac{4bnx \sin^3(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{16b^2n^2 + 1} - \frac{24b^3n^3x \sin(a + b \log(cx^n)) \cos^2(a + b \log(cx^n))}{16b^2n^2 + 1}$$

[Out]  $(24*b^4*n^4*x)/(1 + 20*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x*\text{Cos}[a + b*\text{Log}[c*x^n]])*\text{Sin}[a + b*\text{Log}[c*x^n]]/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^2*x*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/(1 + 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]])*\text{Sin}[a + b*\text{Log}[c*x^n]]^3/(1 + 16*b^2*n^2) + (x*\text{Sin}[a + b*\text{Log}[c*x^n]]^4)/(1 + 16*b^2*n^2)$

**Rubi [A]** time = 0.0511131, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {4477, 8}

$$\frac{12b^2n^2x \sin^2(a + b \log(cx^n))}{64b^4n^4 + 20b^2n^2 + 1} + \frac{x \sin^4(a + b \log(cx^n))}{16b^2n^2 + 1} - \frac{4bnx \sin^3(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{16b^2n^2 + 1} - \frac{24b^3n^3x \sin(a + b \log(cx^n)) \cos^2(a + b \log(cx^n))}{16b^2n^2 + 1}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^4,x]

[Out]  $(24*b^4*n^4*x)/(1 + 20*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x*\text{Cos}[a + b*\text{Log}[c*x^n]])*\text{Sin}[a + b*\text{Log}[c*x^n]]/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^2*x*\text{Sin}[a + b*\text{Log}[c*x^n]]^2)/(1 + 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]])*\text{Sin}[a + b*\text{Log}[c*x^n]]^3/(1 + 16*b^2*n^2) + (x*\text{Sin}[a + b*\text{Log}[c*x^n]]^4)/(1 + 16*b^2*n^2)$

#### Rule 4477

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_), x\_Symbol] := Simp[(x\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*n^2\*p^2 + 1), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + 1), Int[Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*x\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*n^2\*p^2 + 1), x]) /; FreeQ[{a, b, c, d, n}, x] && I GtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + 1, 0]

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \sin^4(a + b \log(cx^n)) dx &= -\frac{4bnx \cos(a + b \log(cx^n)) \sin^3(a + b \log(cx^n))}{1 + 16b^2n^2} + \frac{x \sin^4(a + b \log(cx^n))}{1 + 16b^2n^2} + \frac{(12b^2n^2) \int \sin^2(a + b \log(cx^n)) dx}{1 + 16b^2n^2} \\ &= -\frac{24b^3n^3x \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 20b^2n^2 + 64b^4n^4} + \frac{12b^2n^2x \sin^2(a + b \log(cx^n))}{1 + 20b^2n^2 + 64b^4n^4} - \frac{4bnx \sin^3(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{1 + 16b^2n^2} \\ &= \frac{24b^4n^4x}{1 + 20b^2n^2 + 64b^4n^4} - \frac{24b^3n^3x \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 20b^2n^2 + 64b^4n^4} + \frac{12b^2n^2x \sin^2(a + b \log(cx^n))}{1 + 20b^2n^2 + 64b^4n^4} \end{aligned}$$

**Mathematica [A]** time = 0.400746, size = 168, normalized size = 0.88

$$\frac{x(-128b^3n^3 \sin(2(a + b \log(cx^n))) + 16b^3n^3 \sin(4(a + b \log(cx^n))) - 4(16b^2n^2 + 1) \cos(2(a + b \log(cx^n))) + (4b^2n^2 + 1) \sin(2(a + b \log(cx^n))))}{8(64b^4n^4 + 20b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^4,x]

[Out] (x\*(3 + 60\*b^2\*n^2 + 192\*b^4\*n^4 - 4\*(1 + 16\*b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])]) + (1 + 4\*b^2\*n^2)\*Cos[4\*(a + b\*Log[c\*x^n])] - 8\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])] - 128\*b^3\*n^3\*Sin[2\*(a + b\*Log[c\*x^n])] + 4\*b\*n\*Sin[4\*(a + b\*Log[c\*x^n])] + 16\*b^3\*n^3\*Sin[4\*(a + b\*Log[c\*x^n])])/(8\*(1 + 20\*b^2\*n^2 + 64\*b^4\*n^4))

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int (\sin(a + b \ln(cx^n)))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^4,x)

[Out] int(sin(a+b\*ln(c\*x^n))^4,x)

**Maxima [B]** time = 1.3252, size = 1455, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^4,x, algorithm="maxima")

[Out] 1/16\*((16\*(b^3\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^3\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*sin(4\*b\*log(c)))\*n^3 + 4\*(b^2\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c)))\*n^2 + 4\*(b\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*sin(4\*b\*log(c)))\*n + cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + cos(4\*b\*log(c)))\*x\*cos(4\*b\*log(x^n) + 4\*a) - 4\*(32\*(b^3\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 16\*(b^2\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + 2\*(b\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n + cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*x\*cos(2\*b\*log(x^n) + 2\*a) + (16\*(b^3\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^3\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*cos(4\*b\*log(c)))\*n^3 - 4\*(b^2\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^2\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*sin(4\*b\*log(c)))\*n^2 + 4\*(b\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*cos(4\*b\*log(c)))\*n - cos(4\*b\*log(c))\*sin(8\*b\*log(c)) + cos(8\*b\*log(c))\*sin(4\*b\*log(c)) - sin(4\*b\*log(c)))\*x\*sin(4\*b\*log(x^n) + 4\*a) - 4\*(32\*(b^3\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b^3\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 - 16\*(b^2\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + 2\*(b\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b\*cos(4\*b\*log(c))

```
*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*b*log(c)) - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) + 6*(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*x)/(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + cos(4*b*log(c))^2 + sin(4*b*log(c))^2)
```

**Fricas [A]** time = 0.521285, size = 429, normalized size = 2.25

$$\frac{(4b^2n^2 + 1)x \cos(bn \log(x) + b \log(c) + a)^4 - 2(10b^2n^2 + 1)x \cos(bn \log(x) + b \log(c) + a)^2 + (24b^4n^4 + 16b^2n^2 + 1)x}{64b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] ((4*b^2*n^2 + 1)*x*cos(b*n*log(x) + b*log(c) + a)^4 - 2*(10*b^2*n^2 + 1)*x*cos(b*n*log(x) + b*log(c) + a)^2 + (24*b^4*n^4 + 16*b^2*n^2 + 1)*x + 4*((4*b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 + 20*b^2*n^2 + 1)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.22 \quad \int \frac{\sin^4(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=73

$$-\frac{\sin^3(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{4bn} - \frac{3 \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{8bn} + \frac{3 \log(x)}{8}$$

[Out] (3\*Log[x])/8 - (3\*Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(8\*b\*n) - (Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]]^3)/(4\*b\*n)

**Rubi [A]** time = 0.0494805, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2635, 8}

$$-\frac{\sin^3(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{4bn} - \frac{3 \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{8bn} + \frac{3 \log(x)}{8}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^4/x, x]

[Out] (3\*Log[x])/8 - (3\*Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(8\*b\*n) - (Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]]^3)/(4\*b\*n)

Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x] \* (b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\sin^4(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sin^4(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cos(a+b \log(cx^n)) \sin^3(a+b \log(cx^n))}{4bn} + \frac{3 \text{Subst}\left(\int \sin^2(a+bx) dx, x, \log(cx^n)\right)}{4n} \\ &= -\frac{3 \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{8bn} - \frac{\cos(a+b \log(cx^n)) \sin^3(a+b \log(cx^n))}{4bn} \\ &= \frac{3 \log(x)}{8} - \frac{3 \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{8bn} - \frac{\cos(a+b \log(cx^n)) \sin^3(a+b \log(cx^n))}{4bn} \end{aligned}$$

**Mathematica [A]** time = 0.0909096, size = 51, normalized size = 0.7

$$\frac{12(a+b \log(cx^n)) - 8 \sin(2(a+b \log(cx^n))) + \sin(4(a+b \log(cx^n)))}{32bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^4/x,x]

[Out] (12\*(a + b\*Log[c\*x^n]) - 8\*Sin[2\*(a + b\*Log[c\*x^n])] + Sin[4\*(a + b\*Log[c\*x^n])])/(32\*b\*n)

**Maple [A]** time = 0.027, size = 84, normalized size = 1.2

$$-\frac{\cos(a + b \ln(cx^n)) (\sin(a + b \ln(cx^n)))^3}{4bn} - \frac{3 \cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{8bn} + \frac{3 \ln(cx^n)}{8n} + \frac{3a}{8bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^4/x,x)

[Out] -1/4\*cos(a+b\*ln(c\*x^n))\*sin(a+b\*ln(c\*x^n))^3/b/n-3/8\*cos(a+b\*ln(c\*x^n))\*sin(a+b\*ln(c\*x^n))/b/n+3/8/n\*ln(c\*x^n)+3/8/b/n\*a

**Maxima [A]** time = 1.14208, size = 126, normalized size = 1.73

$$\frac{12bn \log(x) + \cos(4b \log(x^n) + 4a) \sin(4b \log(c)) - 8 \cos(2b \log(x^n) + 2a) \sin(2b \log(c)) + \cos(4b \log(c)) \sin(4b \log(x^n) + 4a) - 8 \cos(2b \log(c)) \sin(2b \log(x^n) + 2a)}{32bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^4/x,x, algorithm="maxima")

[Out] 1/32\*(12\*b\*n\*log(x) + cos(4\*b\*log(x^n) + 4\*a)\*sin(4\*b\*log(c)) - 8\*cos(2\*b\*log(x^n) + 2\*a)\*sin(2\*b\*log(c)) + cos(4\*b\*log(c))\*sin(4\*b\*log(x^n) + 4\*a) - 8\*cos(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a))/(b\*n)

**Fricas [A]** time = 0.500875, size = 177, normalized size = 2.42

$$\frac{3bn \log(x) + (2 \cos(bn \log(x) + b \log(c) + a)^3 - 5 \cos(bn \log(x) + b \log(c) + a)) \sin(bn \log(x) + b \log(c) + a)}{8bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^4/x,x, algorithm="fricas")

[Out] 1/8\*(3\*b\*n\*log(x) + (2\*cos(b\*n\*log(x) + b\*log(c) + a)^3 - 5\*cos(b\*n\*log(x) + b\*log(c) + a))\*sin(b\*n\*log(x) + b\*log(c) + a))/(b\*n)

**Sympy [A]** time = 169.546, size = 110, normalized size = 1.51

$$-\frac{\begin{cases} \log(x) \cos(2a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cos(2a + 2b \log(c)) & \text{for } n = 0 \\ \frac{\sin(2a + 2bn \log(x) + 2b \log(c))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\begin{cases} \log(x) \cos(4a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cos(4a + 4b \log(c)) & \text{for } n = 0 \\ \frac{\sin(4a + 4bn \log(x) + 4b \log(c))}{4bn} & \text{otherwise} \end{cases}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] -Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*n*log(x) + 2*b*log(c))/(2*b*n), True))/2 + Piecewise((log(x)*cos(4*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos(4*a + 4*b*log(c)), Eq(n, 0)), (sin(4*a + 4*b*n*log(x) + 4*b*log(c))/(4*b*n), True))/8 + 3*log(x)/8
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^4/x, x)
```

### 3.23 $\int \frac{\sin^4(a+b \log(cx^n))}{x^2} dx$

**Optimal.** Leaf size=202

$$\frac{12b^2n^2 \sin^2(a+b \log(cx^n))}{x(64b^4n^4+20b^2n^2+1)} - \frac{\sin^4(a+b \log(cx^n))}{x(16b^2n^2+1)} - \frac{4bn \sin^3(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x(16b^2n^2+1)} - \frac{24b^3n^3 \sin(a+b \log(cx^n))}{x(64b^4n^4+20b^2n^2+1)}$$

[Out]  $(-24*b^4*n^4)/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (24*b^3*n^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (12*b^2*n^2*Sin[a + b*Log[c*x^n]]^2)/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (4*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/((1 + 16*b^2*n^2)*x) - Sin[a + b*Log[c*x^n]]^4/((1 + 16*b^2*n^2)*x)$

**Rubi [A]** time = 0.0656843, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 30}

$$\frac{12b^2n^2 \sin^2(a+b \log(cx^n))}{x(64b^4n^4+20b^2n^2+1)} - \frac{\sin^4(a+b \log(cx^n))}{x(16b^2n^2+1)} - \frac{4bn \sin^3(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x(16b^2n^2+1)} - \frac{24b^3n^3 \sin(a+b \log(cx^n))}{x(64b^4n^4+20b^2n^2+1)}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^4/x^2,x]

[Out]  $(-24*b^4*n^4)/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (24*b^3*n^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (12*b^2*n^2*Sin[a + b*Log[c*x^n]]^2)/((1 + 20*b^2*n^2 + 64*b^4*n^4)*x) - (4*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/((1 + 16*b^2*n^2)*x) - Sin[a + b*Log[c*x^n]]^4/((1 + 16*b^2*n^2)*x)$

#### Rule 4487

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps



$$\begin{aligned} \int \frac{\sin^4(a + b \log(cx^n))}{x^2} dx &= -\frac{4bn \cos(a + b \log(cx^n)) \sin^3(a + b \log(cx^n))}{(1 + 16b^2n^2)x} - \frac{\sin^4(a + b \log(cx^n))}{(1 + 16b^2n^2)x} + \frac{(12b^2n^2) \int \frac{\sin^2(a + b \log(cx^n))}{x} dx}{1 + 16b^2n^2} \\ &= -\frac{24b^3n^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{(1 + 20b^2n^2 + 64b^4n^4)x} - \frac{12b^2n^2 \sin^2(a + b \log(cx^n))}{(1 + 20b^2n^2 + 64b^4n^4)x} - \frac{4bn \sin^4(a + b \log(cx^n))}{(1 + 20b^2n^2 + 64b^4n^4)x} \\ &= -\frac{24b^4n^4}{(1 + 20b^2n^2 + 64b^4n^4)x} - \frac{24b^3n^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{(1 + 20b^2n^2 + 64b^4n^4)x} - \frac{12b^2n^2 \sin^2(a + b \log(cx^n))}{(1 + 20b^2n^2 + 64b^4n^4)x} - \frac{4bn \sin^4(a + b \log(cx^n))}{(1 + 20b^2n^2 + 64b^4n^4)x} \end{aligned}$$

**Mathematica [A]** time = 0.500479, size = 170, normalized size = 0.84

$$\frac{128b^3n^3 \sin(2(a + b \log(cx^n))) - 16b^3n^3 \sin(4(a + b \log(cx^n))) - 4(16b^2n^2 + 1) \cos(2(a + b \log(cx^n))) + (4b^2n^2 - 8x(64b^4n^4 + 20b^2n^2 + 1)) \sin(2(a + b \log(cx^n)))}{8x(64b^4n^4 + 20b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^4/x^2,x]

[Out] -(3 + 60\*b^2\*n^2 + 192\*b^4\*n^4 - 4\*(1 + 16\*b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])] + (1 + 4\*b^2\*n^2)\*Cos[4\*(a + b\*Log[c\*x^n])] + 8\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])] + 128\*b^3\*n^3\*Sin[2\*(a + b\*Log[c\*x^n])] - 4\*b\*n\*Sin[4\*(a + b\*Log[c\*x^n])] - 16\*b^3\*n^3\*Sin[4\*(a + b\*Log[c\*x^n])])/(8\*(1 + 20\*b^2\*n^2 + 64\*b^4\*n^4)\*x)

**Maple [F]** time = 0.075, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^4}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^4/x^2,x)

[Out] int(sin(a+b\*ln(c\*x^n))^4/x^2,x)

**Maxima [B]** time = 1.3707, size = 1465, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^4/x^2,x, algorithm="maxima")

[Out] -1/16\*(384\*(b^4\*cos(4\*b\*log(c))^2 + b^4\*sin(4\*b\*log(c))^2)\*n^4 + 120\*(b^2\*cos(4\*b\*log(c))^2 + b^2\*sin(4\*b\*log(c))^2)\*n^2 + 6\*cos(4\*b\*log(c))^2 - (16\*(b^3\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^3\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*sin(4\*b\*log(c)))n^3 - 4\*(b^2\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c)))n^2 + 4\*(b\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*sin(4\*b\*log(c)))n - cos(8\*b\*log(c))\*cos(4\*b\*log(c)) - sin(8\*b\*log(c))\*sin(4\*b\*log(c)) -

```

cos(4*b*log(c))*cos(4*b*log(x^n) + 4*a) + 4*(32*(b^3*cos(4*b*log(c))*sin(6
*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log(c))*sin(
4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 - 16*(b^2*cos(6*b*lo
g(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin(6*b*1
og(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n^2 + 2*(b*co
s(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)) + b*cos(2
*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n - cos(6*b
*log(c))*cos(4*b*log(c)) - cos(4*b*log(c))*cos(2*b*log(c)) - sin(6*b*log(c)
)*sin(4*b*log(c)) - sin(4*b*log(c))*sin(2*b*log(c))*cos(2*b*log(x^n) + 2*a
) + 6*sin(4*b*log(c))^2 - (16*(b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^3*si
n(8*b*log(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 + 4*(b^2*cos(4*b*1
og(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*
log(c)))*n^2 + 4*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin
(4*b*log(c)) + b*cos(4*b*log(c)))*n + cos(4*b*log(c))*sin(8*b*log(c)) - cos
(8*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(4*b*log(x^n) + 4*a) + 4
*(32*(b^3*cos(6*b*log(c))*cos(4*b*log(c)) + b^3*cos(4*b*log(c))*cos(2*b*log
(c)) + b^3*sin(6*b*log(c))*sin(4*b*log(c)) + b^3*sin(4*b*log(c))*sin(2*b*lo
g(c)))*n^3 + 16*(b^2*cos(4*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*
sin(4*b*log(c)) + b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c)
)*sin(2*b*log(c)))*n^2 + 2*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*cos(4*b*lo
g(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c)
))*sin(2*b*log(c))*n + cos(4*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*s
in(4*b*log(c)) + cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*
log(c))*sin(2*b*log(x^n) + 2*a))/((64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b
*log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 +
cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*x)

```

**Fricas [A]** time = 0.523396, size = 420, normalized size = 2.08

$$\frac{24b^4n^4 + (4b^2n^2 + 1)\cos(bn \log(x) + b \log(c) + a)^4 + 16b^2n^2 - 2(10b^2n^2 + 1)\cos(bn \log(x) + b \log(c) + a)^2 - 4((64b^4n^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^2,x, algorithm="fricas")
```

```
[Out] -(24*b^4*n^4 + (4*b^2*n^2 + 1)*cos(b*n*log(x) + b*log(c) + a)^4 + 16*b^2*n^
2 - 2*(10*b^2*n^2 + 1)*cos(b*n*log(x) + b*log(c) + a)^2 - 4*((4*b^3*n^3 + b
*n)*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*n^3 + b*n)*cos(b*n*log(x) +
b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a) + 1)/((64*b^4*n^4 + 20*b^2*n^
2 + 1)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**4/x**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^4}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^4/x^2, x)
```

### 3.24 $\int \frac{\sin^4(a+b \log(cx^n))}{x^3} dx$

**Optimal.** Leaf size=210

$$\frac{3b^2n^2 \sin^2(a+b \log(cx^n))}{2x^2(4b^4n^4+5b^2n^2+1)} - \frac{\sin^4(a+b \log(cx^n))}{2x^2(4b^2n^2+1)} - \frac{bn \sin^3(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x^2(4b^2n^2+1)} - \frac{3b^3n^3 \sin(a+b \log(cx^n))}{2x^2(4b^2n^2+1)}$$

[Out]  $(-3*b^4*n^4)/(4*(1+5*b^2*n^2+4*b^4*n^4)*x^2) - (3*b^3*n^3*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]])/(2*(1+5*b^2*n^2+4*b^4*n^4)*x^2) - (3*b^2*n^2*\text{Sin}[a+b*\text{Log}[c*x^n]]^2)/(2*(1+5*b^2*n^2+4*b^4*n^4)*x^2) - (b*n*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]]^3)/((1+4*b^2*n^2)*x^2) - \text{Sin}[a+b*\text{Log}[c*x^n]]^4/(2*(1+4*b^2*n^2)*x^2)$

**Rubi [A]** time = 0.0628864, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4487, 30}

$$\frac{3b^2n^2 \sin^2(a+b \log(cx^n))}{2x^2(4b^4n^4+5b^2n^2+1)} - \frac{\sin^4(a+b \log(cx^n))}{2x^2(4b^2n^2+1)} - \frac{bn \sin^3(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x^2(4b^2n^2+1)} - \frac{3b^3n^3 \sin(a+b \log(cx^n))}{2x^2(4b^2n^2+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[a+b*\text{Log}[c*x^n]]^4/x^3, x]$

[Out]  $(-3*b^4*n^4)/(4*(1+5*b^2*n^2+4*b^4*n^4)*x^2) - (3*b^3*n^3*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]])/(2*(1+5*b^2*n^2+4*b^4*n^4)*x^2) - (3*b^2*n^2*\text{Sin}[a+b*\text{Log}[c*x^n]]^2)/(2*(1+5*b^2*n^2+4*b^4*n^4)*x^2) - (b*n*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]]^3)/((1+4*b^2*n^2)*x^2) - \text{Sin}[a+b*\text{Log}[c*x^n]]^4/(2*(1+4*b^2*n^2)*x^2)$

#### Rule 4487

$\text{Int}[(e._)*(x._)^{(m._)}*\text{Sin}[(a._)+\text{Log}[(c._)*(x._)^{(n._)}]*(b._)]*(d._)]^{(p._)}, x\_Symbol] \rightarrow \text{Simp}[(m+1)*(e*x)^{(m+1)}*\text{Sin}[d*(a+b*\text{Log}[c*x^n])]^p/(b^2*d^2*e*n^2*p^2+e*(m+1)^2), x] + (\text{Dist}[(b^2*d^2*n^2*p*(p-1))/(b^2*d^2*n^2*p^2+(m+1)^2), \text{Int}[(e*x)^m*\text{Sin}[d*(a+b*\text{Log}[c*x^n])]^{(p-2)}, x], x) - \text{Simp}[(b*d*n*p*(e*x)^{(m+1)}*\text{Cos}[d*(a+b*\text{Log}[c*x^n])]*\text{Sin}[d*(a+b*\text{Log}[c*x^n])]^{(p-1)}]/(b^2*d^2*e*n^2*p^2+e*(m+1)^2), x) /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{IGtQ}[p, 1] \&\& \text{NeQ}[b^2*d^2*n^2*p^2+(m+1)^2, 0]$

#### Rule 30

$\text{Int}[(x._)^{(m._)}, x\_Symbol] \rightarrow \text{Simp}[x^{(m+1)}/(m+1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^4(a + b \log(cx^n))}{x^3} dx &= -\frac{bn \cos(a + b \log(cx^n)) \sin^3(a + b \log(cx^n))}{(1 + 4b^2n^2)x^2} - \frac{\sin^4(a + b \log(cx^n))}{2(1 + 4b^2n^2)x^2} + \frac{(3b^2n^2) \int \frac{\sin^2(a + b \log(cx^n))}{x^3} dx}{1 + 4b^2n^2} \\ &= -\frac{3b^3n^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)x^2} - \frac{3b^2n^2 \sin^2(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)x^2} - \frac{bn \cos(a + b \log(cx^n)) \sin^3(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)x^2} \\ &= -\frac{3b^4n^4}{4(1 + 5b^2n^2 + 4b^4n^4)x^2} - \frac{3b^3n^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)x^2} - \frac{3b^2n^2 \sin^2(a + b \log(cx^n))}{2(1 + 5b^2n^2 + 4b^4n^4)x^2} \end{aligned}$$

**Mathematica [A]** time = 0.456564, size = 169, normalized size = 0.8

$$\frac{16b^3n^3 \sin(2(a + b \log(cx^n))) - 2b^3n^3 \sin(4(a + b \log(cx^n))) - 4(4b^2n^2 + 1) \cos(2(a + b \log(cx^n))) + (b^2n^2 + 1) \cos(4(a + b \log(cx^n)))}{16x^2(4b^4n^4 + 5b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^4/x^3,x]

[Out] -(3 + 15\*b^2\*n^2 + 12\*b^4\*n^4 - 4\*(1 + 4\*b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])] + (1 + b^2\*n^2)\*Cos[4\*(a + b\*Log[c\*x^n])] + 4\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])] + 16\*b^3\*n^3\*Sin[2\*(a + b\*Log[c\*x^n])] - 2\*b\*n\*Sin[4\*(a + b\*Log[c\*x^n])] - 2\*b^3\*n^3\*Sin[4\*(a + b\*Log[c\*x^n])])/(16\*(1 + 5\*b^2\*n^2 + 4\*b^4\*n^4)\*x^2)

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^4}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^4/x^3,x)

[Out] int(sin(a+b\*ln(c\*x^n))^4/x^3,x)

**Maxima [B]** time = 1.32137, size = 1461, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^4/x^3,x, algorithm="maxima")

[Out] -1/32\*(24\*(b^4\*cos(4\*b\*log(c))^2 + b^4\*sin(4\*b\*log(c))^2)\*n^4 + 30\*(b^2\*cos(4\*b\*log(c))^2 + b^2\*sin(4\*b\*log(c))^2)\*n^2 + 6\*cos(4\*b\*log(c))^2 - (2\*(b^3\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^3\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(8\*b\*log(c)) - (b^2\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c))\*sin(8\*b\*log(c))) \* n^2 + 2\*(b\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*sin(4\*b\*log(c))) \* n - cos(8\*b\*log(c))\*cos(4\*b\*log(c)) - sin(8\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*cos(4\*b\*log(x^n) + 4\*a) + 4\*(4\*(b^3\*cos(4\*b\*log(c))\*sin(6\*b\*log(c))

$$\begin{aligned}
& (c)) - b^3 \cos(6b \log(c)) \sin(4b \log(c)) + b^3 \cos(2b \log(c)) \sin(4b \log(c)) \\
& - b^3 \cos(4b \log(c)) \sin(2b \log(c)) \Big) n^3 - 4(b^2 \cos(6b \log(c)) \cos(4b \log(c)) \\
& + b^2 \cos(4b \log(c)) \cos(2b \log(c)) + b^2 \sin(6b \log(c)) \sin(4b \log(c)) \\
& + b^2 \sin(4b \log(c)) \sin(2b \log(c))) n^2 + (b \cos(4b \log(c)) \sin(6b \log(c)) \\
& - b \cos(6b \log(c)) \sin(4b \log(c)) + b \cos(2b \log(c)) \sin(4b \log(c)) \\
& - b \cos(4b \log(c)) \sin(2b \log(c))) n - \cos(6b \log(c)) \cos(4b \log(c)) \\
& - \cos(4b \log(c)) \cos(2b \log(c)) - \sin(6b \log(c)) \sin(4b \log(c)) \\
& - \sin(4b \log(c)) \sin(2b \log(c)) \Big) \cos(2b \log(x^n) + 2a) + 6 \sin(4b \log(c))^2 \\
& - (2(b^3 \cos(8b \log(c)) \cos(4b \log(c)) + b^3 \sin(8b \log(c)) \sin(4b \log(c)) \\
& + b^3 \cos(4b \log(c))) n^3 + (b^2 \cos(4b \log(c)) \sin(8b \log(c)) - b^2 \cos(8b \log(c)) \\
& \sin(4b \log(c)) + b^2 \sin(4b \log(c))) n^2 + 2(b \cos(8b \log(c)) \cos(4b \log(c)) \\
& + b \sin(8b \log(c)) \sin(4b \log(c)) + b \cos(4b \log(c))) n + \cos(4b \log(c)) \sin(8b \log(c)) \\
& - \cos(8b \log(c)) \sin(4b \log(c)) + \sin(4b \log(c)) \sin(4b \log(x^n) + 4a) \\
& + 4(4(b^3 \cos(6b \log(c)) \cos(4b \log(c)) + b^3 \cos(4b \log(c)) \cos(2b \log(c)) \\
& + b^3 \sin(6b \log(c)) \sin(4b \log(c)) + b^3 \sin(4b \log(c)) \sin(2b \log(c))) n^3 \\
& + 4(b^2 \cos(4b \log(c)) \sin(6b \log(c)) - b^2 \cos(6b \log(c)) \sin(4b \log(c)) \\
& + b^2 \cos(2b \log(c)) \sin(4b \log(c)) - b^2 \cos(4b \log(c)) \sin(2b \log(c))) n^2 \\
& + (b \cos(6b \log(c)) \cos(4b \log(c)) + b \cos(4b \log(c)) \cos(2b \log(c)) \\
& + b \sin(6b \log(c)) \sin(4b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c))) n \\
& + \cos(4b \log(c)) \sin(6b \log(c)) - \cos(6b \log(c)) \sin(4b \log(c)) \\
& + \cos(2b \log(c)) \sin(4b \log(c)) - \cos(4b \log(c)) \sin(2b \log(c)) \Big) \sin(2b \log(x^n) \\
& + 2a) / ((4(b^4 \cos(4b \log(c))^2 + b^4 \sin(4b \log(c))^2) n^4 + 5(b^2 \cos(4b \log(c))^2 \\
& + b^2 \sin(4b \log(c))^2) n^2 + \cos(4b \log(c))^2 + \sin(4b \log(c))^2) x^2)
\end{aligned}$$

**Fricas [A]** time = 0.529785, size = 423, normalized size = 2.01

$$\frac{3b^4n^4 + 2(b^2n^2 + 1)\cos(bn \log(x) + b \log(c) + a)^4 + 8b^2n^2 - 2(5b^2n^2 + 2)\cos(bn \log(x) + b \log(c) + a)^2 - 2(2(b^3n^3 + b^n)\cos(bn \log(x) + b \log(c) + a)^3 - (5b^3n^3 + 2bn)\cos(bn \log(x) + b \log(c) + a))\sin(bn \log(x) + b \log(c) + a) + 2}{4(4b^4n^4 + 5b^2n^2 + 1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^4/x^3,x, algorithm="fricas")

[Out]  $-1/4*(3*b^4*n^4 + 2*(b^2*n^2 + 1)*\cos(b*n*\log(x) + b*\log(c) + a)^4 + 8*b^2*n^2 - 2*(5*b^2*n^2 + 2)*\cos(b*n*\log(x) + b*\log(c) + a)^2 - 2*(2*(b^3*n^3 + b*n)*\cos(b*n*\log(x) + b*\log(c) + a)^3 - (5*b^3*n^3 + 2*b*n)*\cos(b*n*\log(x) + b*\log(c) + a))*\sin(b*n*\log(x) + b*\log(c) + a) + 2)/((4*b^4*n^4 + 5*b^2*n^2 + 1)*x^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*ln(c\*x\*\*n))\*\*4/x\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^4}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^4/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^4/x^3, x)
```

### 3.25 $\int \sin(\log(a + bx)) dx$

**Optimal.** Leaf size=39

$$\frac{(a + bx) \sin(\log(a + bx))}{2b} - \frac{(a + bx) \cos(\log(a + bx))}{2b}$$

[Out]  $-\frac{(a + b*x)*\text{Cos}[\text{Log}[a + b*x]]}{(2*b)} + \frac{(a + b*x)*\text{Sin}[\text{Log}[a + b*x]]}{(2*b)}$

**Rubi [A]** time = 0.0142934, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4475}

$$\frac{(a + bx) \sin(\log(a + bx))}{2b} - \frac{(a + bx) \cos(\log(a + bx))}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sin[Log[a + b\*x]],x]

[Out]  $-\frac{(a + b*x)*\text{Cos}[\text{Log}[a + b*x]]}{(2*b)} + \frac{(a + b*x)*\text{Sin}[\text{Log}[a + b*x]]}{(2*b)}$

#### Rule 4475

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.), x\_Symbol] :> Simp[x\*Sin[d\*(a + b\*Log[c\*x^n])]]/(b^2\*d^2\*n^2 + 1), x] - Simp[(b\*d\*n\*x\*Cos[d\*(a + b\*Log[c\*x^n])]]/(b^2\*d^2\*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2\*d^2\*n^2 + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \sin(\log(a + bx)) dx &= \frac{\text{Subst}\left(\int \sin(\log(x)) dx, x, a + bx\right)}{b} \\ &= -\frac{(a + bx) \cos(\log(a + bx))}{2b} + \frac{(a + bx) \sin(\log(a + bx))}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.0155208, size = 29, normalized size = 0.74

$$-\frac{(a + bx)(\cos(\log(a + bx)) - \sin(\log(a + bx)))}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Log[a + b\*x]],x]

[Out]  $-\frac{(a + b*x)*(\text{Cos}[\text{Log}[a + b*x]] - \text{Sin}[\text{Log}[a + b*x]])}{(2*b)}$

**Maple [B]** time = 0.024, size = 76, normalized size = 2.

$$\left(x \tan\left(\frac{\ln(bx + a)}{2}\right) + \frac{a}{b} \tan\left(\frac{\ln(bx + a)}{2}\right) + \frac{a}{b} \left(\tan\left(\frac{\ln(bx + a)}{2}\right)\right)^2 - \frac{x}{2} + \frac{x}{2} \left(\tan\left(\frac{\ln(bx + a)}{2}\right)\right)^2\right) \left(1 + \left(\tan\left(\frac{\ln(bx + a)}{2}\right)\right)^2\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(ln(b*x+a)),x)`

[Out]  $(x \tan(1/2 \ln(bx+a)) + a/b \tan(1/2 \ln(bx+a)) + a/b \tan(1/2 \ln(bx+a))^2 - 1/2 * x + 1/2 * x * \tan(1/2 \ln(bx+a))^2) / (1 + \tan(1/2 \ln(bx+a))^2)$

**Maxima [A]** time = 1.11677, size = 36, normalized size = 0.92

$$\frac{(bx+a)(\cos(\log(bx+a)) - \sin(\log(bx+a)))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(log(b*x+a)),x, algorithm="maxima")`

[Out]  $-1/2 * (bx + a) * (\cos(\log(bx + a)) - \sin(\log(bx + a))) / b$

**Fricas [A]** time = 0.483304, size = 92, normalized size = 2.36

$$\frac{(bx+a)\cos(\log(bx+a)) - (bx+a)\sin(\log(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(log(b*x+a)),x, algorithm="fricas")`

[Out]  $-1/2 * ((bx + a) * \cos(\log(bx + a)) - (bx + a) * \sin(\log(bx + a))) / b$

**Sympy [A]** time = 2.01977, size = 56, normalized size = 1.44

$$\begin{cases} \frac{a \sin(\log(a+bx))}{2b} - \frac{a \cos(\log(a+bx))}{2b} + \frac{x \sin(\log(a+bx))}{2} - \frac{x \cos(\log(a+bx))}{2} & \text{for } b \neq 0 \\ x \sin(\log(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(ln(b*x+a)),x)`

[Out] `Piecewise((a*sin(log(a + b*x))/(2*b) - a*cos(log(a + b*x))/(2*b) + x*sin(log(a + b*x))/2 - x*cos(log(a + b*x))/2, Ne(b, 0)), (x*sin(log(a)), True))`

**Giac [A]** time = 1.14293, size = 47, normalized size = 1.21

$$-\frac{(bx+a)\cos(\log(bx+a))}{2b} + \frac{(bx+a)\sin(\log(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(log(b*x+a)),x, algorithm="giac")`

[Out]  $-1/2 * (bx + a) * \cos(\log(bx + a)) / b + 1/2 * (bx + a) * \sin(\log(bx + a)) / b$

$$3.26 \quad \int x^m \sin \left( a + \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=133

$$\frac{(m+1)x^{m+1} \log(x) e^{\frac{an\sqrt{-(m+1)^2}}{m+1}} (cx^n)^{-\frac{m+1}{n}}}{2n\sqrt{-\frac{(m+1)^2}{n^2}}} - \frac{x^{m+1} e^{\frac{a(m+1)}{n\sqrt{-(m+1)^2}}} (cx^n)^{\frac{m+1}{n}}}{4n\sqrt{-\frac{(m+1)^2}{n^2}}}$$

[Out]  $-(E^{((a*(1+m))/(Sqrt[-((1+m)^2/n^2)]*n)}) * x^{(1+m)} * (c*x^n)^{((1+m)/n)}) / (4*Sqrt[-((1+m)^2/n^2)]*n) + (E^{((a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))* (1+m)*x^{(1+m)}*Log[x]) / (2*Sqrt[-((1+m)^2/n^2)]*n*(c*x^n)^{((1+m)/n)})$

**Rubi [A]** time = 0.277359, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4493, 4489}

$$\frac{(m+1)x^{m+1} \log(x) e^{\frac{an\sqrt{-(m+1)^2}}{m+1}} (cx^n)^{-\frac{m+1}{n}}}{2n\sqrt{-\frac{(m+1)^2}{n^2}}} - \frac{x^{m+1} e^{\frac{a(m+1)}{n\sqrt{-(m+1)^2}}} (cx^n)^{\frac{m+1}{n}}}{4n\sqrt{-\frac{(m+1)^2}{n^2}}}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sin[a + Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n]], x]

[Out]  $-(E^{((a*(1+m))/(Sqrt[-((1+m)^2/n^2)]*n)}) * x^{(1+m)} * (c*x^n)^{((1+m)/n)}) / (4*Sqrt[-((1+m)^2/n^2)]*n) + (E^{((a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))* (1+m)*x^{(1+m)}*Log[x]) / (2*Sqrt[-((1+m)^2/n^2)]*n*(c*x^n)^{((1+m)/n)})$

#### Rule 4493

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] :> Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[x^((m+1)/n-1)\*Sin[d\*(a+b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] :> Dist[(m+1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^{(a\*b\*d^2\*p)/(m+1)})/x^{(m+1)/p} - x^{(m+1)/p}/E^{(a\*b\*d^2\*p)/(m+1)})^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m+1)^2, 0]

#### Rubi steps

$$\begin{aligned}
\int x^m \sin\left(a + \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n)\right) dx &= \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sin\left(a + \sqrt{-\frac{(1+m)^2}{n^2}} \log(x)\right) dx, x, cx^n\right)}{n} \\
&= \frac{\left((1+m)x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \left(\frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}}{1+m}}}{x} - e^{\sqrt{-\frac{(1+m)^2}{n^2}} \frac{a(1+m)}{n}} x^{-1+\frac{2(1+m)}{n}}\right) dx, x, cx^n\right)}{2\sqrt{-\frac{(1+m)^2}{n^2}} n^2} \\
&= -\frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}} n}} x^{1+m} (cx^n)^{\frac{1+m}{n}}}{4\sqrt{-\frac{(1+m)^2}{n^2}} n} + \frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}}{1+m}} (1+m)x^{1+m} (cx^n)^{-\frac{1+m}{n}} \log(x)}{2\sqrt{-\frac{(1+m)^2}{n^2}} n}
\end{aligned}$$

**Mathematica [F]** time = 0.252815, size = 0, normalized size = 0.

$$\int x^m \sin\left(a + \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n)\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m\*Sin[a + Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n]], x]

[Out] Integrate[x^m\*Sin[a + Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n]], x]

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int x^m \sin\left(a + \ln(cx^n) \sqrt{-\frac{(1+m)^2}{n^2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sin(a+ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)), x)

[Out] int(x^m\*sin(a+ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)), x)

**Maxima [A]** time = 1.19864, size = 111, normalized size = 0.83

$$\frac{c^{\frac{2m}{n} + \frac{2}{n}} x e^{\left(m \log(x) + \frac{m \log(x^n)}{n} + \frac{\log(x^n)}{n}\right)} \sin(a) + 2(m \sin(a) + \sin(a)) \log(x)}{4 \left(c^{\frac{m}{n} + \frac{1}{n}} m + c^{\frac{m}{n} + \frac{1}{n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)), x, algorithm="maxima")

[Out] 1/4\*(c^(2\*m/n + 2/n)\*x\*e^(m\*log(x) + m\*log(x^n)/n + log(x^n)/n)\*sin(a) + 2\*(m\*sin(a) + sin(a))\*log(x))/(c^(m/n + 1/n)\*m + c^(m/n + 1/n))

---

**Fricas [C]** time = 0.497432, size = 159, normalized size = 1.2

$$\frac{\left( i x^2 x^{2m} + (-2im - 2i) e^{\left( \frac{2(ian - (m+1)\log(c))}{n} \right)} \log(x) \right) e^{\left( -\frac{ian - (m+1)\log(c)}{n} \right)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)),x, algorithm="fricas")

[Out] 1/4\*(I\*x^2\*x^(2\*m) + (-2\*I\*m - 2\*I)\*e^(2\*(I\*a\*n - (m + 1)\*log(c))/n)\*log(x))\*e^(-(I\*a\*n - (m + 1)\*log(c))/n)/(m + 1)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sin\left(a + \sqrt{-\frac{m^2}{n^2} - \frac{2m}{n^2} - \frac{1}{n^2}} \log(cx^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*sin(a+ln(c\*x\*\*n)\*(-(1+m)\*\*2/n\*\*2)\*\*(1/2)),x)

[Out] Integral(x\*\*m\*sin(a + sqrt(-m\*\*2/n\*\*2 - 2\*m/n\*\*2 - 1/n\*\*2)\*log(c\*x\*\*n)), x)

---

**Giac [C]** time = 1.81321, size = 367, normalized size = 2.76

$$\frac{-i mn^2 x x^m e^{\left( i a - \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2} \right)} + i mn^2 x x^m e^{\left( -i a + \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2} \right)} - i n^2 x x^m e^{\left( i a - \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2} \right)} - i n x x^m e^{\left( -i a + \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2} \right)}}{2(m^2 n^2 + 2 m n^2 - (m n + n)^2 + n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)),x, algorithm="giac")

[Out] 1/2\*(-I\*m\*n^2\*x\*x^m\*e^(I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + I\*m\*n^2\*x\*x^m\*e^(-I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) - I\*n^2\*x\*x^m\*e^(I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) - I\*n\*x\*x^m\*abs(m\*n + n)\*e^(I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + I\*n^2\*x\*x^m\*e^(-I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) - I\*n\*x\*x^m\*abs(m\*n + n)\*e^(-I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2))/(m^2\*n^2 + 2\*m\*n^2 - (m\*n + n)^2 + n^2)

$$3.27 \quad \int x^2 \sin \left( a + 3\sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=88

$$\frac{1}{12} \sqrt{-\frac{1}{n^2}} n x^3 e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{3/n} - \frac{1}{2} \sqrt{-\frac{1}{n^2}} n x^3 e^{a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-3/n}$$

[Out] (Sqrt[-n^(-2)]\*n\*x^3\*(c\*x^n)^(3/n))/(12\*E^(a\*Sqrt[-n^(-2)]\*n)) - (E^(a\*Sqrt[-n^(-2)]\*n)\*Sqrt[-n^(-2)]\*n\*x^3\*Log[x])/(2\*(c\*x^n)^(3/n))

**Rubi [A]** time = 0.0988647, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {4493, 4489}

$$\frac{1}{12} \sqrt{-\frac{1}{n^2}} n x^3 e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{3/n} - \frac{1}{2} \sqrt{-\frac{1}{n^2}} n x^3 e^{a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-3/n}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sin[a + 3\*Sqrt[-n^(-2)]\*Log[c\*x^n]],x]

[Out] (Sqrt[-n^(-2)]\*n\*x^3\*(c\*x^n)^(3/n))/(12\*E^(a\*Sqrt[-n^(-2)]\*n)) - (E^(a\*Sqrt[-n^(-2)]\*n)\*Sqrt[-n^(-2)]\*n\*x^3\*Log[x])/(2\*(c\*x^n)^(3/n))

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1)))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int x^2 \sin \left( a + 3\sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x^3 (cx^n)^{-3/n}) \text{Subst} \left( \int x^{-1+\frac{3}{n}} \sin \left( a + 3\sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= - \left( \frac{1}{2} \left( \sqrt{-\frac{1}{n^2}} x^3 (cx^n)^{-3/n} \right) \text{Subst} \left( \int \left( \frac{e^{a \sqrt{-\frac{1}{n^2}} n}}{x} - e^{-a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{6}{n}} \right) dx, x, cx^n \right) \right) \\ &= \frac{1}{12} e^{-a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x^3 (cx^n)^{3/n} - \frac{1}{2} e^{a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x^3 (cx^n)^{-3/n} \log(x) \end{aligned}$$

**Mathematica [F]** time = 0.145668, size = 0, normalized size = 0.

$$\int x^2 \sin\left(a + 3\sqrt{-\frac{1}{n^2}} \log(cx^n)\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*Sin[a + 3\*Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

[Out] Integrate[x^2\*Sin[a + 3\*Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int x^2 \sin\left(a + 3 \ln(cx^n) \sqrt{-n^{-2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sin(a+3\*ln(c\*x^n)\*(-1/n^2)^(1/2)), x)

[Out] int(x^2\*sin(a+3\*ln(c\*x^n)\*(-1/n^2)^(1/2)), x)

**Maxima [A]** time = 1.09538, size = 42, normalized size = 0.48

$$\frac{c^{\frac{6}{n}} x^6 \sin(a) + 6 \log(x) \sin(a)}{12 c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+3\*log(c\*x^n)\*(-1/n^2)^(1/2)), x, algorithm="maxima")

[Out] 1/12\*(c^(6/n)\*x^6\*sin(a) + 6\*log(x)\*sin(a))/c^(3/n)

**Fricas [C]** time = 0.468447, size = 108, normalized size = 1.23

$$\frac{1}{12} \left( i x^6 - 6i e^{\left(\frac{2(i a n - 3 \log(c))}{n}\right)} \log(x) \right) e^{\left(-\frac{i a n - 3 \log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+3\*log(c\*x^n)\*(-1/n^2)^(1/2)), x, algorithm="fricas")

[Out] 1/12\*(I\*x^6 - 6\*I\*e^(2\*(I\*a\*n - 3\*log(c))/n)\*log(x))\*e^(-(I\*a\*n - 3\*log(c))/n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin\left(a + 3\sqrt{-\frac{1}{n^2}} \log(cx^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+3*ln(c*x**n)*(-1/n**2)**(1/2)),x)
```

```
[Out] Integral(x**2*sin(a + 3*sqrt(-1/n**2)*log(c*x**n)), x)
```

**Giac [A]** time = 1.46388, size = 1, normalized size = 0.01

$+\infty$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+3*log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

### 3.28 $\int x \sin \left( a + 2\sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$

**Optimal.** Leaf size=88

$$\frac{1}{8}\sqrt{-\frac{1}{n^2}}nx^2e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n} - \frac{1}{2}\sqrt{-\frac{1}{n^2}}nx^2e^{a\sqrt{-\frac{1}{n^2}}n}\log(x)(cx^n)^{-2/n}$$

[Out] (Sqrt[-n^(-2)]\*n\*x^2\*(c\*x^n)^(2/n))/(8\*E^(a\*Sqrt[-n^(-2)]\*n)) - (E^(a\*Sqrt[-n^(-2)]\*n)\*Sqrt[-n^(-2)]\*n\*x^2\*Log[x])/(2\*(c\*x^n)^(2/n))

**Rubi [A]** time = 0.0518887, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {4493, 4489}

$$\frac{1}{8}\sqrt{-\frac{1}{n^2}}nx^2e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n} - \frac{1}{2}\sqrt{-\frac{1}{n^2}}nx^2e^{a\sqrt{-\frac{1}{n^2}}n}\log(x)(cx^n)^{-2/n}$$

Antiderivative was successfully verified.

[In] Int[x\*Sin[a + 2\*Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

[Out] (Sqrt[-n^(-2)]\*n\*x^2\*(c\*x^n)^(2/n))/(8\*E^(a\*Sqrt[-n^(-2)]\*n)) - (E^(a\*Sqrt[-n^(-2)]\*n)\*Sqrt[-n^(-2)]\*n\*x^2\*Log[x])/(2\*(c\*x^n)^(2/n))

#### Rule 4493

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_.) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_ .), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_.) + Log[x\_]\*(b\_))\*(d\_)]^(p\_ .), x\_Symbol] := Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int x \sin \left( a + 2\sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst} \left( \int x^{-1+\frac{2}{n}} \sin \left( a + 2\sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= - \left( \frac{1}{2} \left( \sqrt{-\frac{1}{n^2}} x^2 (cx^n)^{-2/n} \right) \text{Subst} \left( \int \left( \frac{e^{a\sqrt{-\frac{1}{n^2}}n}}{x} - e^{-a\sqrt{-\frac{1}{n^2}}n} x^{-1+\frac{4}{n}} \right) dx, x, cx^n \right) \right) \\ &= \frac{1}{8} e^{-a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}} nx^2 (cx^n)^{2/n} - \frac{1}{2} e^{a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}} nx^2 (cx^n)^{-2/n} \log(x) \end{aligned}$$



**Mathematica [F]** time = 0.115529, size = 0, normalized size = 0.

$$\int x \sin \left( a + 2 \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x\*Sin[a + 2\*Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

[Out] Integrate[x\*Sin[a + 2\*Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int x \sin \left( a + 2 \ln(cx^n) \sqrt{-n^{-2}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+2\*ln(c\*x^n)\*(-1/n^2)^(1/2)), x)

[Out] int(x\*sin(a+2\*ln(c\*x^n)\*(-1/n^2)^(1/2)), x)

**Maxima [A]** time = 1.21159, size = 42, normalized size = 0.48

$$\frac{c^{\frac{4}{n}} x^4 \sin(a) + 4 \log(x) \sin(a)}{8 c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+2\*log(c\*x^n)\*(-1/n^2)^(1/2)), x, algorithm="maxima")

[Out] 1/8\*(c^(4/n)\*x^4\*sin(a) + 4\*log(x)\*sin(a))/c^(2/n)

**Fricas [C]** time = 0.471463, size = 107, normalized size = 1.22

$$\frac{1}{8} \left( i x^4 - 4 i e^{\left( \frac{2(i a n - 2 \log(c))}{n} \right)} \log(x) \right) e^{\left( -\frac{i a n - 2 \log(c)}{n} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+2\*log(c\*x^n)\*(-1/n^2)^(1/2)), x, algorithm="fricas")

[Out] 1/8\*(I\*x^4 - 4\*I\*e^(2\*(I\*a\*n - 2\*log(c))/n)\*log(x))\*e^(-(I\*a\*n - 2\*log(c))/n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sin \left( a + 2 \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(a+2*ln(c*x**n)*(-1/n**2)**(1/2)),x)`

[Out] `Integral(x*sin(a + 2*sqrt(-1/n**2)*log(c*x**n)), x)`

**Giac [A]** time = 1.49535, size = 1, normalized size = 0.01

$+\infty$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(a+2*log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="giac")`

[Out] `+Infinity`

$$3.29 \quad \int \sin \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=82

$$\frac{1}{4} \sqrt{-\frac{1}{n^2}} n x e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} - \frac{1}{2} \sqrt{-\frac{1}{n^2}} n x e^{a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n}$$

[Out] (Sqrt[-n^(-2)]\*n\*x\*(c\*x^n)^n^(-1))/(4\*E^(a\*Sqrt[-n^(-2)]\*n)) - (E^(a\*Sqrt[-n^(-2)]\*n)\*Sqrt[-n^(-2)]\*n\*x\*Log[x])/(2\*(c\*x^n)^n^(-1))

**Rubi [A]** time = 0.0517531, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {4483, 4489}

$$\frac{1}{4} \sqrt{-\frac{1}{n^2}} n x e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} - \frac{1}{2} \sqrt{-\frac{1}{n^2}} n x e^{a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

[Out] (Sqrt[-n^(-2)]\*n\*x\*(c\*x^n)^n^(-1))/(4\*E^(a\*Sqrt[-n^(-2)]\*n)) - (E^(a\*Sqrt[-n^(-2)]\*n)\*Sqrt[-n^(-2)]\*n\*x\*Log[x])/(2\*(c\*x^n)^n^(-1))

**Rule 4483**

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

**Rule 4489**

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1)))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

**Rubi steps**

$$\begin{aligned} \int \sin \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x (cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1+\frac{1}{n}} \sin \left( a + \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= - \left( \frac{1}{2} \left( \sqrt{-\frac{1}{n^2}} x (cx^n)^{-1/n} \right) \text{Subst} \left( \int \left( \frac{e^{a \sqrt{-\frac{1}{n^2}} n}}{x} - e^{-a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{2}{n}} \right) dx, x, cx^n \right) \right) \\ &= \frac{1}{4} e^{-a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x (cx^n)^{\frac{1}{n}} - \frac{1}{2} e^{a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x (cx^n)^{-1/n} \log(x) \end{aligned}$$

**Mathematica [F]** time = 0.0858786, size = 0, normalized size = 0.

$$\int \sin \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

[Out] Integrate[Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

**Maple [F]** time = 0.034, size = 0, normalized size = 0.

$$\int \sin\left(a + \ln(cx^n) \sqrt{-n^{-2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2)), x)

[Out] int(sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2)), x)

**Maxima [A]** time = 1.17032, size = 39, normalized size = 0.48

$$\frac{c^{\frac{2}{n}} x^2 \sin(a) + 2 \log(x) \sin(a)}{4 c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+log(c\*x^n)\*(-1/n^2)^(1/2)), x, algorithm="maxima")

[Out] 1/4\*(c^(2/n)\*x^2\*sin(a) + 2\*log(x)\*sin(a))/c^(1/n)

**Fricas [C]** time = 0.466307, size = 101, normalized size = 1.23

$$\frac{1}{4} \left( i x^2 - 2i e^{\left(\frac{2(i a n - \log(c))}{n}\right)} \log(x) \right) e^{\left(-\frac{i a n - \log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+log(c\*x^n)\*(-1/n^2)^(1/2)), x, algorithm="fricas")

[Out] 1/4\*(I\*x^2 - 2\*I\*e^(2\*(I\*a\*n - log(c))/n)\*log(x))\*e^(-(I\*a\*n - log(c))/n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \sqrt{-\frac{1}{n^2}} \log(cx^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2)), x)

```
[Out] Integral(sin(a + sqrt(-1/n**2)*log(c*x**n)), x)
```

---

**Giac [A]** time = 1.27703, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

$$3.30 \quad \int \frac{\sin(a)}{x} dx$$

**Optimal.** Leaf size=5

$$\sin(a) \log(x)$$

[Out] Log[x]\*Sin[a]

**Rubi [A]** time = 0.0044641, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {12, 29}

$$\sin(a) \log(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[a]/x,x]

[Out] Log[x]\*Sin[a]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin(a)}{x} dx &= \sin(a) \int \frac{1}{x} dx \\ &= \log(x) \sin(a) \end{aligned}$$

**Mathematica [A]** time = 0.0008025, size = 5, normalized size = 1.

$$\sin(a) \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a]/x,x]

[Out] Log[x]\*Sin[a]

**Maple [A]** time = 0.012, size = 6, normalized size = 1.2

$$\ln(x) \sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a)/x,x)`

[Out] `ln(x)*sin(a)`

**Maxima [A]** time = 1.0635, size = 7, normalized size = 1.4

$\log(x) \sin(a)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a)/x,x, algorithm="maxima")`

[Out] `log(x)*sin(a)`

**Fricas [A]** time = 0.431015, size = 20, normalized size = 4.

$\log(x) \sin(a)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a)/x,x, algorithm="fricas")`

[Out] `log(x)*sin(a)`

**Sympy [A]** time = 0.27912, size = 5, normalized size = 1.

$\log(x) \sin(a)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a)/x,x)`

[Out] `log(x)*sin(a)`

**Giac [A]** time = 1.14013, size = 8, normalized size = 1.6

$\log(|x|) \sin(a)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a)/x,x, algorithm="giac")`

[Out] `log(abs(x))*sin(a)`

$$3.31 \quad \int \frac{\sin\left(a + \sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^2} dx$$

**Optimal.** Leaf size=86

$$\frac{\sqrt{-\frac{1}{n^2}} n e^{a\sqrt{-\frac{1}{n^2}}n} (cx^n)^{-1/n}}{4x} + \frac{\sqrt{-\frac{1}{n^2}} n e^{-a\sqrt{-\frac{1}{n^2}}n} \log(x) (cx^n)^{\frac{1}{n}}}{2x}$$

[Out] (E^(a\*Sqrt[-n^(-2)]\*n)\*Sqrt[-n^(-2)]\*n)/(4\*x\*(c\*x^n)^n^(-1)) + (Sqrt[-n^(-2)]\*n\*(c\*x^n)^n^(-1)\*Log[x])/(2\*E^(a\*Sqrt[-n^(-2)]\*n)\*x)

**Rubi [A]** time = 0.0611679, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {4493, 4489}

$$\frac{\sqrt{-\frac{1}{n^2}} n e^{a\sqrt{-\frac{1}{n^2}}n} (cx^n)^{-1/n}}{4x} + \frac{\sqrt{-\frac{1}{n^2}} n e^{-a\sqrt{-\frac{1}{n^2}}n} \log(x) (cx^n)^{\frac{1}{n}}}{2x}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]/x^2,x]

[Out] (E^(a\*Sqrt[-n^(-2)]\*n)\*Sqrt[-n^(-2)]\*n)/(4\*x\*(c\*x^n)^n^(-1)) + (Sqrt[-n^(-2)]\*n\*(c\*x^n)^n^(-1)\*Log[x])/(2\*E^(a\*Sqrt[-n^(-2)]\*n)\*x)

#### Rule 4493

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_.) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_.) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] :> Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin\left(a + \sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^2} dx &= \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \sin\left(a + \sqrt{-\frac{1}{n^2}} \log(x)\right) dx, x, cx^n\right)}{nx} \\ &= \frac{\left(\sqrt{-\frac{1}{n^2}} (cx^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \left(\frac{e^{-a\sqrt{-\frac{1}{n^2}}n}}{x} - e^{a\sqrt{-\frac{1}{n^2}}n} x^{-\frac{2+n}{n}}\right) dx, x, cx^n\right)}{2x} \\ &= \frac{e^{a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}} n (cx^n)^{-1/n}}{4x} + \frac{e^{-a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}} n (cx^n)^{\frac{1}{n}} \log(x)}{2x} \end{aligned}$$



**Mathematica [F]** time = 0.0722603, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + \sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]/x^2,x]

[Out] Integrate[Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]/x^2, x]

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sin\left(a + \ln(cx^n) \sqrt{-n^{-2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2))/x^2,x)

[Out] int(sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2))/x^2,x)

**Maxima [A]** time = 1.13135, size = 45, normalized size = 0.52

$$\frac{2 c^{\frac{2}{n}} x^2 \log(x) \sin(a) - \sin(a)}{4 c^{\left(\frac{1}{n}\right)} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))/x^2,x, algorithm="maxima")

[Out] 1/4\*(2\*c^(2/n)\*x^2\*log(x)\*sin(a) - sin(a))/(c^(1/n)\*x^2)

**Fricas [C]** time = 0.472435, size = 107, normalized size = 1.24

$$\frac{\left(2i x^2 \log(x) + i e^{\left(\frac{2(i a n - \log(c))}{n}\right)}\right) e^{-\frac{i a n - \log(c)}{n}}}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))/x^2,x, algorithm="fricas")

[Out] 1/4\*(2\*I\*x^2\*log(x) + I\*e^(2\*(I\*a\*n - log(c))/n))\*e^(-(I\*a\*n - log(c))/n)/x^2

**Sympy [C]** time = 14.2811, size = 214, normalized size = 2.49

$$\frac{i n \sqrt{\frac{1}{n^2}} \log(x) \cos\left(a + i n \sqrt{\frac{1}{n^2}} \log(x) + i \sqrt{\frac{1}{n^2}} \log(c)\right)}{2x} + \frac{i \sqrt{\frac{1}{n^2}} \log(c) \cos\left(a + i n \sqrt{\frac{1}{n^2}} \log(x) + i \sqrt{\frac{1}{n^2}} \log(c)\right)}{2x} + \frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2))/x\*\*2,x)

[Out] I\*n\*sqrt(n\*\*(-2))\*log(x)\*cos(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(2\*x) + I\*sqrt(n\*\*(-2))\*log(c)\*cos(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(2\*x) + log(x)\*sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(2\*x) - sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(2\*x) + log(c)\*sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(2\*n\*x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\sqrt{-\frac{1}{n^2}} \log(cx^n) + a\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))/x^2,x, algorithm="giac")

[Out] integrate(sin(sqrt(-1/n^2)\*log(c\*x^n) + a)/x^2, x)

$$3.32 \quad \int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3} dx$$

**Optimal.** Leaf size=88

$$\frac{\sqrt{-\frac{1}{n^2}}ne^a\sqrt{-\frac{1}{n^2}}(cx^n)^{-2/n}}{8x^2} + \frac{\sqrt{-\frac{1}{n^2}}ne^{-a}\sqrt{-\frac{1}{n^2}}\log(x)(cx^n)^{2/n}}{2x^2}$$

[Out]  $(E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n)/(8*x^2*(c*x^n)^{(2/n)}) + (\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^{(2/n)*\text{Log}[x]})/(2*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*x^2})$

**Rubi [A]** time = 0.0531861, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {4493, 4489}

$$\frac{\sqrt{-\frac{1}{n^2}}ne^a\sqrt{-\frac{1}{n^2}}(cx^n)^{-2/n}}{8x^2} + \frac{\sqrt{-\frac{1}{n^2}}ne^{-a}\sqrt{-\frac{1}{n^2}}\log(x)(cx^n)^{2/n}}{2x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[a + 2*\text{Sqrt}[-n^{(-2)}]*\text{Log}[c*x^n]]/x^3, x]$

[Out]  $(E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n)/(8*x^2*(c*x^n)^{(2/n)}) + (\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^{(2/n)*\text{Log}[x]})/(2*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*x^2})$

#### Rule 4493

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}]*b_{.})*(d_{.})]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)*\text{Sin}[d*(a+b*\text{Log}[x])]}^p, x], x, c*x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\text{NeQ}[c, 1] \parallel \text{NeQ}[n, 1])$

#### Rule 4489

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[x]*b_{.})*(d_{.})]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(m+1)^p/(2^p*b^p*d^p*p^p), \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(E^{((a*b*d^2*p)/(m+1))}/x^{((m+1)/p)} - x^{((m+1)/p)}/E^{((a*b*d^2*p)/(m+1))})^p, x], x] /; \text{FreeQ}\{a, b, d, e, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[b^2*d^2*p^2 + (m+1)^2, 0]$

#### Rubi steps

$$\begin{aligned} \int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3} dx &= \frac{(cx^n)^{2/n} \text{Subst}\left(\int x^{-1-\frac{2}{n}} \sin\left(a+2\sqrt{-\frac{1}{n^2}}\log(x)\right) dx, x, cx^n\right)}{nx^2} \\ &= \frac{\left(\sqrt{-\frac{1}{n^2}}(cx^n)^{2/n}\right) \text{Subst}\left(\int \left(\frac{e^{-a\sqrt{-\frac{1}{n^2}}}}{x} - e^{a\sqrt{-\frac{1}{n^2}}}x^{-\frac{4+n}{n}}\right) dx, x, cx^n\right)}{2x^2} \\ &= \frac{e^{a\sqrt{-\frac{1}{n^2}}}\sqrt{-\frac{1}{n^2}}(cx^n)^{-2/n}}{8x^2} + \frac{e^{-a\sqrt{-\frac{1}{n^2}}}\sqrt{-\frac{1}{n^2}}(cx^n)^{2/n}\log(x)}{2x^2} \end{aligned}$$

**Mathematica [F]** time = 0.0895486, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + 2\sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + 2\*Sqrt[-n^(-2)]\*Log[c\*x^n]]/x^3,x]

[Out] Integrate[Sin[a + 2\*Sqrt[-n^(-2)]\*Log[c\*x^n]]/x^3, x]

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \sin\left(a + 2 \ln(cx^n) \sqrt{-n^{-2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+2\*ln(c\*x^n)\*(-1/n^2)^(1/2))/x^3,x)

[Out] int(sin(a+2\*ln(c\*x^n)\*(-1/n^2)^(1/2))/x^3,x)

**Maxima [A]** time = 1.24162, size = 47, normalized size = 0.53

$$\frac{4c^{\frac{4}{n}}x^4 \log(x) \sin(a) - \sin(a)}{8c^{\frac{2}{n}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2\*log(c\*x^n)\*(-1/n^2)^(1/2))/x^3,x, algorithm="maxima")

[Out] 1/8\*(4\*c^(4/n)\*x^4\*log(x)\*sin(a) - sin(a))/(c^(2/n)\*x^4)

**Fricas [C]** time = 0.471725, size = 112, normalized size = 1.27

$$\frac{\left(4ix^4 \log(x) + ie^{\left(\frac{2(ian-2 \log(c))}{n}\right)}\right)e^{\left(-\frac{ian-2 \log(c)}{n}\right)}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2\*log(c\*x^n)\*(-1/n^2)^(1/2))/x^3,x, algorithm="fricas")

[Out] 1/8\*(4\*I\*x^4\*log(x) + I\*e^(2\*(I\*a\*n - 2\*log(c))/n))\*e^(-(I\*a\*n - 2\*log(c))/n)/x^4

**Sympy [C]** time = 71.6194, size = 240, normalized size = 2.73

$$\frac{i n \sqrt{\frac{1}{n^2}} \log(x) \cos\left(a + 2i n \sqrt{\frac{1}{n^2}} \log(x) + 2i \sqrt{\frac{1}{n^2}} \log(c)\right)}{2x^2} + \frac{i \sqrt{\frac{1}{n^2}} \log(c) \cos\left(a + 2i n \sqrt{\frac{1}{n^2}} \log(x) + 2i \sqrt{\frac{1}{n^2}} \log(c)\right)}{2x^2} + \frac{\log(x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2\*ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2))/x\*\*3,x)

[Out] I\*n\*sqrt(n\*\*(-2))\*log(x)\*cos(a + 2\*I\*n\*sqrt(n\*\*(-2))\*log(x) + 2\*I\*sqrt(n\*\*(-2))\*log(c))/(2\*x\*\*2) + I\*sqrt(n\*\*(-2))\*log(c)\*cos(a + 2\*I\*n\*sqrt(n\*\*(-2))\*log(x) + 2\*I\*sqrt(n\*\*(-2))\*log(c))/(2\*x\*\*2) + log(x)\*sin(a + 2\*I\*n\*sqrt(n\*\*(-2))\*log(x) + 2\*I\*sqrt(n\*\*(-2))\*log(c))/(2\*x\*\*2) - sin(a + 2\*I\*n\*sqrt(n\*\*(-2))\*log(x) + 2\*I\*sqrt(n\*\*(-2))\*log(c))/(4\*x\*\*2) + log(c)\*sin(a + 2\*I\*n\*sqrt(n\*\*(-2))\*log(x) + 2\*I\*sqrt(n\*\*(-2))\*log(c))/(2\*n\*x\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(2\sqrt{-\frac{1}{n^2}}\log(cx^n) + a\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2\*log(c\*x^n)\*(-1/n^2)^(1/2))/x^3,x, algorithm="giac")

[Out] integrate(sin(2\*sqrt(-1/n^2)\*log(c\*x^n) + a)/x^3, x)

$$3.33 \quad \int x^m \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=117

$$-\frac{x^{m+1} e^{-\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{\frac{m+1}{n}}}{8(m+1)} - \frac{1}{4} x^{m+1} \log(x) e^{\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{-\frac{m+1}{n}} + \frac{x^{m+1}}{2(m+1)}$$

[Out]  $x^{(1+m)/(2*(1+m))} - (x^{(1+m)}*(c*x^n)^{((1+m)/n)})/(8*E^{((2*a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))*(1+m)} - (E^{((2*a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))*x^{(1+m)*Log[x]})/(4*(c*x^n)^{((1+m)/n)})}$

**Rubi [A]** time = 0.15885, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$ , Rules used = {4493, 4489}

$$-\frac{x^{m+1} e^{-\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{\frac{m+1}{n}}}{8(m+1)} - \frac{1}{4} x^{m+1} \log(x) e^{\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{-\frac{m+1}{n}} + \frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^m \text{Sin}[a + (\text{Sqrt}[-((1+m)^2/n^2)]) * \text{Log}[c*x^n])/2]^2, x]$

[Out]  $x^{(1+m)/(2*(1+m))} - (x^{(1+m)}*(c*x^n)^{((1+m)/n)})/(8*E^{((2*a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))*(1+m)} - (E^{((2*a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))*x^{(1+m)*Log[x]})/(4*(c*x^n)^{((1+m)/n)})}$

#### Rule 4493

$\text{Int}[(e_*)*(x_*)^{(m_*)} \text{Sin}[(a_*) + \text{Log}[(c_*)*(x_*)^{(n_*)}]*(b_*)*(d_*)]^{(p_*)}, x\_Symbol] \rightarrow \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)} \text{Sin}[d*(a+b*\text{Log}[x])]^p, x], x, c*x^n], x] /;$  FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

$\text{Int}[(e_*)*(x_*)^{(m_*)} \text{Sin}[(a_*) + \text{Log}[x_*]*(b_*)*(d_*)]^{(p_*)}, x\_Symbol] \rightarrow \text{Dist}[(m+1)^p/(2^p*b^p*d^p*p^p), \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(E^{((a*b*d^2*p)/(m+1))}/x^{((m+1)/p)} - x^{((m+1)/p)}/E^{((a*b*d^2*p)/(m+1))})^p, x], x] /;$  FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m+1)^2, 0]

#### Rubi steps

$$\int x^m \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx = \frac{\left( x^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{n}} \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(x) \right) dx, x \right)}{n}$$

$$= \frac{\left( x^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int \left( \frac{e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{n}}}{x} - 2x^{-1+\frac{1+m}{n}} + e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{n}} \right) dx, x \right)}{4n}$$

$$= \frac{x^{1+m}}{2(1+m)} - \frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{n}} x^{1+m} (cx^n)^{\frac{1+m}{n}}}{8(1+m)} - \frac{1}{4} e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{n}} x^{1+m} (cx^n)^{-\frac{1+m}{n}}$$

**Mathematica [F]** time = 0.356133, size = 0, normalized size = 0.

$$\int x^m \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m\*Sin[a + (Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2]^2,x]

[Out] Integrate[x^m\*Sin[a + (Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2]^2, x]

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int x^m \left( \sin \left( a + \frac{\ln(cx^n)}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sin(a+1/2\*ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^2,x)

[Out] int(x^m\*sin(a+1/2\*ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^2,x)

**Maxima [A]** time = 1.37276, size = 234, normalized size = 2.

$$\frac{4 \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{m}{n} + \frac{1}{n}} x x^m - c^{\frac{2m}{n} + \frac{2}{n}} x \cos(2a) e^{\left( m \log(x) + \frac{m \log(x^n)}{n} + \frac{\log(x^n)}{n} \right)} - 2 \left( \cos(2a)^3 + \cos(2a) \sin(2a)^2 \right) + \dots}{8 \left( \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{m}{n} + \frac{1}{n}} m + \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{m}{n} + \frac{1}{n}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/2\*log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="maxima")

[Out] 1/8\*(4\*(cos(2\*a)^2 + sin(2\*a)^2)\*c^(m/n + 1/n)\*x\*x^m - c^(2\*m/n + 2/n)\*x\*cos(2\*a)\*e^(m\*log(x) + m\*log(x^n)/n + log(x^n)/n) - 2\*(cos(2\*a)^3 + cos(2\*a)\*

$$\frac{\sin(2*a)^2 + (\cos(2*a)^3 + \cos(2*a)*\sin(2*a)^2)*m*\log(x)}{((\cos(2*a)^2 + \sin(2*a)^2)*c^{(m/n + 1/n)*m} + (\cos(2*a)^2 + \sin(2*a)^2)*c^{(m/n + 1/n)})}$$

**Fricas [C]** time = 0.495555, size = 316, normalized size = 2.7

$$\frac{\left(2(m+1)e^{\left(\frac{-2((m+1)n\log(x)-2ian+(m+1)\log(c))}{n}\right)}\log(x) - 4e^{\left(\frac{-(m+1)n\log(x)-2ian+(m+1)\log(c)}{n}\right)} + 1\right)e^{\left(\frac{2((m+1)n\log(x)-2ian+(m+1)\log(c))}{n} + \frac{2ian-(m+1)\log(c)}{n}\right)}}{8(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(m+1)*e^(-2*((m+1)*n*log(x) - 2*I*a*n + (m+1)*log(c))/n)*log(x) - 4*e^(-((m+1)*n*log(x) - 2*I*a*n + (m+1)*log(c))/n) + 1)*e^(2*((m+1)*n*log(x) - 2*I*a*n + (m+1)*log(c))/n + (2*I*a*n - (m+1)*log(c))/n)/(m+1)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sin^2 \left( a + \frac{\sqrt{-\frac{m^2}{n^2} - \frac{2m}{n^2} - \frac{1}{n^2}} \log(cx^n)}{2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sin(a+1/2*ln(c*x**n)*(-(1+m)**2/n**2)**(1/2))**2,x)
```

```
[Out] Integral(x**m*sin(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)**2, x)
```

**Giac [C]** time = 2.53023, size = 672, normalized size = 5.74

$$\frac{m^2 n^2 x x^m e^{\left(2i a - \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)} + m^2 n^2 x x^m e^{\left(-2i a + \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)} - 2 m^2 n^2 x x^m + 2 m n^2 x x^m e^{\left(2i a - \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}}{n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sin(a+1/2*log(c*x^n)*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/4*(m^2*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + m^2*n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 2*m^2*n^2*x*x^m + 2*m*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + m*n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 2*m*n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - m*n*x*x^m*abs(m*n + n)*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 4*m*n^2*x*x^m + n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2))
```



$$\begin{aligned}
& g(c)/n^2) + n^2*x*x^m*e^{(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*lo} \\
& g(c))/n^2) - n*x*x^m*abs(m*n + n)*e^{(-2*I*a + (n*abs(m*n + n)*log(x) + abs(} \\
& m*n + n)*log(c))/n^2) + 2*(m*n + n)^2*x*x^m - 2*n^2*x*x^m)/(m^3*n^2 + 3*m^2} \\
& *n^2 - (m*n + n)^2*m + 3*m*n^2 - (m*n + n)^2 + n^2)
\end{aligned}$$

$$3.34 \quad \int x^2 \sin^2 \left( a + \frac{3}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=76

$$-\frac{1}{24} x^3 e^{-2a \sqrt{-\frac{1}{n^2} n}} (cx^n)^{3/n} - \frac{1}{4} x^3 e^{2a \sqrt{-\frac{1}{n^2} n}} \log(x) (cx^n)^{-3/n} + \frac{x^3}{6}$$

[Out]  $x^3/6 - (x^3*(c*x^n)^(3/n))/(24*E^(2*a*Sqrt[-n^(-2)]*n)) - (E^(2*a*Sqrt[-n^(-2)]*n)*x^3*Log[x])/(4*(c*x^n)^(3/n))$

**Rubi [A]** time = 0.0755471, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4493, 4489}

$$-\frac{1}{24} x^3 e^{-2a \sqrt{-\frac{1}{n^2} n}} (cx^n)^{3/n} - \frac{1}{4} x^3 e^{2a \sqrt{-\frac{1}{n^2} n}} \log(x) (cx^n)^{-3/n} + \frac{x^3}{6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{Sin}[a + (3*\text{Sqrt}[-n^(-2)])*\text{Log}[c*x^n])/2]^2, x]$

[Out]  $x^3/6 - (x^3*(c*x^n)^(3/n))/(24*E^(2*a*Sqrt[-n^(-2)]*n)) - (E^(2*a*Sqrt[-n^(-2)]*n)*x^3*Log[x])/(4*(c*x^n)^(3/n))$

#### Rule 4493

$\text{Int}[(e_*)*(x_*)^(m_*)*\text{Sin}[(a_*) + \text{Log}[(c_*)*(x_*)^(n_*)]*(b_*)*(d_*)]^(p_*)]$ , x\_Symbol]  $\rightarrow \text{Dist}[(e*x)^(m+1)/(e*n*(c*x^n)^(m+1/n))]$ , Subst[Int[x^((m+1)/n-1)\*Sin[d\*(a+b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

$\text{Int}[(e_*)*(x_*)^(m_*)*\text{Sin}[(a_*) + \text{Log}[x_]*(b_*)*(d_*)]^(p_*)]$ , x\_Symbol]  $\rightarrow \text{Dist}[(m+1)^p/(2^p*b^p*d^p*p^p)]$ , Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m+1))/x^((m+1)/p) - x^((m+1)/p)/E^((a\*b\*d^2\*p)/(m+1)))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m+1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int x^2 \sin^2 \left( a + \frac{3}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x^3 (cx^n)^{-3/n}) \text{Subst} \left( \int x^{-1+\frac{3}{n}} \sin^2 \left( a + \frac{3}{2} \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= \frac{(x^3 (cx^n)^{-3/n}) \text{Subst} \left( \int \left( \frac{e^{2a \sqrt{-\frac{1}{n^2} n}}}{x} - 2x^{-1+\frac{3}{n}} + e^{-2a \sqrt{-\frac{1}{n^2} n}} x^{-1+\frac{6}{n}} \right) dx, x, cx^n \right)}{4n} \\ &= \frac{x^3}{6} - \frac{1}{24} e^{-2a \sqrt{-\frac{1}{n^2} n}} x^3 (cx^n)^{3/n} - \frac{1}{4} e^{2a \sqrt{-\frac{1}{n^2} n}} x^3 (cx^n)^{-3/n} \log(x) \end{aligned}$$

**Mathematica [F]** time = 0.250722, size = 0, normalized size = 0.

$$\int x^2 \sin^2 \left( a + \frac{3}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*Sin[a + (3\*Sqrt[-n^(-2)]\*Log[c\*x^n])/2]^2,x]

[Out] Integrate[x^2\*Sin[a + (3\*Sqrt[-n^(-2)]\*Log[c\*x^n])/2]^2, x]

**Maple [F]** time = 0.08, size = 0, normalized size = 0.

$$\int x^2 \left( \sin \left( a + \frac{3 \ln(cx^n)}{2} \sqrt{-n^{-2}} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sin(a+3/2\*ln(c\*x^n)\*(-1/n^2)^(1/2))^2,x)

[Out] int(x^2\*sin(a+3/2\*ln(c\*x^n)\*(-1/n^2)^(1/2))^2,x)

**Maxima [A]** time = 1.27296, size = 63, normalized size = 0.83

$$\frac{c^{\frac{6}{n}} x^6 \cos(2a) - 4 c^{\frac{3}{n}} x^3 + 6 \cos(2a) \log(x)}{24 c^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+3/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2,x, algorithm="maxima")

[Out] -1/24\*(c^(6/n)\*x^6\*cos(2\*a) - 4\*c^(3/n)\*x^3 + 6\*cos(2\*a)\*log(x))/c^(3/n)

**Fricas [C]** time = 0.471175, size = 157, normalized size = 2.07

$$-\frac{1}{24} \left( x^6 - 4 x^3 e^{\left( \frac{2i a n - 3 \log(c)}{n} \right)} + 6 e^{\left( \frac{2(2i a n - 3 \log(c))}{n} \right)} \log(x) \right) e^{\left( -\frac{2i a n - 3 \log(c)}{n} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+3/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2,x, algorithm="fricas")

[Out] -1/24\*(x^6 - 4\*x^3\*e^((2\*I\*a\*n - 3\*log(c))/n) + 6\*e^(2\*(2\*I\*a\*n - 3\*log(c))/n)\*log(x))\*e^(-(2\*I\*a\*n - 3\*log(c))/n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+3/2*ln(c*x**n))*(-1/n**2)**(1/2))**2,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 4.35389, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+3/2*log(c*x^n))*(-1/n^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] +Infinity
```

$$3.35 \quad \int x \sin^2 \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=76

$$-\frac{1}{16}x^2e^{-2a\sqrt{-\frac{1}{n^2}}}(cx^n)^{2/n} - \frac{1}{4}x^2e^{2a\sqrt{-\frac{1}{n^2}}}\log(x)(cx^n)^{-2/n} + \frac{x^2}{4}$$

[Out]  $x^2/4 - (x^2*(c*x^n)^(2/n))/(16*E^(2*a*Sqrt[-n^(-2)]*n)) - (E^(2*a*Sqrt[-n^(-2)]*n)*x^2*Log[x])/(4*(c*x^n)^(2/n))$

**Rubi [A]** time = 0.05761, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {4493, 4489}

$$-\frac{1}{16}x^2e^{-2a\sqrt{-\frac{1}{n^2}}}(cx^n)^{2/n} - \frac{1}{4}x^2e^{2a\sqrt{-\frac{1}{n^2}}}\log(x)(cx^n)^{-2/n} + \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]^2,x]

[Out]  $x^2/4 - (x^2*(c*x^n)^(2/n))/(16*E^(2*a*Sqrt[-n^(-2)]*n)) - (E^(2*a*Sqrt[-n^(-2)]*n)*x^2*Log[x])/(4*(c*x^n)^(2/n))$

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1)))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int x \sin^2 \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst} \left( \int x^{-1+\frac{2}{n}} \sin^2 \left( a + \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= -\frac{(x^2 (cx^n)^{-2/n}) \text{Subst} \left( \int \left( \frac{e^{2a\sqrt{-\frac{1}{n^2}}}}{x} - 2x^{-1+\frac{2}{n}} + e^{-2a\sqrt{-\frac{1}{n^2}}} x^{-1+\frac{4}{n}} \right) dx, x, cx^n \right)}{4n} \\ &= \frac{x^2}{4} - \frac{1}{16}e^{-2a\sqrt{-\frac{1}{n^2}}}x^2(cx^n)^{2/n} - \frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}}x^2(cx^n)^{-2/n}\log(x) \end{aligned}$$

**Mathematica [F]** time = 0.151923, size = 0, normalized size = 0.

$$\int x \sin^2 \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x\*Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]^2,x]

[Out] Integrate[x\*Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]^2, x]

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int x \left( \sin \left( a + \ln(cx^n) \sqrt{-n^{-2}} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2))^2,x)

[Out] int(x\*sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2))^2,x)

**Maxima [A]** time = 1.11589, size = 63, normalized size = 0.83

$$\frac{c^{\frac{4}{n}} x^4 \cos(2a) - 4 c^{\frac{2}{n}} x^2 + 4 \cos(2a) \log(x)}{16 c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))^2,x, algorithm="maxima")

[Out] -1/16\*(c^(4/n)\*x^4\*cos(2\*a) - 4\*c^(2/n)\*x^2 + 4\*cos(2\*a)\*log(x))/c^(2/n)

**Fricas [C]** time = 0.475053, size = 146, normalized size = 1.92

$$-\frac{1}{16} \left( x^4 - 4x^2 e^{\left( \frac{2(ian - \log(c))}{n} \right)} + 4 e^{\left( \frac{4(ian - \log(c))}{n} \right)} \log(x) \right) e^{\left( -\frac{2(ian - \log(c))}{n} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))^2,x, algorithm="fricas")

[Out] -1/16\*(x^4 - 4\*x^2\*e^(2\*(I\*a\*n - log(c))/n) + 4\*e^(4\*(I\*a\*n - log(c))/n)\*log(x))\*e^(-2\*(I\*a\*n - log(c))/n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sin^2 \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+ln(c*x**n)*(-1/n**2)**(1/2))**2,x)
```

```
[Out] Integral(x*sin(a + sqrt(-1/n**2)*log(c*x**n))**2, x)
```

**Giac [A]** time = 1.82429, size = 1, normalized size = 0.01

$+\infty$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] +Infinity
```

$$3.36 \quad \int \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=68

$$-\frac{1}{8} x e^{-2a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} - \frac{1}{4} x e^{2a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n} + \frac{x}{2}$$

[Out]  $x/2 - (x*(c*x^n)^n)^{-1}/(8*E^{(2*a*Sqrt[-n^(-2)]*n)}) - (E^{(2*a*Sqrt[-n^(-2)]*n)}*x*Log[x])/(4*(c*x^n)^n)^{-1}$

**Rubi [A]** time = 0.0549811, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {4483, 4489}

$$-\frac{1}{8} x e^{-2a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} - \frac{1}{4} x e^{2a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + (Sqrt[-n^(-2)]\*Log[c\*x^n])/2]^2,x]

[Out]  $x/2 - (x*(c*x^n)^n)^{-1}/(8*E^{(2*a*Sqrt[-n^(-2)]*n)}) - (E^{(2*a*Sqrt[-n^(-2)]*n)}*x*Log[x])/(4*(c*x^n)^n)^{-1}$

#### Rule 4483

Int[Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_.)\*(x\_)^(m\_.))\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x(cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1+\frac{1}{n}} \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= \frac{(x(cx^n)^{-1/n}) \text{Subst} \left( \int \left( \frac{e^{2a \sqrt{-\frac{1}{n^2}} n}}{x} - 2x^{-1+\frac{1}{n}} + e^{-2a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{2}{n}} \right) dx, x, cx^n \right)}{4n} \\ &= \frac{x}{2} - \frac{1}{8} e^{-2a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{\frac{1}{n}} - \frac{1}{4} e^{2a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{-1/n} \log(x) \end{aligned}$$

**Mathematica [F]** time = 0.100833, size = 0, normalized size = 0.

$$\int \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$



Verification is Not applicable to the result.

[In] Integrate[Sin[a + (Sqrt[-n^(-2)])\*Log[c\*x^n])/2]^2, x]

[Out] Integrate[Sin[a + (Sqrt[-n^(-2)])\*Log[c\*x^n])/2]^2, x]

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \left( \sin \left( a + \frac{\ln(cx^n)}{2} \sqrt{-n^{-2}} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+1/2\*ln(c\*x^n)\*(-1/n^2)^(1/2))^2, x)

[Out] int(sin(a+1/2\*ln(c\*x^n)\*(-1/n^2)^(1/2))^2, x)

**Maxima [A]** time = 1.16538, size = 55, normalized size = 0.81

$$\frac{c^{\frac{2}{n}} x^2 \cos(2a) - 4c^{\left(\frac{1}{n}\right)} x + 2 \cos(2a) \log(x)}{8c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2, x, algorithm="maxima")

[Out] -1/8\*(c^(2/n)\*x^2\*cos(2\*a) - 4\*c^(1/n)\*x + 2\*cos(2\*a)\*log(x))/c^(1/n)

**Fricas [C]** time = 0.469305, size = 144, normalized size = 2.12

$$-\frac{1}{8} \left( x^2 - 4x e^{\left(\frac{2ian-\log(c)}{n}\right)} + 2 e^{\left(\frac{2(2ian-\log(c))}{n}\right)} \log(x) \right) e^{\left(\frac{-2ian-\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2, x, algorithm="fricas")

[Out] -1/8\*(x^2 - 4\*x\*e^((2\*I\*a\*n - log(c))/n) + 2\*e^(2\*(2\*I\*a\*n - log(c))/n)\*log(x))\*e^(-(2\*I\*a\*n - log(c))/n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin^2 \left( a + \frac{\sqrt{-\frac{1}{n^2}} \log(cx^n)}{2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*ln(c*x**n)*(-1/n**2)**(1/2))**2,x)
```

```
[Out] Integral(sin(a + sqrt(-1/n**2)*log(c*x**n)/2)**2, x)
```

---

**Giac [A]** time = 1.5854, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] +Infinity
```

$$3.37 \quad \int \frac{\sin^2(a)}{x} dx$$

**Optimal.** Leaf size=7

$$\sin^2(a) \log(x)$$

[Out] Log[x]\*Sin[a]^2

**Rubi [A]** time = 0.006046, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {12, 29}

$$\sin^2(a) \log(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[a]^2/x,x]

[Out] Log[x]\*Sin[a]^2

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :=> Simp[Log[x], x]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(a)}{x} dx &= \sin^2(a) \int \frac{1}{x} dx \\ &= \log(x) \sin^2(a) \end{aligned}$$

**Mathematica [A]** time = 0.0008065, size = 7, normalized size = 1.

$$\sin^2(a) \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a]^2/x,x]

[Out] Log[x]\*Sin[a]^2

**Maple [A]** time = 0.012, size = 8, normalized size = 1.1

$$\ln(x) (\sin(a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a)^2/x,x)
```

```
[Out] ln(x)*sin(a)^2
```

---

**Maxima [A]** time = 1.09338, size = 9, normalized size = 1.29

$$\log(x) \sin(a)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^2/x,x, algorithm="maxima")
```

```
[Out] log(x)*sin(a)^2
```

---

**Fricas [A]** time = 0.436808, size = 32, normalized size = 4.57

$$-(\cos(a)^2 - 1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^2/x,x, algorithm="fricas")
```

```
[Out] -(cos(a)^2 - 1)*log(x)
```

---

**Sympy [A]** time = 0.339272, size = 7, normalized size = 1.

$$\log(x) \sin^2(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)**2/x,x)
```

```
[Out] log(x)*sin(a)**2
```

---

**Giac [A]** time = 1.10733, size = 11, normalized size = 1.57

$$\log(|x|) \sin(a)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a)^2/x,x, algorithm="giac")
```

```
[Out] log(abs(x))*sin(a)^2
```

$$3.38 \quad \int \frac{\sin^2\left(a + \frac{1}{2}\sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^2} dx$$

**Optimal.** Leaf size=74

$$\frac{e^{2a\sqrt{-\frac{1}{n^2}n}} (cx^n)^{-1/n}}{8x} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}n}} \log(x) (cx^n)^{\frac{1}{n}}}{4x} - \frac{1}{2x}$$

[Out]  $-1/(2*x) + E^{(2*a*Sqrt[-n^{(-2)}]*n)/(8*x*(c*x^n)^n^{(-1)})} - ((c*x^n)^n^{(-1)}*Log[x])/(4*E^{(2*a*Sqrt[-n^{(-2)}]*n)*x})$

**Rubi [A]** time = 0.068497, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4493, 4489}

$$\frac{e^{2a\sqrt{-\frac{1}{n^2}n}} (cx^n)^{-1/n}}{8x} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}n}} \log(x) (cx^n)^{\frac{1}{n}}}{4x} - \frac{1}{2x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[a + (\text{Sqrt}[-n^{(-2)}]*\text{Log}[c*x^n])/2]^2/x^2, x]$

[Out]  $-1/(2*x) + E^{(2*a*Sqrt[-n^{(-2)}]*n)/(8*x*(c*x^n)^n^{(-1)})} - ((c*x^n)^n^{(-1)}*Log[x])/(4*E^{(2*a*Sqrt[-n^{(-2)}]*n)*x})$

#### Rule 4493

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}]*b_{.})*(d_{.})]^{(p_{.})}, x\_Symbol] :> \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)*\text{Sin}[d*(a+b*\text{Log}[x])]}^p, x], x, c*x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\text{NeQ}[c, 1] \|\| \text{NeQ}[n, 1])$

#### Rule 4489

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[x_{.}]*b_{.})*(d_{.})]^{(p_{.})}, x\_Symbol] :> \text{Dist}[(m+1)^p/(2^p*b^p*d^p*p^p), \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(E^{((a*b*d^2*p)/(m+1))/x^{(m+1)/p}} - x^{(m+1)/p}/E^{((a*b*d^2*p)/(m+1))})^p, x], x] /; \text{FreeQ}\{a, b, d, e, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[b^2*d^2*p^2 + (m+1)^2, 0]$

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^2\left(a + \frac{1}{2}\sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^2} dx &= \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \sin^2\left(a + \frac{1}{2}\sqrt{-\frac{1}{n^2}} \log(x)\right) dx, x, cx^n\right)}{nx} \\ &= -\frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int \left(\frac{e^{-2a\sqrt{-\frac{1}{n^2}n}}}{x} - 2x^{-\frac{1+n}{n}} + e^{2a\sqrt{-\frac{1}{n^2}n}} x^{-\frac{2+n}{n}}\right) dx, x, cx^n\right)}{4nx} \\ &= -\frac{1}{2x} + \frac{e^{2a\sqrt{-\frac{1}{n^2}n}} (cx^n)^{-1/n}}{8x} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}n}} (cx^n)^{\frac{1}{n}} \log(x)}{4x} \end{aligned}$$

**Mathematica [F]** time = 0.149265, size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a + \frac{1}{2}\sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + (Sqrt[-n^(-2)]\*Log[c\*x^n])/2]^2/x^2, x]

[Out] Integrate[Sin[a + (Sqrt[-n^(-2)]\*Log[c\*x^n])/2]^2/x^2, x]

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left( \sin\left(a + \frac{\ln(cx^n)}{2} \sqrt{-n^{-2}}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+1/2\*ln(c\*x^n)\*(-1/n^2)^(1/2))^2/x^2, x)

[Out] int(sin(a+1/2\*ln(c\*x^n)\*(-1/n^2)^(1/2))^2/x^2, x)

**Maxima [A]** time = 1.1636, size = 65, normalized size = 0.88

$$\frac{2c^{\frac{2}{n}}x^3 \cos(2a) \log(x) + 4c^{\left(\frac{1}{n}\right)}x^2 - x \cos(2a)}{8c^{\left(\frac{1}{n}\right)}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2/x^2, x, algorithm="maxima")

[Out] -1/8\*(2\*c^(2/n)\*x^3\*cos(2\*a)\*log(x) + 4\*c^(1/n)\*x^2 - x\*cos(2\*a))/(c^(1/n)\*x^3)

**Fricas [C]** time = 0.471367, size = 150, normalized size = 2.03

$$\frac{\left(2x^2 \log(x) + 4xe^{\left(\frac{2ian-\log(c)}{n}\right)} - e^{\left(\frac{2(2ian-\log(c))}{n}\right)}\right)e^{\left(\frac{-2ian-\log(c)}{n}\right)}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2/x^2, x, algorithm="fricas")

[Out] -1/8\*(2\*x^2\*log(x) + 4\*x\*e^((2\*I\*a\*n - log(c))/n) - e^(2\*(2\*I\*a\*n - log(c))/n))\*e^(-(2\*I\*a\*n - log(c))/n)/x^2

**Sympy [C]** time = 75.7666, size = 240, normalized size = 3.24

$$\frac{i n \sqrt{\frac{1}{n^2}} \log(x) \sin\left(2a + i n \sqrt{\frac{1}{n^2}} \log(x) + i \sqrt{\frac{1}{n^2}} \log(c)\right)}{4x} + \frac{i n \sqrt{\frac{1}{n^2}} \sin\left(2a + i n \sqrt{\frac{1}{n^2}} \log(x) + i \sqrt{\frac{1}{n^2}} \log(c)\right)}{4x} + \frac{i \sqrt{\frac{1}{n^2}} \log(c)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/2\*ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2))\*\*2/x\*\*2,x)

[Out] I\*n\*sqrt(n\*\*(-2))\*log(x)\*sin(2\*a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(4\*x) + I\*n\*sqrt(n\*\*(-2))\*sin(2\*a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(4\*x) + I\*sqrt(n\*\*(-2))\*log(c)\*sin(2\*a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(4\*x) - log(x)\*cos(2\*a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(4\*x) - 1/(2\*x) - log(c)\*cos(2\*a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(4\*n\*x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) + a\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2/x^2,x, algorithm="giac")

[Out] integrate(sin(1/2\*sqrt(-1/n^2)\*log(c\*x^n) + a)^2/x^2, x)

$$3.39 \quad \int \frac{\sin^2\left(a + \sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^3} dx$$

**Optimal.** Leaf size=76

$$\frac{e^{2a\sqrt{-\frac{1}{n^2}n}}(cx^n)^{-2/n}}{16x^2} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}n}}\log(x)(cx^n)^{2/n}}{4x^2} - \frac{1}{4x^2}$$

[Out]  $-1/(4*x^2) + E^{(2*a*Sqrt[-n^{(-2)}]*n)/(16*x^2*(c*x^n)^{(2/n))} - ((c*x^n)^{(2/n)}*Log[x])/(4*E^{(2*a*Sqrt[-n^{(-2)}]*n)*x^2)}$

**Rubi [A]** time = 0.06197, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {4493, 4489}

$$\frac{e^{2a\sqrt{-\frac{1}{n^2}n}}(cx^n)^{-2/n}}{16x^2} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}n}}\log(x)(cx^n)^{2/n}}{4x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + Sqrt[-n^{(-2)}]\*Log[c\*x^n]]^2/x^3,x]

[Out]  $-1/(4*x^2) + E^{(2*a*Sqrt[-n^{(-2)}]*n)/(16*x^2*(c*x^n)^{(2/n))} - ((c*x^n)^{(2/n)}*Log[x])/(4*E^{(2*a*Sqrt[-n^{(-2)}]*n)*x^2)}$

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^2\left(a + \sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^3} dx &= \frac{(cx^n)^{2/n} \text{Subst}\left(\int x^{-1-\frac{2}{n}} \sin^2\left(a + \sqrt{-\frac{1}{n^2}} \log(x)\right) dx, x, cx^n\right)}{nx^2} \\ &= -\frac{(cx^n)^{2/n} \text{Subst}\left(\int \left(\frac{e^{-2a\sqrt{-\frac{1}{n^2}n}}}{x} - 2x^{-\frac{2+n}{n}} + e^{2a\sqrt{-\frac{1}{n^2}n}} x^{-\frac{4+n}{n}}\right) dx, x, cx^n\right)}{4nx^2} \\ &= -\frac{1}{4x^2} + \frac{e^{2a\sqrt{-\frac{1}{n^2}n}}(cx^n)^{-2/n}}{16x^2} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}n}}(cx^n)^{2/n} \log(x)}{4x^2} \end{aligned}$$



**Mathematica [F]** time = 0.128635, size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a + \sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]^2/x^3, x]

[Out] Integrate[Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]^2/x^3, x]

**Maple [F]** time = 0.061, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \left( \sin\left(a + \ln(cx^n) \sqrt{-n^{-2}}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2))^2/x^3, x)

[Out] int(sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2))^2/x^3, x)

**Maxima [A]** time = 1.10118, size = 73, normalized size = 0.96

$$\frac{4c^{\frac{4}{n}}x^6 \cos(2a) \log(x) + 4c^{\frac{2}{n}}x^4 - x^2 \cos(2a)}{16c^{\frac{2}{n}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))^2/x^3, x, algorithm="maxima")

[Out] -1/16\*(4\*c^(4/n)\*x^6\*cos(2\*a)\*log(x) + 4\*c^(2/n)\*x^4 - x^2\*cos(2\*a))/(c^(2/n)\*x^6)

**Fricas [C]** time = 0.469676, size = 151, normalized size = 1.99

$$\frac{\left(4x^4 \log(x) + 4x^2 e^{\left(\frac{2(ian-\log(c))}{n}\right)} - e^{\left(\frac{4(ian-\log(c))}{n}\right)}\right) e^{\left(-\frac{2(ian-\log(c))}{n}\right)}}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))^2/x^3, x, algorithm="fricas")

[Out] -1/16\*(4\*x^4\*log(x) + 4\*x^2\*e^(2\*(I\*a\*n - log(c))/n) - e^(4\*(I\*a\*n - log(c))/n))\*e^(-2\*(I\*a\*n - log(c))/n)/x^4

**Sympy [C]** time = 55.4519, size = 464, normalized size = 6.11

$$\frac{\log(x) \sin^2\left(a + i n \sqrt{\frac{1}{n^2}} \log(x) + i \sqrt{\frac{1}{n^2}} \log(c)\right)}{4x^2} - \frac{\log(x) \cos^2\left(a + i n \sqrt{\frac{1}{n^2}} \log(x) + i \sqrt{\frac{1}{n^2}} \log(c)\right)}{4x^2} - \frac{\sin^2\left(a + i n \sqrt{\frac{1}{n^2}} \log(x) + i \sqrt{\frac{1}{n^2}} \log(c)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2))\*\*2/x\*\*3,x)

[Out] log(x)\*sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))\*\*2/(4\*x\*\*2) - log(x)\*cos(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))\*\*2/(4\*x\*\*2) - sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))\*\*2/(2\*x\*\*2) + log(c)\*sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))\*\*2/(4\*n\*x\*\*2) - log(c)\*cos(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))\*\*2/(4\*n\*x\*\*2) + I\*log(x)\*sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))\*cos(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(2\*n\*x\*\*2\*sqrt(n\*\*(-2))) - I\*sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))\*cos(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(4\*n\*x\*\*2\*sqrt(n\*\*(-2))) + I\*log(c)\*sin(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))\*cos(a + I\*n\*sqrt(n\*\*(-2))\*log(x) + I\*sqrt(n\*\*(-2))\*log(c))/(2\*n\*\*2\*x\*\*2\*sqrt(n\*\*(-2)))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\sqrt{-\frac{1}{n^2}} \log(cx^n) + a\right)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))^2/x^3,x, algorithm="giac")

[Out] integrate(sin(sqrt(-1/n^2)\*log(c\*x^n) + a)^2/x^3, x)

$$3.40 \quad \int x^m \sin^3 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=226

$$\frac{4x^{m+1} \sin^3 \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right)}{5(m+1)} + \frac{8x^{m+1} \sin \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right)}{5(m+1)} - \frac{4n \sqrt{-\frac{(m+1)^2}{n^2}} x^{m+1} \cos \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right)}{5(m+1)^2}$$

[Out] (-4\*Sqrt[-((1+m)^2/n^2)]\*n\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2])/(5\*(1+m)^2)+(8\*x^(1+m)\*Sin[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2])/(5\*(1+m))+(6\*Sqrt[-((1+m)^2/n^2)]\*n\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2]\*Sin[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2]^2)/(5\*(1+m)^2)-(4\*x^(1+m)\*Sin[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2]^3)/(5\*(1+m))

**Rubi [A]** time = 0.0790366, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$ , Rules used = {4487, 4485}

$$\frac{4x^{m+1} \sin^3 \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right)}{5(m+1)} + \frac{8x^{m+1} \sin \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right)}{5(m+1)} - \frac{4n \sqrt{-\frac{(m+1)^2}{n^2}} x^{m+1} \cos \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right)}{5(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sin[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2]^3,x]

[Out] (-4\*Sqrt[-((1+m)^2/n^2)]\*n\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2])/(5\*(1+m)^2)+(8\*x^(1+m)\*Sin[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2])/(5\*(1+m))+(6\*Sqrt[-((1+m)^2/n^2)]\*n\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2]\*Sin[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2]^2)/(5\*(1+m)^2)-(4\*x^(1+m)\*Sin[a+(Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n])/2]^3)/(5\*(1+m))

#### Rule 4487

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.)+Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] :> Simp[((m+1)\*(e\*x)^(m+1)\*Sin[d\*(a+b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2+e\*(m+1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p-1))/(b^2\*d^2\*n^2\*p^2+(m+1)^2), Int[(e\*x)^m\*Sin[d\*(a+b\*Log[c\*x^n])]^(p-2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m+1)\*Cos[d\*(a+b\*Log[c\*x^n])]\*Sin[d\*(a+b\*Log[c\*x^n])]^(p-1)/(b^2\*d^2\*e\*n^2\*p^2+e\*(m+1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2+(m+1)^2, 0]

#### Rule 4485

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.)+Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] :> Simp[((m+1)\*(e\*x)^(m+1)\*Sin[d\*(a+b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2+e\*(m+1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m+1)\*Cos[d\*(a+b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2+e\*(m+1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2+(m+1)^2, 0]

#### Rubi steps

$$\int x^m \sin^3 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx = \frac{6 \sqrt{-\frac{(1+m)^2}{n^2}} n x^{1+m} \cos \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) \sin^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right)}{5(1+m)^2}$$

$$= -\frac{4 \sqrt{-\frac{(1+m)^2}{n^2}} n x^{1+m} \cos \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right)}{5(1+m)^2} + \frac{8 x^{1+m} \sin \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right)}{5(1+m)^2}$$

**Mathematica [A]** time = 1.1883, size = 169, normalized size = 0.75

$$\frac{x^{m+1} \left( 2(m+1) \left( 5 \sin \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right) + \sin \left( 3a + \frac{3}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right) \right) - 5n \sqrt{-\frac{(m+1)^2}{n^2}} \cos \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right) \right)}{10(m+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Sin[a + (Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2]^3,x]

[Out] (x^(1 + m)\*(-5\*Sqrt[-((1 + m)^2/n^2)]\*n\*Cos[a + (Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2] - 3\*Sqrt[-((1 + m)^2/n^2)]\*n\*Cos[3\*a + (3\*Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2] + 2\*(1 + m)\*(5\*Sin[a + (Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2] + Sin[3\*a + (3\*Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2])))/(10\*(1 + m)^2)

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int x^m \left( \sin \left( a + \frac{\ln(cx^n)}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sin(a+1/2\*ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^3,x)

[Out] int(x^m\*sin(a+1/2\*ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^3,x)

**Maxima [A]** time = 1.29846, size = 263, normalized size = 1.16

$$\frac{\left( c^{\frac{3m}{n} + \frac{3}{n}} x e^{\left( m \log(x) + \frac{3m \log(x^n)}{n} + \frac{3 \log(x^n)}{n} \right)} \sin(3a) - 5 c^{\frac{2m}{n} + \frac{2}{n}} x e^{\left( m \log(x) + \frac{2m \log(x^n)}{n} + \frac{2 \log(x^n)}{n} \right)} \sin(a) - 15 c^{\frac{m}{n} + \frac{1}{n}} x e^{\left( m \log(x) + \frac{m \log(x^n)}{n} \right)} \right)}{20 \left( c^{\frac{3m}{2n} + \frac{3}{2n}} m + c^{\frac{3m}{2n} + \frac{3}{2n}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/2\*log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="maxima")

[Out] -1/20\*(c^(3\*m/n + 3/n)\*x\*e^(m\*log(x) + 3\*m\*log(x^n)/n + 3\*log(x^n)/n)\*sin(3\*a) - 5\*c^(2\*m/n + 2/n)\*x\*e^(m\*log(x) + 2\*m\*log(x^n)/n + 2\*log(x^n)/n)\*sin(a) - 15\*c^(m/n + 1/n)\*x\*e^(m\*log(x) + m\*log(x^n)/n + log(x^n)/n)\*sin(a) - 5\*x\*x^m\*sin(3\*a))\*e^(-3/2\*m\*log(x^n)/n - 3/2\*log(x^n)/n)/(c^(3/2\*m/n + 3/2/n

)<sup>m</sup> + c<sup>(3/2\*m/n + 3/2/n)</sup>)

**Fricas [C]** time = 0.502763, size = 387, normalized size = 1.71

$$\frac{\left(5i e^{\left(-\frac{(m+1)n \log(x)-2ian+(m+1) \log(c)}{n}\right)} - 15i e^{\left(-\frac{2((m+1)n \log(x)-2ian+(m+1) \log(c))}{n}\right)} - 5i e^{\left(-\frac{3((m+1)n \log(x)-2ian+(m+1) \log(c))}{n}\right)} - i\right) e^{\left(\frac{5((m+1)n \log(x)-2ian+(m+1) \log(c))}{2n}\right)}}{20(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*sin(a+1/2\*log(c\*x<sup>n</sup>))\*(-(1+m)<sup>2/n^2</sup>)<sup>(1/2)</sup>)<sup>3</sup>,x, algorithm="fricas")

[Out] 1/20\*(5\*I\*e<sup>-((m + 1)\*n\*log(x) - 2\*I\*a\*n + (m + 1)\*log(c))/n</sup> - 15\*I\*e<sup>-2\*((m + 1)\*n\*log(x) - 2\*I\*a\*n + (m + 1)\*log(c))/n</sup> - 5\*I\*e<sup>-3\*((m + 1)\*n\*log(x) - 2\*I\*a\*n + (m + 1)\*log(c))/n</sup> - I)\*e<sup>(5/2\*((m + 1)\*n\*log(x) - 2\*I\*a\*n + (m + 1)\*log(c))/n + (2\*I\*a\*n - (m + 1)\*log(c))/n)/(m + 1)</sup>

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sin^3 \left( a + \frac{\sqrt{-\frac{m^2}{n^2} - \frac{2m}{n^2} - \frac{1}{n^2}} \log(cx^n)}{2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*sin(a+1/2\*ln(c\*x\*\*n))\*(-(1+m)\*\*2/n\*\*2)\*\*(1/2))\*\*3,x)

[Out] Integral(x\*\*m\*sin(a + sqrt(-m\*\*2/n\*\*2 - 2\*m/n\*\*2 - 1/n\*\*2)\*log(c\*x\*\*n)/2)\*\*3, x)

**Giac [C]** time = 3.68101, size = 2525, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*sin(a+1/2\*log(c\*x<sup>n</sup>))\*(-(1+m)<sup>2/n^2</sup>)<sup>(1/2)</sup>)<sup>3</sup>,x, algorithm="giac")

[Out] 1/4\*(8\*I\*m<sup>3</sup>n<sup>4</sup>\*x\*x<sup>m</sup>\*e<sup>(3\*I\*a - 3/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> - 24\*I\*m<sup>3</sup>n<sup>4</sup>\*x\*x<sup>m</sup>\*e<sup>(I\*a - 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> + 24\*I\*m<sup>3</sup>n<sup>4</sup>\*x\*x<sup>m</sup>\*e<sup>(-I\*a + 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> - 8\*I\*m<sup>3</sup>n<sup>4</sup>\*x\*x<sup>m</sup>\*e<sup>(-3\*I\*a + 3/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> + 24\*I\*m<sup>2</sup>n<sup>4</sup>\*x\*x<sup>m</sup>\*e<sup>(3\*I\*a - 3/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> + 12\*I\*m<sup>2</sup>n<sup>3</sup>\*x\*x<sup>m</sup>\*abs(m\*n + n)\*e<sup>(3\*I\*a - 3/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> - 72\*I\*m<sup>2</sup>n<sup>4</sup>\*x\*x<sup>m</sup>\*e<sup>(I\*a - 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> - 12\*I\*m<sup>2</sup>n<sup>3</sup>\*x\*x<sup>m</sup>\*abs(m\*n + n)\*e<sup>(I\*a - 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> + 72\*I\*m<sup>2</sup>n<sup>4</sup>\*x\*x<sup>m</sup>\*e<sup>(-I\*a + 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup> - 12\*I\*m<sup>2</sup>n<sup>3</sup>\*x\*x<sup>m</sup>\*abs(m\*n + n)\*e<sup>(-I\*a + 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)</sup>

$$\begin{aligned}
& ) - 24*I*m^2*n^4*x*x^m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 12*I*m^2*n^3*x*x^m*abs(m*n + n)*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 2*I*(m*n + n)^2*m*n^2*x*x^m*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 24*I*m*n^4*x*x^m*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 24*I*m*n^3*x*x^m*abs(m*n + n)*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 54*I*(m*n + n)^2*m*n^2*x*x^m*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 72*I*m*n^4*x*x^m*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 24*I*m*n^3*x*x^m*abs(m*n + n)*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 54*I*(m*n + n)^2*m*n^2*x*x^m*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 72*I*m*n^4*x*x^m*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 24*I*m*n^3*x*x^m*abs(m*n + n)*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 2*I*(m*n + n)^2*m*n^2*x*x^m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 24*I*m*n^4*x*x^m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 24*I*m*n^3*x*x^m*abs(m*n + n)*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 2*I*(m*n + n)^2*n^2*x*x^m*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 8*I*n^4*x*x^m*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 3*I*(m*n + n)^2*n*x*x^m*abs(m*n + n)*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 12*I*n^3*x*x^m*abs(m*n + n)*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 54*I*(m*n + n)^2*n^2*x*x^m*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 24*I*n^4*x*x^m*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 27*I*(m*n + n)^2*n*x*x^m*abs(m*n + n)*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 12*I*n^3*x*x^m*abs(m*n + n)*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 54*I*(m*n + n)^2*n^2*x*x^m*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 24*I*n^4*x*x^m*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 27*I*(m*n + n)^2*n*x*x^m*abs(m*n + n)*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 12*I*n^3*x*x^m*abs(m*n + n)*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 2*I*(m*n + n)^2*n^2*x*x^m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 8*I*n^4*x*x^m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 3*I*(m*n + n)^2*n*x*x^m*abs(m*n + n)*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 12*I*n^3*x*x^m*abs(m*n + n)*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)})/(16*m^4*n^4 + 64*m^3*n^4 - 40*(m*n + n)^2*m^2*n^2 + 96*m^2*n^4 - 80*(m*n + n)^2*m*n^2 + 64*m*n^4 + 9*(m*n + n)^4 - 40*(m*n + n)^2*n^2 + 16*n^4)
\end{aligned}$$

### 3.41 $\int x^2 \sin^3 \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$

**Optimal.** Leaf size=172

$$-\frac{3}{16} \sqrt{-\frac{1}{n^2}} nx^3 e^{a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-1/n} - \frac{1}{48} \sqrt{-\frac{1}{n^2}} nx^3 e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{3/n} + \frac{3}{32} \sqrt{-\frac{1}{n^2}} nx^3 e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{1/n} + \frac{1}{8} \sqrt{-\frac{1}{n^2}} nx^3 e^{3a \sqrt{-\frac{1}{n^2}} n}$$

[Out]  $(-3 * E^{(a * \text{Sqrt}[-n^{(-2)}] * n)} * \text{Sqrt}[-n^{(-2)}] * n * x^3) / (16 * (c * x^n)^{n^{(-1)}}) + (3 * \text{Sqrt}[-n^{(-2)}] * n * x^3 * (c * x^n)^{n^{(-1)}}) / (32 * E^{(a * \text{Sqrt}[-n^{(-2)}] * n)}) - (\text{Sqrt}[-n^{(-2)}] * n * x^3 * (c * x^n)^{(3/n)}) / (48 * E^{(3 * a * \text{Sqrt}[-n^{(-2)}] * n)}) + (E^{(3 * a * \text{Sqrt}[-n^{(-2)}] * n)} * \text{Sqrt}[-n^{(-2)}] * n * x^3 * \text{Log}[x]) / (8 * (c * x^n)^{(3/n)})$

**Rubi [A]** time = 0.160594, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$ , Rules used = {4493, 4489}

$$-\frac{3}{16} \sqrt{-\frac{1}{n^2}} nx^3 e^{a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-1/n} - \frac{1}{48} \sqrt{-\frac{1}{n^2}} nx^3 e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{3/n} + \frac{3}{32} \sqrt{-\frac{1}{n^2}} nx^3 e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{1/n} + \frac{1}{8} \sqrt{-\frac{1}{n^2}} nx^3 e^{3a \sqrt{-\frac{1}{n^2}} n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2 * \text{Sin}[a + \text{Sqrt}[-n^{(-2)}] * \text{Log}[c * x^n]]^3, x]$

[Out]  $(-3 * E^{(a * \text{Sqrt}[-n^{(-2)}] * n)} * \text{Sqrt}[-n^{(-2)}] * n * x^3) / (16 * (c * x^n)^{n^{(-1)}}) + (3 * \text{Sqrt}[-n^{(-2)}] * n * x^3 * (c * x^n)^{n^{(-1)}}) / (32 * E^{(a * \text{Sqrt}[-n^{(-2)}] * n)}) - (\text{Sqrt}[-n^{(-2)}] * n * x^3 * (c * x^n)^{(3/n)}) / (48 * E^{(3 * a * \text{Sqrt}[-n^{(-2)}] * n)}) + (E^{(3 * a * \text{Sqrt}[-n^{(-2)}] * n)} * \text{Sqrt}[-n^{(-2)}] * n * x^3 * \text{Log}[x]) / (8 * (c * x^n)^{(3/n)})$

#### Rule 4493

$\text{Int}[(e_{.}) * (x_{.})^{(m_{.})} * \text{Sin}[(a_{.}) + \text{Log}[(c_{.}) * (x_{.})^{(n_{.})}] * (b_{.})] * (d_{.})^{(p_{.})}, x\_Symbol] \rightarrow \text{Dist}[(e * x)^{(m + 1)} / (e * n * (c * x^n)^{((m + 1)/n)}), \text{Subst}[\text{Int}[x^{((m + 1)/n - 1)} * \text{Sin}[d * (a + b * \text{Log}[x])]^p, x], x, c * x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\text{NeQ}[c, 1] \parallel \text{NeQ}[n, 1])$

#### Rule 4489

$\text{Int}[(e_{.}) * (x_{.})^{(m_{.})} * \text{Sin}[(a_{.}) + \text{Log}[x_{.}] * (b_{.})] * (d_{.})^{(p_{.})}, x\_Symbol] \rightarrow \text{Dist}[(m + 1)^p / (2^p * b^p * d^p * p^p), \text{Int}[\text{ExpandIntegrand}[(e * x)^m * (E^{((a * b * d^2 * p)/(m + 1))} / x^{((m + 1)/p)} - x^{((m + 1)/p}) / E^{((a * b * d^2 * p)/(m + 1))})^p, x], x] /; \text{FreeQ}\{a, b, d, e, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[b^2 * d^2 * p^2 + (m + 1)^2, 0]$

#### Rubi steps

$$\begin{aligned} \int x^2 \sin^3 \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x^3 (cx^n)^{-3/n}) \text{Subst} \left( \int x^{-1 + \frac{3}{n}} \sin^3 \left( a + \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= \frac{1}{8} \left( \sqrt{-\frac{1}{n^2}} x^3 (cx^n)^{-3/n} \right) \text{Subst} \left( \int \left( \frac{e^{3a \sqrt{-\frac{1}{n^2}} n}}{x} - 3e^{a \sqrt{-\frac{1}{n^2}} n} x^{-1 + \frac{2}{n}} + 3e^{-a \sqrt{-\frac{1}{n^2}} n} x^{-1} \right) dx, x, cx^n \right) \\ &= -\frac{3}{16} e^{a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} nx^3 (cx^n)^{-1/n} + \frac{3}{32} e^{-a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} nx^3 (cx^n)^{1/n} - \frac{1}{48} e^{-3a \sqrt{-\frac{1}{n^2}} n} \end{aligned}$$

**Mathematica [F]** time = 0.215729, size = 0, normalized size = 0.

$$\int x^2 \sin^3 \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]^3,x]

[Out] Integrate[x^2\*Sin[a + Sqrt[-n^(-2)]\*Log[c\*x^n]]^3, x]

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int x^2 \left( \sin \left( a + \ln(cx^n) \sqrt{-n^{-2}} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2))^3,x)

[Out] int(x^2\*sin(a+ln(c\*x^n)\*(-1/n^2)^(1/2))^3,x)

**Maxima [A]** time = 1.19871, size = 122, normalized size = 0.71

$$\frac{18 c^{\frac{2}{n}} x^3 \sin(a) - 12 (x^n)^{\left(\frac{1}{n}\right)} \log(x) \sin(3a) - \left(2 c^{\frac{6}{n}} x^6 \sin(3a) - 9 c^{\frac{4}{n}} x^4 \sin(a)\right) (x^n)^{\left(\frac{1}{n}\right)}}{96 c^{\frac{3}{n}} (x^n)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))^3,x, algorithm="maxima")

[Out] 1/96\*(18\*c^(2/n)\*x^3\*sin(a) - 12\*(x^n)^(1/n)\*log(x)\*sin(3\*a) - (2\*c^(6/n)\*x^6\*sin(3\*a) - 9\*c^(4/n)\*x^4\*sin(a))\*(x^n)^(1/n))/(c^(3/n)\*(x^n)^(1/n))

**Fricas [C]** time = 0.470377, size = 207, normalized size = 1.2

$$\frac{1}{96} \left( -2i x^6 + 9i x^4 e^{\left(\frac{2(ian-\log(c))}{n}\right)} - 18i x^2 e^{\left(\frac{4(ian-\log(c))}{n}\right)} + 12i e^{\left(\frac{6(ian-\log(c))}{n}\right)} \log(x) \right) e^{\left(-\frac{3(ian-\log(c))}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+log(c\*x^n)\*(-1/n^2)^(1/2))^3,x, algorithm="fricas")

[Out] 1/96\*(-2\*I\*x^6 + 9\*I\*x^4\*e^(2\*(I\*a\*n - log(c))/n) - 18\*I\*x^2\*e^(4\*(I\*a\*n - log(c))/n) + 12\*I\*e^(6\*(I\*a\*n - log(c))/n)\*log(x))\*e^(-3\*(I\*a\*n - log(c))/n)



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+ln(c*x**n))*(-1/n**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+log(c*x^n))*(-1/n^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

### 3.42 $\int x \sin^3 \left( a + \frac{2}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$

**Optimal.** Leaf size=178

$$-\frac{9}{32} \sqrt{-\frac{1}{n^2}} nx^2 e^{a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{2}{3}/n} + \frac{9}{64} \sqrt{-\frac{1}{n^2}} nx^2 e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{2}{3}/n} - \frac{1}{32} \sqrt{-\frac{1}{n^2}} nx^2 e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{2/n} + \frac{1}{8} \sqrt{-\frac{1}{n^2}} nx^2 e^{3a \sqrt{-\frac{1}{n^2}} n}$$

[Out]  $(-9 * E^{(a * \text{Sqrt}[-n^{(-2)}] * n)} * \text{Sqrt}[-n^{(-2)}] * n * x^2) / (32 * (c * x^n)^{(2/(3 * n))}) + (9 * \text{Sqrt}[-n^{(-2)}] * n * x^2 * (c * x^n)^{(2/(3 * n))}) / (64 * E^{(a * \text{Sqrt}[-n^{(-2)}] * n)}) - (\text{Sqrt}[-n^{(-2)}] * n * x^2 * (c * x^n)^{(2/n)}) / (32 * E^{(3 * a * \text{Sqrt}[-n^{(-2)}] * n)}) + (E^{(3 * a * \text{Sqrt}[-n^{(-2)}] * n)} * \text{Sqrt}[-n^{(-2)}] * n * x^2 * \text{Log}[x]) / (8 * (c * x^n)^{(2/n)})$

**Rubi [A]** time = 0.110903, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {4493, 4489}

$$-\frac{9}{32} \sqrt{-\frac{1}{n^2}} nx^2 e^{a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{2}{3}/n} + \frac{9}{64} \sqrt{-\frac{1}{n^2}} nx^2 e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{2}{3}/n} - \frac{1}{32} \sqrt{-\frac{1}{n^2}} nx^2 e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{2/n} + \frac{1}{8} \sqrt{-\frac{1}{n^2}} nx^2 e^{3a \sqrt{-\frac{1}{n^2}} n}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x * \text{Sin}[a + (2 * \text{Sqrt}[-n^{(-2)}]) * \text{Log}[c * x^n]] / 3, x]$

[Out]  $(-9 * E^{(a * \text{Sqrt}[-n^{(-2)}] * n)} * \text{Sqrt}[-n^{(-2)}] * n * x^2) / (32 * (c * x^n)^{(2/(3 * n))}) + (9 * \text{Sqrt}[-n^{(-2)}] * n * x^2 * (c * x^n)^{(2/(3 * n))}) / (64 * E^{(a * \text{Sqrt}[-n^{(-2)}] * n)}) - (\text{Sqrt}[-n^{(-2)}] * n * x^2 * (c * x^n)^{(2/n)}) / (32 * E^{(3 * a * \text{Sqrt}[-n^{(-2)}] * n)}) + (E^{(3 * a * \text{Sqrt}[-n^{(-2)}] * n)} * \text{Sqrt}[-n^{(-2)}] * n * x^2 * \text{Log}[x]) / (8 * (c * x^n)^{(2/n)})$

#### Rule 4493

$\text{Int}[(e_{.}) * (x_{.})^{(m_{.})} * \text{Sin}[(a_{.}) + \text{Log}[(c_{.}) * (x_{.})^{(n_{.})}] * (b_{.}) * (d_{.})]^{(p_{.})}, x\_Symbol] \rightarrow \text{Dist}[(e * x)^{(m + 1)} / (e * n * (c * x^n)^{(m + 1)/n}), \text{Subst}[\text{Int}[x^{(m + 1)/n - 1} * \text{Sin}[d * (a + b * \text{Log}[x])]^p, x], x, c * x^n], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n, p\}, x \ \&\& \ (\text{NeQ}[c, 1] \ \|\ \text{NeQ}[n, 1])$

#### Rule 4489

$\text{Int}[(e_{.}) * (x_{.})^{(m_{.})} * \text{Sin}[(a_{.}) + \text{Log}[x_{.}] * (b_{.}) * (d_{.})]^{(p_{.})}, x\_Symbol] \rightarrow \text{Dist}[(m + 1)^p / (2^p * b^p * d^p * p^p), \text{Int}[\text{ExpandIntegrand}[(e * x)^m * (E^{((a * b * d^{2 * p}) / (m + 1)) / x^{(m + 1)/p} - x^{(m + 1)/p} / E^{((a * b * d^{2 * p}) / (m + 1))})^p, x], x] /;$   $\text{FreeQ}\{a, b, d, e, m\}, x \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[b^2 * d^2 * p^2 + (m + 1)^2, 0]$

#### Rubi steps

$$\begin{aligned} \int x \sin^3 \left( a + \frac{2}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst} \left( \int x^{-1 + \frac{2}{n}} \sin^3 \left( a + \frac{2}{3} \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= \frac{1}{8} \left( \sqrt{-\frac{1}{n^2}} x^2 (cx^n)^{-2/n} \right) \text{Subst} \left( \int \left( \frac{e^{3a \sqrt{-\frac{1}{n^2}} n}}{x} - 3e^{a \sqrt{-\frac{1}{n^2}} n} x^{-1 + \frac{4}{3n}} + 3e^{-a \sqrt{-\frac{1}{n^2}} n} x^{-1 + \frac{2}{3n}} \right) dx, x, cx^n \right) \\ &= -\frac{9}{32} e^{a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} nx^2 (cx^n)^{-\frac{2}{3}/n} + \frac{9}{64} e^{-a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} nx^2 (cx^n)^{\frac{2}{3}/n} - \frac{1}{32} e^{-3a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} nx^2 (cx^n)^{2/n} + \frac{1}{8} e^{3a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} nx^2 (cx^n)^{2/n} \end{aligned}$$

**Mathematica [F]** time = 0.245628, size = 0, normalized size = 0.

$$\int x \sin^3 \left( a + \frac{2}{3} \sqrt{-\frac{1}{n^2} \log(cx^n)} \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x\*Sin[a + (2\*Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3,x]

[Out] Integrate[x\*Sin[a + (2\*Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3, x]

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int x \left( \sin \left( a + \frac{2 \ln(cx^n)}{3} \sqrt{-n^{-2}} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+2/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3,x)

[Out] int(x\*sin(a+2/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3,x)

**Maxima [A]** time = 1.15204, size = 151, normalized size = 0.85

$$\frac{9 c^{\frac{10}{3n}} x^2 (x^n)^{\frac{4}{3n}} \sin(a) - 8 c^{\frac{2}{3n}} (x^n)^{\frac{2}{3n}} \log(x) \sin(3a) + 18 c^{\frac{2}{n}} x^2 \sin(a) - 2 c^{\frac{14}{3n}} e^{\left(\frac{2 \log(x^n)}{3n} + 4 \log(x)\right)} \sin(3a)}{64 c^{\frac{8}{3n}} (x^n)^{\frac{2}{3n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+2/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3,x, algorithm="maxima")

[Out] 1/64\*(9\*c^(10/3/n)\*x^2\*(x^n)^(4/3/n)\*sin(a) - 8\*c^(2/3/n)\*(x^n)^(2/3/n)\*log(x)\*sin(3\*a) + 18\*c^(2/n)\*x^2\*sin(a) - 2\*c^(14/3/n)\*e^(2/3\*log(x^n)/n + 4\*log(x))\*sin(3\*a))/(c^(8/3/n)\*(x^n)^(2/3/n))

**Fricas [C]** time = 0.48014, size = 250, normalized size = 1.4

$$\frac{1}{64} \left( -2i x^4 + 9i x^{\frac{8}{3}} e^{\left(\frac{2(3ian-2 \log(c))}{3n}\right)} - 18i x^{\frac{4}{3}} e^{\left(\frac{4(3ian-2 \log(c))}{3n}\right)} + 24i e^{\left(\frac{2(3ian-2 \log(c))}{n}\right)} \log\left(x^{\frac{1}{3}}\right) \right) e^{\left(-\frac{3ian-2 \log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+2/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3,x, algorithm="fricas")

[Out] 1/64\*(-2\*I\*x^4 + 9\*I\*x^(8/3)\*e^(2/3\*(3\*I\*a\*n - 2\*log(c))/n) - 18\*I\*x^(4/3)\*e^(4/3\*(3\*I\*a\*n - 2\*log(c))/n) + 24\*I\*e^(2\*(3\*I\*a\*n - 2\*log(c))/n)\*log(x^(1/3)))\*e^(-(3\*I\*a\*n - 2\*log(c))/n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2/3*ln(c*x**n)*(-1/n**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+2/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

### 3.43 $\int \sin^3 \left( a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$

**Optimal.** Leaf size=168

$$-\frac{9}{16} \sqrt{-\frac{1}{n^2}} n x e^{a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{1}{3}/n} + \frac{9}{32} \sqrt{-\frac{1}{n^2}} n x e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{3}/n} - \frac{1}{16} \sqrt{-\frac{1}{n^2}} n x e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{8} \sqrt{-\frac{1}{n^2}} n x e^{3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{1}{n}}$$

```
[Out] (-9*E^(a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x)/(16*(c*x^n)^(1/(3*n))) + (9*Sqrt[-n^(-2)]*n*x*(c*x^n)^(1/(3*n)))/(32*E^(a*Sqrt[-n^(-2)]*n)) - (Sqrt[-n^(-2)]*n*x*(c*x^n)^(1/n))/(16*E^(3*a*Sqrt[-n^(-2)]*n)) + (E^(3*a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x*Log[x])/(8*(c*x^n)^(1/n))
```

**Rubi [A]** time = 0.104931, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {4483, 4489}

$$-\frac{9}{16} \sqrt{-\frac{1}{n^2}} n x e^{a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{1}{3}/n} + \frac{9}{32} \sqrt{-\frac{1}{n^2}} n x e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{3}/n} - \frac{1}{16} \sqrt{-\frac{1}{n^2}} n x e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{8} \sqrt{-\frac{1}{n^2}} n x e^{3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{1}{n}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3, x]
```

```
[Out] (-9*E^(a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x)/(16*(c*x^n)^(1/(3*n))) + (9*Sqrt[-n^(-2)]*n*x*(c*x^n)^(1/(3*n)))/(32*E^(a*Sqrt[-n^(-2)]*n)) - (Sqrt[-n^(-2)]*n*x*(c*x^n)^(1/n))/(16*E^(3*a*Sqrt[-n^(-2)]*n)) + (E^(3*a*Sqrt[-n^(-2)]*n)*Sqrt[-n^(-2)]*n*x*Log[x])/(8*(c*x^n)^(1/n))
```

#### Rule 4483

```
Int[Sin[(a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.)]*(d_.)]^(p_.), x_Symbol] := Dist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

#### Rule 4489

```
Int[((e_.)*(x_))^(m_.)*Sin[(a_.) + Log[x_]*(b_.)]*(d_.)]^(p_.), x_Symbol] := Dist[(m + 1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^(m*(E^((a*b*d^2*p)/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a*b*d^2*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

#### Rubi steps

$$\begin{aligned} \int \sin^3 \left( a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x(cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1+\frac{1}{n}} \sin^3 \left( a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= \frac{1}{8} \left( \sqrt{-\frac{1}{n^2}} x (cx^n)^{-1/n} \right) \text{Subst} \left( \int \left( \frac{e^{3a \sqrt{-\frac{1}{n^2}} n}}{x} - 3e^{a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{2}{3n}} + 3e^{-a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{1}{3n}} \right) dx, x, cx^n \right) \\ &= -\frac{9}{16} e^{a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x (cx^n)^{-\frac{1}{3}/n} + \frac{9}{32} e^{-a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x (cx^n)^{\frac{1}{3}/n} - \frac{1}{16} e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{8} e^{3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{1}{n}} \end{aligned}$$

**Mathematica [F]** time = 0.133365, size = 0, normalized size = 0.

$$\int \sin^3 \left( a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + (Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3, x]

[Out] Integrate[Sin[a + (Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3, x]

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \left( \sin \left( a + \frac{\ln(cx^n)}{3} \sqrt{-n^{-2}} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+1/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3,x)

[Out] int(sin(a+1/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3,x)

**Maxima [A]** time = 1.1595, size = 143, normalized size = 0.85

$$\frac{4 c^{\frac{1}{3n}} (x^n)^{\frac{1}{3n}} \log(x) \sin(3a) - 9 c^{\frac{5}{3n}} x (x^n)^{\frac{2}{3n}} \sin(a) + 2 c^{\frac{7}{3n}} e^{\left(\frac{\log(x^n)}{3n} + 2 \log(x)\right)} \sin(3a) - 18 c^{\left(\frac{1}{n}\right)} x \sin(a)}{32 c^{\frac{4}{3n}} (x^n)^{\frac{1}{3n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3,x, algorithm="maxima")

[Out] -1/32\*(4\*c^(1/3/n)\*(x^n)^(1/3/n)\*log(x)\*sin(3\*a) - 9\*c^(5/3/n)\*x\*(x^n)^(2/3/n)\*sin(a) + 2\*c^(7/3/n)\*e^(1/3\*log(x^n)/n + 2\*log(x))\*sin(3\*a) - 18\*c^(1/n)\*x\*sin(a))/(c^(4/3/n)\*(x^n)^(1/3/n))

**Fricas [C]** time = 0.474748, size = 238, normalized size = 1.42

$$\frac{1}{32} \left( 9i x^{\frac{4}{3}} e^{\left(\frac{2(3ian-\log(c))}{3n}\right)} - 2ix^2 + 12ie^{\left(\frac{2(3ian-\log(c))}{n}\right)} \log\left(x^{\frac{1}{3}}\right) - 18ix^{\frac{2}{3}} e^{\left(\frac{4(3ian-\log(c))}{3n}\right)} \right) e^{\left(-\frac{3ian-\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3,x, algorithm="fricas")

[Out] 1/32\*(9\*I\*x^(4/3)\*e^(2/3\*(3\*I\*a\*n - log(c))/n) - 2\*I\*x^2 + 12\*I\*e^(2\*(3\*I\*a\*n - log(c))/n)\*log(x^(1/3)) - 18\*I\*x^(2/3)\*e^(4/3\*(3\*I\*a\*n - log(c))/n))\*e^(-(3\*I\*a\*n - log(c))/n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*ln(c*x**n)*(-1/n**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/3*log(c*x^n)*(-1/n^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.44 \quad \int \frac{\sin^3(a)}{x} dx$$

**Optimal.** Leaf size=7

$$\sin^3(a) \log(x)$$

[Out] Log[x]\*Sin[a]^3

**Rubi [A]** time = 0.0047063, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {12, 29}

$$\sin^3(a) \log(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[a]^3/x,x]

[Out] Log[x]\*Sin[a]^3

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rubi steps

$$\begin{aligned} \int \frac{\sin^3(a)}{x} dx &= \sin^3(a) \int \frac{1}{x} dx \\ &= \log(x) \sin^3(a) \end{aligned}$$

**Mathematica [A]** time = 0.0007848, size = 7, normalized size = 1.

$$\sin^3(a) \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a]^3/x,x]

[Out] Log[x]\*Sin[a]^3

**Maple [A]** time = 0.013, size = 8, normalized size = 1.1

$$\ln(x) (\sin(a))^3$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(sin(a)^3/x,x)`

[Out] `ln(x)*sin(a)^3`

**Maxima [A]** time = 0.994203, size = 9, normalized size = 1.29

$$\log(x) \sin(a)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a)^3/x,x, algorithm="maxima")`

[Out] `log(x)*sin(a)^3`

**Fricas [A]** time = 0.432228, size = 42, normalized size = 6.

$$-(\cos(a)^2 - 1) \log(x) \sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a)^3/x,x, algorithm="fricas")`

[Out] `-(cos(a)^2 - 1)*log(x)*sin(a)`

**Sympy [A]** time = 0.389422, size = 7, normalized size = 1.

$$\log(x) \sin^3(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a)**3/x,x)`

[Out] `log(x)*sin(a)**3`

**Giac [A]** time = 1.1071, size = 11, normalized size = 1.57

$$\log(|x|) \sin(a)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a)^3/x,x, algorithm="giac")`

[Out] `log(abs(x))*sin(a)^3`

$$3.45 \quad \int \frac{\sin^3\left(a + \frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2} dx$$

**Optimal.** Leaf size=176

$$-\frac{\sqrt{-\frac{1}{n^2}}ne^{3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-1/n}}{16x} + \frac{9\sqrt{-\frac{1}{n^2}}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-\frac{1}{3}/n}}{32x} - \frac{9\sqrt{-\frac{1}{n^2}}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{1}{3}/n}}{16x} - \frac{\sqrt{-\frac{1}{n^2}}ne^{-3a\sqrt{-\frac{1}{n^2}}n}\log(x)(cx^n)^{\frac{1}{3}/n}}{8x}$$

[Out]  $-(E^{(3*a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n}/(16*x*(c*x^n)^n^{(-1)}) + (9*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n}/(32*x*(c*x^n)^{(1/(3*n))}) - (9*\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^{(1/(3*n))})/(16*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*x}) - (\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^n^{(-1)}*\text{Log}[x])/(8*E^{(3*a*\text{Sqrt}[-n^{(-2)}]*n)*x}))$

**Rubi [A]** time = 0.13176, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4493, 4489}

$$-\frac{\sqrt{-\frac{1}{n^2}}ne^{3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-1/n}}{16x} + \frac{9\sqrt{-\frac{1}{n^2}}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-\frac{1}{3}/n}}{32x} - \frac{9\sqrt{-\frac{1}{n^2}}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{1}{3}/n}}{16x} - \frac{\sqrt{-\frac{1}{n^2}}ne^{-3a\sqrt{-\frac{1}{n^2}}n}\log(x)(cx^n)^{\frac{1}{3}/n}}{8x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[a + (\text{Sqrt}[-n^{(-2)}]*\text{Log}[c*x^n])/3]^3/x^2, x]$

[Out]  $-(E^{(3*a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n}/(16*x*(c*x^n)^n^{(-1)}) + (9*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n}/(32*x*(c*x^n)^{(1/(3*n))}) - (9*\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^{(1/(3*n))})/(16*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*x}) - (\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^n^{(-1)}*\text{Log}[x])/(8*E^{(3*a*\text{Sqrt}[-n^{(-2)}]*n)*x}))$

#### Rule 4493

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}]*b_{.})*(d_{.})^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)*\text{Sin}[d*(a+b*\text{Log}[x])]}^p, x], x, c*x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\text{NeQ}[c, 1] \parallel \text{NeQ}[n, 1])$

#### Rule 4489

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[x_{.}]*b_{.})*(d_{.})^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(m+1)^p/(2^p*b^p*d^p*p^p), \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(E^{(a*b*d^2*p)/(m+1)})/x^{((m+1)/p)} - x^{((m+1)/p)}/E^{(a*b*d^2*p)/(m+1)}]^p, x], x] /; \text{FreeQ}\{a, b, d, e, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[b^2*d^2*p^2 + (m+1)^2, 0]$

#### Rubi steps

$$\int \frac{\sin^3\left(a + \frac{1}{3}\sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^2} dx = \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \sin^3\left(a + \frac{1}{3}\sqrt{-\frac{1}{n^2}} \log(x)\right) dx, x, cx^n\right)}{nx}$$

$$= -\frac{\left(\sqrt{-\frac{1}{n^2}} (cx^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \left(\frac{e^{-3a\sqrt{-\frac{1}{n^2}}n}}{x} + 3e^{a\sqrt{-\frac{1}{n^2}}n} x^{-1-\frac{4}{3n}} - 3e^{-a\sqrt{-\frac{1}{n^2}}n} x^{-1-\frac{2}{3n}} - e^{3a\sqrt{-\frac{1}{n^2}}n} x^{-1-\frac{2}{3n}}\right) dx, x, cx^n\right)}{8x}$$

$$= -\frac{e^{3a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}}n (cx^n)^{-1/n}}{16x} + \frac{9e^{a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}}n (cx^n)^{-\frac{1}{3}/n}}{32x} - \frac{9e^{-a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}}n (cx^n)^{-1/n}}{16x}$$

**Mathematica [F]** time = 0.170141, size = 0, normalized size = 0.

$$\int \frac{\sin^3\left(a + \frac{1}{3}\sqrt{-\frac{1}{n^2}} \log(cx^n)\right)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + (Sqrt[-n^(-2)])\*Log[c\*x^n])/3]^3/x^2, x]

[Out] Integrate[Sin[a + (Sqrt[-n^(-2)])\*Log[c\*x^n])/3]^3/x^2, x]

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left( \sin\left(a + \frac{\ln(cx^n)}{3} \sqrt{-n^{-2}}\right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+1/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3/x^2, x)

[Out] int(sin(a+1/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3/x^2, x)

**Maxima [A]** time = 1.14978, size = 165, normalized size = 0.94

$$\frac{\left(4 c^{\frac{7}{3n}} x e^{\left(\frac{\log(x^n)}{3n} + 2 \log(x)\right)} \log(x) \sin(3a) - 2 c^{\frac{1}{3n}} x (x^n)^{\frac{1}{3n}} \sin(3a) + 9 c^{\left(\frac{1}{n}\right)} x^2 \sin(a) + 18 c^{\frac{5}{3n}} e^{\left(\frac{2 \log(x^n)}{3n} + 2 \log(x)\right)} \sin(a)\right) e^{\left(-\frac{1}{3} \log(x^n) / n - 2 \log(x)\right)}}{32 c^{\frac{4}{3n}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3/x^2, x, algorithm="maxima")

[Out] -1/32\*(4\*c^(7/3/n)\*x\*e^(1/3\*log(x^n)/n + 2\*log(x))\*log(x)\*sin(3\*a) - 2\*c^(1/3/n)\*x\*(x^n)^(1/3/n)\*sin(3\*a) + 9\*c^(1/n)\*x^2\*sin(a) + 18\*c^(5/3/n)\*e^(2/3\*log(x^n)/n + 2\*log(x))\*sin(a))\*e^(-1/3\*log(x^n)/n - 2\*log(x))/(c^(4/3/n)\*x)

**Fricas [C]** time = 0.478729, size = 244, normalized size = 1.39

$$\frac{\left(-12ix^2 \log\left(x^{\frac{1}{3}}\right) - 18ix^{\frac{4}{3}} e^{\left(\frac{2(3ian-\log(c))}{3n}\right)} + 9ix^{\frac{2}{3}} e^{\left(\frac{4(3ian-\log(c))}{3n}\right)} - 2ie^{\left(\frac{2(3ian-\log(c))}{n}\right)}\right) e^{\left(-\frac{3ian-\log(c)}{n}\right)}}{32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3/x^2,x, algorithm="fricas")

[Out] 1/32\*(-12\*I\*x^2\*log(x^(1/3)) - 18\*I\*x^(4/3)\*e^(2/3\*(3\*I\*a\*n - log(c))/n) + 9\*I\*x^(2/3)\*e^(4/3\*(3\*I\*a\*n - log(c))/n) - 2\*I\*e^(2\*(3\*I\*a\*n - log(c))/n))\*e^(-(3\*I\*a\*n - log(c))/n)/x^2

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/3\*ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2))\*\*3/x\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n) + a\right)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3/x^2,x, algorithm="giac")

[Out] integrate(sin(1/3\*sqrt(-1/n^2)\*log(c\*x^n) + a)^3/x^2, x)

$$3.46 \quad \int \frac{\sin^3\left(a + \frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3} dx$$

**Optimal.** Leaf size=178

$$\frac{\sqrt{-\frac{1}{n^2}}ne^{3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-2/n}}{32x^2} + \frac{9\sqrt{-\frac{1}{n^2}}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-\frac{2}{3}/n}}{64x^2} - \frac{9\sqrt{-\frac{1}{n^2}}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{3}/n}}{32x^2} - \frac{\sqrt{-\frac{1}{n^2}}ne^{-3a\sqrt{-\frac{1}{n^2}}n}\log(x)(cx^n)^{-2/n}}{8x^2}$$

[Out]  $-(E^{(3*a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n}/(32*x^2*(c*x^n)^{(2/n)}) + (9*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n}/(64*x^2*(c*x^n)^{(2/(3*n))}) - (9*\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^{(2/(3*n))})/(32*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*x^2} - (\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^{(2/n)*\text{Log}[x]})/(8*E^{(3*a*\text{Sqrt}[-n^{(-2)}]*n)*x^2}$

**Rubi [A]** time = 0.113835, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4493, 4489}

$$\frac{\sqrt{-\frac{1}{n^2}}ne^{3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-2/n}}{32x^2} + \frac{9\sqrt{-\frac{1}{n^2}}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-\frac{2}{3}/n}}{64x^2} - \frac{9\sqrt{-\frac{1}{n^2}}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{3}/n}}{32x^2} - \frac{\sqrt{-\frac{1}{n^2}}ne^{-3a\sqrt{-\frac{1}{n^2}}n}\log(x)(cx^n)^{-2/n}}{8x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sin}[a + (2*\text{Sqrt}[-n^{(-2)}]*\text{Log}[c*x^n])/3]^3/x^3, x]$

[Out]  $-(E^{(3*a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n}/(32*x^2*(c*x^n)^{(2/n)}) + (9*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*\text{Sqrt}[-n^{(-2)}]*n}/(64*x^2*(c*x^n)^{(2/(3*n))}) - (9*\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^{(2/(3*n))})/(32*E^{(a*\text{Sqrt}[-n^{(-2)}]*n)*x^2} - (\text{Sqrt}[-n^{(-2)}]*n*(c*x^n)^{(2/n)*\text{Log}[x]})/(8*E^{(3*a*\text{Sqrt}[-n^{(-2)}]*n)*x^2}$

#### Rule 4493

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}]*b_{.})*(d_{.})]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)*\text{Sin}[d*(a+b*\text{Log}[x])]}^p, x], x, c*x^n], x] /;$  FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[x_{.}]*b_{.})*(d_{.})]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(m+1)^p/(2^p*b^p*d^p*p^p), \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(E^{(a*b*d^2*p)/(m+1)})/x^{(m+1)/p} - x^{(m+1)/p}/E^{(a*b*d^2*p)/(m+1)}]^p, x], x] /;$  FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m+1)^2, 0]

#### Rubi steps

$$\int \frac{\sin^3\left(a + \frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3} dx = \frac{(cx^n)^{2/n} \text{Subst}\left(\int x^{-1-\frac{2}{n}} \sin^3\left(a + \frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(x)\right) dx, x, cx^n\right)}{nx^2}$$

$$= \frac{\left(\sqrt{-\frac{1}{n^2}}(cx^n)^{2/n}\right) \text{Subst}\left(\int \left(\frac{e^{-3a\sqrt{-\frac{1}{n^2}}}}{x} + 3e^{a\sqrt{-\frac{1}{n^2}}}x^{-1-\frac{8}{3n}} - 3e^{-a\sqrt{-\frac{1}{n^2}}}x^{-1-\frac{4}{3n}} - e^{3a\sqrt{-\frac{1}{n^2}}}\right) dx, x, cx^n\right)}{8x^2}$$

$$= -\frac{e^{3a\sqrt{-\frac{1}{n^2}}}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-2/n}}{32x^2} + \frac{9e^{a\sqrt{-\frac{1}{n^2}}}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-\frac{2}{3}/n}}{64x^2} - \frac{9e^{-a\sqrt{-\frac{1}{n^2}}}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-\frac{2}{3}/n}}{32x^2}$$

**Mathematica [F]** time = 0.205036, size = 0, normalized size = 0.

$$\int \frac{\sin^3\left(a + \frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + (2\*Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3/x^3, x]

[Out] Integrate[Sin[a + (2\*Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3/x^3, x]

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \left( \sin\left(a + \frac{2 \ln(cx^n)}{3} \sqrt{-n^{-2}}\right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+2/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3/x^3, x)

[Out] int(sin(a+2/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3/x^3, x)

**Maxima [A]** time = 1.14344, size = 173, normalized size = 0.97

$$\frac{\left(8c^{\frac{14}{3n}}x^2e^{\left(\frac{2\log(x^n)}{3n}+4\log(x)\right)}\log(x)\sin(3a) + 9c^{\frac{2}{n}}x^4\sin(a) - 2c^{\frac{2}{3n}}x^2(x^n)^{\frac{2}{3n}}\sin(3a) + 18c^{\frac{10}{3n}}e^{\left(\frac{4\log(x^n)}{3n}+4\log(x)\right)}\sin(a)\right)e^{-\frac{2}{3}\log(x^n)/n - 4\log(x)}}{64c^{\frac{8}{3n}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3/x^3, x, algorithm="maxima")

[Out] -1/64\*(8\*c^(14/3/n)\*x^2\*e^(2/3\*log(x^n)/n + 4\*log(x))\*log(x)\*sin(3\*a) + 9\*c^(2/n)\*x^4\*sin(a) - 2\*c^(2/3/n)\*x^2\*(x^n)^(2/3/n)\*sin(3\*a) + 18\*c^(10/3/n)\*e^(4/3\*log(x^n)/n + 4\*log(x))\*sin(a))\*e^(-2/3\*log(x^n)/n - 4\*log(x))/(c^(8/3/n)\*x^2)

**Fricas [C]** time = 0.480342, size = 255, normalized size = 1.43

$$\frac{\left(-24ix^4 \log\left(x^{\frac{1}{3}}\right) - 18ix^{\frac{8}{3}} e^{\left(\frac{2(3ian-2 \log(c))}{3n}\right)} + 9ix^{\frac{4}{3}} e^{\left(\frac{4(3ian-2 \log(c))}{3n}\right)} - 2ie^{\left(\frac{2(3ian-2 \log(c))}{n}\right)}\right) e^{\left(-\frac{3ian-2 \log(c)}{n}\right)}}{64x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3/x^3,x, algorithm="fricas")

[Out] 1/64\*(-24\*I\*x^4\*log(x^(1/3)) - 18\*I\*x^(8/3)\*e^(2/3\*(3\*I\*a\*n - 2\*log(c))/n) + 9\*I\*x^(4/3)\*e^(4/3\*(3\*I\*a\*n - 2\*log(c))/n) - 2\*I\*e^(2\*(3\*I\*a\*n - 2\*log(c))/n))\*e^(-(3\*I\*a\*n - 2\*log(c))/n)/x^4

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2/3\*ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2))\*\*3/x\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{2}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) + a\right)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+2/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3/x^3,x, algorithm="giac")

[Out] integrate(sin(2/3\*sqrt(-1/n^2)\*log(c\*x^n) + a)^3/x^3, x)

$$3.47 \quad \int x^m \sin\left(a + \frac{1}{2}\sqrt{-(1+m)^2} \log(cx^2)\right) dx$$

**Optimal.** Leaf size=112

$$\frac{(m+1)e^{\frac{a\sqrt{-(m+1)^2}}{m+1}} x^{m+1} \log(x) (cx^2)^{\frac{1}{2}(-m-1)}}{2\sqrt{-(m+1)^2}} - \frac{e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}}} x^{m+1} (cx^2)^{\frac{m+1}{2}}}{4\sqrt{-(m+1)^2}}$$

[Out]  $-(E^{((a*(1+m))/\text{Sqrt}[-(1+m)^2])}*x^{(1+m)*(c*x^2)^{((1+m)/2)}})/(4*\text{Sqrt}[-(1+m)^2]) + (E^{((a*\text{Sqrt}[-(1+m)^2])/(1+m))*(1+m)*x^{(1+m)*(c*x^2)^{(-1-m)/2}}*\text{Log}[x])}/(2*\text{Sqrt}[-(1+m)^2])$

**Rubi [A]** time = 0.194204, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4493, 4489}

$$\frac{(m+1)e^{\frac{a\sqrt{-(m+1)^2}}{m+1}} x^{m+1} \log(x) (cx^2)^{\frac{1}{2}(-m-1)}}{2\sqrt{-(m+1)^2}} - \frac{e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}}} x^{m+1} (cx^2)^{\frac{m+1}{2}}}{4\sqrt{-(m+1)^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^m*\text{Sin}[a + (\text{Sqrt}[-(1+m)^2]*\text{Log}[c*x^2])/2], x]$

[Out]  $-(E^{((a*(1+m))/\text{Sqrt}[-(1+m)^2])}*x^{(1+m)*(c*x^2)^{((1+m)/2)}})/(4*\text{Sqrt}[-(1+m)^2]) + (E^{((a*\text{Sqrt}[-(1+m)^2])/(1+m))*(1+m)*x^{(1+m)*(c*x^2)^{(-1-m)/2}}*\text{Log}[x])}/(2*\text{Sqrt}[-(1+m)^2])$

#### Rule 4493

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}]* (b_{.})*(d_{.})]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)*\text{Sin}[d*(a+b*\text{Log}[x])]}^p, x], x, c*x^n], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n, p\}, x \ \&\& \ (\text{NeQ}[c, 1] \ || \ \text{NeQ}[n, 1])$

#### Rule 4489

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sin}[(a_{.}) + \text{Log}[x_{.}]* (b_{.})*(d_{.})]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \text{Dist}[(m+1)^p/(2^p*b^p*d^p*p^p), \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(E^{((a*b*d^2*p)/(m+1))}/x^{((m+1)/p}) - x^{((m+1)/p)}/E^{((a*b*d^2*p)/(m+1))})^p, x], x] /;$   $\text{FreeQ}\{a, b, d, e, m\}, x \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[b^2*d^2*p^2 + (m+1)^2, 0]$

#### Rubi steps



$$\begin{aligned} \int x^m \sin\left(a + \frac{1}{2}\sqrt{-(1+m)^2} \log(cx^2)\right) dx &= \frac{1}{2} \left(x^{1+m} (cx^2)^{\frac{1}{2}(-1-m)}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{2}} \sin\left(a + \frac{1}{2}\sqrt{-(1+m)^2} \log(x)\right) dx, x, cx\right) \\ &= \frac{\left((1+m)x^{1+m} (cx^2)^{\frac{1}{2}(-1-m)}\right) \text{Subst}\left(\int \left(\frac{e^{\frac{a\sqrt{-(1+m)^2}}{1+m}}}{x} - e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}} x^m}\right) dx, x, cx\right)}{4\sqrt{-(1+m)^2}} \\ &= -\frac{e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}} x^{1+m} (cx^2)^{\frac{1+m}{2}}}}{4\sqrt{-(1+m)^2}} + \frac{e^{\frac{a\sqrt{-(1+m)^2}}{1+m}} (1+m)x^{1+m} (cx^2)^{\frac{1}{2}(-1-m)} \log(x)}{2\sqrt{-(1+m)^2}} \end{aligned}$$

**Mathematica [F]** time = 0.230448, size = 0, normalized size = 0.

$$\int x^m \sin\left(a + \frac{1}{2}\sqrt{-(1+m)^2} \log(cx^2)\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m\*Sin[a + (Sqrt[-(1 + m)^2]\*Log[c\*x^2])/2], x]

[Out] Integrate[x^m\*Sin[a + (Sqrt[-(1 + m)^2]\*Log[c\*x^2])/2], x]

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int x^m \sin\left(a + \frac{\ln(cx^2)}{2} \sqrt{-(1+m)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sin(a+1/2\*ln(c\*x^2)\*(-(1+m)^2)^(1/2)), x)

[Out] int(x^m\*sin(a+1/2\*ln(c\*x^2)\*(-(1+m)^2)^(1/2)), x)

**Maxima [A]** time = 1.06175, size = 65, normalized size = 0.58

$$\frac{c^{m+1} x^2 x^{2m} \sin(a) + 2(m \sin(a) + \sin(a)) \log(x)}{4\left(c^{\frac{1}{2}m} m + c^{\frac{1}{2}m}\right) \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/2\*log(c\*x^2)\*(-(1+m)^2)^(1/2)), x, algorithm="maxima")

[Out] 1/4\*(c^(m + 1)\*x^2\*x^(2\*m)\*sin(a) + 2\*(m\*sin(a) + sin(a))\*log(x))/((c^(1/2\*m)\*m + c^(1/2\*m))\*sqrt(c))

**Fricas [C]** time = 0.486322, size = 149, normalized size = 1.33

$$\frac{(i x^2 x^{2m} + (-2i m - 2i) e^{-(m+1) \log(c) + 2i a} \log(x)) e^{\frac{1}{2}(m+1) \log(c) - i a}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/2\*log(c\*x^2)\*(-(1+m)^2)^(1/2)),x, algorithm="fricas")

[Out] 1/4\*(I\*x^2\*x^(2\*m) + (-2\*I\*m - 2\*I)\*e^(-(m + 1)\*log(c) + 2\*I\*a)\*log(x))\*e^(1/2\*(m + 1)\*log(c) - I\*a)/(m + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sin\left(a + \frac{\sqrt{-m^2 - 2m - 1} \log(cx^2)}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*sin(a+1/2\*ln(c\*x\*\*2)\*(-(1+m)\*\*2)\*\*(1/2)),x)

[Out] Integral(x\*\*m\*sin(a + sqrt(-m\*\*2 - 2\*m - 1)\*log(c\*x\*\*2)/2), x)

**Giac [C]** time = 1.4687, size = 255, normalized size = 2.28

$$\frac{i m x x^m e^{\left(\frac{1}{2} |m+1| \log(c) + |m+1| \log(x) - i a\right)} - i x x^m |m+1| e^{\left(\frac{1}{2} |m+1| \log(c) + |m+1| \log(x) - i a\right)} - i m x x^m e^{\left(-\frac{1}{2} |m+1| \log(c) - |m+1| \log(x) + i a\right)} - i x x^m e^{\left(-\frac{1}{2} |m+1| \log(c) - |m+1| \log(x) + i a\right)}}{2 \left( (m+1)^2 - m^2 - 2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/2\*log(c\*x^2)\*(-(1+m)^2)^(1/2)),x, algorithm="giac")

[Out] -1/2\*(I\*m\*x\*x^m\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - I\*a) - I\*x\*x^m\*abs(m + 1)\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - I\*a) - I\*m\*x\*x^m\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + I\*a) - I\*x\*x^m\*abs(m + 1)\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + I\*a) + I\*x\*x^m\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - I\*a) - I\*x\*x^m\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + I\*a))/((m + 1)^2 - m^2 - 2)

$$3.48 \quad \int \sin \left( a + \frac{1}{2}i \log \left( cx^2 \right) \right) dx$$

**Optimal.** Leaf size=52

$$\frac{ie^{-ia}cx^3}{4\sqrt{cx^2}} - \frac{ie^{ia}x \log(x)}{2\sqrt{cx^2}}$$

[Out]  $((I/4)*c*x^3)/(E^{(I*a)*Sqrt[c*x^2]}) - ((I/2)*E^{(I*a)*x*Log[x]})/Sqrt[c*x^2]$

**Rubi [A]** time = 0.0354132, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {4483, 4489}

$$\frac{ie^{-ia}cx^3}{4\sqrt{cx^2}} - \frac{ie^{ia}x \log(x)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + (I/2)\*Log[c\*x^2]], x]

[Out]  $((I/4)*c*x^3)/(E^{(I*a)*Sqrt[c*x^2]}) - ((I/2)*E^{(I*a)*x*Log[x]})/Sqrt[c*x^2]$

#### Rule 4483

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(m + 1)^(1/p)/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^(m\*(E^((a\*b\*d^2\*p)/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \sin \left( a + \frac{1}{2}i \log \left( cx^2 \right) \right) dx &= \frac{x \operatorname{Subst} \left( \int \frac{\sin \left( a + \frac{1}{2}i \log(x) \right)}{\sqrt{x}} dx, x, cx^2 \right)}{2\sqrt{cx^2}} \\ &= -\frac{(ix) \operatorname{Subst} \left( \int \left( -e^{-ia} + \frac{e^{ia}}{x} \right) dx, x, cx^2 \right)}{4\sqrt{cx^2}} \\ &= \frac{ice^{-ia}x^3}{4\sqrt{cx^2}} - \frac{ie^{ia}x \log(x)}{2\sqrt{cx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.058512, size = 44, normalized size = 0.85

$$\frac{x \left( \sin(a) \left( cx^2 + 2 \log(x) \right) + i \cos(a) \left( cx^2 - 2 \log(x) \right) \right)}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + (I/2)\*Log[c\*x^2]],x]

[Out] (x\*(I\*Cos[a]\*(c\*x^2 - 2\*Log[x]) + (c\*x^2 + 2\*Log[x])\*Sin[a]))/(4\*Sqrt[c\*x^2])

**Maple [B]** time = 0.039, size = 106, normalized size = 2.

$$\left(\frac{i}{2}x - \frac{i}{2}x \left(\tan\left(\frac{a}{2} + \frac{i}{4}\ln(cx^2)\right)\right)\right)^2 + \frac{x \ln(cx^2)}{2} \tan\left(\frac{a}{2} + \frac{i}{4}\ln(cx^2)\right) - \frac{i}{4}x \ln(cx^2) + \frac{i}{4}x \ln(cx^2) \left(\tan\left(\frac{a}{2} + \frac{i}{4}\ln(cx^2)\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+1/2\*I\*ln(c\*x^2)),x)

[Out] (1/2\*I\*x-1/2\*I\*x\*tan(1/2\*a+1/4\*I\*ln(c\*x^2))^2+1/2\*x\*ln(c\*x^2)\*tan(1/2\*a+1/4\*I\*ln(c\*x^2))-1/4\*I\*x\*ln(c\*x^2)+1/4\*I\*x\*ln(c\*x^2)\*tan(1/2\*a+1/4\*I\*ln(c\*x^2))^2)/(1+tan(1/2\*a+1/4\*I\*ln(c\*x^2))^2)

**Maxima [A]** time = 1.00925, size = 42, normalized size = 0.81

$$\frac{cx^2(i \cos(a) + \sin(a)) - 2(i \cos(a) - \sin(a)) \log(x)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/2\*I\*log(c\*x^2)),x, algorithm="maxima")

[Out] 1/4\*(c\*x^2\*(I\*cos(a) + sin(a)) - 2\*(I\*cos(a) - sin(a))\*log(x))/sqrt(c)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/2\*I\*log(c\*x^2)),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \frac{i \log(cx^2)}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*I*ln(c*x**2)),x)
```

```
[Out] Integral(sin(a + I*log(c*x**2)/2), x)
```

**Giac [A]** time = 1.18503, size = 39, normalized size = 0.75

$$-\frac{-i c^{\frac{3}{2}} x^2 e^{-i a} + 2i \sqrt{c} e^{i a} \log(x)}{4 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/2*I*log(c*x^2)),x, algorithm="giac")
```

```
[Out] -1/4*(-I*c^(3/2)*x^2*e^(-I*a) + 2*I*sqrt(c)*e^(I*a)*log(x))/c
```

### 3.49 $\int x^m \sin^2 \left( a + \frac{1}{4} \sqrt{-(1+m)^2} \log(cx^2) \right) dx$

**Optimal.** Leaf size=106

$$-\frac{e^{\frac{2a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} (cx^2)^{\frac{m+1}{2}}}}{8(m+1)} - \frac{1}{4} e^{-\frac{2a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} \log(x) (cx^2)^{\frac{1}{2}(-m-1)}} + \frac{x^{m+1}}{2(m+1)}$$

[Out]  $x^{(1+m)/(2*(1+m))} - (E^{((2*a*(1+m))/Sqrt[-(1+m)^2])} * x^{(1+m)} * (c*x^2)^{((1+m)/2)}) / (8*(1+m)) - (x^{(1+m)} * (c*x^2)^{((-1-m)/2)} * \text{Log}[x]) / (4 * E^{((2*a*(1+m))/Sqrt[-(1+m)^2])})$

**Rubi [A]** time = 0.14467, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {4493, 4489}

$$-\frac{e^{\frac{2a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} (cx^2)^{\frac{m+1}{2}}}}{8(m+1)} - \frac{1}{4} e^{-\frac{2a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} \log(x) (cx^2)^{\frac{1}{2}(-m-1)}} + \frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^m * \text{Sin}[a + (\text{Sqrt}[-(1+m)^2] * \text{Log}[c*x^2]) / 4]^2, x]$

[Out]  $x^{(1+m)/(2*(1+m))} - (E^{((2*a*(1+m))/Sqrt[-(1+m)^2])} * x^{(1+m)} * (c*x^2)^{((1+m)/2)}) / (8*(1+m)) - (x^{(1+m)} * (c*x^2)^{((-1-m)/2)} * \text{Log}[x]) / (4 * E^{((2*a*(1+m))/Sqrt[-(1+m)^2])})$

#### Rule 4493

$\text{Int}[(e^{\cdot}) * (x^{\cdot})^{(m^{\cdot})} * \text{Sin}[(a^{\cdot}) + \text{Log}[(c^{\cdot}) * (x^{\cdot})^{(n^{\cdot})}] * (b^{\cdot})] * (d^{\cdot})]^{(p^{\cdot})}, x\_Symbol] \rightarrow \text{Dist}[(e*x)^{(m+1)} / (e*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)} * \text{Sin}[d*(a+b*\text{Log}[x])]^p, x], x, c*x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\text{NeQ}[c, 1] \mid\mid \text{NeQ}[n, 1])$

#### Rule 4489

$\text{Int}[(e^{\cdot}) * (x^{\cdot})^{(m^{\cdot})} * \text{Sin}[(a^{\cdot}) + \text{Log}[x^{\cdot}] * (b^{\cdot})] * (d^{\cdot})]^{(p^{\cdot})}, x\_Symbol] \rightarrow \text{Dist}[(m+1)^p / (2^p * b^p * d^p * p^p), \text{Int}[\text{ExpandIntegrand}[(e*x)^m * (E^{(a*b*d^2*p)/(m+1)}) / x^{((m+1)/p)} - x^{((m+1)/p)} / E^{(a*b*d^2*p)/(m+1)}]^p, x], x] /; \text{FreeQ}\{a, b, d, e, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[b^2*d^2*p^2 + (m+1)^2, 0]$

#### Rubi steps

$$\begin{aligned} \int x^m \sin^2 \left( a + \frac{1}{4} \sqrt{-(1+m)^2} \log(cx^2) \right) dx &= \frac{1}{2} \left( x^{1+m} (cx^2)^{\frac{1}{2}(-1-m)} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{2}} \sin^2 \left( a + \frac{1}{4} \sqrt{-(1+m)^2} \log(x) \right) dx, x \right) \\ &= - \left( \frac{1}{8} \left( x^{1+m} (cx^2)^{\frac{1}{2}(-1-m)} \right) \text{Subst} \left( \int \left( \frac{e^{-\frac{2a(1+m)}{\sqrt{-(1+m)^2}}}}{x} - 2x^{\frac{1}{2}(-1+m)} + e^{\frac{2a(1+m)}{\sqrt{-(1+m)^2}}} x^m \right) dx, x \right) \right) \\ &= \frac{x^{1+m}}{2(1+m)} - \frac{e^{\frac{2a(1+m)}{\sqrt{-(1+m)^2}} x^{1+m} (cx^2)^{\frac{1+m}{2}}}}{8(1+m)} - \frac{1}{4} e^{-\frac{2a(1+m)}{\sqrt{-(1+m)^2}} x^{1+m} (cx^2)^{\frac{1}{2}(-1-m)}} \log(x) \end{aligned}$$

**Mathematica [F]** time = 0.3165, size = 0, normalized size = 0.

$$\int x^m \sin^2 \left( a + \frac{1}{4} \sqrt{-(1+m)^2} \log(cx^2) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m\*Sin[a + (Sqrt[-(1 + m)^2]\*Log[c\*x^2])/4]^2,x]

[Out] Integrate[x^m\*Sin[a + (Sqrt[-(1 + m)^2]\*Log[c\*x^2])/4]^2, x]

**Maple [F]** time = 0.061, size = 0, normalized size = 0.

$$\int x^m \left( \sin \left( a + \frac{\ln(cx^2)}{4} \sqrt{-(1+m)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sin(a+1/4\*ln(c\*x^2)\*(-(1+m)^2)^(1/2))^2,x)

[Out] int(x^m\*sin(a+1/4\*ln(c\*x^2)\*(-(1+m)^2)^(1/2))^2,x)

**Maxima [A]** time = 1.06182, size = 181, normalized size = 1.71

$$\frac{c^{m+1} x^2 x^{2m} \cos(2a) - 4 \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{1}{2}m + \frac{1}{2}} x x^m + 2 \left( \cos(2a)^3 + \cos(2a) \sin(2a)^2 + \left( \cos(2a)^3 + \cos(2a) \sin(2a)^2 \right) \log(x) \right)}{8 \left( \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{1}{2}m} m + \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{1}{2}m} \right) \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/4\*log(c\*x^2)\*(-(1+m)^2)^(1/2))^2,x, algorithm="maxima")

[Out] -1/8\*(c^(m + 1)\*x^2\*x^(2\*m)\*cos(2\*a) - 4\*(cos(2\*a)^2 + sin(2\*a)^2)\*c^(1/2\*m + 1/2)\*x\*x^m + 2\*(cos(2\*a)^3 + cos(2\*a)\*sin(2\*a)^2 + (cos(2\*a)^3 + cos(2\*a)\*sin(2\*a)^2)\*m\*log(x))/(((cos(2\*a)^2 + sin(2\*a)^2)\*c^(1/2\*m)\*m + (cos(2\*a)^2 + sin(2\*a)^2)\*c^(1/2\*m))\*sqrt(c))

**Fricas [C]** time = 0.487449, size = 252, normalized size = 2.38

$$\frac{\left( 2(m+1)e^{-(m+1)\log(c)-2(m+1)\log(x)+4ia} \log(x) - 4e^{\left(-\frac{1}{2}(m+1)\log(c)-(m+1)\log(x)+2ia\right)} + 1 \right) e^{\left(\frac{1}{2}(m+1)\log(c)+2(m+1)\log(x)-2ia\right)}}{8(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/4\*log(c\*x^2)\*(-(1+m)^2)^(1/2))^2,x, algorithm="fricas")

[Out] -1/8\*(2\*(m + 1)\*e^(-(m + 1)\*log(c) - 2\*(m + 1)\*log(x) + 4\*I\*a)\*log(x) - 4\*e^(-1/2\*(m + 1)\*log(c) - (m + 1)\*log(x) + 2\*I\*a) + 1)\*e^(1/2\*(m + 1)\*log(c))

$$+ 2*(m + 1)*\log(x) - 2*I*a)/(m + 1)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sin^2 \left( a + \frac{\sqrt{-m^2 - 2m - 1} \log(cx^2)}{4} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*sin(a+1/4\*ln(c\*x\*\*2)\*(-(1+m)\*\*2)\*\*(1/2))\*\*2,x)

[Out] Integral(x\*\*m\*sin(a + sqrt(-m\*\*2 - 2\*m - 1)\*log(c\*x\*\*2)/4)\*\*2, x)

**Giac [C]** time = 1.89452, size = 473, normalized size = 4.46

$$m^2 x x^m e^{\left(\frac{1}{2} |m+1| \log(c) + |m+1| \log(x) - 2i a\right)} - m x x^m |m+1| e^{\left(\frac{1}{2} |m+1| \log(c) + |m+1| \log(x) - 2i a\right)} + m^2 x x^m e^{\left(-\frac{1}{2} |m+1| \log(c) - |m+1| \log(x) + 2i a\right)} + m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/4\*log(c\*x^2)\*(-(1+m)^2)^(1/2))^2,x, algorithm="giac")

[Out] 1/4\*(m^2\*x\*x^m\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 2\*I\*a) - m\*x\*x^m\*abs(m + 1)\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 2\*I\*a) + m^2\*x\*x^m\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + 2\*I\*a) + m\*x\*x^m\*abs(m + 1)\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + 2\*I\*a) + 2\*(m + 1)^2\*x\*x^m - 2\*m^2\*x\*x^m + 2\*m\*x\*x^m\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 2\*I\*a) - x\*x^m\*abs(m + 1)\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 2\*I\*a) + 2\*m\*x\*x^m\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + 2\*I\*a) + x\*x^m\*abs(m + 1)\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + 2\*I\*a) - 4\*m\*x\*x^m + x\*x^m\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 2\*I\*a) + x\*x^m\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + 2\*I\*a) - 2\*x\*x^m)/(m + 1)^2\*m - m^3 + (m + 1)^2 - 3\*m^2 - 3\*m - 1)



### 3.50 $\int \sin^2 \left( a + \frac{1}{4}i \log(cx^2) \right) dx$

**Optimal.** Leaf size=53

$$-\frac{e^{-2ia}cx^3}{8\sqrt{cx^2}} - \frac{e^{2ia}x \log(x)}{4\sqrt{cx^2}} + \frac{x}{2}$$

[Out]  $x/2 - (c*x^3)/(8*E^{((2*I)*a)*Sqrt[c*x^2]} - (E^{((2*I)*a)*x*Log[x]})/(4*Sqrt[c*x^2])$

**Rubi [A]** time = 0.0452294, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4483, 4489}

$$-\frac{e^{-2ia}cx^3}{8\sqrt{cx^2}} - \frac{e^{2ia}x \log(x)}{4\sqrt{cx^2}} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + (I/4)\*Log[c\*x^2]]^2, x]

[Out]  $x/2 - (c*x^3)/(8*E^{((2*I)*a)*Sqrt[c*x^2]} - (E^{((2*I)*a)*x*Log[x]})/(4*Sqrt[c*x^2])$

#### Rule 4483

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1)))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \sin^2 \left( a + \frac{1}{4}i \log(cx^2) \right) dx &= \frac{x \operatorname{Subst} \left( \int \frac{\sin^2 \left( a + \frac{1}{4}i \log(x) \right)}{\sqrt{x}} dx, x, cx^2 \right)}{2\sqrt{cx^2}} \\ &= -\frac{x \operatorname{Subst} \left( \int \left( e^{-2ia} + \frac{e^{2ia}}{x} - \frac{2}{\sqrt{x}} \right) dx, x, cx^2 \right)}{8\sqrt{cx^2}} \\ &= \frac{x}{2} - \frac{ce^{-2ia}x^3}{8\sqrt{cx^2}} - \frac{e^{2ia}x \log(x)}{4\sqrt{cx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.0995453, size = 60, normalized size = 1.13

$$\frac{x \left( i \sin(2a) (cx^2 - 2 \log(x)) - \cos(2a) (cx^2 + 2 \log(x)) + 4\sqrt{cx^2} \right)}{8\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + (I/4)\*Log[c\*x^2]]^2,x]

[Out] (x\*(4\*Sqrt[c\*x^2] - Cos[2\*a]\*(c\*x^2 + 2\*Log[x]) + I\*(c\*x^2 - 2\*Log[x])\*Sin[2\*a]))/(8\*Sqrt[c\*x^2])

**Maple [B]** time = 0.069, size = 173, normalized size = 3.3

$$\left(\frac{x}{4} + \frac{5x}{2} \left(\tan\left(\frac{a}{2} + \frac{i}{8} \ln(cx^2)\right)\right)\right)^2 + \frac{x}{4} \left(\tan\left(\frac{a}{2} + \frac{i}{8} \ln(cx^2)\right)\right)^4 - \frac{x \ln(cx^2)}{8} + \frac{3x \ln(cx^2)}{4} \left(\tan\left(\frac{a}{2} + \frac{i}{8} \ln(cx^2)\right)\right)^2 - \frac{x \ln^2(cx^2)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+1/4\*I\*ln(c\*x^2))^2,x)

[Out] (1/4\*x+5/2\*x\*tan(1/2\*a+1/8\*I\*ln(c\*x^2))^2+1/4\*x\*tan(1/2\*a+1/8\*I\*ln(c\*x^2))^4-1/8\*x\*ln(c\*x^2)+3/4\*x\*ln(c\*x^2)\*tan(1/2\*a+1/8\*I\*ln(c\*x^2))^2-1/8\*x\*ln(c\*x^2)\*tan(1/2\*a+1/8\*I\*ln(c\*x^2))^4-1/2\*I\*x\*ln(c\*x^2)\*tan(1/2\*a+1/8\*I\*ln(c\*x^2))^3)/(1+tan(1/2\*a+1/8\*I\*ln(c\*x^2))^2)^2

**Maxima [A]** time = 1.02164, size = 65, normalized size = 1.23

$$\frac{4cx - (cx^2(\cos(2a) - i \sin(2a)) + (2 \cos(2a) + 2i \sin(2a)) \log(x))\sqrt{c}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/4\*I\*log(c\*x^2))^2,x, algorithm="maxima")

[Out] 1/8\*(4\*c\*x - (c\*x^2\*(cos(2\*a) - I\*sin(2\*a)) + (2\*cos(2\*a) + 2\*I\*sin(2\*a))\*log(x))\*sqrt(c))/c

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/4\*I\*log(c\*x^2))^2,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin^2\left(a + \frac{i \log(cx^2)}{4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/4\*I\*ln(c\*x\*\*2))\*\*2,x)

[Out] Integral(sin(a + I\*log(c\*x\*\*2)/4)\*\*2, x)

**Giac [A]** time = 1.26885, size = 43, normalized size = 0.81

$$\frac{1}{2}x - \frac{c^{\frac{3}{2}}x^2e^{-2ia} + 2\sqrt{c}e^{2ia}\log(x)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/4\*I\*log(c\*x^2))^2,x, algorithm="giac")

[Out] 1/2\*x - 1/8\*(c^(3/2)\*x^2\*e^(-2\*I\*a) + 2\*sqrt(c)\*e^(2\*I\*a)\*log(x))/c

$$3.51 \quad \int x^m \sin^3 \left( a + \frac{1}{6} \sqrt{-(1+m)^2} \log(cx^2) \right) dx$$

**Optimal.** Leaf size=218

$$\frac{9e^{\frac{a\sqrt{-(m+1)^2}}{m+1}} x^{m+1} (cx^2)^{\frac{1}{6}(-m-1)}}{16\sqrt{-(m+1)^2}} - \frac{9e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} (cx^2)^{\frac{m+1}{6}}}}{32\sqrt{-(m+1)^2}} + \frac{e^{\frac{3a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} (cx^2)^{\frac{m+1}{2}}}}{16\sqrt{-(m+1)^2}} - \frac{(m+1)e^{-\frac{3a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} \log(x) (cx^2)^{\frac{1}{2}}}}{8\sqrt{-(m+1)^2}}$$

```
[Out] (9*E^((a*Sqrt[-(1+m)^2])/(1+m))*x^(1+m)*(c*x^2)^((-1-m)/6))/(16*Sqr
t[-(1+m)^2]) - (9*E^((a*(1+m))/Sqrt[-(1+m)^2])*x^(1+m)*(c*x^2)^((1
+m)/6))/(32*Sqrt[-(1+m)^2]) + (E^((3*a*(1+m))/Sqrt[-(1+m)^2])*x^(1+
m)*(c*x^2)^((1+m)/2))/(16*Sqrt[-(1+m)^2]) - ((1+m)*x^(1+m)*(c*x^2)
^((-1-m)/2)*Log[x])/(8*E^((3*a*(1+m))/Sqrt[-(1+m)^2])*Sqrt[-(1+m)^2
])
```

**Rubi [A]** time = 0.304595, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {4493, 4489}

$$\frac{9e^{\frac{a\sqrt{-(m+1)^2}}{m+1}} x^{m+1} (cx^2)^{\frac{1}{6}(-m-1)}}{16\sqrt{-(m+1)^2}} - \frac{9e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} (cx^2)^{\frac{m+1}{6}}}}{32\sqrt{-(m+1)^2}} + \frac{e^{\frac{3a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} (cx^2)^{\frac{m+1}{2}}}}{16\sqrt{-(m+1)^2}} - \frac{(m+1)e^{-\frac{3a(m+1)}{\sqrt{-(m+1)^2}} x^{m+1} \log(x) (cx^2)^{\frac{1}{2}}}}{8\sqrt{-(m+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sin[a + (Sqrt[-(1+m)^2]*Log[c*x^2])/6]^3,x]
```

```
[Out] (9*E^((a*Sqrt[-(1+m)^2])/(1+m))*x^(1+m)*(c*x^2)^((-1-m)/6))/(16*Sqr
t[-(1+m)^2]) - (9*E^((a*(1+m))/Sqrt[-(1+m)^2])*x^(1+m)*(c*x^2)^((1
+m)/6))/(32*Sqrt[-(1+m)^2]) + (E^((3*a*(1+m))/Sqrt[-(1+m)^2])*x^(1+
m)*(c*x^2)^((1+m)/2))/(16*Sqrt[-(1+m)^2]) - ((1+m)*x^(1+m)*(c*x^2)
^((-1-m)/2)*Log[x])/(8*E^((3*a*(1+m))/Sqrt[-(1+m)^2])*Sqrt[-(1+m)^2
])
```

#### Rule 4493

```
Int[((e_)*(x_))^(m_)*Sin[((a_) + Log[(c_)*(x_)^(n_)]*(b_))*(d_)]^(p_
_), x_Symbol] := Dist[(e*x)^(m+1)/(e*n*(c*x^n)^((m+1)/n)), Subst[Int[x^
((m+1)/n-1)*Sin[d*(a+b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

#### Rule 4489

```
Int[((e_)*(x_))^(m_)*Sin[((a_) + Log[x_]*(b_))*(d_)]^(p_), x_Symbol]
:= Dist[(m+1)^p/(2^p*b^p*d^p*p^p), Int[ExpandIntegrand[(e*x)^m*(E^((a*b*d
^2*p)/(m+1))/x^((m+1)/p) - x^((m+1)/p)/E^((a*b*d^2*p)/(m+1))]^p, x]
, x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m
+ 1)^2, 0]
```

#### Rubi steps

$$\int x^m \sin^3 \left( a + \frac{1}{6} \sqrt{-(1+m)^2} \log(cx^2) \right) dx = \frac{1}{2} \left( x^{1+m} (cx^2)^{\frac{1}{2}(-1-m)} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{2}} \sin^3 \left( a + \frac{1}{6} \sqrt{-(1+m)^2} \log(x) \right) dx \right)$$

$$= \frac{\left( \sqrt{-(1+m)^2} x^{1+m} (cx^2)^{\frac{1}{2}(-1-m)} \right) \text{Subst} \left( \int \left( \frac{e^{-\frac{3a(1+m)}{\sqrt{-(1+m)^2}}}}{x} - 3e^{\frac{a\sqrt{-(1+m)^2}}{1+m}} x^{\frac{1}{3}(-2)} \right) dx \right)}{16(1+m)}$$

$$= \frac{9e^{\frac{a\sqrt{-(1+m)^2}}{1+m}} x^{1+m} (cx^2)^{\frac{1}{6}(-1-m)}}{16\sqrt{-(1+m)^2}} - \frac{9e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}} x^{1+m} (cx^2)^{\frac{1+m}{6}}}}{32\sqrt{-(1+m)^2}} + \frac{e^{\frac{3a(1+m)}{\sqrt{-(1+m)^2}} x^{1+m}}}{16\sqrt{-(1+m)^2}}$$

**Mathematica [F]** time = 0.429437, size = 0, normalized size = 0.

$$\int x^m \sin^3 \left( a + \frac{1}{6} \sqrt{-(1+m)^2} \log(cx^2) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m\*Sin[a + (Sqrt[-(1 + m)^2]\*Log[c\*x^2])/6]^3,x]

[Out] Integrate[x^m\*Sin[a + (Sqrt[-(1 + m)^2]\*Log[c\*x^2])/6]^3, x]

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int x^m \left( \sin \left( a + \frac{\ln(cx^2)}{6} \sqrt{-(1+m)^2} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sin(a+1/6\*ln(c\*x^2)\*(-(1+m)^2)^(1/2))^3,x)

[Out] int(x^m\*sin(a+1/6\*ln(c\*x^2)\*(-(1+m)^2)^(1/2))^3,x)

**Maxima [A]** time = 1.08219, size = 278, normalized size = 1.28

$$\frac{9(\cos(2a)\sin(3a) - \cos(3a)\sin(2a))c^{\frac{5}{6}m + \frac{5}{6}}x^{\frac{5}{3}}x^{\frac{4}{3}m} + 18(\cos(3a)\sin(4a) - \cos(4a)\sin(3a))c^{\frac{1}{2}m + \frac{1}{2}}xx^{\frac{2}{3}m} - 2\left(c^{\frac{7}{6}}\right)}{32\left((\cos(3a))^2 + \sin(3a)^2\right)c^{\frac{2}{3}m}m + \dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/6\*log(c\*x^2)\*(-(1+m)^2)^(1/2))^3,x, algorithm="maxima")

[Out] 1/32\*(9\*(cos(2\*a)\*sin(3\*a) - cos(3\*a)\*sin(2\*a))\*c^(5/6\*m + 5/6)\*x^(5/3)\*x^(4/3\*m) + 18\*(cos(3\*a)\*sin(4\*a) - cos(4\*a)\*sin(3\*a))\*c^(1/2\*m + 1/2)\*x\*x^(2/3\*m) - 2\*(c^(7/6\*m + 1)\*x^2\*x^(2\*m)\*sin(3\*a) + 2\*((cos(3\*a)^2\*sin(3\*a) + sin(3\*a)^3)\*c^(1/6\*m)\*m + (cos(3\*a)^2\*sin(3\*a) + sin(3\*a)^3)\*c^(1/6\*m))\*log(x))\*c^(1/6)\*x^(1/3))/(((cos(3\*a)^2 + sin(3\*a)^2)\*c^(2/3\*m)\*m + (cos(3\*a)^2 +

$$\sin(3a)^2 * c^{(2/3*m)} * c^{(2/3)} * x^{(1/3)}$$

**Fricas [C]** time = 0.503488, size = 350, normalized size = 1.61

$$\frac{\left( (4im + 4i)e^{-(m+1)\log(c)-2(m+1)\log(x)+6ia} \log(x) + 9ie^{\left(-\frac{1}{3}(m+1)\log(c)-\frac{2}{3}(m+1)\log(x)+2ia\right)} - 18ie^{\left(-\frac{2}{3}(m+1)\log(c)-\frac{4}{3}(m+1)\log(x)+4ia\right)} \right)}{32(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/6\*log(c\*x^2)\*(-(1+m)^2)^(1/2))^3,x, algorithm="fricas")

[Out] 1/32\*((4\*I\*m + 4\*I)\*e^(-(m + 1)\*log(c) - 2\*(m + 1)\*log(x) + 6\*I\*a)\*log(x) + 9\*I\*e^(-1/3\*(m + 1)\*log(c) - 2/3\*(m + 1)\*log(x) + 2\*I\*a) - 18\*I\*e^(-2/3\*(m + 1)\*log(c) - 4/3\*(m + 1)\*log(x) + 4\*I\*a) - 2\*I)\*e^(1/2\*(m + 1)\*log(c) + 2\*(m + 1)\*log(x) - 3\*I\*a)/(m + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sin^3 \left( a + \frac{\sqrt{-m^2 - 2m - 1} \log(cx^2)}{6} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*sin(a+1/6\*ln(c\*x\*\*2)\*(-(1+m)\*\*2)\*\*(1/2))\*\*3,x)

[Out] Integral(x\*\*m\*sin(a + sqrt(-m\*\*2 - 2\*m - 1)\*log(c\*x\*\*2)/6)\*\*3, x)

**Giac [C]** time = 2.45635, size = 1751, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sin(a+1/6\*log(c\*x^2)\*(-(1+m)^2)^(1/2))^3,x, algorithm="giac")

[Out] 1/8\*(I\*(m + 1)^2\*m\*x\*x^m\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 3\*I\*a) - 9\*I\*m^3\*x\*x^m\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 3\*I\*a) - I\*(m + 1)^2\*x\*x^m\*abs(m + 1)\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 3\*I\*a) + 9\*I\*m^2\*x\*x^m\*abs(m + 1)\*e^(1/2\*abs(m + 1)\*log(c) + abs(m + 1)\*log(x) - 3\*I\*a) - 27\*I\*(m + 1)^2\*m\*x\*x^m\*e^(1/6\*abs(m + 1)\*log(c) + 1/3\*abs(m + 1)\*log(x) - I\*a) + 27\*I\*m^3\*x\*x^m\*e^(1/6\*abs(m + 1)\*log(c) + 1/3\*abs(m + 1)\*log(x) - I\*a) + 9\*I\*(m + 1)^2\*x\*x^m\*abs(m + 1)\*e^(1/6\*abs(m + 1)\*log(c) + 1/3\*abs(m + 1)\*log(x) - I\*a) - 9\*I\*m^2\*x\*x^m\*abs(m + 1)\*e^(1/6\*abs(m + 1)\*log(c) + 1/3\*abs(m + 1)\*log(x) - I\*a) + 27\*I\*(m + 1)^2\*m\*x\*x^m\*e^(-1/6\*abs(m + 1)\*log(c) - 1/3\*abs(m + 1)\*log(x) + I\*a) - 27\*I\*m^3\*x\*x^m\*e^(-1/6\*abs(m + 1)\*log(c) - 1/3\*abs(m + 1)\*log(x) + I\*a) + 9\*I\*(m + 1)^2\*x\*x^m\*abs(m + 1)\*e^(-1/6\*abs(m + 1)\*log(c) - 1/3\*abs(m + 1)\*log(x) + I\*a) - 9\*I\*m^2\*x\*x^m\*abs(m + 1)\*e^(-1/6\*abs(m + 1)\*log(c) - 1/3\*abs(m + 1)\*log(x) + I\*a) - I\*(m + 1)^2\*m\*x\*x^m\*e^(-1/2\*abs(m + 1)\*log(c) - abs(m + 1)\*log(x) + 3\*I\*a) +

$$\begin{aligned}
& 9*I*m^3*x*x^m*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a) - I*(m+1)^2*x*x^m*abs(m+1)*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a)} \\
& + 9*I*m^2*x*x^m*abs(m+1)*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a)} \\
& + I*(m+1)^2*x*x^m*e^{(1/2*abs(m+1)*log(c) + abs(m+1)*log(x) - 3*I*a)} \\
& - 27*I*m^2*x*x^m*e^{(1/2*abs(m+1)*log(c) + abs(m+1)*log(x) - 3*I*a)} \\
& + 18*I*m*x*x^m*abs(m+1)*e^{(1/2*abs(m+1)*log(c) + abs(m+1)*log(x) - 3*I*a)} \\
& - 27*I*(m+1)^2*x*x^m*e^{(1/6*abs(m+1)*log(c) + 1/3*abs(m+1)*log(x) - I*a)} \\
& + 81*I*m^2*x*x^m*e^{(1/6*abs(m+1)*log(c) + 1/3*abs(m+1)*log(x) - I*a)} \\
& - 18*I*m*x*x^m*abs(m+1)*e^{(1/6*abs(m+1)*log(c) + 1/3*abs(m+1)*log(x) - I*a)} \\
& + 27*I*(m+1)^2*x*x^m*e^{(-1/6*abs(m+1)*log(c) - 1/3*abs(m+1)*log(x) + I*a)} \\
& - 81*I*m^2*x*x^m*e^{(-1/6*abs(m+1)*log(c) - 1/3*abs(m+1)*log(x) + I*a)} \\
& - 18*I*m*x*x^m*abs(m+1)*e^{(-1/6*abs(m+1)*log(c) - 1/3*abs(m+1)*log(x) + I*a)} \\
& - I*(m+1)^2*x*x^m*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a)} \\
& + 27*I*m^2*x*x^m*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a)} \\
& + 18*I*m*x*x^m*abs(m+1)*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a)} \\
& - 27*I*m*x*x^m*e^{(1/2*abs(m+1)*log(c) + abs(m+1)*log(x) - 3*I*a)} \\
& + 9*I*x*x^m*abs(m+1)*e^{(1/2*abs(m+1)*log(c) + abs(m+1)*log(x) - 3*I*a)} \\
& + 81*I*m*x*x^m*e^{(1/6*abs(m+1)*log(c) + 1/3*abs(m+1)*log(x) - I*a)} \\
& - 9*I*x*x^m*abs(m+1)*e^{(1/6*abs(m+1)*log(c) + 1/3*abs(m+1)*log(x) - I*a)} \\
& - 81*I*m*x*x^m*e^{(-1/6*abs(m+1)*log(c) - 1/3*abs(m+1)*log(x) + I*a)} \\
& - 9*I*x*x^m*abs(m+1)*e^{(-1/6*abs(m+1)*log(c) - 1/3*abs(m+1)*log(x) + I*a)} \\
& + 27*I*m*x*x^m*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a)} \\
& + 9*I*x*x^m*abs(m+1)*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a)} \\
& - 9*I*x*x^m*e^{(1/2*abs(m+1)*log(c) + abs(m+1)*log(x) - 3*I*a)} \\
& + 27*I*x*x^m*e^{(1/6*abs(m+1)*log(c) + 1/3*abs(m+1)*log(x) - I*a)} \\
& - 27*I*x*x^m*e^{(-1/6*abs(m+1)*log(c) - 1/3*abs(m+1)*log(x) + I*a)} \\
& + 9*I*x*x^m*e^{(-1/2*abs(m+1)*log(c) - abs(m+1)*log(x) + 3*I*a)} \\
& ) / ((m+1)^4 - 10*(m+1)^2*m^2 + 9*m^4 - 20*(m+1)^2*m + 36*m^3 - 10*(m+1)^2 + 54*m^2 + 36*m + 9)
\end{aligned}$$

### 3.52 $\int \sin^3 \left( a + \frac{1}{6}i \log(cx^2) \right) dx$

**Optimal.** Leaf size=98

$$-\frac{ie^{-3ia}cx^3}{16\sqrt{cx^2}} + \frac{9}{32}ie^{-ia}x\sqrt[6]{cx^2} - \frac{9ie^{ia}x}{16\sqrt[6]{cx^2}} + \frac{ie^{3ia}x \log(x)}{8\sqrt{cx^2}}$$

[Out]  $((-I/16)*c*x^3)/(E^((3*I)*a)*Sqrt[c*x^2]) - (((9*I)/16)*E^(I*a)*x)/(c*x^2)^(1/6) + (((9*I)/32)*x*(c*x^2)^(1/6))/E^(I*a) + ((I/8)*E^((3*I)*a)*x*Log[x])/Sqrt[c*x^2]$

**Rubi [A]** time = 0.0609948, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4483, 4489}

$$-\frac{ie^{-3ia}cx^3}{16\sqrt{cx^2}} + \frac{9}{32}ie^{-ia}x\sqrt[6]{cx^2} - \frac{9ie^{ia}x}{16\sqrt[6]{cx^2}} + \frac{ie^{3ia}x \log(x)}{8\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + (I/6)\*Log[c\*x^2]]^3,x]

[Out]  $((-I/16)*c*x^3)/(E^((3*I)*a)*Sqrt[c*x^2]) - (((9*I)/16)*E^(I*a)*x)/(c*x^2)^(1/6) + (((9*I)/32)*x*(c*x^2)^(1/6))/E^(I*a) + ((I/8)*E^((3*I)*a)*x*Log[x])/Sqrt[c*x^2]$

#### Rule 4483

Int[Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4489

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(m + 1)^p/(2^p\*b^p\*d^p\*p^p), Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \sin^3 \left( a + \frac{1}{6}i \log(cx^2) \right) dx &= \frac{x \operatorname{Subst} \left( \int \frac{\sin^3 \left( a + \frac{1}{6}i \log(x) \right)}{\sqrt{x}} dx, x, cx^2 \right)}{2\sqrt{cx^2}} \\ &= \frac{(ix) \operatorname{Subst} \left( \int \left( -e^{-3ia} + \frac{e^{3ia}}{x} - \frac{3e^{ia}}{x^{2/3}} + \frac{3e^{-ia}}{\sqrt[3]{x}} \right) dx, x, cx^2 \right)}{16\sqrt{cx^2}} \\ &= -\frac{ice^{-3ia}x^3}{16\sqrt{cx^2}} - \frac{9ie^{ia}x}{16\sqrt[6]{cx^2}} + \frac{9}{32}ie^{-ia}x\sqrt[6]{cx^2} + \frac{ie^{3ia}x \log(x)}{8\sqrt{cx^2}} \end{aligned}$$



**Mathematica [A]** time = 0.121187, size = 103, normalized size = 1.05

$$\frac{x \left( -2cx^2 \sin(3a) + 9 \sin(a) (cx^2)^{2/3} + 18 \sin(a) \sqrt[3]{cx^2} + 9i \cos(a) \sqrt[3]{cx^2} \left( \sqrt[3]{cx^2} - 2 \right) - 2i \cos(3a) (cx^2 - 2 \log(x)) - 4 \sin(3a) \right)}{32 \sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + (I/6)\*Log[c\*x^2]]^3,x]

[Out] (x\*((9\*I)\*(c\*x^2)^(1/3)\*(-2 + (c\*x^2)^(1/3))\*Cos[a] - (2\*I)\*Cos[3\*a]\*(c\*x^2 - 2\*Log[x]) + 18\*(c\*x^2)^(1/3)\*Sin[a] + 9\*(c\*x^2)^(2/3)\*Sin[a] - 2\*c\*x^2\*Sin[3\*a] - 4\*Log[x]\*Sin[3\*a]))/(32\*Sqrt[c\*x^2])

**Maple [B]** time = 0.081, size = 284, normalized size = 2.9

$$\left( -\frac{23i}{40}x + \frac{27x}{10} \tan\left(\frac{a}{2} + \frac{i}{12} \ln(cx^2)\right) + \frac{27x}{10} \left( \tan\left(\frac{a}{2} + \frac{i}{12} \ln(cx^2)\right) \right)^5 + \frac{33i}{8}x \left( \tan\left(\frac{a}{2} + \frac{i}{12} \ln(cx^2)\right) \right)^2 + \frac{23i}{40}x \left( \tan\left(\frac{a}{2} + \frac{i}{12} \ln(cx^2)\right) \right)^4 \right) / (32 \sqrt{cx^2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+1/6\*I\*ln(c\*x^2))^3,x)

[Out] (-23/40\*I\*x+27/10\*x\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))+27/10\*x\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^5+33/8\*I\*x\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^2+23/40\*I\*x\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^4-33/8\*I\*x\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^6-3/8\*x\*ln(c\*x^2)\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))+5/4\*x\*ln(c\*x^2)\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^3-3/8\*x\*ln(c\*x^2)\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^5+1/16\*I\*x\*ln(c\*x^2)-15/16\*I\*x\*ln(c\*x^2)\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^2+15/16\*I\*x\*ln(c\*x^2)\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^4-1/16\*I\*x\*ln(c\*x^2)\*tan(1/2\*a+1/12\*I\*ln(c\*x^2))^6)/(1+tan(1/2\*a+1/12\*I\*ln(c\*x^2))^2)^3

**Maxima [A]** time = 1.08594, size = 101, normalized size = 1.03

$$\frac{9c^{\frac{4}{3}}x^{\frac{4}{3}}(-i \cos(a) - \sin(a)) + 18cx^{\frac{2}{3}}(i \cos(a) - \sin(a)) + 2(cx^2(i \cos(3a) + \sin(3a)) + 2(-i \cos(3a) + \sin(3a)))}{32c^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+1/6\*I\*log(c\*x^2))^3,x, algorithm="maxima")

[Out] -1/32\*(9\*c^(4/3)\*x^(4/3)\*(-I\*cos(a) - sin(a)) + 18\*c\*x^(2/3)\*(I\*cos(a) - sin(a)) + 2\*(c\*x^2\*(I\*cos(3\*a) + sin(3\*a)) + 2\*(-I\*cos(3\*a) + sin(3\*a))\*log(x)))\*c^(2/3)/c^(7/6)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/6*I*log(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/6*I*ln(c*x**2))**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin\left(a + \frac{1}{6}i \log(cx^2)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+1/6*I*log(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(a + 1/6*I*log(c*x^2))^3, x)
```

### 3.53 $\int x \sqrt{\sin(a + b \log(cx^n))} dx$

**Optimal.** Leaf size=111

$$\frac{{}_2x^2\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}\left(-1 - \frac{4i}{bn}\right), \frac{1}{4}\left(3 - \frac{4i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sin(a + b \log(cx^n))}}{(4 - ibn) \sqrt{1 - e^{2ia}(cx^n)^{2ib}}}$$

[Out] (2\*x^2\*Hypergeometric2F1[-1/2, (-1 - (4\*I)/(b\*n))/4, (3 - (4\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/((4 - I\*b\*n)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

**Rubi [A]** time = 0.0860572, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4493, 4491, 364}

$$\frac{{}_2x^2 {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-1 - \frac{4i}{bn}\right); \frac{1}{4}\left(3 - \frac{4i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sin(a + b \log(cx^n))}}{(4 - ibn) \sqrt{1 - e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

[In] Int[x\*Sqrt[Sin[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x^2\*Hypergeometric2F1[-1/2, (-1 - (4\*I)/(b\*n))/4, (3 - (4\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/((4 - I\*b\*n)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x \sqrt{\sin(a + b \log(cx^n))} dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int x^{-1+\frac{2}{n}} \sqrt{\sin(a + b \log(x))} dx, x, cx^n\right)}{n} \\ &= \frac{\left(x^2 (cx^n)^{\frac{ib}{2}-\frac{2}{n}} \sqrt{\sin(a + b \log(cx^n))}\right) \text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{2}{n}} \sqrt{1 - e^{2ia} x^{2ib}} dx, x, cx^n\right)}{n \sqrt{1 - e^{2ia} (cx^n)^{2ib}}} \\ &= \frac{2x^2 {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-1 - \frac{4i}{bn}\right); \frac{1}{4}\left(3 - \frac{4i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right) \sqrt{\sin(a + b \log(cx^n))}}{(4 - ibn) \sqrt{1 - e^{2ia} (cx^n)^{2ib}}} \end{aligned}$$

**Mathematica [A]** time = 1.39445, size = 94, normalized size = 0.85

$$\frac{2x^2 \left(-1 + e^{2i(a+b \log(cx^n))}\right) \sqrt{\sin(a + b \log(cx^n))} \text{Hypergeometric2F1}\left(1, \frac{5}{4} - \frac{i}{bn}, \frac{3}{4} - \frac{i}{bn}, e^{2i(a+b \log(cx^n))}\right)}{-4 + ibn}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sqrt[Sin[a + b\*Log[c\*x^n]]], x]

[Out] (2\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x^2 \* Hypergeometric2F1[1, 5/4 - I/(b\*n), 3/4 - I/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))] \* Sqrt[Sin[a + b\*Log[c\*x^n]]]) / (-4 + I\*b\*n)

**Maple [F]** time = 0.349, size = 0, normalized size = 0.

$$\int x \sqrt{\sin(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(x\*sin(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{\sin(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(x\*sqrt(sin(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sin(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x*sqrt(sin(a + b*log(c*x**n))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sin(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(sin(b*log(c*x^n) + a)), x)
```

### 3.54 $\int \sqrt{\sin(a + b \log(cx^n))} dx$

**Optimal.** Leaf size=110

$$\frac{{}_2F_1\left(-\frac{1}{2}, -\frac{bn+2i}{4bn}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sin(a + b \log(cx^n))}}{(2 - ibn) \sqrt{1 - e^{2ia}(cx^n)^{2ib}}}$$

[Out] (2\*x\*Hypergeometric2F1[-1/2, -(2\*I + b\*n)/(4\*b\*n), (3 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/((2 - I\*b\*n)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

**Rubi [A]** time = 0.0749734, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4483, 4491, 364}

$$\frac{{}_2F_1\left(-\frac{1}{2}, -\frac{bn+2i}{4bn}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sin(a + b \log(cx^n))}}{(2 - ibn) \sqrt{1 - e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sin[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x\*Hypergeometric2F1[-1/2, -(2\*I + b\*n)/(4\*b\*n), (3 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/((2 - I\*b\*n)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

#### Rule 4483

Int[Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_)^(m\_.)\*Sin[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))]^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sqrt{\sin(a + b \log(cx^n))} dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sqrt{\sin(a + b \log(x))} dx, x, cx^n\right)}{n} \\ &= \frac{(x(cx^n)^{\frac{ib}{2}-\frac{1}{n}} \sqrt{\sin(a + b \log(cx^n))}) \operatorname{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1}{n}} \sqrt{1 - e^{2ia} x^{2ib}} dx, x, cx^n\right)}{n\sqrt{1 - e^{2ia} (cx^n)^{2ib}}} \\ &= \frac{2x {}_2F_1\left(-\frac{1}{2}, -\frac{2i+bn}{4bn}; \frac{1}{4}\left(3 - \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right) \sqrt{\sin(a + b \log(cx^n))}}{(2 - ibn)\sqrt{1 - e^{2ia} (cx^n)^{2ib}}} \end{aligned}$$

**Mathematica [A]** time = 1.32914, size = 96, normalized size = 0.87

$$\frac{2x \left(-1 + e^{2i(a+b \log(cx^n))}\right) \sqrt{\sin(a + b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(1, \frac{5}{4} - \frac{i}{2bn}, \frac{3}{4} - \frac{i}{2bn}, e^{2i(a+b \log(cx^n))}\right)}{-2 + ibn}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Sin[a + b\*Log[c\*x^n]]], x]

[Out] (2\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x \* Hypergeometric2F1[1, 5/4 - (I/2)/(b\*n), 3/4 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))] \* Sqrt[Sin[a + b\*Log[c\*x^n]]]) / (-2 + I\*b\*n)

**Maple [F]** time = 0.169, size = 0, normalized size = 0.

$$\int \sqrt{\sin(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(sin(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(sin(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(a + b*log(c*x**n))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a)), x)
```



$$3.55 \quad \int \frac{\sqrt{\sin(a+b \log(cx^n))}}{x} dx$$

**Optimal.** Leaf size=29

$$\frac{2E\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

[Out] (2\*EllipticE[(a - Pi/2 + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Rubi [A]** time = 0.0273205, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2639}

$$\frac{2E\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sin[a + b\*Log[c\*x^n]]]/x,x]

[Out] (2\*EllipticE[(a - Pi/2 + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Rule 2639**

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticE[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt{\sin(a+b \log(cx^n))}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{\sin(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b \log(cx^n)\right)\middle|2\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0834006, size = 32, normalized size = 1.1

$$-\frac{2E\left(\frac{1}{2}\left(-a-b \log(cx^n)+\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sin[a + b\*Log[c\*x^n]]]/x,x]

[Out] (-2\*EllipticE[(-a + Pi/2 - b\*Log[c\*x^n])/2, 2])/(b\*n)

**Maple [A]** time = 0.938, size = 129, normalized size = 4.5

$$-\frac{1}{bn \cos(a+b \ln(cx^n))} \sqrt{\sin(a+b \ln(cx^n))+1} \sqrt{-2 \sin(a+b \ln(cx^n))+2} \sqrt{-\sin(a+b \ln(cx^n))} \left( 2 \text{EllipticE}\left(\sqrt{\sin(a+b \ln(cx^n))}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a+b*ln(c*x^n))^(1/2)/x,x)`

[Out]  $-1/n*(\sin(a+b*\ln(c*x^n))+1)^{(1/2)}*(-2*\sin(a+b*\ln(c*x^n))+2)^{(1/2)}*(-\sin(a+b*\ln(c*x^n)))^{(1/2)}*(2*\text{EllipticE}((\sin(a+b*\ln(c*x^n))+1)^{(1/2)},1/2*2^{(1/2)})-\text{EllipticF}((\sin(a+b*\ln(c*x^n))+1)^{(1/2)},1/2*2^{(1/2)}))/\cos(a+b*\ln(c*x^n))/\sin(a+b*\ln(c*x^n))^{(1/2)}/b$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt(sin(b*log(c*x^n) + a))/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sin(b \log(cx^n) + a)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")`

[Out] `integral(sqrt(sin(b*log(c*x^n) + a))/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(a + b \log(cx^n))}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*ln(c*x**n))**(1/2)/x,x)`

[Out] `Integral(sqrt(sin(a + b*log(c*x**n)))/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")`

[Out] `integrate(sqrt(sin(b*log(c*x^n) + a))/x, x)`

$$3.56 \quad \int \frac{\sqrt{\sin(a+b \log(cx^n))}}{x^2} dx$$

**Optimal.** Leaf size=111

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}\left(-1 + \frac{2i}{bn}\right), \frac{1}{4}\left(3 + \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sin(a+b \log(cx^n))}}{x(2+ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

[Out] (-2\*Hypergeometric2F1[-1/2, (-1 + (2\*I)/(b\*n))/4, (3 + (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sin[a + b\*Log[c\*x^n]]]/((2 + I\*b\*n)\*x\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

**Rubi [A]** time = 0.0880578, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4493, 4491, 364}

$$\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(\frac{2i}{bn} - 1\right); \frac{1}{4}\left(3 + \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sin(a+b \log(cx^n))}}{x(2+ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sin[a + b\*Log[c\*x^n]]]/x^2, x]

[Out] (-2\*Hypergeometric2F1[-1/2, (-1 + (2\*I)/(b\*n))/4, (3 + (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sin[a + b\*Log[c\*x^n]]]/((2 + I\*b\*n)\*x\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[(e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{\sqrt{\sin(a + b \log(cx^n))}}{x^2} dx = \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \sqrt{\sin(a + b \log(x))} dx, x, cx^n\right)}{nx}$$

$$= \frac{\left((cx^n)^{\frac{ib}{2} + \frac{1}{n}} \sqrt{\sin(a + b \log(cx^n))}\right) \text{Subst}\left(\int x^{-1-\frac{ib}{2}-\frac{1}{n}} \sqrt{1 - e^{2ia} x^{2ib}} dx, x, cx^n\right)}{nx \sqrt{1 - e^{2ia} (cx^n)^{2ib}}}$$

$$= -\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-1 + \frac{2i}{bn}\right); \frac{1}{4}\left(3 + \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right) \sqrt{\sin(a + b \log(cx^n))}}{(2 + ibn)x \sqrt{1 - e^{2ia} (cx^n)^{2ib}}}$$

**Mathematica [A]** time = 1.40171, size = 99, normalized size = 0.89

$$\frac{2i(-1 + e^{2i(a+b \log(cx^n))}) \sqrt{\sin(a + b \log(cx^n))} \text{Hypergeometric2F1}\left(1, \frac{5}{4} + \frac{i}{2bn}, \frac{3}{4} + \frac{i}{2bn}, e^{2i(a+b \log(cx^n))}\right)}{x(bn - 2i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Sin[a + b\*Log[c\*x^n]]]/x^2,x]

[Out] ((-2\*I)\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n])))\*Hypergeometric2F1[1, 5/4 + (I/2)/(b\*n), 3/4 + (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/((-2\*I + b\*n)\*x)

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sqrt{\sin(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^(1/2)/x^2,x)

[Out] int(sin(a+b\*ln(c\*x^n))^(1/2)/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(b \log(cx^n) + a)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(sin(b\*log(c\*x^n) + a))/x^2, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(a + b \log(cx^n))}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(sin(a + b*log(c*x**n)))/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(b \log(cx^n) + a)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a))/x^2, x)
```

$$3.57 \quad \int \frac{\sqrt{\sin(a+b \log(cx^n))}}{x^3} dx$$

**Optimal.** Leaf size=111

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}\left(-1 + \frac{4i}{bn}\right), \frac{1}{4}\left(3 + \frac{4i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sin(a+b \log(cx^n))}}{x^2(4+ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

[Out] (-2\*Hypergeometric2F1[-1/2, (-1 + (4\*I)/(b\*n))/4, (3 + (4\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sin[a + b\*Log[c\*x^n]])/((4 + I\*b\*n)\*x^2\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

**Rubi [A]** time = 0.0897216, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4493, 4491, 364}

$$\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(\frac{4i}{bn} - 1\right); \frac{1}{4}\left(3 + \frac{4i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sin(a+b \log(cx^n))}}{x^2(4+ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sin[a + b\*Log[c\*x^n]]]/x^3,x]

[Out] (-2\*Hypergeometric2F1[-1/2, (-1 + (4\*I)/(b\*n))/4, (3 + (4\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sin[a + b\*Log[c\*x^n]])/((4 + I\*b\*n)\*x^2\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

#### Rule 4493

Int[((e\_)\*(x\_))^(m\_)\*Sin[(a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_)]\*(d\_)^(p\_), x\_Symbol] := Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[x^((m+1)/n-1)\*Sin[d\*(a+b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_)\*(x\_))^(m\_)\*Sin[(a\_) + Log[x\_]\*(b\_)]\*(d\_)^(p\_), x\_Symbol] := Dist[(Sin[d\*(a+b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/((c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{\sqrt{\sin(a + b \log(cx^n))}}{x^3} dx = \frac{(cx^n)^{2/n} \text{Subst}\left(\int x^{-1-\frac{2}{n}} \sqrt{\sin(a + b \log(x))} dx, x, cx^n\right)}{nx^2}$$

$$= \frac{\left((cx^n)^{\frac{ib}{2} + \frac{2}{n}} \sqrt{\sin(a + b \log(cx^n))}\right) \text{Subst}\left(\int x^{-1-\frac{ib}{2}-\frac{2}{n}} \sqrt{1 - e^{2ia} x^{2ib}} dx, x, cx^n\right)}{nx^2 \sqrt{1 - e^{2ia} (cx^n)^{2ib}}}$$

$$= \frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-1 + \frac{4i}{bn}\right); \frac{1}{4}\left(3 + \frac{4i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right) \sqrt{\sin(a + b \log(cx^n))}}{(4 + ibn)x^2 \sqrt{1 - e^{2ia} (cx^n)^{2ib}}}$$

**Mathematica [A]** time = 1.36427, size = 95, normalized size = 0.86

$$\frac{2i\left(-1 + e^{2i(a+b \log(cx^n))}\right) \sqrt{\sin(a + b \log(cx^n))} \text{Hypergeometric2F1}\left(1, \frac{5}{4} + \frac{i}{bn}, \frac{3}{4} + \frac{i}{bn}, e^{2i(a+b \log(cx^n))}\right)}{x^2(bn - 4i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Sin[a + b\*Log[c\*x^n]]]/x^3,x]

[Out] ((-2\*I)\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n])))\*Hypergeometric2F1[1, 5/4 + I/(b\*n), 3/4 + I/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/((-4\*I + b\*n)\*x^2)

**Maple [F]** time = 0.169, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \sqrt{\sin(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^(1/2)/x^3,x)

[Out] int(sin(a+b\*ln(c\*x^n))^(1/2)/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(b \log(cx^n) + a)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(sin(b\*log(c\*x^n) + a))/x^3, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(a + b \log(cx^n))}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(sin(a + b*log(c*x**n)))/x**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sin(b \log(cx^n) + a)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*log(c*x^n) + a))/x^3, x)
```



### 3.58 $\int x \sin^{\frac{3}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=111

$$\frac{2x^2 \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{4i}{bn}\right), \frac{1}{4}\left(1 - \frac{4i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b \log(cx^n))}{(4 - 3ibn)(1 - e^{2ia}(cx^n)^{2ib})^{3/2}}$$

[Out] (2\*x^2\*Hypergeometric2F1[-3/2, (-3 - (4\*I)/(b\*n))/4, (1 - (4\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^(3/2))/((4 - (3\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(3/2))

**Rubi [A]** time = 0.0780123, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4493, 4491, 364}

$$\frac{2x^2 {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{4i}{bn}\right); \frac{1}{4}\left(1 - \frac{4i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b \log(cx^n))}{(4 - 3ibn)(1 - e^{2ia}(cx^n)^{2ib})^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x\*Sin[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^2\*Hypergeometric2F1[-3/2, (-3 - (4\*I)/(b\*n))/4, (1 - (4\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^(3/2))/((4 - (3\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(3/2))

#### Rule 4493

Int[((e\_)\*(x\_))^(m\_)\*Sin[(a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_)]\*(d\_)]^(p\_), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_)\*(x\_))^(m\_)\*Sin[(a\_) + Log[x]\*(b\_)]\*(d\_)]^(p\_), x\_Symbol] :> Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[(e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x \sin^{\frac{3}{2}}(a + b \log(cx^n)) dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int x^{-1+\frac{2}{n}} \sin^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(x^2 (cx^n)^{\frac{3ib}{2}-\frac{2}{n}} \sin^{\frac{3}{2}}(a + b \log(cx^n))) \text{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{2}{n}} (1 - e^{2ia} x^{2ib})^{3/2} dx, x, cx^n\right)}{n (1 - e^{2ia} (cx^n)^{2ib})^{3/2}} \\ &= \frac{2x^2 {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{4i}{bn}\right); \frac{1}{4}\left(1 - \frac{4i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b \log(cx^n))}{(4 - 3ibn) (1 - e^{2ia} (cx^n)^{2ib})^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 1.77717, size = 159, normalized size = 1.43

$$\frac{x^2 \left(6b^2 n^2 (-1 + e^{2ia} (cx^n)^{2ib}) \text{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{bn}, \frac{5}{4} - \frac{i}{bn}, e^{2i(a+b \log(cx^n))}\right) + (4 + ibn) (3bn \sin(2(a + b \log(cx^n))))\right)}{(-4 - ibn) (9b^2 n^2 + 16) \sqrt{\sin(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sin[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (x^2\*(6\*b^2\*n^2\*(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, 3/4 - I/(b\*n), 5/4 - I/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))] + (4 + I\*b\*n)\*(-8\*Sin[a + b\*Log[c\*x^n]]^2 + 3\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])]))/((-4 - I\*b\*n)\*(16 + 9\*b^2\*n^2)\*Sqrt[Sin[a + b\*Log[c\*x^n]]])

**Maple [F]** time = 0.167, size = 0, normalized size = 0.

$$\int x (\sin(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(x\*sin(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sin(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(x\*sin(b\*log(c\*x^n) + a)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sin(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*log(c*x^n) + a)^(3/2), x)
```

### 3.59 $\int \sin^{\frac{3}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=109

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right), \frac{1}{4}\left(1 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b \log(cx^n))}{(2 - 3ibn)(1 - e^{2ia}(cx^n)^{2ib})^{3/2}}$$

[Out] (2\*x\*Hypergeometric2F1[-3/2, (-3 - (2\*I)/(b\*n))/4, (1 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^(3/2)/((2 - (3\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^3/2)

**Rubi [A]** time = 0.0727374, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4483, 4491, 364}

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right); \frac{1}{4}\left(1 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b \log(cx^n))}{(2 - 3ibn)(1 - e^{2ia}(cx^n)^{2ib})^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x\*Hypergeometric2F1[-3/2, (-3 - (2\*I)/(b\*n))/4, (1 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^(3/2)/((2 - (3\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^3/2)

#### Rule 4483

Int[Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sin^{\frac{3}{2}}(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sin^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{\left(x(cx^n)^{\frac{3ib}{2}-\frac{1}{n}} \sin^{\frac{3}{2}}(a + b \log(cx^n))\right) \operatorname{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1}{n}} (1 - e^{2ia}x^{2ib})^{3/2} dx, x, cx^n\right)}{n(1 - e^{2ia}(cx^n)^{2ib})^{3/2}} \\ &= \frac{2x {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right); \frac{1}{4}\left(1 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b \log(cx^n))}{(2 - 3ibn)(1 - e^{2ia}(cx^n)^{2ib})^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 1.79785, size = 161, normalized size = 1.48

$$\frac{x \left(6ib^2n^2 (-1 + e^{2ia}(cx^n)^{2ib}) \operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, e^{2i(a+b \log(cx^n))}\right) + (bn - 2i) (4 \sin^2(a + b \log(cx^n)))\right)}{(bn - 2i) (9b^2n^2 + 4) \sqrt{\sin(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (x\*((6\*I)\*b^2\*n^2\*(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, 3/4 - (I/2)/(b\*n), 5/4 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))] + (-2\*I + b\*n)\*(4\*Sin[a + b\*Log[c\*x^n]]^2 - 3\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])])))/((-2\*I + b\*n)\*(4 + 9\*b^2\*n^2)\*Sqrt[Sin[a + b\*Log[c\*x^n]]])

**Maple [F]** time = 0.166, size = 0, normalized size = 0.

$$\int (\sin(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(sin(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2), x)
```

$$3.60 \quad \int \frac{\sin^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=68

$$\frac{2\text{EllipticF}\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right), 2\right)}{3bn} - \frac{2\sqrt{\sin(a+b \log(cx^n))} \cos(a+b \log(cx^n))}{3bn}$$

[Out] (2\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2])/(3\*b\*n) - (2\*Cos[a + b\*Log[c\*x^n]]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(3\*b\*n)

**Rubi [A]** time = 0.0418833, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2635, 2641}

$$\frac{2F\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{3bn} - \frac{2\sqrt{\sin(a+b \log(cx^n))} \cos(a+b \log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^(3/2)/x, x]

[Out] (2\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2])/(3\*b\*n) - (2\*Cos[a + b\*Log[c\*x^n]]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(3\*b\*n)

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x] \* (b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^3(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sin^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{2 \cos(a+b \log(cx^n)) \sqrt{\sin(a+b \log(cx^n))}}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sin(a+bx)}} dx, x, \log(cx^n)\right)}{3n} \\ &= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b \log(cx^n)\right)\middle|2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n)) \sqrt{\sin(a+b \log(cx^n))}}{3bn} \end{aligned}$$

**Mathematica [A]** time = 0.141985, size = 58, normalized size = 0.85

$$\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(-2a-2b \log(cx^n)+\pi), 2\right) + \sqrt{\sin(a+b \log(cx^n))} \cos(a+b \log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^(3/2)/x,x]

[Out] (-2\*(EllipticF[(-2\*a + Pi - 2\*b\*Log[c\*x^n])/4, 2] + Cos[a + b\*Log[c\*x^n]]\*Sqrt[Sin[a + b\*Log[c\*x^n]]]))/(3\*b\*n)

**Maple [A]** time = 1.328, size = 131, normalized size = 1.9

$$\frac{1}{bn \cos(a + b \ln(cx^n))} \left( \frac{1}{3} \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n)) + 2} \sqrt{-\sin(a + b \ln(cx^n))} \text{EllipticF} \left( \sqrt{\sin(a + b \ln(cx^n))} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^(3/2)/x,x)

[Out] 1/n\*(1/3\*(sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticF((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))-2/3\*cos(a+b\*ln(c\*x^n))^2\*sin(a+b\*ln(c\*x^n))/cos(a+b\*ln(c\*x^n))/sin(a+b\*ln(c\*x^n))^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^(3/2)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sin(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="fricas")

[Out] integral(sin(b\*log(c\*x^n) + a)^(3/2)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(sin(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x, x)
```

$$3.61 \quad \int \frac{\sin^2(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=111

$$\frac{{}_2\text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}\left(-3 + \frac{2i}{bn}\right), \frac{1}{4}\left(1 + \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a+b \log(cx^n))}{x(2+3ibn)\left(1 - e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

[Out] (-2\*Hypergeometric2F1[-3/2, (-3 + (2\*I)/(b\*n))/4, (1 + (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^(3/2))/((2 + (3\*I)\*b\*n)\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)

**Rubi [A]** time = 0.0875124, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4493, 4491, 364}

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(\frac{2i}{bn} - 3\right); \frac{1}{4}\left(1 + \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a+b \log(cx^n))}{x(2+3ibn)\left(1 - e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^(3/2)/x^2,x]

[Out] (-2\*Hypergeometric2F1[-3/2, (-3 + (2\*I)/(b\*n))/4, (1 + (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^(3/2))/((2 + (3\*I)\*b\*n)\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)

#### Rule 4493

Int[((e\_)\*(x\_))^(m\_)\*Sin[(a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_)]\*(d\_)]^(p\_), x\_Symbol] := Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[x^((m+1)/n-1)\*Sin[d\*(a+b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_)\*(x\_))^(m\_)\*Sin[(a\_) + Log[x\_]\*(b\_)]\*(d\_)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a+b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/((c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{\sin^{\frac{3}{2}}(a + b \log(cx^n))}{x^2} dx = \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \sin^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{nx}$$

$$= \frac{\left((cx^n)^{\frac{3ib}{2} + \frac{1}{n}} \sin^{\frac{3}{2}}(a + b \log(cx^n))\right) \text{Subst}\left(\int x^{-1-\frac{3ib}{2}-\frac{1}{n}} (1 - e^{2ia} x^{2ib})^{3/2} dx, x, cx^n\right)}{nx (1 - e^{2ia} (cx^n)^{2ib})^{3/2}}$$

$$= -\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 + \frac{2i}{bn}\right); \frac{1}{4}\left(1 + \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b \log(cx^n))}{(2 + 3ibn)x (1 - e^{2ia} (cx^n)^{2ib})^{3/2}}$$

**Mathematica [A]** time = 1.15841, size = 172, normalized size = 1.55

$$\frac{6ib^2n^2(-1 + e^{2ia}(cx^n)^{2ib}) \text{Hypergeometric2F1}\left(1, \frac{3}{4} + \frac{i}{2bn}, \frac{5}{4} + \frac{i}{2bn}, e^{2i(a+b \log(cx^n))}\right) - (bn + 2i)(4 \sin^2(a + b \log(cx^n)))}{x(bn + 2i)(3bn - 2i)(3bn + 2i)\sqrt{\sin(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^(3/2)/x^2, x]

[Out] ((6\*I)\*b^2\*n^2\*(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, 3/4 + (I/2)/(b\*n), 5/4 + (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))] - (2\*I + b\*n)\*(4\*Sin[a + b\*Log[c\*x^n]]^2 + 3\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])]))/((2\*I + b\*n)\*(-2\*I + 3\*b\*n)\*(2\*I + 3\*b\*n)\*x\*Sqrt[Sin[a + b\*Log[c\*x^n]]])

**Maple [F]** time = 0.166, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\sin(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^(3/2)/x^2, x)

[Out] int(sin(a+b\*ln(c\*x^n))^(3/2)/x^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^(3/2)/x^2, x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^(3/2)/x^2, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x^2, x)
```

$$3.62 \quad \int \frac{\sin^2(a+b \log(cx^n))}{x^3} dx$$

**Optimal.** Leaf size=111

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}\left(-3 + \frac{4i}{bn}\right), \frac{1}{4}\left(1 + \frac{4i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a+b \log(cx^n))}{x^2(4+3ibn)(1-e^{2ia}(cx^n)^{2ib})^{3/2}}$$

[Out] (-2\*Hypergeometric2F1[-3/2, (-3 + (4\*I)/(b\*n))/4, (1 + (4\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^(3/2))/((4 + (3\*I)\*b\*n)\*x^2\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)

**Rubi [A]** time = 0.0869713, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4493, 4491, 364}

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(\frac{4i}{bn} - 3\right); \frac{1}{4}\left(1 + \frac{4i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a+b \log(cx^n))}{x^2(4+3ibn)(1-e^{2ia}(cx^n)^{2ib})^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^(3/2)/x^3, x]

[Out] (-2\*Hypergeometric2F1[-3/2, (-3 + (4\*I)/(b\*n))/4, (1 + (4\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^(3/2))/((4 + (3\*I)\*b\*n)\*x^2\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[x^((m+1)/n-1)\*Sin[d\*(a+b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] :> Dist[(Sin[d\*(a+b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/((c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{\sin^{\frac{3}{2}}(a + b \log(cx^n))}{x^3} dx = \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int x^{-1-\frac{2}{n}} \sin^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{nx^2}$$

$$= \frac{\left((cx^n)^{\frac{3ib}{2} + \frac{2}{n}} \sin^{\frac{3}{2}}(a + b \log(cx^n))\right) \operatorname{Subst}\left(\int x^{-1-\frac{3ib}{2}-\frac{2}{n}} (1 - e^{2ia} x^{2ib})^{3/2} dx, x, cx^n\right)}{nx^2 (1 - e^{2ia} (cx^n)^{2ib})^{3/2}}$$

$$= -\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 + \frac{4i}{bn}\right); \frac{1}{4}\left(1 + \frac{4i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b \log(cx^n))}{(4 + 3ibn)x^2 (1 - e^{2ia} (cx^n)^{2ib})^{3/2}}$$

**Mathematica [A]** time = 1.19889, size = 168, normalized size = 1.51

$$\frac{6ib^2n^2(-1 + e^{2ia}(cx^n)^{2ib}) \operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} + \frac{i}{bn}, \frac{5}{4} + \frac{i}{bn}, e^{2i(a+b \log(cx^n))}\right) - (bn + 4i)(8 \sin^2(a + b \log(cx^n)) + 3)}{x^2(bn + 4i)(3bn - 4i)(3bn + 4i)\sqrt{\sin(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^(3/2)/x^3,x]

[Out] ((6\*I)\*b^2\*n^2\*(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, 3/4 + I/(b\*n), 5/4 + I/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))] - (4\*I + b\*n)\*(8\*Sin[a + b\*Log[c\*x^n]]^2 + 3\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])]))/((4\*I + b\*n)\*(-4\*I + 3\*b\*n)\*(4\*I + 3\*b\*n)\*x^2\*Sqrt[Sin[a + b\*Log[c\*x^n]])]

**Maple [F]** time = 0.16, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (\sin(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^(3/2)/x^3,x)

[Out] int(sin(a+b\*ln(c\*x^n))^(3/2)/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^(3/2)/x^3, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^(3/2)/x^3, x)
```

### 3.63 $\int \frac{1}{\sqrt{\sin(a+b \log(cx^n))}} dx$

**Optimal.** Leaf size=109

$$\frac{2x\sqrt{1 - e^{2ia} (cx^n)^{2ib}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right), \frac{1}{4}\left(5 - \frac{2i}{bn}\right), e^{2ia} (cx^n)^{2ib}\right)}{(2 + ibn)\sqrt{\sin(a + b \log(cx^n))}}$$

[Out] (2\*x\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, (1 - (2\*I)/(b\*n))/4, (5 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 + I\*b\*n)\*Sqrt[Sin[a + b\*Log[c\*x^n]]])

**Rubi [A]** time = 0.0695574, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4483, 4491, 364}

$$\frac{2x\sqrt{1 - e^{2ia} (cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right); \frac{1}{4}\left(5 - \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{(2 + ibn)\sqrt{\sin(a + b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Sin[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, (1 - (2\*I)/(b\*n))/4, (5 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 + I\*b\*n)\*Sqrt[Sin[a + b\*Log[c\*x^n]]])

#### Rule 4483

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_), x\_Symbol] :> Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\int \frac{1}{\sqrt{\sin(a+b \log(cx^n))}} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sqrt{\sin(a+b \log(x))}} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{ib}{2}-\frac{1}{n}} \sqrt{1-e^{2ia}(cx^n)^{2ib}}\right) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1}{n}}}{\sqrt{1-e^{2ia}x^{2ib}}} dx, x, cx^n\right)}{n\sqrt{\sin(a+b \log(cx^n))}}$$

$$= \frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}\left(1-\frac{2i}{bn}\right); \frac{1}{4}\left(5-\frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{(2+ibn)\sqrt{\sin(a+b \log(cx^n))}}$$

**Mathematica [A]** time = 0.382072, size = 96, normalized size = 0.88

$$\frac{2x(-1 + e^{2i(a+b \log(cx^n))}) \operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, e^{2i(a+b \log(cx^n))}\right)}{(-2 - ibn)\sqrt{\sin(a+b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[Sin[a + b\*Log[c\*x^n]]], x]

[Out] (2\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x \* Hypergeometric2F1[1, 3/4 - (I/2)/(b\*n), 5/4 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))]) / ((-2 - I\*b\*n) \* Sqrt[Sin[a + b\*Log[c\*x^n]]])

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(a+b \ln(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(1/sin(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(1/sqrt(sin(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sin(a + b*log(c*x**n))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sin(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sin(b*log(c*x^n) + a)), x)
```

$$3.64 \quad \int \frac{1}{x\sqrt{\sin(a+b \log(cx^n))}} dx$$

**Optimal.** Leaf size=29

$$\frac{2\text{EllipticF}\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right), 2\right)}{bn}$$

[Out] (2\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Rubi [A]** time = 0.0265995, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2641}

$$\frac{2F\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[Sin[a + b\*Log[c\*x^n]]]), x]

[Out] (2\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Rule 2641**

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{1}{x\sqrt{\sin(a+b \log(cx^n))}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sin(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b \log(cx^n)\right)\middle|2\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0906466, size = 32, normalized size = 1.1

$$-\frac{2\text{EllipticF}\left(\frac{1}{2}\left(-a-b \log(cx^n)+\frac{\pi}{2}\right), 2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[Sin[a + b\*Log[c\*x^n]]]), x]

[Out] (-2\*EllipticF[(-a + Pi/2 - b\*Log[c\*x^n])/2, 2])/(b\*n)

**Maple [A]** time = 0.692, size = 102, normalized size = 3.5

$$\frac{1}{bn \cos(a+b \ln(cx^n))} \sqrt{\sin(a+b \ln(cx^n))+1} \sqrt{-2 \sin(a+b \ln(cx^n))+2} \sqrt{-\sin(a+b \ln(cx^n))} \text{EllipticF}\left(\sqrt{\sin(a+b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/sin(a+b*ln(c*x^n))^(1/2),x)`

[Out] `1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))/cos(a+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\sin(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(x*sqrt(sin(b*log(c*x^n) + a))), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\sqrt{\sin(b \log(cx^n) + a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")`

[Out] `integral(1/(x*sqrt(sin(b*log(c*x^n) + a))), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\sin(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/sin(a+b*ln(c*x**n))**(1/2),x)`

[Out] `Integral(1/(x*sqrt(sin(a + b*log(c*x**n))))), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\sin(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(x*sqrt(sin(b*log(c*x^n) + a))), x)`

$$3.65 \quad \int \frac{1}{\sin^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=109

$$\frac{2x \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{3/2} \text{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), \frac{1}{4}\left(7 - \frac{2i}{bn}\right), e^{2ia} (cx^n)^{2ib}\right)}{(2 + 3ibn) \sin^{\frac{3}{2}}(a + b \log(cx^n))}$$

[Out] (2\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Hypergeometric2F1[3/2, (3 - (2\*I)/(b\*n))/4, (7 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/((2 + (3\*I)\*b\*n)\*Sin[a + b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.0703636, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4483, 4491, 364}

$$\frac{2x \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right); \frac{1}{4}\left(7 - \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{(2 + 3ibn) \sin^{\frac{3}{2}}(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^(-3/2), x]

[Out] (2\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Hypergeometric2F1[3/2, (3 - (2\*I)/(b\*n))/4, (7 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/((2 + (3\*I)\*b\*n)\*Sin[a + b\*Log[c\*x^n]]^(3/2))

#### Rule 4483

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sin^{\frac{3}{2}}(a + b \log(cx^n))} dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sin^{\frac{3}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n} \\
&= \frac{\left(x(cx^n)^{-\frac{3ib}{2}-\frac{1}{n}} (1 - e^{2ia} (cx^n)^{2ib})^{3/2}\right) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1}{n}}}{(1-e^{2ia}x^{2ib})^{3/2}} dx, x, cx^n\right)}{n \sin^{\frac{3}{2}}(a + b \log(cx^n))} \\
&= \frac{2x (1 - e^{2ia} (cx^n)^{2ib})^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4} \left(3 - \frac{2i}{bn}\right); \frac{1}{4} \left(7 - \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{(2 + 3ibn) \sin^{\frac{3}{2}}(a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.917895, size = 96, normalized size = 0.88

$$\frac{2x \left(-1 + e^{2i(a+b \log(cx^n))}\right) \operatorname{Hypergeometric2F1}\left(1, \frac{1}{4} - \frac{i}{2bn}, \frac{7}{4} - \frac{i}{2bn}, e^{2i(a+b \log(cx^n))}\right)}{(-2 - 3ibn) \sin^{\frac{3}{2}}(a + b \log(cx^n))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^(-3/2), x]

[Out] (2\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x \* Hypergeometric2F1[1, 1/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))]) / ((-2 - (3\*I)\*b\*n) \* Sin[a + b\*Log[c\*x^n]]^(3/2))

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int (\sin(a + b \ln(cx^n)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(1/sin(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^(-3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Integral(sin(a + b\*log(c\*x\*\*n))\*\*(-3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] integrate(sin(b\*log(c\*x^n) + a)^(-3/2), x)

$$3.66 \quad \int \frac{1}{x \sin^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=64

$$-\frac{2E\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn} - \frac{2 \cos(a+b \log(cx^n))}{bn \sqrt{\sin(a+b \log(cx^n))}}$$

[Out]  $(-2*\text{EllipticE}[(a - \text{Pi}/2 + b*\text{Log}[c*x^n])/2, 2])/(b*n) - (2*\text{Cos}[a + b*\text{Log}[c*x^n]])/(b*n*\text{Sqrt}[\text{Sin}[a + b*\text{Log}[c*x^n]]])$

**Rubi [A]** time = 0.0423166, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2636, 2639}

$$-\frac{2E\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn} - \frac{2 \cos(a+b \log(cx^n))}{bn \sqrt{\sin(a+b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{Sin}[a + b*\text{Log}[c*x^n]]^{(3/2)}), x]$

[Out]  $(-2*\text{EllipticE}[(a - \text{Pi}/2 + b*\text{Log}[c*x^n])/2, 2])/(b*n) - (2*\text{Cos}[a + b*\text{Log}[c*x^n]])/(b*n*\text{Sqrt}[\text{Sin}[a + b*\text{Log}[c*x^n]]])$

#### Rule 2636

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)})/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n+2)}, x], x] /;$  FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 2639

$\text{Int}[\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_*)]], x\_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /;$  FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x \sin^{\frac{3}{2}}(a+b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{2 \cos(a+b \log(cx^n))}{bn \sqrt{\sin(a+b \log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\sin(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b \log(cx^n)\right)\middle|2\right)}{bn} - \frac{2 \cos(a+b \log(cx^n))}{bn \sqrt{\sin(a+b \log(cx^n))}} \end{aligned}$$

**Mathematica [A]** time = 0.183536, size = 57, normalized size = 0.89

$$\frac{2\left(E\left(\frac{1}{4}(-2a-2b \log(cx^n)+\pi)\middle|2\right)-\frac{\cos(a+b \log(cx^n))}{\sqrt{\sin(a+b \log(cx^n))}}\right)}{bn}$$



Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sin[a + b\*Log[c\*x^n]]^(3/2)),x]

[Out] (2\*(EllipticE[(-2\*a + Pi - 2\*b\*Log[c\*x^n])/4, 2] - Cos[a + b\*Log[c\*x^n]]/Sqrt[Sin[a + b\*Log[c\*x^n]]]))/(b\*n)

**Maple [A]** time = 1.102, size = 190, normalized size = 3.

$$\frac{1}{bn \cos(a + b \ln(cx^n))} \left( 2 \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n)) + 2} \sqrt{-\sin(a + b \ln(cx^n))} \text{EllipticE} \left( \sqrt{\sin(a + b \ln(cx^n))} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sin(a+b\*ln(c\*x^n))^(3/2),x)

[Out] 1/n\*(2\*(sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticE((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))-sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticF((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))-2\*cos(a+b\*ln(c\*x^n))^2)/cos(a+b\*ln(c\*x^n))/sin(a+b\*ln(c\*x^n))^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sin(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sin(a+b\*log(c\*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x\*sin(b\*log(c\*x^n) + a)^(3/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\sqrt{\sin(b \log(cx^n) + a)}}{x \cos(b \log(cx^n) + a)^2 - x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sin(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] integral(-sqrt(sin(b\*log(c\*x^n) + a))/(x\*cos(b\*log(c\*x^n) + a)^2 - x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sin(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sin(b*log(c*x^n) + a)^(3/2)), x)
```

$$3.67 \quad \int \frac{1}{\sin^2(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=109

$$\frac{2x \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{5/2} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right), \frac{1}{4}\left(9 - \frac{2i}{bn}\right), e^{2ia} (cx^n)^{2ib}\right)}{(2 + 5ibn) \sin^2(a + b \log(cx^n))}$$

[Out] (2\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(5/2)\*Hypergeometric2F1[5/2, (5 - (2\*I)/(b\*n))/4, (9 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 + (5\*I)\*b\*n)\*Sin[a + b\*Log[c\*x^n]]^(5/2))

**Rubi [A]** time = 0.0738603, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4483, 4491, 364}

$$\frac{2x \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right); \frac{1}{4}\left(9 - \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{(2 + 5ibn) \sin^2(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^(-5/2), x]

[Out] (2\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(5/2)\*Hypergeometric2F1[5/2, (5 - (2\*I)/(b\*n))/4, (9 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 + (5\*I)\*b\*n)\*Sin[a + b\*Log[c\*x^n]]^(5/2))

#### Rule 4483

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{\sin^{\frac{5}{2}}(a + b \log(cx^n))} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sin^{\frac{5}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{(x(cx^n)^{-\frac{5ib}{2}-\frac{1}{n}} (1 - e^{2ia}(cx^n)^{2ib})^{5/2}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1}{n}}}{(1-e^{2ia}x^{2ib})^{5/2}} dx, x, cx^n\right)}{n \sin^{\frac{5}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x(1 - e^{2ia}(cx^n)^{2ib})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right); \frac{1}{4}\left(9 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{(2 + 5ibn) \sin^{\frac{5}{2}}(a + b \log(cx^n))}$$

**Mathematica [A]** time = 1.50711, size = 125, normalized size = 1.15

$$\frac{2x \left( i(bn + 2i) (-1 + e^{2ia}(cx^n)^{2ib}) \operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, e^{2i(a+b \log(cx^n))}\right) - bn \cot(a + b \log(cx^n)) - 2 \right)}{3b^2 n^2 \sqrt{\sin(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^(-5/2), x]

[Out] (2\*x\*(-2 - b\*n\*Cot[a + b\*Log[c\*x^n]] + I\*(2\*I + b\*n)\*(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, 3/4 - (I/2)/(b\*n), 5/4 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))])/(3\*b^2\*n^2\*Sqrt[Sin[a + b\*Log[c\*x^n]]])

**Maple [F]** time = 0.167, size = 0, normalized size = 0.

$$\int (\sin(a + b \ln(cx^n)))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(a+b\*ln(c\*x^n))^(5/2), x)

[Out] int(1/sin(a+b\*ln(c\*x^n))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*log(c\*x^n))^(5/2), x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^(-5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*log(c\*x^n))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a+b\*log(c\*x^n))^(5/2),x, algorithm="giac")

[Out] integrate(sin(b\*log(c\*x^n) + a)^(-5/2), x)

$$3.68 \quad \int \frac{1}{x \sin^{\frac{5}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=68

$$\frac{2\text{EllipticF}\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right), 2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n))}{3bn \sin^{\frac{3}{2}}(a+b \log(cx^n))}$$

[Out] (2\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2])/(3\*b\*n) - (2\*Cos[a + b\*Log[c\*x^n]])/(3\*b\*n\*Sin[a + b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.0421785, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2636, 2641}

$$\frac{2F\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n))}{3bn \sin^{\frac{3}{2}}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sin[a + b\*Log[c\*x^n]]^(5/2)),x]

[Out] (2\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2])/(3\*b\*n) - (2\*Cos[a + b\*Log[c\*x^n]])/(3\*b\*n\*Sin[a + b\*Log[c\*x^n]]^(3/2))

#### Rule 2636

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1))/(b\*d\*(n + 1)), x] + Dist[(n + 2)/(b^2\*(n + 1)), Int[(b\*Sin[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x \sin^{\frac{5}{2}}(a+b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{2 \cos(a+b \log(cx^n))}{3bn \sin^{\frac{3}{2}}(a+b \log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sin(a+bx)}} dx, x, \log(cx^n)\right)}{3n} \\ &= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b \log(cx^n)\right)\middle|2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n))}{3bn \sin^{\frac{3}{2}}(a+b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.214424, size = 61, normalized size = 0.9

$$\frac{2 \left( \text{EllipticF} \left( \frac{1}{4} (2a + 2b \log(cx^n) - \pi), 2 \right) - \frac{\cos(a+b \log(cx^n))}{\sin^2(a+b \log(cx^n))} \right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sin[a + b\*Log[c\*x^n]]^(5/2)),x]

[Out] (2\*(EllipticF[(2\*a - Pi + 2\*b\*Log[c\*x^n])/4, 2] - Cos[a + b\*Log[c\*x^n]]/Sin[a + b\*Log[c\*x^n]]^(3/2)))/(3\*b\*n)

**Maple [A]** time = 1.235, size = 131, normalized size = 1.9

$$\frac{1}{3bn \cos(a + b \ln(cx^n))} \left( \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n)) + 2} \sqrt{-\sin(a + b \ln(cx^n))} \text{EllipticF} \left( \sqrt{\sin(a + b \ln(cx^n)) + 1}, \frac{1}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sin(a+b\*ln(c\*x^n))^(5/2),x)

[Out] 1/3/n/sin(a+b\*ln(c\*x^n))^(3/2)\*((sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticF((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))\*sin(a+b\*ln(c\*x^n))-2\*cos(a+b\*ln(c\*x^n))^2)/cos(a+b\*ln(c\*x^n))/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sin(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sin(a+b\*log(c\*x^n))^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x\*sin(b\*log(c\*x^n) + a)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{1}{(x \cos(b \log(cx^n) + a)^2 - x) \sqrt{\sin(b \log(cx^n) + a)}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sin(a+b\*log(c\*x^n))^(5/2),x, algorithm="fricas")

[Out] integral(-1/((x\*cos(b\*log(c\*x^n) + a)^2 - x)\*sqrt(sin(b\*log(c\*x^n) + a))), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sin(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sin(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sin(a+b\*log(c\*x^n))^(5/2),x, algorithm="giac")

[Out] integrate(1/(x\*sin(b\*log(c\*x^n) + a)^(5/2)), x)



$$3.69 \quad \int \frac{1}{\sin^{\frac{3}{2}}(a-2i \log(cx))} dx$$

**Optimal.** Leaf size=49

$$\frac{e^{-2ia} (1 - e^{2ia} c^4 x^4)}{2c^4 x^3 \sin^{\frac{3}{2}}(a - 2i \log(cx))}$$

[Out] (1 - c^4\*E^((2\*I)\*a)\*x^4)/(2\*c^4\*E^((2\*I)\*a)\*x^3\*Sin[a - (2\*I)\*Log[c\*x]]^(3/2))

**Rubi [A]** time = 0.0389924, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4483, 4481, 261}

$$\frac{e^{-2ia} (1 - e^{2ia} c^4 x^4)}{2c^4 x^3 \sin^{\frac{3}{2}}(a - 2i \log(cx))}$$

Antiderivative was successfully verified.

[In] Int[Sin[a - (2\*I)\*Log[c\*x]]^(-3/2), x]

[Out] (1 - c^4\*E^((2\*I)\*a)\*x^4)/(2\*c^4\*E^((2\*I)\*a)\*x^3\*Sin[a - (2\*I)\*Log[c\*x]]^(3/2))

#### Rule 4483

Int[Sin[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4481

Int[Sin[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, p}, x] && !IntegerQ[p]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\int \frac{1}{\sin^{\frac{3}{2}}(a - 2i \log(cx))} dx = \frac{\text{Subst}\left(\int \frac{1}{\sin^{\frac{3}{2}}(a - 2i \log(x))} dx, x, cx\right)}{c}$$

$$= \frac{(1 - c^4 e^{2ia} x^4)^{3/2} \text{Subst}\left(\int \frac{x^3}{(1 - e^{2ia} x^4)^{3/2}} dx, x, cx\right)}{c^4 x^3 \sin^{\frac{3}{2}}(a - 2i \log(cx))}$$

$$= \frac{e^{-2ia} (1 - c^4 e^{2ia} x^4)}{2c^4 x^3 \sin^{\frac{3}{2}}(a - 2i \log(cx))}$$

**Mathematica [A]** time = 0.131781, size = 81, normalized size = 1.65

$$\frac{x(\cos(a) - i \sin(a)) \sqrt{\frac{2 \sin(a)(c^4 x^4 + 1) - 2i \cos(a)(c^4 x^4 - 1)}{c^2 x^2}}}{\cos(a)(c^4 x^4 - 1) + i \sin(a)(c^4 x^4 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a - (2\*I)\*Log[c\*x]]^(-3/2), x]

[Out] (x\*(Cos[a] - I\*Sin[a])\*Sqrt[((-2\*I)\*(-1 + c^4\*x^4)\*Cos[a] + 2\*(1 + c^4\*x^4)\*Sin[a])/(c^2\*x^2)]/((-1 + c^4\*x^4)\*Cos[a] + I\*(1 + c^4\*x^4)\*Sin[a])

**Maple [F]** time = 0.415, size = 0, normalized size = 0.

$$\int (\sin(a - 2i \ln(cx)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(a-2\*I\*ln(c\*x))^(3/2), x)

[Out] int(1/sin(a-2\*I\*ln(c\*x))^(3/2), x)

**Maxima [B]** time = 2.44169, size = 543, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a-2\*I\*log(c\*x))^(3/2), x, algorithm="maxima")

[Out] ((cos(a)^2 + sin(a)^2)\*c^4\*x^4 + 2\*c^2\*x^2\*cos(a) + 1)^(1/4)\*((cos(a)^2 + sin(a)^2)\*c^4\*x^4 - 2\*c^2\*x^2\*cos(a) + 1)^(1/4)\*(((c^4\*x^4\*((I + 1)\*cos(3/2\*a) + (I - 1)\*sin(3/2\*a)) - (I + 1)\*cos(1/2\*a) + (I - 1)\*sin(1/2\*a))\*cos(3/2\*arctan2(c^2\*x^2\*sin(a), -c^2\*x^2\*cos(a) + 1)) + (c^4\*x^4\*((I - 1)\*cos(3/2\*a) - (I + 1)\*sin(3/2\*a)) - (I - 1)\*cos(1/2\*a) - (I + 1)\*sin(1/2\*a))\*sin(3/2\*arctan2(c^2\*x^2\*sin(a), -c^2\*x^2\*cos(a) + 1)))\*cos(3/2\*arctan2(c^2\*x^2\*sin(a), c^2\*x^2\*cos(a) + 1)) + ((c^4\*x^4\*(-(I - 1)\*cos(3/2\*a) + (I + 1)\*sin(3/2\*a)) - (I + 1)\*cos(1/2\*a) + (I - 1)\*sin(1/2\*a))\*sin(3/2\*arctan2(c^2\*x^2\*sin(a), -c^2\*x^2\*cos(a) + 1)) + (c^4\*x^4\*((I + 1)\*cos(3/2\*a) + (I - 1)\*sin(3/2\*a)) - (I + 1)\*cos(1/2\*a) - (I - 1)\*sin(1/2\*a))\*cos(3/2\*arctan2(c^2\*x^2\*sin(a), c^2\*x^2\*cos(a) + 1)))

$2*a)) + (I - 1)*\cos(1/2*a) + (I + 1)*\sin(1/2*a))*\cos(3/2*\arctan2(c^2*x^2*\sin(a), -c^2*x^2*\cos(a) + 1)) + (c^4*x^4*((I + 1)*\cos(3/2*a) + (I - 1)*\sin(3/2*a)) - (I + 1)*\cos(1/2*a) + (I - 1)*\sin(1/2*a))*\sin(3/2*\arctan2(c^2*x^2*\sin(a), -c^2*x^2*\cos(a) + 1)))*\sin(3/2*\arctan2(c^2*x^2*\sin(a), c^2*x^2*\cos(a) + 1)))/(((\cos(a)^4 + 2*\cos(a)^2*\sin(a)^2 + \sin(a)^4)*c^8*x^8 - 2*(\cos(a)^2 - \sin(a)^2)*c^4*x^4 + 1)*c)$

**Fricas [A]** time = 0.458693, size = 146, normalized size = 2.98

$$\frac{2\sqrt{\frac{1}{2}}x\sqrt{ie^{(-2ia-4\log(cx))} - ie^{(-\frac{3}{2}ia-3\log(cx))}}}{e^{(-2ia-4\log(cx))} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a-2\*I\*log(c\*x))^(3/2),x, algorithm="fricas")

[Out] -2\*sqrt(1/2)\*x\*sqrt(I\*e^(-2\*I\*a - 4\*log(c\*x)) - I)\*e^(-3/2\*I\*a - 3\*log(c\*x))/(e^(-2\*I\*a - 4\*log(c\*x)) - 1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a-2\*I\*ln(c\*x))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(a - 2i \log(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(a-2\*I\*log(c\*x))^(3/2),x, algorithm="giac")

[Out] integrate(sin(a - 2\*I\*log(c\*x))^(3/2), x)

### 3.70 $\int (ex)^m \sin^4(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=337

$$\frac{(m+1)(ex)^{m+1} \sin^4(d(a + b \log(cx^n)))}{e(16b^2d^2n^2 + (m+1)^2)} + \frac{12b^2d^2(m+1)n^2(ex)^{m+1} \sin^2(d(a + b \log(cx^n)))}{e(4b^2d^2n^2 + (m+1)^2)(16b^2d^2n^2 + (m+1)^2)} - \frac{4bdn(ex)^{m+1} \sin^3(d(a + b \log(cx^n)))}{e(16b^2d^2n^2 + (m+1)^2)}$$

[Out]  $(24*b^4*d^4*n^4*(e*x)^{(1+m)})/(e*(1+m)*((1+m)^2 + 4*b^2*d^2*n^2)*((1+m)^2 + 16*b^2*d^2*n^2)) - (24*b^3*d^3*n^3*(e*x)^{(1+m)}*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])])/(e*((1+m)^2 + 4*b^2*d^2*n^2)*((1+m)^2 + 16*b^2*d^2*n^2)) + (12*b^2*d^2*(1+m)*n^2*(e*x)^{(1+m)}*Sin[d*(a + b*Log[c*x^n])]^2)/(e*((1+m)^2 + 4*b^2*d^2*n^2)*((1+m)^2 + 16*b^2*d^2*n^2)) - (4*b*d*n*(e*x)^{(1+m)}*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])]^3)/(e*((1+m)^2 + 16*b^2*d^2*n^2)) + ((1+m)*(e*x)^{(1+m)}*Sin[d*(a + b*Log[c*x^n])]^4)/(e*((1+m)^2 + 16*b^2*d^2*n^2))$

**Rubi [A]** time = 0.16951, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {4487, 32}

$$\frac{(m+1)(ex)^{m+1} \sin^4(d(a + b \log(cx^n)))}{e(16b^2d^2n^2 + (m+1)^2)} + \frac{12b^2d^2(m+1)n^2(ex)^{m+1} \sin^2(d(a + b \log(cx^n)))}{e(4b^2d^2n^2 + (m+1)^2)(16b^2d^2n^2 + (m+1)^2)} - \frac{4bdn(ex)^{m+1} \sin^3(d(a + b \log(cx^n)))}{e(16b^2d^2n^2 + (m+1)^2)}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^4,x]

[Out]  $(24*b^4*d^4*n^4*(e*x)^{(1+m)})/(e*(1+m)*((1+m)^2 + 4*b^2*d^2*n^2)*((1+m)^2 + 16*b^2*d^2*n^2)) - (24*b^3*d^3*n^3*(e*x)^{(1+m)}*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])])/(e*((1+m)^2 + 4*b^2*d^2*n^2)*((1+m)^2 + 16*b^2*d^2*n^2)) + (12*b^2*d^2*(1+m)*n^2*(e*x)^{(1+m)}*Sin[d*(a + b*Log[c*x^n])]^2)/(e*((1+m)^2 + 4*b^2*d^2*n^2)*((1+m)^2 + 16*b^2*d^2*n^2)) - (4*b*d*n*(e*x)^{(1+m)}*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])]^3)/(e*((1+m)^2 + 16*b^2*d^2*n^2)) + ((1+m)*(e*x)^{(1+m)}*Sin[d*(a + b*Log[c*x^n])]^4)/(e*((1+m)^2 + 16*b^2*d^2*n^2))$

#### Rule 4487

Int[((e.\_)\*(x.\_))^(m.\_)\*Sin[((a.\_) + Log[(c.\_)\*(x.\_)^(n.\_)]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] :> Simp[((m+1)\*(e\*x)^(m+1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p-1))/(b^2\*d^2\*n^2\*p^2 + (m+1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p-2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m+1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p-1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m+1)^2, 0]

#### Rule 32

Int[((a.\_) + (b.\_)\*(x.\_))^(m.\_), x\_Symbol] :> Simp[(a + b\*x)^(m+1)/(b\*(m+1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (ex)^m \sin^4(d(a + b \log(cx^n))) dx &= -\frac{4bdn(ex)^{1+m} \cos(d(a + b \log(cx^n))) \sin^3(d(a + b \log(cx^n)))}{e((1+m)^2 + 16b^2d^2n^2)} + \frac{(1+m)(ex)^{1+m}}{e((1+m)^2 + 16b^2d^2n^2)} \\ &= -\frac{24b^3d^3n^3(ex)^{1+m} \cos(d(a + b \log(cx^n))) \sin(d(a + b \log(cx^n)))}{e((1+m)^2 + 4b^2d^2n^2)((1+m)^2 + 16b^2d^2n^2)} + \frac{12b^2d^2(1+m)}{e((1+m)^2 + 16b^2d^2n^2)} \\ &= \frac{24b^4d^4n^4(ex)^{1+m}}{e(1+m)((1+m)^4 + 20b^2d^2(1+m)^2n^2 + 64b^4d^4n^4)} - \frac{24b^3d^3n^3(ex)^{1+m} \cos(d(a + b \log(cx^n)))}{e((1+m)^2 + 16b^2d^2n^2)} \end{aligned}$$

**Mathematica [A]** time = 1.90931, size = 341, normalized size = 1.01

$$\frac{1}{8}x(ex)^m \left( \frac{4 \sin(2bdn \log(x)) ((m+1) \sin(2d(a + b \log(cx^n) - bn \log(x))) - 2bdn \cos(2d(a + b \log(cx^n) - bn \log(x))))}{4b^2d^2n^2 + m^2 + 2m + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^4,x]

[Out] (x\*(e\*x)^m\*(3/(1+m) + (4\*Sin[2\*b\*d\*n\*Log[x]]\*(-2\*b\*d\*n\*Cos[2\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] + (1+m)\*Sin[2\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])])))/(1 + 2\*m + m^2 + 4\*b^2\*d^2\*n^2) - (4\*Cos[2\*b\*d\*n\*Log[x]]\*((1+m)\*Cos[2\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] + 2\*b\*d\*n\*Sin[2\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]))/(1 + 2\*m + m^2 + 4\*b^2\*d^2\*n^2) - (Sin[4\*b\*d\*n\*Log[x]]\*(-4\*b\*d\*n\*Cos[4\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] + (1+m)\*Sin[4\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]))/(1 + 2\*m + m^2 + 16\*b^2\*d^2\*n^2) + (Cos[4\*b\*d\*n\*Log[x]]\*((1+m)\*Cos[4\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] + 4\*b\*d\*n\*Sin[4\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]))/(1 + 2\*m + m^2 + 16\*b^2\*d^2\*n^2))/8

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int (ex)^m (\sin(d(a + b \ln(cx^n))))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^4,x)

[Out] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^4,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n)))^4,x, algorithm="maxima")

[Out] Timed out

**Fricas [A]** time = 0.590223, size = 1083, normalized size = 3.21

$$4 \left( (4(b^3 d^3 m + b^3 d^3) n^3 + (bdm^3 + 3 bdm^2 + 3 bdm + bd)n) x \cos(bdn \log(x) + bd \log(c) + ad)^3 - (10(b^3 d^3 m + b^3 d^3) n^3 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n)))^4,x, algorithm="fricas")

[Out] (4\*((4\*(b^3\*d^3\*m + b^3\*d^3)\*n^3 + (b\*d\*m^3 + 3\*b\*d\*m^2 + 3\*b\*d\*m + b\*d)\*n) \*x\*cos(b\*d\*n\*log(x) + b\*d\*log(c) + a\*d)^3 - (10\*(b^3\*d^3\*m + b^3\*d^3)\*n^3 + (b\*d\*m^3 + 3\*b\*d\*m^2 + 3\*b\*d\*m + b\*d)\*n)\*x\*cos(b\*d\*n\*log(x) + b\*d\*log(c) + a\*d))\*e^(m\*log(e) + m\*log(x))\*sin(b\*d\*n\*log(x) + b\*d\*log(c) + a\*d) + ((m^4 + 4\*m^3 + 4\*(b^2\*d^2\*m^2 + 2\*b^2\*d^2\*m + b^2\*d^2)\*n^2 + 6\*m^2 + 4\*m + 1)\*x\*cos(b\*d\*n\*log(x) + b\*d\*log(c) + a\*d)^4 - 2\*(m^4 + 4\*m^3 + 10\*(b^2\*d^2\*m^2 + 2\*b^2\*d^2\*m + b^2\*d^2)\*n^2 + 6\*m^2 + 4\*m + 1)\*x\*cos(b\*d\*n\*log(x) + b\*d\*log(c) + a\*d)^2 + (24\*b^4\*d^4\*n^4 + m^4 + 4\*m^3 + 16\*(b^2\*d^2\*m^2 + 2\*b^2\*d^2\*m + b^2\*d^2)\*n^2 + 6\*m^2 + 4\*m + 1)\*x)\*e^(m\*log(e) + m\*log(x)))/(m^5 + 64\*(b^4\*d^4\*m + b^4\*d^4)\*n^4 + 5\*m^4 + 10\*m^3 + 20\*(b^2\*d^2\*m^3 + 3\*b^2\*d^2\*m^2 + 3\*b^2\*d^2\*m + b^2\*d^2)\*n^2 + 10\*m^2 + 5\*m + 1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*sin(d\*(a+b\*ln(c\*x\*\*n)))\*\*4,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n)))^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

### 3.71 $\int (ex)^m \sin^3(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=256

$$\frac{(m+1)(ex)^{m+1} \sin^3(d(a + b \log(cx^n)))}{e(9b^2d^2n^2 + (m+1)^2)} + \frac{6b^2d^2(m+1)n^2(ex)^{m+1} \sin(d(a + b \log(cx^n)))}{e(b^2d^2n^2 + (m+1)^2)(9b^2d^2n^2 + (m+1)^2)} - \frac{6b^3d^3n^3(ex)^{m+1} \cos(d(a + b \log(cx^n)))}{e(b^2d^2n^2 + (m+1)^2)(9b^2d^2n^2 + (m+1)^2)}$$

[Out]  $(-6*b^3*d^3*n^3*(e*x)^{(1+m)}*\text{Cos}[d*(a + b*\text{Log}[c*x^n])])/(e*((1+m)^2 + b^2*d^2*n^2)*((1+m)^2 + 9*b^2*d^2*n^2)) + (6*b^2*d^2*(1+m)*n^2*(e*x)^{(1+m)}*\text{Sin}[d*(a + b*\text{Log}[c*x^n])])/(e*((1+m)^2 + b^2*d^2*n^2)*((1+m)^2 + 9*b^2*d^2*n^2)) - (3*b*d*n*(e*x)^{(1+m)}*\text{Cos}[d*(a + b*\text{Log}[c*x^n])]*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^2)/(e*((1+m)^2 + 9*b^2*d^2*n^2)) + ((1+m)*(e*x)^{(1+m)}*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^3)/(e*((1+m)^2 + 9*b^2*d^2*n^2))$

**Rubi [A]** time = 0.117907, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {4487, 4485}

$$\frac{(m+1)(ex)^{m+1} \sin^3(d(a + b \log(cx^n)))}{e(9b^2d^2n^2 + (m+1)^2)} + \frac{6b^2d^2(m+1)n^2(ex)^{m+1} \sin(d(a + b \log(cx^n)))}{e(b^2d^2n^2 + (m+1)^2)(9b^2d^2n^2 + (m+1)^2)} - \frac{6b^3d^3n^3(ex)^{m+1} \cos(d(a + b \log(cx^n)))}{e(b^2d^2n^2 + (m+1)^2)(9b^2d^2n^2 + (m+1)^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e*x)^m*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^3, x]$

[Out]  $(-6*b^3*d^3*n^3*(e*x)^{(1+m)}*\text{Cos}[d*(a + b*\text{Log}[c*x^n])])/(e*((1+m)^2 + b^2*d^2*n^2)*((1+m)^2 + 9*b^2*d^2*n^2)) + (6*b^2*d^2*(1+m)*n^2*(e*x)^{(1+m)}*\text{Sin}[d*(a + b*\text{Log}[c*x^n])])/(e*((1+m)^2 + b^2*d^2*n^2)*((1+m)^2 + 9*b^2*d^2*n^2)) - (3*b*d*n*(e*x)^{(1+m)}*\text{Cos}[d*(a + b*\text{Log}[c*x^n])]*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^2)/(e*((1+m)^2 + 9*b^2*d^2*n^2)) + ((1+m)*(e*x)^{(1+m)}*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^3)/(e*((1+m)^2 + 9*b^2*d^2*n^2))$

#### Rule 4487

$\text{Int}[(e_*)*(x_*)^{(m_*)}*\text{Sin}[(a_*) + \text{Log}[(c_*)*(x_*)^{(n_*)}]]*(b_*)*(d_*)]^{(p_*)}, x\_Symbol] := \text{Simp}[(m+1)*(e*x)^{(m+1)}*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^p/(b^2*d^2*e*n^2*p^2 + e*(m+1)^2), x] + (\text{Dist}[(b^2*d^2*n^2*p*(p-1))/(b^2*d^2*n^2*p^2 + (m+1)^2), \text{Int}[(e*x)^m*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^{(p-2)}, x], x] - \text{Simp}[(b*d*n*p*(e*x)^{(m+1)}*\text{Cos}[d*(a + b*\text{Log}[c*x^n])]*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^{(p-1)}]/(b^2*d^2*e*n^2*p^2 + e*(m+1)^2), x]) /;$  FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m+1)^2, 0]

#### Rule 4485

$\text{Int}[(e_*)*(x_*)^{(m_*)}*\text{Sin}[(a_*) + \text{Log}[(c_*)*(x_*)^{(n_*)}]]*(b_*)*(d_*)], x\_Symbol] := \text{Simp}[(m+1)*(e*x)^{(m+1)}*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]/(b^2*d^2*e*n^2 + e*(m+1)^2), x] - \text{Simp}[(b*d*n*(e*x)^{(m+1)}*\text{Cos}[d*(a + b*\text{Log}[c*x^n])]*\text{Sin}[d*(a + b*\text{Log}[c*x^n])]^2)/(b^2*d^2*e*n^2 + e*(m+1)^2), x] /;$  FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2 + (m+1)^2, 0]

#### Rubi steps

$$\int (ex)^m \sin^3(d(a + b \log(cx^n))) dx = -\frac{3bdn(ex)^{1+m} \cos(d(a + b \log(cx^n))) \sin^2(d(a + b \log(cx^n)))}{e((1+m)^2 + 9b^2d^2n^2)} + \frac{(1+m)(ex)^{1+m} \sin^3(d(a + b \log(cx^n)))}{e((1+m)^2 + 9b^2d^2n^2)}$$

$$= -\frac{6b^3d^3n^3(ex)^{1+m} \cos(d(a + b \log(cx^n)))}{e((1+m)^2 + b^2d^2n^2)((1+m)^2 + 9b^2d^2n^2)} + \frac{6b^2d^2(1+m)n^2(ex)^{1+m} \sin(d(a + b \log(cx^n)))}{e((1+m)^2 + b^2d^2n^2)((1+m)^2 + 9b^2d^2n^2)}$$

**Mathematica [A]** time = 1.20835, size = 326, normalized size = 1.27

$$\frac{1}{4}x(ex)^m \left( \frac{3 \cos(bdn \log(x)) ((m+1) \sin(d(a + b \log(cx^n) - bn \log(x))) - bdn \cos(d(a + b \log(cx^n) - bn \log(x))))}{b^2d^2n^2 + m^2 + 2m + 1} + \frac{3 \sin(bdn \log(x)) ((m+1) \cos(d(a + b \log(cx^n) - bn \log(x))) + bdn \sin(d(a + b \log(cx^n) - bn \log(x))))}{b^2d^2n^2 + m^2 + 2m + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^3,x]

[Out] (x\*(e\*x)^m\*((3\*Cos[b\*d\*n\*Log[x]]\*(-(b\*d\*n\*Cos[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]) + (1+m)\*Sin[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]))/(1+2\*m+m^2+b^2\*d^2\*n^2) + (3\*Sin[b\*d\*n\*Log[x]]\*((1+m)\*Cos[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]) + b\*d\*n\*Sin[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]))/(1+2\*m+m^2+b^2\*d^2\*n^2) - (Cos[3\*b\*d\*n\*Log[x]]\*(-3\*b\*d\*n\*Cos[3\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]) + (1+m)\*Sin[3\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]))/(1+2\*m+m^2+9\*b^2\*d^2\*n^2) - (Sin[3\*b\*d\*n\*Log[x]]\*((1+m)\*Cos[3\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]) + 3\*b\*d\*n\*Sin[3\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]))/(1+2\*m+m^2+9\*b^2\*d^2\*n^2))/4

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int (ex)^m (\sin(d(a + b \ln(cx^n))))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^3,x)

[Out] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^3,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n)))^3,x, algorithm="maxima")

[Out] Timed out

**Fricas [A]** time = 0.548708, size = 695, normalized size = 2.71

$$\frac{((m^3 + (b^2d^2m + b^2d^2))n^2 + 3m^2 + 3m + 1)x \cos(bdn \log(x) + bd \log(c) + ad)^2 - (m^3 + 7(b^2d^2m + b^2d^2))n^2 + 3m^2 + 3m + 1}{(b^2d^2n^2 + m^2 + 2m + 1)^2}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] -(((m^3 + (b^2*d^2*m + b^2*d^2)*n^2 + 3*m^2 + 3*m + 1)*x*cos(b*d*n*log(x) +
b*d*log(c) + a*d)^2 - (m^3 + 7*(b^2*d^2*m + b^2*d^2)*n^2 + 3*m^2 + 3*m + 1
)*x)*e^(m*log(e) + m*log(x))*sin(b*d*n*log(x) + b*d*log(c) + a*d) - 3*((b^3
*d^3*n^3 + (b*d*m^2 + 2*b*d*m + b*d)*n)*x*cos(b*d*n*log(x) + b*d*log(c) + a
*d)^3 - (3*b^3*d^3*n^3 + (b*d*m^2 + 2*b*d*m + b*d)*n)*x*cos(b*d*n*log(x) +
b*d*log(c) + a*d))*e^(m*log(e) + m*log(x)))/(9*b^4*d^4*n^4 + m^4 + 4*m^3 +
10*(b^2*d^2*m^2 + 2*b^2*d^2*m + b^2*d^2)*n^2 + 6*m^2 + 4*m + 1)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] Timed out
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

### 3.72 $\int (ex)^m \sin^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=154

$$\frac{(m+1)(ex)^{m+1} \sin^2(d(a + b \log(cx^n)))}{e(4b^2d^2n^2 + (m+1)^2)} - \frac{2bdn(ex)^{m+1} \sin(d(a + b \log(cx^n))) \cos(d(a + b \log(cx^n)))}{e(4b^2d^2n^2 + (m+1)^2)} + \frac{2b^2d^2n^2}{e(m+1)(4b^2d^2n^2)}$$

[Out]  $(2*b^2*d^2*n^2*(e*x)^(1+m))/(e*(1+m)*((1+m)^2 + 4*b^2*d^2*n^2)) - (2*b*d*n*(e*x)^(1+m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])]/(e*((1+m)^2 + 4*b^2*d^2*n^2)) + ((1+m)*(e*x)^(1+m)*Sin[d*(a + b*Log[c*x^n])])^2/(e*((1+m)^2 + 4*b^2*d^2*n^2))$

**Rubi [A]** time = 0.055039, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {4487, 32}

$$\frac{(m+1)(ex)^{m+1} \sin^2(d(a + b \log(cx^n)))}{e(4b^2d^2n^2 + (m+1)^2)} - \frac{2bdn(ex)^{m+1} \sin(d(a + b \log(cx^n))) \cos(d(a + b \log(cx^n)))}{e(4b^2d^2n^2 + (m+1)^2)} + \frac{2b^2d^2n^2}{e(m+1)(4b^2d^2n^2)}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out]  $(2*b^2*d^2*n^2*(e*x)^(1+m))/(e*(1+m)*((1+m)^2 + 4*b^2*d^2*n^2)) - (2*b*d*n*(e*x)^(1+m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])]/(e*((1+m)^2 + 4*b^2*d^2*n^2)) + ((1+m)*(e*x)^(1+m)*Sin[d*(a + b*Log[c*x^n])])^2/(e*((1+m)^2 + 4*b^2*d^2*n^2))$

#### Rule 4487

Int[((e.\_)\*(x.\_))^(m.\_)\*Sin[((a.\_) + Log[(c.\_)\*(x.\_)^(n.\_)]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] :> Simp[((m+1)\*(e\*x)^(m+1)\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p-1))/(b^2\*d^2\*n^2\*p^2 + (m+1)^2), Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(p-2), x], x] - Simp[(b\*d\*n\*p\*(e\*x)^(m+1)\*Cos[d\*(a + b\*Log[c\*x^n])]\*Sin[d\*(a + b\*Log[c\*x^n])]^(p-1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m+1)^2, 0]

#### Rule 32

Int[((a.\_) + (b.\_)\*(x.\_))^(m.\_), x\_Symbol] :> Simp[(a + b\*x)^(m+1)/(b\*(m+1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (ex)^m \sin^2(d(a + b \log(cx^n))) dx &= -\frac{2bdn(ex)^{1+m} \cos(d(a + b \log(cx^n))) \sin(d(a + b \log(cx^n)))}{e((1+m)^2 + 4b^2d^2n^2)} + \frac{(1+m)(ex)^{1+m} \sin^2(d(a + b \log(cx^n)))}{e((1+m)^2 + 4b^2d^2n^2)} \\ &= \frac{2b^2d^2n^2(ex)^{1+m}}{e(1+m)((1+m)^2 + 4b^2d^2n^2)} - \frac{2bdn(ex)^{1+m} \cos(d(a + b \log(cx^n))) \sin(d(a + b \log(cx^n)))}{e((1+m)^2 + 4b^2d^2n^2)} \end{aligned}$$

**Mathematica [C]** time = 0.287066, size = 102, normalized size = 0.66

$$\frac{x(ex)^m (2bd(m+1)n \sin(2d(a + b \log(cx^n))) + (m+1)^2 \cos(2d(a + b \log(cx^n))) - 4b^2d^2n^2 - m^2 - 2m - 1)}{2(m+1)(-2ibdn + m+1)(2ibdn + m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -(x*(e*x)^m*(-1 - 2*m - m^2 - 4*b^2*d^2*n^2 + (1 + m)^2*Cos[2*d*(a + b*Log[
c*x^n]]) + 2*b*d*(1 + m)*n*Sin[2*d*(a + b*Log[c*x^n]])))/(2*(1 + m)*(1 + m
- (2*I)*b*d*n)*(1 + m + (2*I)*b*d*n))
```

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int (ex)^m (\sin(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^2,x)
```

**Maxima [B]** time = 1.53686, size = 3444, normalized size = 22.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c))
+ (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos(4*
b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) - ((cos(2*a*d)*sin(4*a*d) - cos(
4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*
sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)) - sin(2*b*d*log(c))*sin(2*
a*d))*e^m*m^2 + 2*(((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*cos(2*b
*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(
c)))*cos(4*b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) - ((cos(2*a*d)*sin(4*
a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos(4*a*d)*cos(2*a*d) +
sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)) - sin(2*b*d*log
(c))*sin(2*a*d))*e^m + (((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*
cos(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b
*d*log(c)))*cos(4*b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) - ((cos(2*a*d)
*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos(4*a*d)*cos(2*
a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)) - sin(2*
b*d*log(c))*sin(2*a*d))*e^m + 2*((b*d*cos(2*a*d)*sin(2*b*d*log(c)) + b*d*cos
(2*b*d*log(c))*sin(2*a*d) + ((b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*s
in(2*a*d))*cos(2*b*d*log(c)) - (b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*
sin(2*a*d))*sin(2*b*d*log(c)))*cos(4*b*d*log(c)) + ((b*d*cos(4*a*d)*cos(2*a
*d) + b*d*sin(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) + (b*d*cos(2*a*d)*sin(4*
a*d) - b*d*cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c))*e^m
+ (b*d*cos(2*a*d)*sin(2*b*d*log(c)) + b*d*cos(2*b*d*log(c))*sin(2*a*d) +
((b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c))
- (b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)
))*cos(4*b*d*log(c)) + ((b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a
*d))*cos(2*b*d*log(c)) + (b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*
a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c))*e^m)*n)*x*x^m*cos(2*b*d*log(x^n
```

```

)) - (((cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) -
(cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos(4*b
*d*log(c)) + ((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*cos(2*b*d*log
(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c))*s
in(4*b*d*log(c)) + cos(2*a*d)*sin(2*b*d*log(c)) + cos(2*b*d*log(c))*sin(2*a
*d))*e^m*m^2 + 2*(((cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*
d*log(c)) - (cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c
)))*cos(4*b*d*log(c)) + ((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*co
s(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b*d
*log(c)))*sin(4*b*d*log(c)) + cos(2*a*d)*sin(2*b*d*log(c)) + cos(2*b*d*log(
c))*sin(2*a*d))*e^m*m + (((cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*c
os(2*b*d*log(c)) - (cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*
d*log(c)))*cos(4*b*d*log(c)) + ((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a
*d))*cos(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*si
n(2*b*d*log(c)))*sin(4*b*d*log(c)) + cos(2*a*d)*sin(2*b*d*log(c)) + cos(2*b
*d*log(c))*sin(2*a*d))*e^m - 2*((b*d*cos(2*b*d*log(c))*cos(2*a*d) - b*d*sin
(2*b*d*log(c))*sin(2*a*d) + ((b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*si
n(2*a*d))*cos(2*b*d*log(c)) + (b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*s
in(2*a*d))*sin(2*b*d*log(c)))*cos(4*b*d*log(c)) - ((b*d*cos(2*a*d)*sin(4*a*
d) - b*d*cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (b*d*cos(4*a*d)*cos(2*a
*d) + b*d*sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)))*e^m*
m + (b*d*cos(2*b*d*log(c))*cos(2*a*d) - b*d*sin(2*b*d*log(c))*sin(2*a*d) +
((b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c))
+ (b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c))
)*cos(4*b*d*log(c)) - ((b*d*cos(2*a*d)*sin(4*a*d) - b*d*cos(4*a*d)*sin(2*a*
d))*cos(2*b*d*log(c)) - (b*d*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a
*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)))*e^m)*n)*x*x^m*sin(2*b*d*log(x^n
)) - 2*(((cos(2*a*d)^2 + sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a*d)^2 +
sin(2*a*d)^2)*sin(2*b*d*log(c))^2)*e^m*m^2 + 4*((b^2*d^2*cos(2*a*d)^2 + b^
2*d^2*sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (b^2*d^2*cos(2*a*d)^2 + b^2*d^2*s
in(2*a*d)^2)*sin(2*b*d*log(c))^2)*e^m*n^2 + 2*((cos(2*a*d)^2 + sin(2*a*d)^2
)*cos(2*b*d*log(c))^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*sin(2*b*d*log(c))^2)*
e^m*m + ((cos(2*a*d)^2 + sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a*d)^2
+ sin(2*a*d)^2)*sin(2*b*d*log(c))^2)*e^m)*x*x^m)/(((cos(2*a*d)^2 + sin(2*a*
d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*sin(2*b*d*log(c))
^2)*m^3 + 3*((cos(2*a*d)^2 + sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a*d
)^2 + sin(2*a*d)^2)*sin(2*b*d*log(c))^2)*m^2 + 4*((b^2*d^2*cos(2*a*d)^2 + b
^2*d^2*sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (b^2*d^2*cos(2*a*d)^2 + b^2*d^2*
sin(2*a*d)^2)*sin(2*b*d*log(c))^2 + ((b^2*d^2*cos(2*a*d)^2 + b^2*d^2*sin(2*
a*d)^2)*cos(2*b*d*log(c))^2 + (b^2*d^2*cos(2*a*d)^2 + b^2*d^2*sin(2*a*d)^2
)*sin(2*b*d*log(c))^2)*m)*n^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*cos(2*b*d*log(
c))^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*sin(2*b*d*log(c))^2 + 3*((cos(2*a*d)^
2 + sin(2*a*d)^2)*cos(2*b*d*log(c))^2 + (cos(2*a*d)^2 + sin(2*a*d)^2)*sin(2
*b*d*log(c))^2)*m)

```

---

**Fricas [A]** time = 0.51077, size = 401, normalized size = 2.6

$$\frac{2(bdm + bd)nx \cos(bdn \log(x) + bd \log(c) + ad) e^{(m \log(e) + m \log(x))} \sin(bdn \log(x) + bd \log(c) + ad) + ((m^2 + 2m + 1) m^3 + 4(b^2 d^2 m + b^2 d^2) n^2 + 3m^2 + 3m)}{m^3 + 4(b^2 d^2 m + b^2 d^2) n^2 + 3m^2 + 3m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] -(2*(b*d*m + b*d)*n*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)*e^(m*log(e) + m*
log(x))*sin(b*d*n*log(x) + b*d*log(c) + a*d) + ((m^2 + 2*m + 1)*x*cos(b*d*n
*log(x) + b*d*log(c) + a*d)^2 - (2*b^2*d^2*n^2 + m^2 + 2*m + 1)*x)*e^(m*log
```

$(e + m \log(x)) / (m^3 + 4(b^2 d^2 m + b^2 d^2) n^2 + 3m^2 + 3m + 1)$

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*sin(d\*(a+b\*ln(c\*x\*\*n)))\*\*2,x)

[Out] Exception raised: TypeError

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n)))^2,x, algorithm="giac")

[Out] Timed out

### 3.73 $\int (ex)^m \sin(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=92

$$\frac{(m+1)(ex)^{m+1} \sin(d(a + b \log(cx^n)))}{e(b^2 d^2 n^2 + (m+1)^2)} - \frac{bdn(ex)^{m+1} \cos(d(a + b \log(cx^n)))}{e(b^2 d^2 n^2 + (m+1)^2)}$$

[Out]  $-\left(\frac{b*d*n*(e*x)^{(1+m)*Cos[d*(a + b*Log[c*x^n])]}{e*((1+m)^2 + b^2*d^2*n^2)}\right) + \left(\frac{(1+m)*(e*x)^{(1+m)*Sin[d*(a + b*Log[c*x^n])]}{e*((1+m)^2 + b^2*d^2*n^2)}\right)$

**Rubi [A]** time = 0.0245137, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {4485}

$$\frac{(m+1)(ex)^{m+1} \sin(d(a + b \log(cx^n)))}{e(b^2 d^2 n^2 + (m+1)^2)} - \frac{bdn(ex)^{m+1} \cos(d(a + b \log(cx^n)))}{e(b^2 d^2 n^2 + (m+1)^2)}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m\*SIN[d\*(a + b\*Log[c\*x^n])],x]

[Out]  $-\left(\frac{b*d*n*(e*x)^{(1+m)*Cos[d*(a + b*Log[c*x^n])]}{e*((1+m)^2 + b^2*d^2*n^2)}\right) + \left(\frac{(1+m)*(e*x)^{(1+m)*Sin[d*(a + b*Log[c*x^n])]}{e*((1+m)^2 + b^2*d^2*n^2)}\right)$

#### Rule 4485

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] - Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\int (ex)^m \sin(d(a + b \log(cx^n))) dx = -\frac{bdn(ex)^{1+m} \cos(d(a + b \log(cx^n)))}{e((1+m)^2 + b^2 d^2 n^2)} + \frac{(1+m)(ex)^{1+m} \sin(d(a + b \log(cx^n)))}{e((1+m)^2 + b^2 d^2 n^2)}$$

**Mathematica [A]** time = 0.13483, size = 63, normalized size = 0.68

$$\frac{x(ex)^m ((m+1) \sin(d(a + b \log(cx^n))) - bdn \cos(d(a + b \log(cx^n))))}{b^2 d^2 n^2 + m^2 + 2m + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*SIN[d\*(a + b\*Log[c\*x^n])],x]

[Out]  $(x*(e*x)^m*(-(b*d*n*COS[d*(a + b*Log[c*x^n])]) + (1+m)*SIN[d*(a + b*Log[c*x^n])]))/(1 + 2*m + m^2 + b^2*d^2*n^2)$

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int (ex)^m \sin(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n))),x)

[Out] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n))),x)

**Maxima [B]** time = 1.26394, size = 1705, normalized size = 18.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n))),x, algorithm="maxima")

[Out] 1/2\*(((cos(a\*d)\*sin(2\*a\*d) - cos(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) - (cos(2\*a\*d)\*cos(a\*d) + sin(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*cos(2\*b\*d\*log(c)) + ((cos(2\*a\*d)\*cos(a\*d) + sin(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) + (cos(a\*d)\*sin(2\*a\*d) - cos(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*sin(2\*b\*d\*log(c)) + cos(a\*d)\*sin(b\*d\*log(c)) + cos(b\*d\*log(c))\*sin(a\*d))\*e^m\*m - (b\*d\*cos(b\*d\*log(c))\*cos(a\*d) - b\*d\*sin(b\*d\*log(c))\*sin(a\*d) + ((b\*d\*cos(2\*a\*d)\*cos(a\*d) + b\*d\*sin(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) + (b\*d\*cos(a\*d)\*sin(2\*a\*d) - b\*d\*cos(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*cos(2\*b\*d\*log(c)) - ((b\*d\*cos(a\*d)\*sin(2\*a\*d) - b\*d\*cos(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) - (b\*d\*cos(2\*a\*d)\*cos(a\*d) + b\*d\*sin(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*sin(2\*b\*d\*log(c)))\*e^m\*n + (((cos(a\*d)\*sin(2\*a\*d) - cos(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) - (cos(2\*a\*d)\*cos(a\*d) + sin(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*cos(2\*b\*d\*log(c)) + ((cos(2\*a\*d)\*cos(a\*d) + sin(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) + (cos(a\*d)\*sin(2\*a\*d) - cos(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*sin(2\*b\*d\*log(c)) + cos(a\*d)\*sin(b\*d\*log(c)) + cos(b\*d\*log(c))\*sin(a\*d))\*e^m\*x\*x^m\*cos(b\*d\*log(x^n)) + (((cos(2\*a\*d)\*cos(a\*d) + sin(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) + (cos(a\*d)\*sin(2\*a\*d) - cos(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*cos(2\*b\*d\*log(c)) + cos(b\*d\*log(c))\*cos(a\*d) - ((cos(a\*d)\*sin(2\*a\*d) - cos(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) - (cos(2\*a\*d)\*cos(a\*d) + sin(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*sin(2\*b\*d\*log(c)) - sin(b\*d\*log(c))\*sin(a\*d))\*e^m\*m + (b\*d\*cos(a\*d)\*sin(b\*d\*log(c)) + b\*d\*cos(b\*d\*log(c))\*sin(a\*d) + ((b\*d\*cos(a\*d)\*sin(2\*a\*d) - b\*d\*cos(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) - (b\*d\*cos(2\*a\*d)\*cos(a\*d) + b\*d\*sin(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*cos(2\*b\*d\*log(c)) + ((b\*d\*cos(2\*a\*d)\*cos(a\*d) + b\*d\*sin(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) + (b\*d\*cos(a\*d)\*sin(2\*a\*d) - b\*d\*cos(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*sin(2\*b\*d\*log(c)))\*e^m\*n + (((cos(2\*a\*d)\*cos(a\*d) + sin(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) + (cos(a\*d)\*sin(2\*a\*d) - cos(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*cos(2\*b\*d\*log(c)) + cos(b\*d\*log(c))\*cos(a\*d) - ((cos(a\*d)\*sin(2\*a\*d) - cos(2\*a\*d)\*sin(a\*d))\*cos(b\*d\*log(c)) - (cos(2\*a\*d)\*cos(a\*d) + sin(2\*a\*d)\*sin(a\*d))\*sin(b\*d\*log(c)))\*sin(2\*b\*d\*log(c)) - sin(b\*d\*log(c))\*sin(a\*d))\*e^m\*x\*x^m\*sin(b\*d\*log(x^n)))/(((cos(a\*d)^2 + sin(a\*d)^2)\*cos(b\*d\*log(c))^2 + (cos(a\*d)^2 + sin(a\*d)^2)\*sin(b\*d\*log(c))^2)\*m^2 + ((b^2\*d^2\*cos(a\*d)^2 + b^2\*d^2\*sin(a\*d)^2)\*cos(b\*d\*log(c))^2 + (b^2\*d^2\*cos(a\*d)^2 + b^2\*d^2\*sin(a\*d)^2)\*sin(b\*d\*log(c))^2)\*n^2 + (cos(a\*d)^2 + sin(a\*d)^2)\*cos(b\*d\*log(c))^2 + (cos(a\*d)^2 + sin(a\*d)^2)\*sin(b\*d\*log(c))^2 + 2\*((cos(a\*d)^2 + sin(a\*d)^2)\*cos(b\*d\*log(c))^2 + (cos(a\*d)^2 + sin(a\*d)^2)\*sin(b\*d\*log(c))^2)\*m)

**Fricas [A]** time = 0.496398, size = 238, normalized size = 2.59

$$\frac{bdnx \cos(bdn \log(x) + bd \log(c) + ad) e^{(m \log(e) + m \log(x))} - (m + 1) x e^{(m \log(e) + m \log(x))} \sin(bdn \log(x) + bd \log(c) + ad)}{b^2 d^2 n^2 + m^2 + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n))),x, algorithm="fricas")

[Out]  $-(b*d*n*x*\cos(b*d*n*\log(x) + b*d*\log(c) + a*d)*e^{(m*\log(e) + m*\log(x))} - (m + 1)*x*e^{(m*\log(e) + m*\log(x))*\sin(b*d*n*\log(x) + b*d*\log(c) + a*d)})/(b^2*d^2*n^2 + m^2 + 2*m + 1)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sin(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*sin(d\*(a+b\*ln(c\*x\*\*n))),x)

[Out] Integral((e\*x)\*\*m\*sin(a\*d + b\*d\*log(c\*x\*\*n)), x)

**Giac [B]** time = 1.75122, size = 7776, normalized size = 84.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n))),x, algorithm="giac")

[Out]  $\frac{1}{2}*(b*d*n*x*\abs(x)^m*e^{(1/2*\pi*b*d*n*\operatorname{sgn}(x) - 1/2*\pi*b*d*n + 1/2*\pi*b*d*\operatorname{sgn}(c) - 1/2*\pi*b*d + m)*\tan(1/2*b*d*n*\log(\abs(x)) + 1/2*b*d*\log(\abs(c)))^2*\tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)^2*\tan(1/2*a*d)^2 + b*d*n*x*\abs(x)^m*e^{(-1/2*\pi*b*d*n*\operatorname{sgn}(x) + 1/2*\pi*b*d*n - 1/2*\pi*b*d*\operatorname{sgn}(c) + 1/2*\pi*b*d + m)*\tan(1/2*b*d*n*\log(\abs(x)) + 1/2*b*d*\log(\abs(c)))^2*\tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)^2*\tan(1/2*a*d)^2 - b*d*n*x*\abs(x)^m*e^{(1/2*\pi*b*d*n*\operatorname{sgn}(x) - 1/2*\pi*b*d*n + 1/2*\pi*b*d*\operatorname{sgn}(c) - 1/2*\pi*b*d + m)*\tan(1/2*b*d*n*\log(\abs(x)) + 1/2*b*d*\log(\abs(c)))^2*\tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)^2 - b*d*n*x*\abs(x)^m*e^{(-1/2*\pi*b*d*n*\operatorname{sgn}(x) + 1/2*\pi*b*d*n - 1/2*\pi*b*d*\operatorname{sgn}(c) + 1/2*\pi*b*d + m)*\tan(1/2*b*d*n*\log(\abs(x)) + 1/2*b*d*\log(\abs(c)))^2*\tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)^2 + 4*b*d*n*x*\abs(x)^m*e^{(1/2*\pi*b*d*n*\operatorname{sgn}(x) - 1/2*\pi*b*d*n + 1/2*\pi*b*d*\operatorname{sgn}(c) - 1/2*\pi*b*d + m)*\tan(1/2*b*d*n*\log(\abs(x)) + 1/2*b*d*\log(\abs(c)))^2*\tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)*\tan(1/2*a*d) - 4*b*d*n*x*\abs(x)^m*e^{(-1/2*\pi*b*d*n*\operatorname{sgn}(x) + 1/2*\pi*b*d*n - 1/2*\pi*b*d*\operatorname{sgn}(c) + 1/2*\pi*b*d + m)*\tan(1/2*b*d*n*\log(\abs(x)) + 1/2*b*d*\log(\abs(c)))^2*\tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)*\tan(1/2*a*d) - 4*b*d*n*x*\abs(x)^m*e^{(1/2*\pi*b*d*n*\operatorname{sgn}(x) - 1/2*\pi*b*d*n + 1/2*\pi*b*d*\operatorname{sgn}(c) - 1/2*\pi*b*d + m)*\tan(1/2*b*d*n*\log(\abs(x)) + 1/2*b*d*\log(\abs(c)))^2*\tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)^2*\tan(1/2*a*d) - b*d*n*x*\abs(x)^m*e^{(1/2*\pi*b*d*n*\operatorname{sgn}(x) - 1/2*\pi*b*d*n + 1/2*\pi*b*d*\operatorname{sgn}(c) - 1/2*\pi*b*d + m)*\tan(1/2*b*d*n*\log(\abs(x)) + 1/2*b*d*\log(\abs(c)))^2*\tan(1/2*a*d)^2 - b*d*n*x*\abs(x)^m*e^{(-1/2*\pi*b*d*n}}$









### 3.74 $\int (ex)^m \sin^2 \left( d(a + b \log(cx^n)) \right) dx$

**Optimal.** Leaf size=150

$$\frac{2(ex)^{m+1} \sin^2 \left( d(a + b \log(cx^n)) \right) \text{Hypergeometric2F1} \left( -\frac{3}{2}, -\frac{3bdn+2im+2i}{4bdn}, -\frac{-bdn+2im+2i}{4bdn}, e^{2iad} (cx^n)^{2ibd} \right)}{e^{-3ibdn + 2m + 2} \left( 1 - e^{2iad} (cx^n)^{2ibd} \right)^{3/2}}$$

[Out] (2\*(e\*x)^(1 + m)\*Hypergeometric2F1[-3/2, -(2\*I + (2\*I)\*m + 3\*b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Sin[d\*(a + b\*Log[c\*x^n])]^(3/2))/(e\*(2 + 2\*m - (3\*I)\*b\*d\*n)\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^(3/2))

**Rubi [A]** time = 0.125528, antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4493, 4491, 364}

$$\frac{2(ex)^{m+1} {}_2F_1 \left( -\frac{3}{2}, \frac{1}{4} \left( -\frac{2i(m+1)}{bdn} - 3 \right); -\frac{2im-bdn+2i}{4bdn}; e^{2iad} (cx^n)^{2ibd} \right) \sin^2 \left( d(a + b \log(cx^n)) \right)}{e^{-3ibdn + 2m + 2} \left( 1 - e^{2iad} (cx^n)^{2ibd} \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(3/2),x]

[Out] (2\*(e\*x)^(1 + m)\*Hypergeometric2F1[-3/2, (-3 - ((2\*I)\*(1 + m))/(b\*d\*n))/4, -(2\*I + (2\*I)\*m - b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Sin[d\*(a + b\*Log[c\*x^n])]^(3/2))/(e\*(2 + 2\*m - (3\*I)\*b\*d\*n)\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^(3/2))

#### Rule 4493

Int[((e.\_)\*(x.\_))^(m.\_)\*Sin[((a.\_) + Log[(c.\_)\*(x.\_)^(n.\_)]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e.\_)\*(x.\_))^(m.\_)\*Sin[((a.\_) + Log[x\_]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c.\_)\*(x.\_))^(m.\_)\*((a.\_) + (b.\_)\*(x.\_)^(n.\_))^(p.\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/ (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a + b \log(cx^n))) dx = \frac{\left( (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{n}} \sin^{\frac{3}{2}}(d(a + b \log(x))) dx, x, cx^n \right)}{en}$$

$$= \frac{\left( (ex)^{1+m} (cx^n)^{\frac{3ibd}{2} - \frac{1+m}{n}} \sin^{\frac{3}{2}}(d(a + b \log(cx^n))) \right) \text{Subst} \left( \int x^{-1-\frac{3ibd}{2} + \frac{1+m}{n}} (1 - e^{2iad})^{3/2} \right)}{en (1 - e^{2iad} (cx^n)^{2ibd})^{3/2}}$$

$$= \frac{2(ex)^{1+m} {}_2F_1 \left( -\frac{3}{2}, \frac{1}{4} \left( -3 - \frac{2i(1+m)}{bdn} \right); -\frac{2i+2im-bdn}{4bdn}; e^{2iad} (cx^n)^{2ibd} \right) \sin^{\frac{3}{2}}(d(a + b \log(cx^n)))}{e(2 + 2m - 3ibdn) (1 - e^{2iad} (cx^n)^{2ibd})^{3/2}}$$

**Mathematica [A]** time = 1.97296, size = 235, normalized size = 1.57

$$\frac{2(ex)^m \left( x(ibdn + 2m + 2) \sin(d(a + b \log(cx^n))) (2(m + 1) \sin(d(a + b \log(cx^n))) - 3bdn \cos(d(a + b \log(cx^n)))) - 3bdn \right)}{(ibdn + 2m + 2)(-3ibdn + 2m + 2)(3ibdn + 2m + 2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^(3/2), x]

[Out] (2\*(e\*x)^m\*(-3\*b^2\*d^2\*(-1 + E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))))\*n^2\*x\*Hypergeometric2F1[1, -(2\*I + (2\*I)\*m - 3\*b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 5\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))] + (2 + 2\*m + I\*b\*d\*n)\*x\*Sin[d\*(a + b\*Log[c\*x^n])]\*(-3\*b\*d\*n\*Cos[d\*(a + b\*Log[c\*x^n])] + 2\*(1 + m)\*Sin[d\*(a + b\*Log[c\*x^n])]))/((2 + 2\*m + I\*b\*d\*n)\*(2 + 2\*m - (3\*I)\*b\*d\*n)\*(2 + 2\*m + (3\*I)\*b\*d\*n)\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]])

**Maple [F]** time = 0.431, size = 0, normalized size = 0.

$$\int (ex)^m (\sin(d(a + b \ln(cx^n))))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^(3/2), x)

[Out] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sin((b \log(cx^n) + a)d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n)))^(3/2), x, algorithm="maxima")

[Out] integrate((e\*x)^m\*sin((b\*log(c\*x^n) + a)\*d)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.75 $\int (ex)^m \sqrt{\sin(d(a + b \log(cx^n)))} dx$

**Optimal.** Leaf size=149

$$\frac{2(ex)^{m+1} \sqrt{\sin(d(a + b \log(cx^n)))} \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{bdn+2im+2i}{4bdn}, -\frac{-3bdn+2im+2i}{4bdn}, e^{2iad}(cx^n)^{2ibd}\right)}{e(-ibdn + 2m + 2) \sqrt{1 - e^{2iad}(cx^n)^{2ibd}}}$$

[Out] (2\*(e\*x)^(1 + m)\*Hypergeometric2F1[-1/2, -(2\*I + (2\*I)\*m + b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 3\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]])/(e\*(2 + 2\*m - I\*b\*d\*n)\*Sqrt[1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)])

**Rubi [A]** time = 0.11187, antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4493, 4491, 364}

$$\frac{2(ex)^{m+1} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-\frac{2i(m+1)}{bdn} - 1\right); -\frac{2im-3bdn+2i}{4bdn}; e^{2iad}(cx^n)^{2ibd}\right) \sqrt{\sin(d(a + b \log(cx^n)))}}{e(-ibdn + 2m + 2) \sqrt{1 - e^{2iad}(cx^n)^{2ibd}}}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]], x]

[Out] (2\*(e\*x)^(1 + m)\*Hypergeometric2F1[-1/2, (-1 - ((2\*I)\*(1 + m))/(b\*d\*n))/4, -(2\*I + (2\*I)\*m - 3\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]])/(e\*(2 + 2\*m - I\*b\*d\*n)\*Sqrt[1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)])

#### Rule 4493

Int[((e.\_)\*(x.\_))^(m.\_)\*Sin[((a.\_) + Log[(c.\_)\*(x.\_)^(n.\_)]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e.\_)\*(x.\_))^(m.\_)\*Sin[((a.\_) + Log[x]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] :> Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[(e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c.\_)\*(x.\_))^(m.\_)\*((a.\_) + (b.\_)\*(x.\_)^(n.\_))^(p.\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/ (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int (ex)^m \sqrt{\sin(d(a+b \log(cx^n)))} dx &= \frac{\left( (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{n}} \sqrt{\sin(d(a+b \log(x)))} dx, x, cx^n \right)}{en} \\ &= \frac{\left( (ex)^{1+m} (cx^n)^{\frac{ibd}{2}-\frac{1+m}{n}} \sqrt{\sin(d(a+b \log(cx^n)))} \right) \text{Subst} \left( \int x^{-1-\frac{ibd}{2}+\frac{1+m}{n}} \sqrt{1-e^{2iad}x^{2ibd}} dx, x, cx^n \right)}{en\sqrt{1-e^{2iad}(cx^n)^{2ibd}}} \\ &= \frac{2(ex)^{1+m} {}_2F_1 \left( -\frac{1}{2}, \frac{1}{4} \left( -1 - \frac{2i(1+m)}{bdn} \right); -\frac{2i+2im-3bdn}{4bdn}; e^{2iad}(cx^n)^{2ibd} \right) \sqrt{\sin(d(a+b \log(cx^n)))}}{e(2+2m-ibdn)\sqrt{1-e^{2iad}(cx^n)^{2ibd}}} \end{aligned}$$

**Mathematica [B]** time = 5.46119, size = 488, normalized size = 3.28

$$2x(ex)^m \left( \frac{\sqrt{\sin(d(a+b \log(cx^n)))} \sin(d(a+b \log(cx^n) - bn \log(x)))}{2(m+1) \sin(d(a+b \log(cx^n) - bn \log(x))) + bdn \cos(d(a+b \log(cx^n) - bn \log(x)))} - \frac{bdnx^{-ibdn} \sqrt{2-2e^{2iad}}}{\dots} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]],x]

[Out] 2\*x\*(e\*x)^m\*(-((b\*d\*E^(I\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])))\*n\*Sqrt[2 - 2\*E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*((2\*I + (2\*I)\*m + b\*d\*n)\*x^((2\*I)\*b\*d\*n)\*Hypergeometric2F1[1/2, ((-I/2)\*(1 + m + ((3\*I)/2)\*b\*d\*n))/(b\*d\*n), -(2\*I + (2\*I)\*m - 7\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)] + (-2\*I - (2\*I)\*m + 3\*b\*d\*n)\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m + b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 3\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]))/((2 + 2\*m - I\*b\*d\*n)\*(2 + 2\*m + (3\*I)\*b\*d\*n)\*(2\*I + (2\*I)\*m + b\*d\*n + E^((2\*I)\*d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))\*(-2\*I - (2\*I)\*m + b\*d\*n))\*x^(I\*b\*d\*n)\*Sqrt[((-I)\*(-1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))/(E^(I\*a\*d)\*(c\*x^n)^(I\*b\*d))]) + (Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]]\*Sin[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])])/(b\*d\*n\*Cos[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] + 2\*(1 + m)\*Sin[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]))

**Maple [F]** time = 0.269, size = 0, normalized size = 0.

$$\int (ex)^m \sqrt{\sin(d(a+b \ln(cx^n)))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^(1/2),x)

[Out] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sqrt{\sin((b \log(cx^n) + a)d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.76 \quad \int \frac{(ex)^m}{\sqrt{\sin(d(a+b \log(cx^n)))}} dx$$

**Optimal.** Leaf size=150

$$\frac{2(ex)^{m+1} \sqrt{1 - e^{2iad} (cx^n)^{2ibd}} \operatorname{Hypergeometric2F1} \left( \frac{1}{2}, -\frac{-bdn+2im+2i}{4bdn}, -\frac{-5bdn+2im+2i}{4bdn}, e^{2iad} (cx^n)^{2ibd} \right)}{e(ibdn + 2m + 2) \sqrt{\sin(d(a + b \log(cx^n)))}}$$

[Out] (2\*(e\*x)^(1 + m)\*Sqrt[1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m - b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 5\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]/(e\*(2 + 2\*m + I\*b\*d\*n)\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]])

**Rubi [A]** time = 0.109999, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4493, 4491, 364}

$$\frac{2(ex)^{m+1} \sqrt{1 - e^{2iad} (cx^n)^{2ibd}} {}_2F_1 \left( \frac{1}{2}, -\frac{2im-bdn+2i}{4bdn}; -\frac{2im-5bdn+2i}{4bdn}; e^{2iad} (cx^n)^{2ibd} \right)}{e(ibdn + 2m + 2) \sqrt{\sin(d(a + b \log(cx^n)))}}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m/Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]], x]

[Out] (2\*(e\*x)^(1 + m)\*Sqrt[1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m - b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 5\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]/(e\*(2 + 2\*m + I\*b\*d\*n)\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]])

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\int \frac{(ex)^m}{\sqrt{\sin(d(a+b \log(cx^n)))}} dx = \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sqrt{\sin(d(a+b \log(x)))}} dx, x, cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1}{2}ibd-\frac{1+m}{n}} \sqrt{1-e^{2iad} (cx^n)^{2ibd}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{ibd}{2}+\frac{1+m}{n}}}{\sqrt{1-e^{2iad} x^{2ibd}}} dx, x, cx^n\right)}{en\sqrt{\sin(d(a+b \log(cx^n)))}}$$

$$= \frac{2(ex)^{1+m} \sqrt{1-e^{2iad} (cx^n)^{2ibd}} {}_2F_1\left(\frac{1}{2}, -\frac{2i+2im-bdn}{4bdn}; -\frac{2i+2im-5bdn}{4bdn}; e^{2iad} (cx^n)^{2ibd}\right)}{e(2+2m+ibdn)\sqrt{\sin(d(a+b \log(cx^n)))}}$$

**Mathematica [A]** time = 0.517193, size = 131, normalized size = 0.87

$$\frac{2x(ex)^m \left(-1 + e^{2id(a+b \log(cx^n))}\right) \text{Hypergeometric2F1}\left(1, -\frac{-3bdn+2im+2i}{4bdn}, -\frac{-5bdn+2im+2i}{4bdn}, e^{2id(a+b \log(cx^n))}\right)}{(ibdn + 2m + 2)\sqrt{\sin(d(a+b \log(cx^n)))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m/Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]], x]

[Out]  $(-2*(-1 + E^{((2*I)*d*(a + b*Log[c*x^n])})) * x * (e*x)^m * \text{Hypergeometric2F1}[1, -(2*I + (2*I)*m - 3*b*d*n)/(4*b*d*n), -(2*I + (2*I)*m - 5*b*d*n)/(4*b*d*n), E^{((2*I)*d*(a + b*Log[c*x^n])})]) / ((2 + 2*m + I*b*d*n) * \text{Sqrt}[\text{Sin}[d*(a + b*Log[c*x^n])]])$

**Maple [F]** time = 0.263, size = 0, normalized size = 0.

$$\int (ex)^m \frac{1}{\sqrt{\sin(d(a+b \ln(cx^n)))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m/sin(d\*(a+b\*ln(c\*x^n)))^(1/2), x)

[Out] int((e\*x)^m/sin(d\*(a+b\*ln(c\*x^n)))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sqrt{\sin((b \log(cx^n) + a)d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m/sin(d\*(a+b\*log(c\*x^n)))^(1/2), x, algorithm="maxima")

[Out] integrate((e\*x)^m/sqrt(sin((b\*log(c\*x^n) + a)\*d)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m/sin(d\*(a+b\*log(c\*x^n)))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sqrt{\sin(ad + bd \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m/sin(d\*(a+b\*ln(c\*x\*\*n))))\*\*(1/2),x)

[Out] Integral((e\*x)\*\*m/sqrt(sin(a\*d + b\*d\*log(c\*x\*\*n))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sqrt{\sin((b \log(cx^n) + a)d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m/sin(d\*(a+b\*log(c\*x^n)))^(1/2),x, algorithm="giac")

[Out] integrate((e\*x)^m/sqrt(sin((b\*log(c\*x^n) + a)\*d)), x)

$$3.77 \quad \int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b \log(cx^n)))} dx$$

**Optimal.** Leaf size=150

$$\frac{2(ex)^{m+1} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^{3/2} \text{Hypergeometric2F1}\left(\frac{3}{2}, -\frac{3bdn+2im+2i}{4bdn}, -\frac{-7bdn+2im+2i}{4bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{e(3ibd n + 2m + 2) \sin^{\frac{3}{2}}(d(a + b \log(cx^n)))}$$

[Out] (2\*(e\*x)^(1 + m)\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))^(3/2)\*Hypergeometric2F1[3/2, -(2\*I + (2\*I)\*m - 3\*b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 7\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]/(e\*(2 + 2\*m + (3\*I)\*b\*d\*n)\*Sin[d\*(a + b\*Log[c\*x^n])]^(3/2))

**Rubi [A]** time = 0.113057, antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4493, 4491, 364}

$$\frac{2(ex)^{m+1} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4} \left(3 - \frac{2i(m+1)}{bdn}\right); -\frac{2im-7bdn+2i}{4bdn}; e^{2iad} (cx^n)^{2ibd}\right)}{e(3ibd n + 2m + 2) \sin^{\frac{3}{2}}(d(a + b \log(cx^n)))}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m/Sin[d\*(a + b\*Log[c\*x^n])]^(3/2), x]

[Out] (2\*(e\*x)^(1 + m)\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))^(3/2)\*Hypergeometric2F1[3/2, (3 - ((2\*I)\*(1 + m))/(b\*d\*n))/4, -(2\*I + (2\*I)\*m - 7\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]/(e\*(2 + 2\*m + (3\*I)\*b\*d\*n)\*Sin[d\*(a + b\*Log[c\*x^n])]^(3/2))

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[(e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b \log(cx^n)))} dx = \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sin^{\frac{3}{2}}(d(a+b \log(x)))} dx, x, cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m} (cx^n)^{-\frac{3}{2}ibd-\frac{1+m}{n}} (1 - e^{2iad} (cx^n)^{2ibd}\right)^{3/2} \text{Subst}\left(\int \frac{x^{-1+\frac{3ibd}{2}+\frac{1+m}{n}}}{(1-e^{2iad}x^{2ibd})^{3/2}} dx, x, cx^n\right)}{en \sin^{\frac{3}{2}}(d(a+b \log(cx^n)))}$$

$$= \frac{2(ex)^{1+m} (1 - e^{2iad} (cx^n)^{2ibd})^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i(1+m)}{bdn}\right); -\frac{2i+2im-7bdn}{4bdn}; e^{2iad} (cx^n)^{2ibd}\right)}{e(2+2m+3ibd) \sin^{\frac{3}{2}}(d(a+b \log(cx^n)))}$$

**Mathematica [B]** time = 5.17482, size = 544, normalized size = 3.63

$$(ex)^m (b^2 d^2 n^2 + 4m^2 + 8m + 4) x^{1+ibdn} \sqrt{2 - 2e^{2iad} (cx^n)^{2ibd}} \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{i\left(\frac{3}{2}ibd+n+m+1\right)}{2bdn}, -\frac{-7bdn+2im+2i}{4bdn}, e^{2iad} (cx^n)^{-1}\right) / bdn(3bdn - 2im - 2i) \sqrt{-ie^{-iad} (cx^n)^{-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m/Sin[d\*(a + b\*Log[c\*x^n])]^(3/2), x]

[Out] ((4 + 8\*m + 4\*m^2 + b^2\*d^2\*n^2)\*x^(1 + I\*b\*d\*n)\*(e\*x)^m\*Sqrt[2 - 2\*E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Hypergeometric2F1[1/2, ((-I/2)\*(1 + m + ((3\*I)/2)\*b\*d\*n))/(b\*d\*n), -(2\*I + (2\*I)\*m - 7\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)] + ((-2\*I - (2\*I)\*m + 3\*b\*d\*n)\*x^(1 - I\*b\*d\*n)\*(e\*x)^m\*(-2\*x^(I\*b\*d\*n)\*Sqrt[((-I)\*(-1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))/(E^(I\*a\*d)\*(c\*x^n)^(I\*b\*d))]\*(b\*d\*n\*Cos[b\*d\*n\*Log[x]] - 2\*(1 + m)\*Sin[b\*d\*n\*Log[x]]) + (-2\*I - (2\*I)\*m + b\*d\*n)\*Sqrt[2 - 2\*E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m + b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 3\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]]))/Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]]/(b\*d\*n\*(-2\*I - (2\*I)\*m + 3\*b\*d\*n)\*Sqrt[((-I)\*(-1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))/(E^(I\*a\*d)\*(c\*x^n)^(I\*b\*d))]\*(b\*d\*n\*Cos[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] + 2\*(1 + m)\*Sin[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])])

**Maple [F]** time = 0.261, size = 0, normalized size = 0.

$$\int (ex)^m (\sin(d(a+b \ln(cx^n))))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m/sin(d\*(a+b\*ln(c\*x^n)))^(3/2), x)

[Out] int((e\*x)^m/sin(d\*(a+b\*ln(c\*x^n)))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sin((b \log(cx^n) + a)d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/sin((b*log(c*x^n) + a)*d)^(3/2), x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sin(d*(a+b*ln(c*x**n)))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sin((b \log(cx^n) + a)d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sin((b*log(c*x^n) + a)*d)^(3/2), x)
```

$$3.78 \quad \int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b \log(cx^n)))} dx$$

**Optimal.** Leaf size=150

$$\frac{2(ex)^{m+1} (1 - e^{2iad} (cx^n)^{2ibd})^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{2}, -\frac{-5bdn+2im+2i}{4bdn}, -\frac{-9bdn+2im+2i}{4bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{e(5ibdn + 2m + 2) \sin^{\frac{5}{2}}(d(a + b \log(cx^n)))}$$

[Out] (2\*(e\*x)^(1 + m)\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^(5/2)\*Hypergeometric2F1[5/2, -(2\*I + (2\*I)\*m - 5\*b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 9\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)])/((e\*(2 + 2\*m + (5\*I)\*b\*d\*n)\*Sin[d\*(a + b\*Log[c\*x^n])])^(5/2))

**Rubi [A]** time = 0.113741, antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4493, 4491, 364}

$$\frac{2(ex)^{m+1} (1 - e^{2iad} (cx^n)^{2ibd})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i(m+1)}{bdn}\right); -\frac{2im-9bdn+2i}{4bdn}; e^{2iad} (cx^n)^{2ibd}\right)}{e(5ibdn + 2m + 2) \sin^{\frac{5}{2}}(d(a + b \log(cx^n)))}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m/Sin[d\*(a + b\*Log[c\*x^n])])^(5/2), x]

[Out] (2\*(e\*x)^(1 + m)\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^(5/2)\*Hypergeometric2F1[5/2, (5 - ((2\*I)\*(1 + m))/(b\*d\*n))/4, -(2\*I + (2\*I)\*m - 9\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)])/((e\*(2 + 2\*m + (5\*I)\*b\*d\*n)\*Sin[d\*(a + b\*Log[c\*x^n])])^(5/2))

#### Rule 4493

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b \log(cx^n)))} dx = \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sin^{\frac{5}{2}}(d(a+b \log(x)))} dx, x, cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m} (cx^n)^{-\frac{5}{2}ibd-\frac{1+m}{n}} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^{5/2}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{5ibd}{2}+\frac{1+m}{n}}}{(1-e^{2iad}x^{2ibd})^{5/2}} dx, x, cx^n\right)}{en \sin^{\frac{5}{2}}(d(a+b \log(cx^n)))}$$

$$= \frac{2(ex)^{1+m} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i(1+m)}{bdn}\right); -\frac{2i+2im-9bdn}{4bdn}; e^{2iad} (cx^n)^{2ibd}\right)}{e(2+2m+5ibdn) \sin^{\frac{5}{2}}(d(a+b \log(cx^n)))}$$

**Mathematica [A]** time = 2.41237, size = 214, normalized size = 1.43

$$2x(ex)^m \left(-(-ibdn + 2m + 2) \left(-1 + e^{2id(a+b \log(cx^n))}\right)\right) \text{Hypergeometric2F1}\left(1, -\frac{-3bdn+2im+2i}{4bdn}, -\frac{-5bdn+2im+2i}{4bdn}, e^{2id(a+b \log(cx^n))}\right)$$

$3b^2d^2n$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m/Sin[d\*(a + b\*Log[c\*x^n])]^(5/2), x]

[Out] (2\*x\*(e\*x)^m\*(-2 - 2\*m - b\*d\*n\*Cot[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]) - (-1 + E^((2\*I)\*d\*(a + b\*Log[c\*x^n])))\*(2 + 2\*m - I\*b\*d\*n)\*Hypergeometric2F1[1, -(2\*I + (2\*I)\*m - 3\*b\*d\*n)/(4\*b\*d\*n), -(2\*I + (2\*I)\*m - 5\*b\*d\*n)/(4\*b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))] + b\*d\*n\*Csc[d\*(a + b\*Log[c\*x^n])]\*Csc[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]\*Sin[b\*d\*n\*Log[x]])/(3\*b^2\*d^2\*n^2\*Sqrt[Sin[d\*(a + b\*Log[c\*x^n])]])

**Maple [F]** time = 0.261, size = 0, normalized size = 0.

$$\int (ex)^m (\sin(d(a+b \ln(cx^n))))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m/sin(d\*(a+b\*ln(c\*x^n)))^(5/2), x)

[Out] int((e\*x)^m/sin(d\*(a+b\*ln(c\*x^n)))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sin((b \log(cx^n) + a)d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m/sin(d\*(a+b\*log(c\*x^n)))^(5/2), x, algorithm="maxima")

[Out] integrate((e\*x)^m/sin((b\*log(c\*x^n) + a)\*d)^(5/2), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m/sin(d\*(a+b\*log(c\*x^n)))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m/sin(d\*(a+b\*ln(c\*x\*\*n))))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex)^m}{\sin((b \log(cx^n) + a)d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m/sin(d\*(a+b\*log(c\*x^n)))^(5/2),x, algorithm="giac")

[Out] integrate((e\*x)^m/sin((b\*log(c\*x^n) + a)\*d)^(5/2), x)

### 3.79 $\int (ex)^m \sin^p(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=144

$$\frac{(ex)^{m+1} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^{-p} \sin^p(d(a + b \log(cx^n))) \operatorname{Hypergeometric2F1}\left(-p, -\frac{bdnp+im+i}{2bdn}, \frac{1}{2}\left(-\frac{i(m+1)}{bdn} - p + 2\right), e^{2iaad}\right)}{e(-ibdnp + m + 1)}$$

[Out] ((e\*x)^(1 + m)\*Hypergeometric2F1[-p, -(I + I\*m + b\*d\*n\*p)/(2\*b\*d\*n), (2 - (I\*(1 + m))/(b\*d\*n) - p)/2, E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(e\*(1 + m - I\*b\*d\*n\*p)\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^p)

**Rubi [A]** time = 0.122529, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4493, 4491, 364}

$$\frac{(ex)^{m+1} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^{-p} {}_2F_1\left(-p, -\frac{im+bdnp+i}{2bdn}; \frac{1}{2}\left(-\frac{i(m+1)}{bdn} - p + 2\right); e^{2iaad} (cx^n)^{2ibd}\right) \sin^p(d(a + b \log(cx^n)))}{e(-ibdnp + m + 1)}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out] ((e\*x)^(1 + m)\*Hypergeometric2F1[-p, -(I + I\*m + b\*d\*n\*p)/(2\*b\*d\*n), (2 - (I\*(1 + m))/(b\*d\*n) - p)/2, E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(e\*(1 + m - I\*b\*d\*n\*p)\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^p)

#### Rule 4493

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^p, x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int (ex)^m \sin^p(d(a + b \log(cx^n))) dx = \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sin^p(d(a + b \log(x))) dx, x, cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}+ibdp} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^{-p} \sin^p(d(a + b \log(cx^n)))\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sin^p(d(a + b \log(x))) dx, x, cx^n\right)}{en}$$

$$= \frac{(ex)^{1+m} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^{-p} {}_2F_1\left(-p, -\frac{i+im+bdnp}{2bdn}; \frac{1}{2}\left(2 - \frac{i(1+m)}{bdn} - p\right); e^{2iad} (cx^n)^{2ibd}\right)}{e(1+m-ibdn)}$$

**Mathematica [A]** time = 0.932089, size = 122, normalized size = 0.85

$$\frac{x(ex)^m \left(-1 + e^{2id(a+b \log(cx^n))}\right) \sin^p(d(a + b \log(cx^n))) \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-\frac{i(m+1)}{bdn} + p + 2\right), -\frac{i(m+1)}{2bdn} - \frac{p}{2} + 1, e^{2id(a+b \log(cx^n))}\right)}{-ibdn + m + 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m\*Sin[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out] -((( -1 + E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))))\*x\*(e\*x)^m\*Hypergeometric2F1[1, (2 - (I\*(1 + m))/(b\*d\*n) + p)/2, 1 - ((I/2)\*(1 + m))/(b\*d\*n) - p/2, E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]\*Sin[d\*(a + b\*Log[c\*x^n])]^p)/(1 + m - I\*b\*d\*n\*p))

**Maple [F]** time = 0.283, size = 0, normalized size = 0.

$$\int (ex)^m (\sin(d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^p,x)

[Out] int((e\*x)^m\*sin(d\*(a+b\*ln(c\*x^n)))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sin((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sin(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="maxima")

[Out] integrate((e\*x)^m\*sin((b\*log(c\*x^n) + a)\*d)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \sin(bd \log(cx^n) + ad)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sin(b*d*log(c*x^n) + a*d)^p, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.80 $\int x^2 \sin^p(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=114

$$\frac{x^3 (1 - e^{2ia} (cx^n)^{2ib})^{-p} \operatorname{Hypergeometric2F1}\left(-p, -\frac{bnp+3i}{2bn}, \frac{1}{2}\left(-\frac{3i}{bn} - p + 2\right), e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{3 - ibnp}$$

[Out]  $(x^3 \operatorname{Hypergeometric2F1}[-p, -(3I + b*n*p)/(2*b*n), (2 - (3I)/(b*n) - p)/2, E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}] * \operatorname{Sin}[a + b*\operatorname{Log}[c*x^n]]^p) / ((3 - I*b*n*p)*(1 - E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}))^p$

**Rubi [A]** time = 0.0950033, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4493, 4491, 364}

$$\frac{x^3 (1 - e^{2ia} (cx^n)^{2ib})^{-p} {}_2F_1\left(-p, -\frac{bnp+3i}{2bn}; \frac{1}{2}\left(-p - \frac{3i}{bn} + 2\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{3 - ibnp}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 * \operatorname{Sin}[a + b * \operatorname{Log}[c * x^n]]^p, x]$

[Out]  $(x^3 \operatorname{Hypergeometric2F1}[-p, -(3I + b*n*p)/(2*b*n), (2 - (3I)/(b*n) - p)/2, E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}] * \operatorname{Sin}[a + b*\operatorname{Log}[c*x^n]]^p) / ((3 - I*b*n*p)*(1 - E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}))^p$

#### Rule 4493

$\operatorname{Int}[(e^.) * (x^.)^{(m_.)} * \operatorname{Sin}[(a_.) + \operatorname{Log}[(c_.) * (x^.)^{(n_.)}] * (b_.)] * (d_.)]^{(p_.)}, x\_Symbol] := \operatorname{Dist}[(e*x)^{(m+1)} / (e*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[x^{(m+1)/n - 1} * \operatorname{Sin}[d*(a + b*\operatorname{Log}[x])]^p, x], x, c*x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\operatorname{NeQ}[c, 1] \mid \mid \operatorname{NeQ}[n, 1])$

#### Rule 4491

$\operatorname{Int}[(e^.) * (x^.)^{(m_.)} * \operatorname{Sin}[(a_.) + \operatorname{Log}[x] * (b_.)] * (d_.)]^{(p_.)}, x\_Symbol] := \operatorname{Dist}[(\operatorname{Sin}[d*(a + b*\operatorname{Log}[x])]^p * x^{(I*b*d*p)}) / (1 - E^{(2*I*a*d)} * x^{(2*I*b*d)})^p, \operatorname{Int}[(e*x)^m * (1 - E^{(2*I*a*d)} * x^{(2*I*b*d)})^p / x^{(I*b*d*p)}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, m, p\}, x] \&\& !\operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c^.) * (x^.)^{(m_.)} * ((a_.) + (b_.) * (x^.)^{(n_.)})^{(p_.)}, x\_Symbol] := \operatorname{Simp}[(a^p * (c*x)^{(m+1)} * \operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -(b*x^n)/a]) / (c*(m+1)), x] /; \operatorname{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \mid \mid \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\int x^2 \sin^p(a + b \log(cx^n)) dx = \frac{(x^3 (cx^n)^{-3/n}) \text{Subst}\left(\int x^{-1+\frac{3}{n}} \sin^p(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{(x^3 (cx^n)^{-\frac{3}{n}+ibp} (1 - e^{2ia} (cx^n)^{2ib})^{-p} \sin^p(a + b \log(cx^n))) \text{Subst}\left(\int x^{-1+\frac{3}{n}-ibp} (1 - e^{2ia} (cx^n)^{2ib})^{-p} \sin^p(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{x^3 (1 - e^{2ia} (cx^n)^{2ib})^{-p} {}_2F_1\left(-p, -\frac{3i+ibp}{2bn}; \frac{1}{2}\left(2 - \frac{3i}{bn} - p\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{3 - ibnp}$$

**Mathematica [A]** time = 0.647153, size = 100, normalized size = 0.88

$$\frac{x^3 (-1 + e^{2i(a+b \log(cx^n))}) \sin^p(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-\frac{3i}{bn} + p + 2\right), -\frac{3i}{2bn} - \frac{p}{2} + 1, e^{2i(a+b \log(cx^n))}\right)}{-3 + ibnp}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*Sin[a + b\*Log[c\*x^n]]^p,x]

[Out] ((-1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x^3 \* Hypergeometric2F1[1, (2 - (3\*I)/(b\*n) + p)/2, 1 - ((3\*I)/2)/(b\*n) - p/2, E^((2\*I)\*(a + b\*Log[c\*x^n]))] \* Sin[a + b\*Log[c\*x^n]]^p) / (-3 + I\*b\*n\*p)

**Maple [F]** time = 0.146, size = 0, normalized size = 0.

$$\int x^2 (\sin(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sin(a+b\*ln(c\*x^n))^p,x)

[Out] int(x^2\*sin(a+b\*ln(c\*x^n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sin(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] integrate(x^2\*sin(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \sin(b \log(cx^n) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(x^2*sin(b*log(c*x^n) + a)^p, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(a+b*ln(c*x**n))**p,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(b*log(c*x^n) + a)^p, x)
```



### 3.81 $\int x \sin^p (a + b \log (cx^n)) dx$

**Optimal.** Leaf size=114

$$\frac{x^2 (1 - e^{2ia} (cx^n)^{2ib})^{-p} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}\left(-p - \frac{2i}{bn}\right), -p, \frac{1}{2}\left(-\frac{2i}{bn} - p + 2\right), e^{2ia} (cx^n)^{2ib}\right) \sin^p (a + b \log (cx^n))}{2 - ibnp}$$

[Out]  $(x^2 \operatorname{Hypergeometric2F1}[\frac{(-2*I)/(b*n) - p}{2}, -p, \frac{2 - (2*I)/(b*n) - p}{2}, E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}] * \operatorname{Sin}[a + b * \operatorname{Log}[c*x^n]]^p) / ((2 - I*b*n*p) * (1 - E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}))^p$

**Rubi [A]** time = 0.0831404, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4493, 4491, 364}

$$\frac{x^2 (1 - e^{2ia} (cx^n)^{2ib})^{-p} {}_2F_1\left(\frac{1}{2}\left(-p - \frac{2i}{bn}\right), -p; \frac{1}{2}\left(-p - \frac{2i}{bn} + 2\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p (a + b \log (cx^n))}{2 - ibnp}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x * \operatorname{Sin}[a + b * \operatorname{Log}[c * x^n]]^p, x]$

[Out]  $(x^2 \operatorname{Hypergeometric2F1}[\frac{(-2*I)/(b*n) - p}{2}, -p, \frac{2 - (2*I)/(b*n) - p}{2}, E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}] * \operatorname{Sin}[a + b * \operatorname{Log}[c*x^n]]^p) / ((2 - I*b*n*p) * (1 - E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}))^p$

#### Rule 4493

$\operatorname{Int}[(e^.) * (x^.)^{(m_.)} * \operatorname{Sin}[(a_.) + \operatorname{Log}[(c_.) * (x^.)^{(n_.)}] * (b_.)] * (d_.)]^{(p_.)}, x\_Symbol] :> \operatorname{Dist}[(e*x)^{(m+1)} / (e*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[x^{((m+1)/n-1)} * \operatorname{Sin}[d*(a+b*\operatorname{Log}[x])]^p, x], x, c*x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\operatorname{NeQ}[c, 1] \|\| \operatorname{NeQ}[n, 1])$

#### Rule 4491

$\operatorname{Int}[(e^.) * (x^.)^{(m_.)} * \operatorname{Sin}[(a_.) + \operatorname{Log}[x] * (b_.)] * (d_.)]^{(p_.)}, x\_Symbol] :> \operatorname{Dist}[(\operatorname{Sin}[d*(a+b*\operatorname{Log}[x])]^p * x^{(I*b*d*p)}) / (1 - E^{(2*I*a*d)} * x^{(2*I*b*d)})^p, \operatorname{Int}[(e*x)^m * (1 - E^{(2*I*a*d)} * x^{(2*I*b*d)})^p / x^{(I*b*d*p)}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, m, p\}, x] \&\& !\operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c^.) * (x^.)^{(m_.)} * ((a_.) + (b_.) * (x^.)^{(n_.)})^{(p_.)}, x\_Symbol] :> \operatorname{Simp}[(a^p * (c*x)^{(m+1)} * \operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -((b*x^n)/a)]) / (c*(m+1)), x] /; \operatorname{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\operatorname{IntegerQ}[p, 0] \&\& (\operatorname{IntegerQ}[p, 0] \|\| \operatorname{IntegerQ}[a, 0])$

#### Rubi steps

$$\begin{aligned} \int x \sin^p(a + b \log(cx^n)) dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int x^{-1+\frac{2}{n}} \sin^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(x^2 (cx^n)^{-\frac{2}{n}+ibp} (1 - e^{2ia} (cx^n)^{2ib})^{-p} \sin^p(a + b \log(cx^n))) \text{Subst}\left(\int x^{-1+\frac{2}{n}-ibp} (1 - e^{2ia} x^{2ib})\right)}{n} \\ &= \frac{x^2 (1 - e^{2ia} (cx^n)^{2ib})^{-p} {}_2F_1\left(\frac{1}{2}\left(-\frac{2i}{bn} - p\right), -p; \frac{1}{2}\left(2 - \frac{2i}{bn} - p\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{2 - ibnp} \end{aligned}$$

**Mathematica [A]** time = 0.590007, size = 98, normalized size = 0.86

$$\frac{x^2 (-1 + e^{2i(a+b \log(cx^n))}) \sin^p(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, -\frac{i}{bn} + \frac{p}{2} + 1, -\frac{i}{bn} - \frac{p}{2} + 1, e^{2i(a+b \log(cx^n))}\right)}{-2 + ibnp}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sin[a + b\*Log[c\*x^n]]^p, x]

[Out] ((-1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x^2 \* Hypergeometric2F1[1, 1 - I/(b\*n) + p/2, 1 - I/(b\*n) - p/2, E^((2\*I)\*(a + b\*Log[c\*x^n]))] \* Sin[a + b\*Log[c\*x^n]]^p) / (-2 + I\*b\*n\*p)

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int x (\sin(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sin(a+b\*ln(c\*x^n))^p, x)

[Out] int(x\*sin(a+b\*ln(c\*x^n))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sin(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sin(a+b\*log(c\*x^n))^p, x, algorithm="maxima")

[Out] integrate(x\*sin(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \sin(b \log(cx^n) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(x*sin(b*log(c*x^n) + a)^p, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sin^p(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(x*sin(a + b*log(c*x**n))**p, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sin(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*log(c*x^n) + a)^p, x)
```

### 3.82 $\int \sin^p (a + b \log (cx^n)) dx$

**Optimal.** Leaf size=112

$$\frac{x \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{-p} \operatorname{Hypergeometric2F1}\left(-p, -\frac{bnp+i}{2bn}, \frac{1}{2}\left(-\frac{i}{bn} - p + 2\right), e^{2ia} (cx^n)^{2ib}\right) \sin^p (a + b \log (cx^n))}{1 - ibnp}$$

[Out] (x\*Hypergeometric2F1[-p, -(I + b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) - p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^p)/((1 - I\*b\*n\*p)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p

**Rubi [A]** time = 0.0747292, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4483, 4491, 364}

$$\frac{x \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{-p} {}_2F_1\left(-p, -\frac{bnp+i}{2bn}; \frac{1}{2}\left(-p - \frac{i}{bn} + 2\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p (a + b \log (cx^n))}{1 - ibnp}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^p, x]

[Out] (x\*Hypergeometric2F1[-p, -(I + b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) - p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^p)/((1 - I\*b\*n\*p)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p

#### Rule 4483

Int[Sin[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_.)\*(x\_))^(m\_.)\*Sin[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^(m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d)))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sin^p(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int x^{-1+\frac{1}{n}} \sin^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(x(cx^n)^{-\frac{1}{n}+ibp} (1 - e^{2ia} (cx^n)^{2ib})^{-p} \sin^p(a + b \log(cx^n))) \text{Subst}\left(\int x^{-1+\frac{1}{n}-ibp} (1 - e^{2ia} x^{2ib})^{-p} \sin^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{x(1 - e^{2ia} (cx^n)^{2ib})^{-p} {}_2F_1\left(-p, -\frac{i+bnp}{2bn}; \frac{1}{2}\left(2 - \frac{i}{bn} - p\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{1 - ibnp} \end{aligned}$$

**Mathematica [A]** time = 0.537741, size = 98, normalized size = 0.88

$$\frac{x(-1 + e^{2i(a+b \log(cx^n))}) \sin^p(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-\frac{i}{bn} + p + 2\right), -\frac{i}{2bn} - \frac{p}{2} + 1, e^{2i(a+b \log(cx^n))}\right)}{-1 + ibnp}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^p, x]

[Out] ((-1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x \* Hypergeometric2F1[1, (2 - I/(b\*n) + p)/2, 1 - (I/2)/(b\*n) - p/2, E^((2\*I)\*(a + b\*Log[c\*x^n]))] \* Sin[a + b\*Log[c\*x^n]]^p) / (-1 + I\*b\*n\*p)

**Maple [F]** time = 0.125, size = 0, normalized size = 0.

$$\int (\sin(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^p, x)

[Out] int(sin(a+b\*ln(c\*x^n))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^p, x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sin(b \log(cx^n) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(sin(b*log(c*x^n) + a)^p, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin^p(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(sin(a + b*log(c*x**n))**p, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sin(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p, x)
```

$$3.83 \quad \int \frac{\sin^p(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=86

$$\frac{\cos(a+b \log(cx^n)) \sin^{p+1}(a+b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{p+1}{2}, \frac{p+3}{2}, \sin^2(a+b \log(cx^n))\right)}{bn(p+1)\sqrt{\cos^2(a+b \log(cx^n))}}$$

[Out] (Cos[a + b\*Log[c\*x^n]]\*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[a + b\*Log[c\*x^n]]^2]\*Sin[a + b\*Log[c\*x^n]]^(1 + p))/(b\*n\*(1 + p)\*Sqrt[Cos[a + b\*Log[c\*x^n]]^2])

**Rubi [A]** time = 0.0601127, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2643}

$$\frac{\cos(a+b \log(cx^n)) \sin^{p+1}(a+b \log(cx^n)) {}_2F_1\left(\frac{1}{2}, \frac{p+1}{2}; \frac{p+3}{2}; \sin^2(a+b \log(cx^n))\right)}{bn(p+1)\sqrt{\cos^2(a+b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^p/x, x]

[Out] (Cos[a + b\*Log[c\*x^n]]\*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[a + b\*Log[c\*x^n]]^2]\*Sin[a + b\*Log[c\*x^n]]^(1 + p))/(b\*n\*(1 + p)\*Sqrt[Cos[a + b\*Log[c\*x^n]]^2])

#### Rule 2643

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1)\*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d\*x]^2])/(b\*d\*(n + 1)\*Sqrt[Cos[c + d\*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2\*n]

#### Rubi steps

$$\begin{aligned} \int \frac{\sin^p(a+b \log(cx^n))}{x} dx &= \frac{\operatorname{Subst}\left(\int \sin^p(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\cos(a+b \log(cx^n)) {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \sin^2(a+b \log(cx^n))\right) \sin^{1+p}(a+b \log(cx^n))}{bn(1+p)\sqrt{\cos^2(a+b \log(cx^n))}} \end{aligned}$$

**Mathematica [A]** time = 0.149624, size = 86, normalized size = 1.

$$\frac{\sec(a+b \log(cx^n)) \sqrt{\cos^2(a+b \log(cx^n))} \sin^{p+1}(a+b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{p+1}{2}, \frac{p+3}{2}, \sin^2(a+b \log(cx^n))\right)}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^p/x, x]

[Out] (Sqrt[Cos[a + b\*Log[c\*x^n]]^2]\*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[a + b\*Log[c\*x^n]]^2]\*Sec[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]]^(1 + p))/(b\*n\*(1 + p))

**Maple [F]** time = 0.13, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^p/x,x)

[Out] int(sin(a+b\*ln(c\*x^n))^p/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^p/x,x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^p/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(b \log(cx^n) + a)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^p/x,x, algorithm="fricas")

[Out] integral(sin(b\*log(c\*x^n) + a)^p/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^p(a + b \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*ln(c\*x\*\*n))\*\*p/x,x)

[Out] Integral(sin(a + b\*log(c\*x\*\*n))\*\*p/x, x)



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p/x, x)
```

$$3.84 \quad \int \frac{\sin^p(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=115

$$\frac{(1 - e^{2ia} (cx^n)^{2ib})^{-p} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}\left(-p + \frac{i}{bn}\right), -p, \frac{1}{2}\left(\frac{i}{bn} - p + 2\right), e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{x(1 + ibnp)}$$

[Out] -((Hypergeometric2F1[(I/(b\*n) - p)/2, -p, (2 + I/(b\*n) - p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^p)/((1 + I\*b\*n\*p)\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^p))

**Rubi [A]** time = 0.0948704, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4493, 4491, 364}

$$\frac{(1 - e^{2ia} (cx^n)^{2ib})^{-p} {}_2F_1\left(\frac{1}{2}\left(\frac{i}{bn} - p\right), -p; \frac{1}{2}\left(-p + \frac{i}{bn} + 2\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{x(1 + ibnp)}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b\*Log[c\*x^n]]^p/x^2, x]

[Out] -((Hypergeometric2F1[(I/(b\*n) - p)/2, -p, (2 + I/(b\*n) - p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sin[a + b\*Log[c\*x^n]]^p)/((1 + I\*b\*n\*p)\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^p))

#### Rule 4493

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sin[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4491

Int[((e\_)\*(x\_))^(m\_)\*Sin[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[(Sin[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{\sin^p(a + b \log(cx^n))}{x^2} dx = \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \sin^p(a + b \log(x)) dx, x, cx^n\right)}{nx}$$

$$= \frac{\left((cx^n)^{\frac{1}{n}+ibp} (1 - e^{2ia} (cx^n)^{2ib})^{-p} \sin^p(a + b \log(cx^n))\right) \text{Subst}\left(\int x^{-1-\frac{1}{n}-ibp} (1 - e^{2ia} x^{2ib})^p dx, x, cx^n\right)}{nx}$$

$$= -\frac{\left(1 - e^{2ia} (cx^n)^{2ib}\right)^{-p} {}_2F_1\left(\frac{1}{2}\left(\frac{i}{bn} - p\right), -p; \frac{1}{2}\left(2 + \frac{i}{bn} - p\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{(1 + ibnp)x}$$

**Mathematica [A]** time = 0.593845, size = 102, normalized size = 0.89

$$\frac{i(-1 + e^{2i(a+b \log(cx^n))}) \sin^p(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(\frac{i}{bn} + p + 2\right), \frac{i}{2bn} - \frac{p}{2} + 1, e^{2i(a+b \log(cx^n))}\right)}{x(bnp - i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^p/x^2, x]

[Out] ((-I)\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n])))\*Hypergeometric2F1[1, (2 + I/(b\*n) + p)/2, 1 + (I/2)/(b\*n) - p/2, E^((2\*I)\*(a + b\*Log[c\*x^n]))]\*Sin[a + b\*Log[c\*x^n]]^p)/((-I + b\*n\*p)\*x)

**Maple [F]** time = 0.135, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^p/x^2, x)

[Out] int(sin(a+b\*ln(c\*x^n))^p/x^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^p/x^2, x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^p/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(b \log(cx^n) + a)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*log(c*x^n) + a)^p/x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^p(a + b \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**p/x**2,x)
```

```
[Out] Integral(sin(a + b*log(c*x**n))**p/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p/x^2, x)
```

$$3.85 \quad \int \frac{\sin^p(a+b \log(cx^n))}{x^3} dx$$

**Optimal.** Leaf size=115

$$\frac{(1 - e^{2ia} (cx^n)^{2ib})^{-p} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}\left(-p + \frac{2i}{bn}\right), -p, \frac{1}{2}\left(\frac{2i}{bn} - p + 2\right), e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{x^2(2 + ibnp)}$$

[Out]  $-\left(\operatorname{Hypergeometric2F1}\left[\left(\frac{(2*I)}{(b*n)} - p\right)/2, -p, \left(2 + \frac{(2*I)}{(b*n)} - p\right)/2, E^{\left((2*I)*a\right)*\left(c*x^n\right)^{\left((2*I)*b\right)}\right]*\operatorname{Sin}\left[a + b*\operatorname{Log}\left[c*x^n\right]\right]^p\right)/\left(\left(2 + I*b*n*p\right)*x^2*\left(1 - E^{\left((2*I)*a\right)*\left(c*x^n\right)^{\left((2*I)*b\right)}\right)^p\right)$

**Rubi [A]** time = 0.0924682, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4493, 4491, 364}

$$\frac{(1 - e^{2ia} (cx^n)^{2ib})^{-p} {}_2F_1\left(\frac{1}{2}\left(\frac{2i}{bn} - p\right), -p; \frac{1}{2}\left(-p + \frac{2i}{bn} + 2\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{x^2(2 + ibnp)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sin}[a + b*\operatorname{Log}[c*x^n]]^p/x^3, x]$

[Out]  $-\left(\operatorname{Hypergeometric2F1}\left[\left(\frac{(2*I)}{(b*n)} - p\right)/2, -p, \left(2 + \frac{(2*I)}{(b*n)} - p\right)/2, E^{\left((2*I)*a\right)*\left(c*x^n\right)^{\left((2*I)*b\right)}\right]*\operatorname{Sin}\left[a + b*\operatorname{Log}\left[c*x^n\right]\right]^p\right)/\left(\left(2 + I*b*n*p\right)*x^2*\left(1 - E^{\left((2*I)*a\right)*\left(c*x^n\right)^{\left((2*I)*b\right)}\right)^p\right)$

#### Rule 4493

$\operatorname{Int}[\left((e_{.})*(x_{.})\right)^{(m_{.})}*\operatorname{Sin}[\left((a_{.}) + \operatorname{Log}[(c_{.})*(x_{.})^{(n_{.})}]\right)*\left(b_{.}\right)*\left(d_{.}\right)]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[x^{((m+1)/n-1)*\operatorname{Sin}[d*(a+b*\operatorname{Log}[x])]}]^p, x], x, c*x^n], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, m, n, p\}, x] \&\& (\operatorname{NeQ}[c, 1] \parallel \operatorname{NeQ}[n, 1])$

#### Rule 4491

$\operatorname{Int}[\left((e_{.})*(x_{.})\right)^{(m_{.})}*\operatorname{Sin}[\left((a_{.}) + \operatorname{Log}[x_{.}]*\left(b_{.}\right)*\left(d_{.}\right)\right)]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(\operatorname{Sin}[d*(a+b*\operatorname{Log}[x])]]^p*x^{(I*b*d*p)})/(1 - E^{(2*I*a*d)*x^{(2*I*b*d)}})^p, \operatorname{Int}[\left((e*x)^m*(1 - E^{(2*I*a*d)*x^{(2*I*b*d)}})^p\right)/x^{(I*b*d*p)}, x], x] /; \operatorname{FreeQ}[\{a, b, d, e, m, p\}, x] \&\& !\operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[\left((c_{.})*(x_{.})\right)^{(m_{.})}*\left((a_{.}) + \left(b_{.}\right)*(x_{.})^{(n_{.})}\right)^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(a^p*(c*x)^{(m+1)}*\operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/ (c*(m+1)), x] /; \operatorname{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \parallel \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\int \frac{\sin^p(a + b \log(cx^n))}{x^3} dx = \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int x^{-1-\frac{2}{n}} \sin^p(a + b \log(x)) dx, x, cx^n\right)}{nx^2}$$

$$= \frac{\left((cx^n)^{\frac{2}{n}+ibp} (1 - e^{2ia} (cx^n)^{2ib})^{-p} \sin^p(a + b \log(cx^n))\right) \operatorname{Subst}\left(\int x^{-1-\frac{2}{n}-ibp} (1 - e^{2ia} x^{2ib})^p dx, x, cx^n\right)}{nx^2}$$

$$= -\frac{\left(1 - e^{2ia} (cx^n)^{2ib}\right)^{-p} {}_2F_1\left(\frac{1}{2}\left(\frac{2i}{bn} - p\right), -p; \frac{1}{2}\left(2 + \frac{2i}{bn} - p\right); e^{2ia} (cx^n)^{2ib}\right) \sin^p(a + b \log(cx^n))}{(2 + ibnp)x^2}$$

**Mathematica [A]** time = 0.627963, size = 100, normalized size = 0.87

$$\frac{i(-1 + e^{2i(a+b \log(cx^n))}) \sin^p(a + b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bn} + \frac{p}{2} + 1, \frac{i}{bn} - \frac{p}{2} + 1, e^{2i(a+b \log(cx^n))}\right)}{x^2(bnp - 2i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b\*Log[c\*x^n]]^p/x^3,x]

[Out] ((-I)\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))))\*Hypergeometric2F1[1, 1 + I/(b\*n) + p/2, 1 + I/(b\*n) - p/2, E^((2\*I)\*(a + b\*Log[c\*x^n]))]\*Sin[a + b\*Log[c\*x^n]]^p)/((-2\*I + b\*n\*p)\*x^2)

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int \frac{(\sin(a + b \ln(cx^n)))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a+b\*ln(c\*x^n))^p/x^3,x)

[Out] int(sin(a+b\*ln(c\*x^n))^p/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a+b\*log(c\*x^n))^p/x^3,x, algorithm="maxima")

[Out] integrate(sin(b\*log(c\*x^n) + a)^p/x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sin(b \log(cx^n) + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^3,x, algorithm="fricas")
```

```
[Out] integral(sin(b*log(c*x^n) + a)^p/x^3, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*ln(c*x**n))**p/x**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(b \log(cx^n) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a+b*log(c*x^n))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*log(c*x^n) + a)^p/x^3, x)
```

### 3.86 $\int x^2 \cos(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=56

$$\frac{bnx^3 \sin(a + b \log(cx^n))}{b^2n^2 + 9} + \frac{3x^3 \cos(a + b \log(cx^n))}{b^2n^2 + 9}$$

[Out] (3\*x^3\*Cos[a + b\*Log[c\*x^n]])/(9 + b^2\*n^2) + (b\*n\*x^3\*Sin[a + b\*Log[c\*x^n]])/(9 + b^2\*n^2)

**Rubi [A]** time = 0.0175289, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {4486}

$$\frac{bnx^3 \sin(a + b \log(cx^n))}{b^2n^2 + 9} + \frac{3x^3 \cos(a + b \log(cx^n))}{b^2n^2 + 9}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Cos[a + b\*Log[c\*x^n]],x]

[Out] (3\*x^3\*Cos[a + b\*Log[c\*x^n]])/(9 + b^2\*n^2) + (b\*n\*x^3\*Sin[a + b\*Log[c\*x^n]])/(9 + b^2\*n^2)

#### Rule 4486

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)])\*(b\_.)]\*(d\_.)]\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] + Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\int x^2 \cos(a + b \log(cx^n)) dx = \frac{3x^3 \cos(a + b \log(cx^n))}{9 + b^2n^2} + \frac{bnx^3 \sin(a + b \log(cx^n))}{9 + b^2n^2}$$

**Mathematica [A]** time = 0.0721208, size = 43, normalized size = 0.77

$$\frac{x^3 (bn \sin(a + b \log(cx^n)) + 3 \cos(a + b \log(cx^n)))}{b^2n^2 + 9}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cos[a + b\*Log[c\*x^n]],x]

[Out] (x^3\*(3\*Cos[a + b\*Log[c\*x^n]] + b\*n\*Sin[a + b\*Log[c\*x^n]]))/(9 + b^2\*n^2)

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int x^2 \cos(a + b \ln(cx^n)) dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(a+b*ln(c*x^n)),x)`

[Out] `int(x^2*cos(a+b*ln(c*x^n)),x)`

**Maxima [B]** time = 1.05383, size = 294, normalized size = 5.25

$$\frac{((b \cos(b \log(c)) \sin(2b \log(c)) - b \cos(2b \log(c)) \sin(b \log(c)) + b \sin(b \log(c)))n + 3 \cos(2b \log(c)) \cos(b \log(c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{2} * (((b * \cos(b * \log(c)) * \sin(2 * b * \log(c)) - b * \cos(2 * b * \log(c)) * \sin(b * \log(c)) + b * \sin(b * \log(c))) * n + 3 * \cos(2 * b * \log(c)) * \cos(b * \log(c)) + 3 * \sin(2 * b * \log(c)) * \sin(b * \log(c)) + 3 * \cos(b * \log(c))) * x^3 * \cos(b * \log(x^n) + a) + ((b * \cos(2 * b * \log(c)) * \cos(b * \log(c)) + b * \sin(2 * b * \log(c)) * \sin(b * \log(c)) + b * \cos(b * \log(c))) * n - 3 * \cos(b * \log(c)) * \sin(2 * b * \log(c)) + 3 * \cos(2 * b * \log(c)) * \sin(b * \log(c)) - 3 * \sin(b * \log(c))) * x^3 * \sin(b * \log(x^n) + a)) / ((b^2 * \cos(b * \log(c))^2 + b^2 * \sin(b * \log(c))^2) * n^2 + 9 * \cos(b * \log(c))^2 + 9 * \sin(b * \log(c))^2)$$

**Fricas [A]** time = 0.495998, size = 128, normalized size = 2.29

$$\frac{bnx^3 \sin(bn \log(x) + b \log(c) + a) + 3x^3 \cos(bn \log(x) + b \log(c) + a)}{b^2n^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out] 
$$(b * n * x^3 * \sin(b * n * \log(x) + b * \log(c) + a) + 3 * x^3 * \cos(b * n * \log(x) + b * \log(c) + a)) / (b^2 * n^2 + 9)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(a+b*ln(c*x**n)),x)`

[Out] Timed out

**Giac [B]** time = 1.30412, size = 1246, normalized size = 22.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 
$$\begin{aligned}
& -1/2*(2*b*n*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a) + 2*b*n*x^3* \\
& e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} * \tan(1/2*b* \\
& n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a) + 2*b*n*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) * \tan(1/2*a)^2 + 2*b*n*x^3*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) * \tan(1/2*a)^2 - 3*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 - 2*b*n*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) - 2*b*n*x^3*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\
& * \tan(1/2*a) - 2*b*n*x^3*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\
& * \tan(1/2*a) + 3*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + 3*x^3*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + 12*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) * \tan(1/2*a) + 12*x^3*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) * \tan(1/2*a) + 3*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& * \tan(1/2*a)^2 + 3*x^3*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\
& * \tan(1/2*a)^2 - 3*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& - 3*x^3*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\
& * \tan(1/2*a) + 1/2*\pi*b) * \tan(1/2*a) + 3*x^3*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\
& * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 + b^2*n^2 * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 + b^2*n^2 * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 + b^2*n^2 * \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 + 9*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 * \tan(1/2*a)^2 + 9*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + 9*\tan(1/2*a)^2 + 9)
\end{aligned}$$

### 3.87 $\int x \cos(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=56

$$\frac{bnx^2 \sin(a + b \log(cx^n))}{b^2n^2 + 4} + \frac{2x^2 \cos(a + b \log(cx^n))}{b^2n^2 + 4}$$

[Out] (2\*x^2\*Cos[a + b\*Log[c\*x^n]])/(4 + b^2\*n^2) + (b\*n\*x^2\*Sin[a + b\*Log[c\*x^n]])/(4 + b^2\*n^2)

**Rubi [A]** time = 0.0118961, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {4486}

$$\frac{bnx^2 \sin(a + b \log(cx^n))}{b^2n^2 + 4} + \frac{2x^2 \cos(a + b \log(cx^n))}{b^2n^2 + 4}$$

Antiderivative was successfully verified.

[In] Int[x\*Cos[a + b\*Log[c\*x^n]],x]

[Out] (2\*x^2\*Cos[a + b\*Log[c\*x^n]])/(4 + b^2\*n^2) + (b\*n\*x^2\*Sin[a + b\*Log[c\*x^n]])/(4 + b^2\*n^2)

**Rule 4486**

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] + Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

**Rubi steps**

$$\int x \cos(a + b \log(cx^n)) dx = \frac{2x^2 \cos(a + b \log(cx^n))}{4 + b^2n^2} + \frac{bnx^2 \sin(a + b \log(cx^n))}{4 + b^2n^2}$$

**Mathematica [A]** time = 0.0641587, size = 43, normalized size = 0.77

$$\frac{x^2 (bn \sin(a + b \log(cx^n)) + 2 \cos(a + b \log(cx^n)))}{b^2n^2 + 4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cos[a + b\*Log[c\*x^n]],x]

[Out] (x^2\*(2\*Cos[a + b\*Log[c\*x^n]] + b\*n\*Sin[a + b\*Log[c\*x^n]]))/(4 + b^2\*n^2)

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x \cos(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(a+b*ln(c*x^n)),x)`

[Out] `int(x*cos(a+b*ln(c*x^n)),x)`

**Maxima [B]** time = 1.05336, size = 294, normalized size = 5.25

$$\frac{((b \cos(b \log(c)) \sin(2 b \log(c)) - b \cos(2 b \log(c)) \sin(b \log(c)) + b \sin(b \log(c)))n + 2 \cos(2 b \log(c)) \cos(b \log(c)) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{2} * (((b \cos(b \log(c)) \sin(2 b \log(c)) - b \cos(2 b \log(c)) \sin(b \log(c)) + b \sin(b \log(c)))n + 2 \cos(2 b \log(c)) \cos(b \log(c)) + 2 \sin(2 b \log(c)) \sin(b \log(c)) + 2 \cos(b \log(c)) \sin(b \log(c)))x^2 \cos(b \log(x^n) + a) + ((b \cos(2 b \log(c)) \cos(b \log(c)) + b \sin(2 b \log(c)) \sin(b \log(c)) + b \cos(b \log(c)))n - 2 \cos(b \log(c)) \sin(2 b \log(c)) + 2 \cos(2 b \log(c)) \sin(b \log(c)) - 2 \sin(b \log(c)) \sin(2 b \log(c)))x^2 \sin(b \log(x^n) + a) / ((b^2 \cos(b \log(c))^2 + b^2 \sin(b \log(c))^2)n^2 + 4 \cos(b \log(c))^2 + 4 \sin(b \log(c))^2)$$

**Fricas [A]** time = 0.485436, size = 128, normalized size = 2.29

$$\frac{bnx^2 \sin(bn \log(x) + b \log(c) + a) + 2x^2 \cos(bn \log(x) + b \log(c) + a)}{b^2n^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out] 
$$(b*n*x^2*\sin(b*n*\log(x) + b*\log(c) + a) + 2*x^2*\cos(b*n*\log(x) + b*\log(c) + a))/(b^2*n^2 + 4)$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(a+b*ln(c*x**n)),x)`

[Out] Exception raised: TypeError

**Giac [B]** time = 1.27723, size = 1235, normalized size = 22.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 
$$-(b^n x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 \tan(1/2 a) + b^n x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 \tan(1/2 a) + b^n x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c))) \tan(1/2 a)^2 + b^n x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c))) \tan(1/2 a)^2 - x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 \tan(1/2 a)^2 - x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 \tan(1/2 a)^2 - b^n x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c))) - b^n x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c))) - b^n x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} \tan(1/2 a) - b^n x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} \tan(1/2 a) + x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 + x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 + 4 x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c))) \tan(1/2 a) + 4 x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c))) \tan(1/2 a) + x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} \tan(1/2 a)^2 + x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} \tan(1/2 a)^2 - x^2 e^{(1/2 \pi b n \operatorname{sgn}(x) - 1/2 \pi b n + 1/2 \pi b \operatorname{sgn}(c) - 1/2 \pi b)} - x^2 e^{(-1/2 \pi b n \operatorname{sgn}(x) + 1/2 \pi b n - 1/2 \pi b \operatorname{sgn}(c) + 1/2 \pi b)} / (b^2 n^2 \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 \tan(1/2 a)^2 + b^2 n^2 \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 + b^2 n^2 \tan(1/2 a)^2 + b^2 n^2 + 4 \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 \tan(1/2 a)^2 + 4 \tan(1/2 b n \log(\operatorname{abs}(x)) + 1/2 b \log(\operatorname{abs}(c)))^2 + 4 \tan(1/2 a)^2 + 4)$$

### 3.88 $\int \cos(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=51

$$\frac{bnx \sin(a + b \log(cx^n))}{b^2n^2 + 1} + \frac{x \cos(a + b \log(cx^n))}{b^2n^2 + 1}$$

[Out] (x\*Cos[a + b\*Log[c\*x^n]])/(1 + b^2\*n^2) + (b\*n\*x\*Sin[a + b\*Log[c\*x^n]])/(1 + b^2\*n^2)

**Rubi [A]** time = 0.0091753, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {4476}

$$\frac{bnx \sin(a + b \log(cx^n))}{b^2n^2 + 1} + \frac{x \cos(a + b \log(cx^n))}{b^2n^2 + 1}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]],x]

[Out] (x\*Cos[a + b\*Log[c\*x^n]])/(1 + b^2\*n^2) + (b\*n\*x\*Sin[a + b\*Log[c\*x^n]])/(1 + b^2\*n^2)

#### Rule 4476

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] :> Simp[(x\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] + Simp[(b\*d\*n\*x\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2\*d^2\*n^2 + 1, 0]

#### Rubi steps

$$\int \cos(a + b \log(cx^n)) dx = \frac{x \cos(a + b \log(cx^n))}{1 + b^2n^2} + \frac{bnx \sin(a + b \log(cx^n))}{1 + b^2n^2}$$

**Mathematica [A]** time = 0.04705, size = 39, normalized size = 0.76

$$\frac{x(bn \sin(a + b \log(cx^n)) + \cos(a + b \log(cx^n)))}{b^2n^2 + 1}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]],x]

[Out] (x\*(Cos[a + b\*Log[c\*x^n]] + b\*n\*Sin[a + b\*Log[c\*x^n]]))/(1 + b^2\*n^2)

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int \cos(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n)),x)
```

```
[Out] int(cos(a+b*ln(c*x^n)),x)
```

**Maxima [B]** time = 1.06308, size = 277, normalized size = 5.43

$$\frac{((b \cos(b \log(c)) \sin(2b \log(c)) - b \cos(2b \log(c)) \sin(b \log(c)) + b \sin(b \log(c)))n + \cos(2b \log(c)) \cos(b \log(c)))}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
b*sin(b*log(c)))*n + cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*log(c))*sin(b*
log(c)) + cos(b*log(c)))*x*cos(b*log(x^n) + a) + ((b*cos(2*b*log(c))*cos(b*
log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n - cos(b*log(
c))*sin(2*b*log(c)) + cos(2*b*log(c))*sin(b*log(c)) - sin(b*log(c)))*x*sin(
b*log(x^n) + a))/(b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + cos(b*log(
c))^2 + sin(b*log(c))^2)
```

**Fricas [A]** time = 0.483584, size = 120, normalized size = 2.35

$$\frac{bnx \sin(bn \log(x) + b \log(c) + a) + x \cos(bn \log(x) + b \log(c) + a)}{b^2n^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (b*n*x*sin(b*n*log(x) + b*log(c) + a) + x*cos(b*n*log(x) + b*log(c) + a))/(
b^2*n^2 + 1)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n)),x)
```

```
[Out] Exception raised: TypeError
```

**Giac [B]** time = 1.1702, size = 1185, normalized size = 23.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*(2*b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a) + 2*b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a) + 2*b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 + 2*b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 - x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 - x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 - 2*b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) - 2*b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c))) - 2*b*n*x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*a) - 2*b*n*x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*a) + x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + 4*x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a) + 4*x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a) + x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*a)^2 + x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*a)^2 - x*e^{(1/2*\pi*b*n*\text{sgn}(x) - 1/2*\pi*b*n + 1/2*\pi*b*\text{sgn}(c) - 1/2*\pi*b)} \\ & \tan(1/2*a) - x*e^{(-1/2*\pi*b*n*\text{sgn}(x) + 1/2*\pi*b*n - 1/2*\pi*b*\text{sgn}(c) + 1/2*\pi*b)} \\ & \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2*\tan(1/2*a)^2 + b^2*n^2*\tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 \\ & + b^2*n^2*\tan(1/2*a)^2 + b^2*n^2 + \tan(1/2*b*n*\log(\text{abs}(x)) + 1/2*b*\log(\text{abs}(c)))^2 + \tan(1/2*a)^2 + 1) \end{aligned}$$



$$3.89 \quad \int \frac{\cos(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=18

$$\frac{\sin(a+b \log(cx^n))}{bn}$$

[Out] Sin[a + b\*Log[c\*x^n]]/(b\*n)

**Rubi [A]** time = 0.0147802, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2637}

$$\frac{\sin(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]/x,x]

[Out] Sin[a + b\*Log[c\*x^n]]/(b\*n)

**Rule 2637**

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

**Rubi steps**

$$\int \frac{\cos(a+b \log(cx^n))}{x} dx = \frac{\text{Subst}\left(\int \cos(a+bx) dx, x, \log(cx^n)\right)}{n} = \frac{\sin(a+b \log(cx^n))}{bn}$$

**Mathematica [B]** time = 0.0269324, size = 37, normalized size = 2.06

$$\frac{\sin(a) \cos(b \log(cx^n))}{bn} + \frac{\cos(a) \sin(b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]/x,x]

[Out] (Cos[b\*Log[c\*x^n]]\*Sin[a])/(b\*n) + (Cos[a]\*Sin[b\*Log[c\*x^n]])/(b\*n)

**Maple [A]** time = 0.021, size = 19, normalized size = 1.1

$$\frac{\sin(a+b \ln(cx^n))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*ln(c*x^n))/x,x)`

[Out] `sin(a+b*ln(c*x^n))/b/n`

**Maxima [A]** time = 0.989078, size = 24, normalized size = 1.33

$$\frac{\sin(b \log(cx^n) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))/x,x, algorithm="maxima")`

[Out] `sin(b*log(c*x^n) + a)/(b*n)`

**Fricas [A]** time = 0.48318, size = 51, normalized size = 2.83

$$\frac{\sin(bn \log(x) + b \log(c) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))/x,x, algorithm="fricas")`

[Out] `sin(b*n*log(x) + b*log(c) + a)/(b*n)`

**Sympy [A]** time = 1.39403, size = 37, normalized size = 2.06

$$\begin{cases} \log(x) \cos(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cos(a + b \log(c)) & \text{for } n = 0 \\ \frac{\sin(a + bn \log(x) + b \log(c))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*ln(c*x**n))/x,x)`

[Out] `Piecewise((log(x)*cos(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos(a + b*log(c)), Eq(n, 0)), (sin(a + b*n*log(x) + b*log(c))/(b*n), True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))/x,x, algorithm="giac")`

[Out] `integrate(cos(b*log(c*x^n) + a)/x, x)`

$$3.90 \quad \int \frac{\cos(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=56

$$\frac{bn \sin(a + b \log(cx^n))}{x(b^2n^2 + 1)} - \frac{\cos(a + b \log(cx^n))}{x(b^2n^2 + 1)}$$

[Out]  $-(\text{Cos}[a + b \cdot \text{Log}[c \cdot x^n]] / ((1 + b^2 \cdot n^2) \cdot x)) + (b \cdot n \cdot \text{Sin}[a + b \cdot \text{Log}[c \cdot x^n]]) / ((1 + b^2 \cdot n^2) \cdot x)$

**Rubi [A]** time = 0.015031, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {4486}

$$\frac{bn \sin(a + b \log(cx^n))}{x(b^2n^2 + 1)} - \frac{\cos(a + b \log(cx^n))}{x(b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]/x^2,x]

[Out]  $-(\text{Cos}[a + b \cdot \text{Log}[c \cdot x^n]] / ((1 + b^2 \cdot n^2) \cdot x)) + (b \cdot n \cdot \text{Sin}[a + b \cdot \text{Log}[c \cdot x^n]]) / ((1 + b^2 \cdot n^2) \cdot x)$

**Rule 4486**

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] + Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

**Rubi steps**

$$\int \frac{\cos(a + b \log(cx^n))}{x^2} dx = -\frac{\cos(a + b \log(cx^n))}{(1 + b^2n^2)x} + \frac{bn \sin(a + b \log(cx^n))}{(1 + b^2n^2)x}$$

**Mathematica [A]** time = 0.0565324, size = 41, normalized size = 0.73

$$\frac{bn \sin(a + b \log(cx^n)) - \cos(a + b \log(cx^n))}{b^2n^2x + x}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]/x^2,x]

[Out]  $(-\text{Cos}[a + b \cdot \text{Log}[c \cdot x^n]] + b \cdot n \cdot \text{Sin}[a + b \cdot \text{Log}[c \cdot x^n]]) / (x + b^2 \cdot n^2 \cdot x)$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\cos(a + b \ln(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*ln(c*x^n))/x^2,x)`

[Out] `int(cos(a+b*ln(c*x^n))/x^2,x)`

**Maxima [B]** time = 1.09827, size = 281, normalized size = 5.02

$$\frac{((b \cos(b \log(c)) \sin(2 b \log(c)) - b \cos(2 b \log(c)) \sin(b \log(c)) + b \sin(b \log(c)))n - \cos(2 b \log(c)) \cos(b \log(c)) - \sin(2 b \log(c)) \sin(b \log(c)) + \cos(b \log(c)) \cos(2 b \log(c)) \sin(b \log(c)) - \sin(b \log(c)) \cos(2 b \log(c)) \sin(b \log(c)) + \cos(b \log(c)) \sin(2 b \log(c)) \sin(b \log(c)) - \cos(b \log(c)) \sin(2 b \log(c)) \sin(b \log(c)) + \sin(b \log(c)) \sin(b \log(c)) \cos(2 b \log(c)) - \cos(2 b \log(c)) \sin(b \log(c)) + \sin(b \log(c)) \sin(b \log(c)))}{((b^2 \cos(b \log(c))^2 + b^2 \sin(b \log(c))^2)n^2 + \cos(b \log(c))^2 + \sin(b \log(c))^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="maxima")`

[Out] 
$$\frac{1/2 * ((b \cos(b \log(c)) \sin(2 b \log(c)) - b \cos(2 b \log(c)) \sin(b \log(c)) + b \sin(b \log(c)))n - \cos(2 b \log(c)) \cos(b \log(c)) - \sin(2 b \log(c)) \sin(b \log(c)) + \cos(b \log(c)) \cos(2 b \log(c)) \sin(b \log(c)) - \sin(b \log(c)) \cos(2 b \log(c)) \sin(b \log(c)) + \cos(b \log(c)) \sin(2 b \log(c)) \sin(b \log(c)) - \cos(b \log(c)) \sin(2 b \log(c)) \sin(b \log(c)) + \sin(b \log(c)) \sin(b \log(c)) \cos(2 b \log(c)) - \cos(2 b \log(c)) \sin(b \log(c)) + \sin(b \log(c)) \sin(b \log(c)))}{((b^2 \cos(b \log(c))^2 + b^2 \sin(b \log(c))^2)n^2 + \cos(b \log(c))^2 + \sin(b \log(c))^2)x}$$

**Fricas [A]** time = 0.487415, size = 120, normalized size = 2.14

$$\frac{bn \sin(bn \log(x) + b \log(c) + a) - \cos(bn \log(x) + b \log(c) + a)}{(b^2 n^2 + 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="fricas")`

[Out] 
$$(b*n*\sin(b*n*\log(x) + b*\log(c) + a) - \cos(b*n*\log(x) + b*\log(c) + a))/((b^2*n^2 + 1)*x)$$

**Sympy [A]** time = 25.1753, size = 287, normalized size = 5.12

$$\left\{ \begin{array}{l} -\frac{i \log(x) \sin\left(-a+i \log(x)+\frac{i \log(c)}{n}\right)}{2x} + \frac{\log(x) \cos\left(-a+i \log(x)+\frac{i \log(c)}{n}\right)}{2x} - \frac{i \sin\left(-a+i \log(x)+\frac{i \log(c)}{n}\right)}{2x} - \frac{i \log(c) \sin\left(-a+i \log(x)+\frac{i \log(c)}{n}\right)}{2nx} + \frac{\log(c) \cos\left(-a+i \log(x)+\frac{i \log(c)}{n}\right)}{2nx} \\ -\frac{i \log(x) \sin\left(a+i \log(x)+\frac{i \log(c)}{n}\right)}{2x} + \frac{\log(x) \cos\left(a+i \log(x)+\frac{i \log(c)}{n}\right)}{2x} - \frac{i \sin\left(a+i \log(x)+\frac{i \log(c)}{n}\right)}{2x} - \frac{i \log(c) \sin\left(a+i \log(x)+\frac{i \log(c)}{n}\right)}{2nx} + \frac{\log(c) \cos\left(a+i \log(x)+\frac{i \log(c)}{n}\right)}{2nx} \\ \frac{bn \sin(a+bn \log(x)+b \log(c))}{b^2 n^2 x+x} - \frac{\cos(a+bn \log(x)+b \log(c))}{b^2 n^2 x+x} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*ln(c*x**n))/x**2,x)`

[Out] 
$$\text{Piecewise}\left(\left(-I*\log(x)*\sin(-a + I*\log(x) + I*\log(c)/n)/(2*x) + \log(x)*\cos(-a + I*\log(x) + I*\log(c)/n)/(2*x) - I*\sin(-a + I*\log(x) + I*\log(c)/n)/(2*x) - I*\log(c)*\sin(-a + I*\log(x) + I*\log(c)/n)/(2*n*x) + \log(c)*\cos(-a + I*\log(x) + I*\log(c)/n)/(2*n*x), \text{Eq}(b, -I/n)\right), \left(-I*\log(x)*\sin(a + I*\log(x) + I*\log(c)/n)/(2*x) + \log(x)*\cos(a + I*\log(x) + I*\log(c)/n)/(2*x) - I*\sin(a + I*\log(x) + I*\log(c)/n)/(2*x) - I*\log(c)*\sin(a + I*\log(x) + I*\log(c)/n)/(2*n*x) + \log(c)*\cos(a + I*\log(x) + I*\log(c)/n)/(2*n*x)\right)$$

```
)/n)/(2*x) + log(x)*cos(a + I*log(x) + I*log(c)/n)/(2*x) - I*sin(a + I*log(x) + I*log(c)/n)/(2*x) - I*log(c)*sin(a + I*log(x) + I*log(c)/n)/(2*n*x) + log(c)*cos(a + I*log(x) + I*log(c)/n)/(2*n*x), Eq(b, I/n)), (b*n*sin(a + b*n*log(x) + b*log(c))/(b**2*n**2*x + x) - cos(a + b*n*log(x) + b*log(c))/(b**2*n**2*x + x), True))
```

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)/x^2, x)
```

### 3.91 $\int x^2 \cos^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=97

$$\frac{3x^3 \cos^2(a + b \log(cx^n))}{4b^2n^2 + 9} + \frac{2bnx^3 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 9} + \frac{2b^2n^2x^3}{3(4b^2n^2 + 9)}$$

[Out]  $(2*b^2*n^2*x^3)/(3*(9 + 4*b^2*n^2)) + (3*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]^2)/(9 + 4*b^2*n^2) + (2*b*n*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(9 + 4*b^2*n^2)$

**Rubi [A]** time = 0.0300538, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4488, 30}

$$\frac{3x^3 \cos^2(a + b \log(cx^n))}{4b^2n^2 + 9} + \frac{2bnx^3 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 9} + \frac{2b^2n^2x^3}{3(4b^2n^2 + 9)}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Cos[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(2*b^2*n^2*x^3)/(3*(9 + 4*b^2*n^2)) + (3*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]^2)/(9 + 4*b^2*n^2) + (2*b*n*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(9 + 4*b^2*n^2)$

#### Rule 4488

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 \cos^2(a + b \log(cx^n)) dx &= \frac{3x^3 \cos^2(a + b \log(cx^n))}{9 + 4b^2n^2} + \frac{2bnx^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{9 + 4b^2n^2} + \frac{(2b^2n^2) \int x^2 \cos^2(a + b \log(cx^n)) dx}{9 + 4b^2n^2} \\ &= \frac{2b^2n^2x^3}{3(9 + 4b^2n^2)} + \frac{3x^3 \cos^2(a + b \log(cx^n))}{9 + 4b^2n^2} + \frac{2bnx^3 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{9 + 4b^2n^2} \end{aligned}$$

**Mathematica [A]** time = 0.143587, size = 61, normalized size = 0.63

$$\frac{x^3 (6bn \sin(2(a + b \log(cx^n))) + 9 \cos(2(a + b \log(cx^n))) + 4b^2n^2 + 9)}{6(4b^2n^2 + 9)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cos[a + b\*Log[c\*x^n]]^2,x]

[Out] (x^3\*(9 + 4\*b^2\*n^2 + 9\*Cos[2\*(a + b\*Log[c\*x^n])] + 6\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n]])))/(6\*(9 + 4\*b^2\*n^2))

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int x^2 (\cos(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cos(a+b\*ln(c\*x^n))^2,x)

[Out] int(x^2\*cos(a+b\*ln(c\*x^n))^2,x)

**Maxima [B]** time = 1.11067, size = 406, normalized size = 4.19

$$\frac{3(2(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(2b \log(c)) + b \sin(2b \log(c)))n + 3 \cos(4b \log(c)) \cos(2b \log(c)))x^3 + 3 \cos(4b \log(c)) \cos(2b \log(c))n^2 + 9 \cos(2b \log(c)) \cos(2b \log(c))n^2 + 9 \sin(2b \log(c)) \sin(2b \log(c))n^2}{3(4b^2n^2 + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 
$$\frac{1}{12} \cdot \frac{3(2(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(2b \log(c)) + b \sin(2b \log(c)))n + 3 \cos(4b \log(c)) \cos(2b \log(c))n^2 + 9 \cos(2b \log(c)) \cos(2b \log(c))n^2 + 9 \sin(2b \log(c)) \sin(2b \log(c))n^2)}{3(4b^2n^2 + 9)}$$

**Fricas [A]** time = 0.491411, size = 205, normalized size = 2.11

$$\frac{2b^2n^2x^3 + 6bnx^3 \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) + 9x^3 \cos(bn \log(x) + b \log(c) + a)^2}{3(4b^2n^2 + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{3} \cdot \frac{2b^2n^2x^3 + 6bnx^3 \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) + 9x^3 \cos(bn \log(x) + b \log(c) + a)^2}{3(4b^2n^2 + 9)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cos(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.48767, size = 1125, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$\frac{1}{6}x^3 - \frac{1}{4}(4b^2n^2x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4b^2n^2x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4b^2n^2x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4b^2n^2x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) - 3x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 3x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 4b^2n^2x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4b^2n^2x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4b^2n^2x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(a) - 4b^2n^2x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(a) + 3x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 3x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 12x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 12x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 3x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(a)^2 + 3x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(a)^2 - 3x^3e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b) - 3x^3e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)}}/(4b^2n^2tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4b^2n^2tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4b^2n^2tan(a)^2 + 4b^2n^2 + 9tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 9tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 9tan(a)^2 + 9)$$



### 3.92 $\int x \cos^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=98

$$\frac{x^2 \cos^2(a + b \log(cx^n))}{2(b^2 n^2 + 1)} + \frac{bnx^2 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{2(b^2 n^2 + 1)} + \frac{b^2 n^2 x^2}{4(b^2 n^2 + 1)}$$

[Out] (b^2\*n^2\*x^2)/(4\*(1 + b^2\*n^2)) + (x^2\*Cos[a + b\*Log[c\*x^n]]^2)/(2\*(1 + b^2\*n^2)) + (b\*n\*x^2\*Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(2\*(1 + b^2\*n^2))

**Rubi [A]** time = 0.0231482, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {4488, 30}

$$\frac{x^2 \cos^2(a + b \log(cx^n))}{2(b^2 n^2 + 1)} + \frac{bnx^2 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{2(b^2 n^2 + 1)} + \frac{b^2 n^2 x^2}{4(b^2 n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[x\*Cos[a + b\*Log[c\*x^n]]^2,x]

[Out] (b^2\*n^2\*x^2)/(4\*(1 + b^2\*n^2)) + (x^2\*Cos[a + b\*Log[c\*x^n]]^2)/(2\*(1 + b^2\*n^2)) + (b\*n\*x^2\*Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(2\*(1 + b^2\*n^2))

#### Rule 4488

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x \cos^2(a + b \log(cx^n)) dx &= \frac{x^2 \cos^2(a + b \log(cx^n))}{2(1 + b^2 n^2)} + \frac{bnx^2 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{2(1 + b^2 n^2)} + \frac{(b^2 n^2) \int x}{2(1 + b^2 n^2)} \\ &= \frac{b^2 n^2 x^2}{4(1 + b^2 n^2)} + \frac{x^2 \cos^2(a + b \log(cx^n))}{2(1 + b^2 n^2)} + \frac{bnx^2 \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{2(1 + b^2 n^2)} \end{aligned}$$

**Mathematica [A]** time = 0.096574, size = 54, normalized size = 0.55

$$\frac{x^2 (bn \sin(2(a + b \log(cx^n))) + \cos(2(a + b \log(cx^n))) + b^2 n^2 + 1)}{4b^2 n^2 + 4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^2*(1 + b^2*n^2 + Cos[2*(a + b*Log[c*x^n]]) + b*n*Sin[2*(a + b*Log[c*x^n]
)))/(4 + 4*b^2*n^2)
```

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int x (\cos(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a+b*ln(c*x^n))^2,x)
```

```
[Out] int(x*cos(a+b*ln(c*x^n))^2,x)
```

**Maxima [B]** time = 1.11276, size = 381, normalized size = 3.89

```
((b cos(2 b log(c)) sin(4 b log(c)) - b cos(4 b log(c)) sin(2 b log(c)) + b sin(2 b log(c)))n + cos(4 b log(c)) cos(2 b log
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(((b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)
) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c)
)*sin(2*b*log(c)) + cos(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) + ((b*cos(
4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*cos(2*b
*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*log
(c)) - sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) + 2*a) + 2*((b^2*cos(2*b*log(c)
))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*
x^2)/((b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c)
)^2 + sin(2*b*log(c))^2)
```

**Fricas [A]** time = 0.492698, size = 200, normalized size = 2.04

$$\frac{b^2 n^2 x^2 + 2 b n x^2 \cos(b n \log(x) + b \log(c) + a) \sin(b n \log(x) + b \log(c) + a) + 2 x^2 \cos(b n \log(x) + b \log(c) + a)^2}{4(b^2 n^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*n^2*x^2 + 2*b*n*x^2*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x)
+ b*log(c) + a) + 2*x^2*cos(b*n*log(x) + b*log(c) + a)^2)/(b^2*n^2 + 1)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.52641, size = 1107, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$\frac{1}{4}x^2 - \frac{1}{8}(2b^2n^2x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 2b^2n^2x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 2b^2n^2x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 2b^2n^2x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - 2b^2n^2x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - 2b^2n^2x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - 2b^2n^2x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} - 2b^2n^2x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 4x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + 4x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2} + x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b)*tan(a)^2} + x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)*tan(a)^2} - x^2e^{(pi*bn*sgn(x) - pi*bn + pi*b*sgn(c) - pi*b) - x^2e^{(-pi*bn*sgn(x) + pi*bn - pi*b*sgn(c) + pi*b)})/(b^2n^2tan(b*n*log(abs(x)) + b*log(abs(c)))^2tan(a)^2 + b^2n^2tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + b^2n^2tan(a)^2 + b^2n^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2tan(a)^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + tan(a)^2 + 1)$$

### 3.93 $\int \cos^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=88

$$\frac{x \cos^2(a + b \log(cx^n))}{4b^2n^2 + 1} + \frac{2bnx \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 1} + \frac{2b^2n^2x}{4b^2n^2 + 1}$$

[Out]  $(2*b^2*n^2*x)/(1 + 4*b^2*n^2) + (x*\text{Cos}[a + b*\text{Log}[c*x^n]]^2)/(1 + 4*b^2*n^2) + (2*b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(1 + 4*b^2*n^2)$

**Rubi [A]** time = 0.0162598, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {4478, 8}

$$\frac{x \cos^2(a + b \log(cx^n))}{4b^2n^2 + 1} + \frac{2bnx \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + 1} + \frac{2b^2n^2x}{4b^2n^2 + 1}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(2*b^2*n^2*x)/(1 + 4*b^2*n^2) + (x*\text{Cos}[a + b*\text{Log}[c*x^n]]^2)/(1 + 4*b^2*n^2) + (2*b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(1 + 4*b^2*n^2)$

#### Rule 4478

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_), x\_Symbol] :> Sim p[(x\*Cos[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*n^2\*p^2 + 1), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + 1), Int[Cos[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] + Simp[(b\*d\*n\*p\*x\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2\*p^2 + 1), x]) /; FreeQ[{a, b, c, d, n}, x] && I GtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + 1, 0]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \cos^2(a + b \log(cx^n)) dx &= \frac{x \cos^2(a + b \log(cx^n))}{1 + 4b^2n^2} + \frac{2bnx \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 4b^2n^2} + \frac{(2b^2n^2) \int 1 dx}{1 + 4b^2n^2} \\ &= \frac{2b^2n^2x}{1 + 4b^2n^2} + \frac{x \cos^2(a + b \log(cx^n))}{1 + 4b^2n^2} + \frac{2bnx \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 4b^2n^2} \end{aligned}$$

**Mathematica [A]** time = 0.0804297, size = 54, normalized size = 0.61

$$\frac{x(2bn \sin(2(a + b \log(cx^n))) + \cos(2(a + b \log(cx^n))) + 4b^2n^2 + 1)}{8b^2n^2 + 2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(x*(1 + 4*b^2*n^2 + \text{Cos}[2*(a + b*\text{Log}[c*x^n])]) + 2*b*n*\text{Sin}[2*(a + b*\text{Log}[c*x^n])]))/(2 + 8*b^2*n^2)$

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int (\cos(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*ln(c*x^n))^2,x)`

[Out] `int(cos(a+b*ln(c*x^n))^2,x)`

**Maxima [B]** time = 1.11729, size = 378, normalized size = 4.3

$(2(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(2b \log(c)) + b \sin(2b \log(c)))n + \cos(4b \log(c)) \cos(2b \log(c)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{4}((2*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)) + b*\sin(2*b*\log(c)))*n + \cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)) + \cos(2*b*\log(c)))*x*\cos(2*b*\log(x^n) + 2*a) + (2*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)) + b*\cos(2*b*\log(c)))*n - \cos(2*b*\log(c))*\sin(4*b*\log(c)) + \cos(4*b*\log(c))*\sin(2*b*\log(c)) - \sin(2*b*\log(c)))*x*\sin(2*b*\log(x^n) + 2*a) + 2*(4*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + \cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*x)/(4*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + \cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)$

**Fricas [A]** time = 0.489928, size = 189, normalized size = 2.15

$\frac{2b^2n^2x + 2bnx \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) + x \cos(bn \log(x) + b \log(c) + a)^2}{4b^2n^2 + 1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))^2,x, algorithm="fricas")`

[Out]  $(2*b^2*n^2*x + 2*b*n*x*\cos(b*n*\log(x) + b*\log(c) + a)*\sin(b*n*\log(x) + b*\log(c) + a) + x*\cos(b*n*\log(x) + b*\log(c) + a)^2)/(4*b^2*n^2 + 1)$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**2,x)
```

```
[Out] Exception raised: TypeError
```

**Giac [B]** time = 1.37772, size = 1061, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*(4*b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*
n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x*e^(-pi*b*n*sgn(x) + pi*b*
n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b
*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) +
b*log(abs(c)))*tan(a)^2 + 4*b*n*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c)
+ pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x*e^(pi*b*n*sgn(x)
- pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a
)^2 - x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)
) + b*log(abs(c)))^2*tan(a)^2 - 4*b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sg
n(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b*n*x*e^(-pi*b*n*sgn(
x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*
b*n*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a) - 4*b*n*x*e^(-
pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(a) + x*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + x*e^(
-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(a
bs(c)))^2 + 4*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log
(abs(x)) + b*log(abs(c)))*tan(a) + 4*x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sg
n(c) + pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + x*e^(pi*b*n*sgn(
x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a)^2 + x*e^(-pi*b*n*sgn(x) + pi*b*n -
pi*b*sgn(c) + pi*b)*tan(a)^2 - x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) -
pi*b) - x*e^(-pi*b*n*sgn(x) + pi*b*n - pi*b*sgn(c) + pi*b))/(4*b^2*n^2*tan
(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*tan(b*n*log(abs(x)
) + b*log(abs(c)))^2 + 4*b^2*n^2*tan(a)^2 + 4*b^2*n^2 + tan(b*n*log(abs(x))
+ b*log(abs(c)))^2*tan(a)^2 + tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + tan
(a)^2 + 1)
```

$$3.94 \quad \int \frac{\cos^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=39

$$\frac{\sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{2bn} + \frac{\log(x)}{2}$$

[Out] Log[x]/2 + (Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(2\*b\*n)

**Rubi [A]** time = 0.0288512, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2635, 8}

$$\frac{\sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{2bn} + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^2/x, x]

[Out] Log[x]/2 + (Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(2\*b\*n)

Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x] \* (b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cos^2(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int 1 dx, x, \log(cx^n)\right)}{2n} \\ &= \frac{\log(x)}{2} + \frac{\cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{2bn} \end{aligned}$$

**Mathematica [A]** time = 0.067156, size = 36, normalized size = 0.92

$$\frac{2(a+b \log(cx^n)) + \sin(2(a+b \log(cx^n)))}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^2/x, x]

[Out] (2\*(a + b\*Log[c\*x^n]) + Sin[2\*(a + b\*Log[c\*x^n]]))/(4\*b\*n)

---

**Maple [A]** time = 0.027, size = 52, normalized size = 1.3

$$\frac{\cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{2bn} + \frac{\ln(cx^n)}{2n} + \frac{a}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^2/x,x)

[Out] 1/2\*cos(a+b\*ln(c\*x^n))\*sin(a+b\*ln(c\*x^n))/b/n+1/2/n\*ln(c\*x^n)+1/2/b/n\*a

---

**Maxima [A]** time = 1.10687, size = 72, normalized size = 1.85

$$\frac{2bn \log(x) + \cos(2b \log(x^n) + 2a) \sin(2b \log(c)) + \cos(2b \log(c)) \sin(2b \log(x^n) + 2a)}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^2/x,x, algorithm="maxima")

[Out] 1/4\*(2\*b\*n\*log(x) + cos(2\*b\*log(x^n) + 2\*a))\*sin(2\*b\*log(c)) + cos(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a))/(b\*n)

---

**Fricas [A]** time = 0.495247, size = 119, normalized size = 3.05

$$\frac{bn \log(x) + \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^2/x,x, algorithm="fricas")

[Out] 1/2\*(b\*n\*log(x) + cos(b\*n\*log(x) + b\*log(c) + a))\*sin(b\*n\*log(x) + b\*log(c) + a))/(b\*n)

---

**Sympy [A]** time = 10.1328, size = 56, normalized size = 1.44

$$\frac{\begin{cases} \log(x) \cos(2a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cos(2a + 2b \log(c)) & \text{for } n = 0 \\ \frac{\sin(2a + 2bn \log(x) + 2b \log(c))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\log(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*ln(c\*x\*\*n))\*\*2/x,x)

[Out] Piecewise((log(x)\*cos(2\*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)\*cos(2\*a + 2\*b\*log(c)), Eq(n, 0)), (sin(2\*a + 2\*b\*n\*log(x) + 2\*b\*log(c))/(2\*b\*n), True))/2 + log(x)/2



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^2/x, x)
```

$$3.95 \quad \int \frac{\cos^2(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=95

$$-\frac{\cos^2(a+b \log(cx^n))}{x(4b^2n^2+1)} + \frac{2bn \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x(4b^2n^2+1)} - \frac{2b^2n^2}{x(4b^2n^2+1)}$$

[Out]  $(-2*b^2*n^2)/((1+4*b^2*n^2)*x) - \text{Cos}[a+b*\text{Log}[c*x^n]]^2/((1+4*b^2*n^2)*x) + (2*b*n*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]])/((1+4*b^2*n^2)*x)$

**Rubi [A]** time = 0.026947, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4488, 30}

$$-\frac{\cos^2(a+b \log(cx^n))}{x(4b^2n^2+1)} + \frac{2bn \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{x(4b^2n^2+1)} - \frac{2b^2n^2}{x(4b^2n^2+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^2/x^2, x]

[Out]  $(-2*b^2*n^2)/((1+4*b^2*n^2)*x) - \text{Cos}[a+b*\text{Log}[c*x^n]]^2/((1+4*b^2*n^2)*x) + (2*b*n*\text{Cos}[a+b*\text{Log}[c*x^n]]*\text{Sin}[a+b*\text{Log}[c*x^n]])/((1+4*b^2*n^2)*x)$

#### Rule 4488

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a+b \log(cx^n))}{x^2} dx &= -\frac{\cos^2(a+b \log(cx^n))}{(1+4b^2n^2)x} + \frac{2bn \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{(1+4b^2n^2)x} + \frac{(2b^2n^2) \int \frac{1}{x^2} dx}{1+4b^2n^2} \\ &= -\frac{2b^2n^2}{(1+4b^2n^2)x} - \frac{\cos^2(a+b \log(cx^n))}{(1+4b^2n^2)x} + \frac{2bn \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{(1+4b^2n^2)x} \end{aligned}$$

**Mathematica [A]** time = 0.117084, size = 57, normalized size = 0.6

$$-\frac{-2bn \sin(2(a+b \log(cx^n))) + \cos(2(a+b \log(cx^n))) + 4b^2n^2 + 1}{2(4b^2n^2x + x)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^2/x^2,x]

[Out]  $-(1 + 4b^2n^2 + \cos[2(a + b\log[cx^n])]) - 2bn\sin[2(a + b\log[cx^n])]) / (2(x + 4b^2n^2x))$

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int \frac{(\cos(a + b \ln(cx^n)))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^2/x^2,x)

[Out] int(cos(a+b\*ln(c\*x^n))^2/x^2,x)

**Maxima [B]** time = 1.37307, size = 385, normalized size = 4.05

$$\frac{8(b^2 \cos(2b \log(c))^2 + b^2 \sin(2b \log(c))^2)n^2 + 2 \cos(2b \log(c))^2 - (2(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)))n - \cos(4b \log(c)) \cos(2b \log(c)) - \sin(4b \log(c)) \sin(2b \log(c)) + b \sin(2b \log(c)))n - \cos(4b \log(c)) \cos(2b \log(c)) - \sin(4b \log(c)) \sin(2b \log(c)) - \cos(2b \log(c)) \cos(2b \log(x^n) + 2a) + 2 \sin(2b \log(c))^2 - (2(b \cos(4b \log(c)) \cos(2b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c)) + b \cos(2b \log(c)))n + \cos(2b \log(c)) \sin(4b \log(c)) - \cos(4b \log(c)) \sin(2b \log(c)) + \sin(2b \log(c)) \sin(2b \log(x^n) + 2a)) / ((4(b^2 \cos(2b \log(c))^2 + b^2 \sin(2b \log(c))^2)n^2 + \cos(2b \log(c))^2 + \sin(2b \log(c))^2)x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^2/x^2,x, algorithm="maxima")

[Out]  $-1/4*(8*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + 2*\cos(2*b*\log(c))^2 - (2*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)) + b*\sin(2*b*\log(c)))n - \cos(4*b*\log(c))*\cos(2*b*\log(c)) - \sin(4*b*\log(c))*\sin(2*b*\log(c)) - \cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 2*\sin(2*b*\log(c))^2 - (2*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)) + b*\cos(2*b*\log(c)))n + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)) + \sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a)) / ((4*(b^2*\cos(2*b*\log(c))^2 + b^2*\sin(2*b*\log(c))^2)*n^2 + \cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*x)$

**Fricas [A]** time = 0.504295, size = 188, normalized size = 1.98

$$\frac{2b^2n^2 - 2bn \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) + \cos(bn \log(x) + b \log(c) + a)^2}{(4b^2n^2 + 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^2/x^2,x, algorithm="fricas")

[Out]  $-(2*b^2*n^2 - 2*b*n*\cos(b*n*\log(x) + b*\log(c) + a)*\sin(b*n*\log(x) + b*\log(c) + a) + \cos(b*n*\log(x) + b*\log(c) + a)^2) / ((4*b^2*n^2 + 1)*x)$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*ln(c\*x\*\*n))\*\*2/x\*\*2,x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^2/x^2,x, algorithm="giac")

[Out] integrate(cos(b\*log(c\*x^n) + a)^2/x^2, x)

### 3.96 $\int x^2 \cos^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=160

$$\frac{2b^3n^3x^3 \sin(a + b \log(cx^n))}{3(b^4n^4 + 10b^2n^2 + 9)} + \frac{x^3 \cos^3(a + b \log(cx^n))}{3(b^2n^2 + 1)} + \frac{2b^2n^2x^3 \cos(a + b \log(cx^n))}{b^4n^4 + 10b^2n^2 + 9} + \frac{bnx^3 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{3(b^2n^2 + 1)}$$

[Out]  $(2*b^2*n^2*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]])/(9 + 10*b^2*n^2 + b^4*n^4) + (x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]^3)/(3*(1 + b^2*n^2)) + (2*b^3*n^3*x^3*\text{Sin}[a + b*\text{Log}[c*x^n]])/(3*(9 + 10*b^2*n^2 + b^4*n^4)) + (b*n*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]^2*\text{Sin}[a + b*\text{Log}[c*x^n]])/(3*(1 + b^2*n^2))$

**Rubi [A]** time = 0.0511728, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4488, 4486}

$$\frac{2b^3n^3x^3 \sin(a + b \log(cx^n))}{3(b^4n^4 + 10b^2n^2 + 9)} + \frac{x^3 \cos^3(a + b \log(cx^n))}{3(b^2n^2 + 1)} + \frac{2b^2n^2x^3 \cos(a + b \log(cx^n))}{b^4n^4 + 10b^2n^2 + 9} + \frac{bnx^3 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{3(b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{Cos}[a + b*\text{Log}[c*x^n]]^3, x]$

[Out]  $(2*b^2*n^2*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]])/(9 + 10*b^2*n^2 + b^4*n^4) + (x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]^3)/(3*(1 + b^2*n^2)) + (2*b^3*n^3*x^3*\text{Sin}[a + b*\text{Log}[c*x^n]])/(3*(9 + 10*b^2*n^2 + b^4*n^4)) + (b*n*x^3*\text{Cos}[a + b*\text{Log}[c*x^n]]^2*\text{Sin}[a + b*\text{Log}[c*x^n]])/(3*(1 + b^2*n^2))$

#### Rule 4488

$\text{Int}[\text{Cos}[(a + \text{Log}[(c \cdot x)^n]) \cdot (b \cdot d)]^{(p)} \cdot ((e \cdot x)^m), x\_Symbol] := \text{Simp}[(m + 1) \cdot (e \cdot x)^{(m + 1)} \cdot \text{Cos}[d \cdot (a + b \cdot \text{Log}[c \cdot x^n])]^p / (b^2 \cdot d^2 \cdot e \cdot n^2 \cdot p^2 + e \cdot (m + 1)^2), x] + (\text{Dist}[b^2 \cdot d^2 \cdot n^2 \cdot p \cdot (p - 1) / (b^2 \cdot d^2 \cdot n^2 \cdot p^2 + (m + 1)^2), \text{Int}[(e \cdot x)^m \cdot \text{Cos}[d \cdot (a + b \cdot \text{Log}[c \cdot x^n])]^{(p - 2)}, x], x] + \text{Simp}[(b \cdot d \cdot n \cdot p \cdot (e \cdot x)^{(m + 1)} \cdot \text{Sin}[d \cdot (a + b \cdot \text{Log}[c \cdot x^n])] \cdot \text{Cos}[d \cdot (a + b \cdot \text{Log}[c \cdot x^n])]^{(p - 1)} / (b^2 \cdot d^2 \cdot e \cdot n^2 \cdot p^2 + e \cdot (m + 1)^2), x]) / ; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{IGtQ}[p, 1] \&\& \text{NeQ}[b^2 \cdot d^2 \cdot n^2 \cdot p^2 + (m + 1)^2, 0]$

#### Rule 4486

$\text{Int}[\text{Cos}[(a + \text{Log}[(c \cdot x)^n]) \cdot (b \cdot d)] \cdot ((e \cdot x)^m), x\_Symbol] := \text{Simp}[(m + 1) \cdot (e \cdot x)^{(m + 1)} \cdot \text{Cos}[d \cdot (a + b \cdot \text{Log}[c \cdot x^n])] / (b^2 \cdot d^2 \cdot e \cdot n^2 + e \cdot (m + 1)^2), x] + \text{Simp}[(b \cdot d \cdot n \cdot (e \cdot x)^{(m + 1)} \cdot \text{Sin}[d \cdot (a + b \cdot \text{Log}[c \cdot x^n])]) / (b^2 \cdot d^2 \cdot e \cdot n^2 + e \cdot (m + 1)^2), x] / ; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \& \& \text{NeQ}[b^2 \cdot d^2 \cdot n^2 + (m + 1)^2, 0]$

#### Rubi steps

$$\begin{aligned} \int x^2 \cos^3(a + b \log(cx^n)) dx &= \frac{x^3 \cos^3(a + b \log(cx^n))}{3(1 + b^2n^2)} + \frac{bnx^3 \cos^2(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{3(1 + b^2n^2)} + \frac{(2b^2n^2)}{3(9 + 10b^2n^2 + b^4n^4)} \\ &= \frac{2b^2n^2x^3 \cos(a + b \log(cx^n))}{9 + 10b^2n^2 + b^4n^4} + \frac{x^3 \cos^3(a + b \log(cx^n))}{3(1 + b^2n^2)} + \frac{2b^3n^3x^3 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{3(9 + 10b^2n^2 + b^4n^4)} \end{aligned}$$

**Mathematica [A]** time = 0.525288, size = 120, normalized size = 0.75

$$\frac{x^3 \left( 27(b^2 n^2 + 1) \cos(a + b \log(cx^n)) + (b^2 n^2 + 9) \cos(3(a + b \log(cx^n))) + 2bn \sin(a + b \log(cx^n)) \left( (b^2 n^2 + 9) \cos(2(a + b \log(cx^n))) + \sin(2(a + b \log(cx^n))) \right) \right)}{12(b^4 n^4 + 10b^2 n^2 + 9)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cos[a + b\*Log[c\*x^n]]^3,x]

[Out] (x^3\*(27\*(1 + b^2\*n^2)\*Cos[a + b\*Log[c\*x^n]] + (9 + b^2\*n^2)\*Cos[3\*(a + b\*Log[c\*x^n])] + 2\*b\*n\*(9 + 5\*b^2\*n^2 + (9 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n]])\*Sin[a + b\*Log[c\*x^n]]))/(12\*(9 + 10\*b^2\*n^2 + b^4\*n^4))

**Maple [F]** time = 0.073, size = 0, normalized size = 0.

$$\int x^2 (\cos(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cos(a+b\*ln(c\*x^n))^3,x)

[Out] int(x^2\*cos(a+b\*ln(c\*x^n))^3,x)

**Maxima [B]** time = 1.26603, size = 1359, normalized size = 8.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] 1/24\*(((b^3\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c)))\*n^3 + (b^2\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c)))\*n^2 + 9\*(b\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c)))\*n + 9\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + 9\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + 9\*cos(3\*b\*log(c)))\*x^3\*cos(3\*b\*log(x^n) + 3\*a) + 9\*((b^3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^3\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 3\*(b^2\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n + 3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + 3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + 3\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + 3\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*x^3\*cos(b\*log(x^n) + a) + ((b^3\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c)))\*n^3 - (b^2\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c)))\*n^2 + 9\*(b\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(3\*b\*log(c)))\*n - 9\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) + 9\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) - 9\*sin(3\*b\*log(c)))\*x^3\*sin(3\*b\*log(x^n) + 3\*a) + 9\*((b^3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c))\*sin(2\*b\*log(c))

$$\begin{aligned} & c)) * n^3 - 3 * (b^2 * \cos(3 * b * \log(c)) * \sin(4 * b * \log(c)) - b^2 * \cos(4 * b * \log(c)) * \sin( \\ & (3 * b * \log(c)) + b^2 * \cos(2 * b * \log(c)) * \sin(3 * b * \log(c)) - b^2 * \cos(3 * b * \log(c)) * \sin( \\ & n(2 * b * \log(c))) * n^2 + (b * \cos(4 * b * \log(c)) * \cos(3 * b * \log(c)) + b * \cos(3 * b * \log(c)) \\ & * \cos(2 * b * \log(c)) + b * \sin(4 * b * \log(c)) * \sin(3 * b * \log(c)) + b * \sin(3 * b * \log(c)) * \sin( \\ & n(2 * b * \log(c))) * n - 3 * \cos(3 * b * \log(c)) * \sin(4 * b * \log(c)) + 3 * \cos(4 * b * \log(c)) * \sin( \\ & n(3 * b * \log(c)) - 3 * \cos(2 * b * \log(c)) * \sin(3 * b * \log(c)) + 3 * \cos(3 * b * \log(c)) * \sin(2 \\ & * b * \log(c))) * x^3 * \sin(b * \log(x^n) + a) / ((b^4 * \cos(3 * b * \log(c))^2 + b^4 * \sin(3 * b * \\ & \log(c))^2) * n^4 + 10 * (b^2 * \cos(3 * b * \log(c))^2 + b^2 * \sin(3 * b * \log(c))^2) * n^2 + 9 \\ & * \cos(3 * b * \log(c))^2 + 9 * \sin(3 * b * \log(c))^2) \end{aligned}$$

**Fricas [A]** time = 0.510899, size = 321, normalized size = 2.01

$$\frac{6 b^2 n^2 x^3 \cos(b n \log(x) + b \log(c) + a) + (b^2 n^2 + 9) x^3 \cos(b n \log(x) + b \log(c) + a)^3 + (2 b^3 n^3 x^3 + (b^3 n^3 + 9 b n) x^3)}{3 (b^4 n^4 + 10 b^2 n^2 + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/3\*(6\*b^2\*n^2\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a) + (b^2\*n^2 + 9)\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a)^3 + (2\*b^3\*n^3\*x^3 + (b^3\*n^3 + 9\*b\*n)\*x^3\*cos(b\*n\*log(x) + b\*log(c) + a)^2)\*sin(b\*n\*log(x) + b\*log(c) + a))/(b^4\*n^4 + 10\*b^2\*n^2 + 9)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cos(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cos(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] Timed out

### 3.97 $\int x \cos^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=158

$$\frac{6b^3n^3x^2 \sin(a + b \log(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} + \frac{2x^2 \cos^3(a + b \log(cx^n))}{9b^2n^2 + 4} + \frac{12b^2n^2x^2 \cos(a + b \log(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} + \frac{3bnx^2 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{9b^2n^2 + 4}$$

[Out]  $(12b^2n^2x^2\text{Cos}[a + b\text{Log}[cx^n]])/(16 + 40b^2n^2 + 9b^4n^4) + (2x^2\text{Cos}[a + b\text{Log}[cx^n]]^3)/(4 + 9b^2n^2) + (6b^3n^3x^2\text{Sin}[a + b\text{Log}[cx^n]])/(16 + 40b^2n^2 + 9b^4n^4) + (3bnx^2\text{Cos}[a + b\text{Log}[cx^n]]^2\text{Sin}[a + b\text{Log}[cx^n]])/(4 + 9b^2n^2)$

**Rubi [A]** time = 0.0451337, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {4488, 4486}

$$\frac{6b^3n^3x^2 \sin(a + b \log(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} + \frac{2x^2 \cos^3(a + b \log(cx^n))}{9b^2n^2 + 4} + \frac{12b^2n^2x^2 \cos(a + b \log(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} + \frac{3bnx^2 \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{9b^2n^2 + 4}$$

Antiderivative was successfully verified.

[In] Int[x\*Cos[a + b\*Log[c\*x^n]]^3,x]

[Out]  $(12b^2n^2x^2\text{Cos}[a + b\text{Log}[cx^n]])/(16 + 40b^2n^2 + 9b^4n^4) + (2x^2\text{Cos}[a + b\text{Log}[cx^n]]^3)/(4 + 9b^2n^2) + (6b^3n^3x^2\text{Sin}[a + b\text{Log}[cx^n]])/(16 + 40b^2n^2 + 9b^4n^4) + (3bnx^2\text{Cos}[a + b\text{Log}[cx^n]]^2\text{Sin}[a + b\text{Log}[cx^n]])/(4 + 9b^2n^2)$

#### Rule 4488

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 4486

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] + Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int x \cos^3(a + b \log(cx^n)) dx &= \frac{2x^2 \cos^3(a + b \log(cx^n))}{4 + 9b^2n^2} + \frac{3bnx^2 \cos^2(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{4 + 9b^2n^2} + \frac{(6b^2n^2) \int x \cos^2(a + b \log(cx^n)) dx}{16 + 40b^2n^2 + 9b^4n^4} \\ &= \frac{12b^2n^2x^2 \cos(a + b \log(cx^n))}{16 + 40b^2n^2 + 9b^4n^4} + \frac{2x^2 \cos^3(a + b \log(cx^n))}{4 + 9b^2n^2} + \frac{6b^3n^3x^2 \sin(a + b \log(cx^n))}{16 + 40b^2n^2 + 9b^4n^4} \end{aligned}$$



**Mathematica [A]** time = 0.487056, size = 123, normalized size = 0.78

$$\frac{x^2 \left( 6 \left( 9b^2n^2 + 4 \right) \cos(a + b \log(cx^n)) + 2 \left( b^2n^2 + 4 \right) \cos(3(a + b \log(cx^n))) + 6bn \sin(a + b \log(cx^n)) \left( (b^2n^2 + 4) \cos(a + b \log(cx^n)) \right) \right)}{4 \left( 9b^4n^4 + 40b^2n^2 + 16 \right)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cos[a + b\*Log[c\*x^n]]^3,x]

[Out] (x^2\*(6\*(4 + 9\*b^2\*n^2)\*Cos[a + b\*Log[c\*x^n]] + 2\*(4 + b^2\*n^2)\*Cos[3\*(a + b\*Log[c\*x^n]]) + 6\*b\*n\*(4 + 5\*b^2\*n^2 + (4 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n]])\*Sin[a + b\*Log[c\*x^n]]))/(4\*(16 + 40\*b^2\*n^2 + 9\*b^4\*n^4))

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int x (\cos(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cos(a+b\*ln(c\*x^n))^3,x)

[Out] int(x\*cos(a+b\*ln(c\*x^n))^3,x)

**Maxima [B]** time = 1.23019, size = 1370, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] 1/8\*((3\*(b^3\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c)))\*n^3 + 2\*(b^2\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c)))\*n^2 + 12\*(b\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c)))\*n + 8\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + 8\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + 8\*cos(3\*b\*log(c))\*x^2\*cos(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^3\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 18\*(b^2\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + 4\*(b\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n + 8\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + 8\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + 8\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + 8\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*x^2\*cos(b\*log(x^n) + a) + (3\*(b^3\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c)))\*n^3 - 2\*(b^2\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c)))\*n^2 + 12\*(b\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(3\*b\*log(c)))\*n - 8\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) + 8\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) - 8\*sin(3\*b\*log(c)))\*x^2\*sin(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c))

$$\begin{aligned} & c)) \sin(2b \log(c)) n^3 - 18(b^2 \cos(3b \log(c)) \sin(4b \log(c)) - b^2 \cos(4b \log(c)) \sin(3b \log(c)) + b^2 \cos(2b \log(c)) \sin(3b \log(c)) - b^2 \cos(3b \log(c)) \sin(2b \log(c))) n^2 \\ & + 4(b \cos(4b \log(c)) \cos(3b \log(c)) + b \cos(3b \log(c)) \cos(2b \log(c)) + b \sin(4b \log(c)) \sin(3b \log(c)) + b \sin(3b \log(c)) \sin(2b \log(c))) n \\ & - 8 \cos(3b \log(c)) \sin(4b \log(c)) + 8 \cos(4b \log(c)) \sin(3b \log(c)) - 8 \cos(2b \log(c)) \sin(3b \log(c)) + 8 \cos(3b \log(c)) \sin(2b \log(c)) \\ & \times x^2 \sin(b \log(x^n) + a) / (9(b^4 \cos(3b \log(c))^2 + b^4 \sin(3b \log(c))^2) n^4 + 40(b^2 \cos(3b \log(c))^2 + b^2 \sin(3b \log(c))^2) n^2 + 16 \cos(3b \log(c))^2 + 16 \sin(3b \log(c))^2) \end{aligned}$$

**Fricas [A]** time = 0.507962, size = 327, normalized size = 2.07

$$\frac{12b^2n^2x^2 \cos(bn \log(x) + b \log(c) + a) + 2(b^2n^2 + 4)x^2 \cos(bn \log(x) + b \log(c) + a)^3 + 3(2b^3n^3x^2 + (b^3n^3 + 4bn)x^2)}{9b^4n^4 + 40b^2n^2 + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] (12\*b^2\*n^2\*x^2\*cos(b\*n\*log(x) + b\*log(c) + a) + 2\*(b^2\*n^2 + 4)\*x^2\*cos(b\*n\*log(x) + b\*log(c) + a)^3 + 3\*(2\*b^3\*n^3\*x^2 + (b^3\*n^3 + 4\*b\*n)\*x^2\*cos(b\*n\*log(x) + b\*log(c) + a)^2)\*sin(b\*n\*log(x) + b\*log(c) + a))/(9\*b^4\*n^4 + 40\*b^2\*n^2 + 16)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] Timed out

### 3.98 $\int \cos^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=149

$$\frac{6b^3n^3x \sin(a + b \log(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} + \frac{x \cos^3(a + b \log(cx^n))}{9b^2n^2 + 1} + \frac{6b^2n^2x \cos(a + b \log(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} + \frac{3bnx \sin(a + b \log(cx^n)) \cos^2(a + b \log(cx^n))}{9b^2n^2 + 1}$$

[Out]  $(6*b^2*n^2*x*Cos[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) + (x*Cos[a + b*Log[c*x^n]]^3)/(1 + 9*b^2*n^2) + (6*b^3*n^3*x*Sin[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) + (3*b*n*x*Cos[a + b*Log[c*x^n]]^2*Sin[a + b*Log[c*x^n]])/(1 + 9*b^2*n^2)$

**Rubi [A]** time = 0.036033, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {4478, 4476}

$$\frac{6b^3n^3x \sin(a + b \log(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} + \frac{x \cos^3(a + b \log(cx^n))}{9b^2n^2 + 1} + \frac{6b^2n^2x \cos(a + b \log(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} + \frac{3bnx \sin(a + b \log(cx^n)) \cos^2(a + b \log(cx^n))}{9b^2n^2 + 1}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^3,x]

[Out]  $(6*b^2*n^2*x*Cos[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) + (x*Cos[a + b*Log[c*x^n]]^3)/(1 + 9*b^2*n^2) + (6*b^3*n^3*x*Sin[a + b*Log[c*x^n]])/(1 + 10*b^2*n^2 + 9*b^4*n^4) + (3*b*n*x*Cos[a + b*Log[c*x^n]]^2*Sin[a + b*Log[c*x^n]])/(1 + 9*b^2*n^2)$

#### Rule 4478

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_), x\_Symbol] := Simp[(x\*Cos[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*n^2\*p^2 + 1), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + 1), Int[Cos[d\*(a + b\*Log[c\*x^n])]^(p - 2), x], x] + Simp[(b\*d\*n\*p\*x\*Cos[d\*(a + b\*Log[c\*x^n])]^(p - 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2\*p^2 + 1), x]) /; FreeQ[{a, b, c, d, n}, x] && I GtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + 1, 0]

#### Rule 4476

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] := Simp[(x\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] + Simp[(b\*d\*n\*x\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2\*d^2\*n^2 + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \cos^3(a + b \log(cx^n)) dx &= \frac{x \cos^3(a + b \log(cx^n))}{1 + 9b^2n^2} + \frac{3bnx \cos^2(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 9b^2n^2} + \frac{(6b^2n^2) \int \cos^2(a + b \log(cx^n)) dx}{1 + 9b^2n^2} \\ &= \frac{6b^2n^2x \cos(a + b \log(cx^n))}{1 + 10b^2n^2 + 9b^4n^4} + \frac{x \cos^3(a + b \log(cx^n))}{1 + 9b^2n^2} + \frac{6b^3n^3x \sin(a + b \log(cx^n))}{1 + 10b^2n^2 + 9b^4n^4} + \dots \end{aligned}$$

**Mathematica [A]** time = 0.41392, size = 117, normalized size = 0.79

$$\frac{x(3(9b^2n^2 + 1) \cos(a + b \log(cx^n)) + (b^2n^2 + 1) \cos(3(a + b \log(cx^n)))) + 6bn \sin(a + b \log(cx^n))((b^2n^2 + 1) \cos(2(a + b \log(cx^n))))}{36b^4n^4 + 40b^2n^2 + 4}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^3,x]

[Out] (x\*(3\*(1 + 9\*b^2\*n^2)\*Cos[a + b\*Log[c\*x^n]] + (1 + b^2\*n^2)\*Cos[3\*(a + b\*Log[c\*x^n])]) + 6\*b\*n\*(1 + 5\*b^2\*n^2 + (1 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])])\*Sin[a + b\*Log[c\*x^n]])/(4 + 40\*b^2\*n^2 + 36\*b^4\*n^4)

**Maple [F]** time = 0.095, size = 0, normalized size = 0.

$$\int (\cos(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^3,x)

[Out] int(cos(a+b\*ln(c\*x^n))^3,x)

**Maxima [B]** time = 1.22745, size = 1335, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] 1/8\*((3\*(b^3\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c)))\*n^3 + (b^2\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c)))\*n^2 + 3\*(b\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c)))\*n + cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + cos(3\*b\*log(c))\*x\*cos(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^3\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 9\*(b^2\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n + cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*x\*cos(b\*log(x^n) + a) + (3\*(b^3\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c)))\*n^3 - (b^2\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c)))\*n^2 + 3\*(b\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(3\*b\*log(c)))\*n - cos(3\*b\*log(c))\*sin(6\*b\*log(c)) + cos(6\*b\*log(c))\*sin(3\*b\*log(c)) - sin(3\*b\*log(c)))\*x\*sin(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 - 9\*(b^2\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^2\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n - cos(3\*b\*log(c))\*sin(4\*b\*log(c)) + cos(4\*b\*log(c))\*sin(3\*b\*log(c)) - cos(2\*b

```
*log(c))*sin(3*b*log(c)) + cos(3*b*log(c))*sin(2*b*log(c)))*x*sin(b*log(x^n
) + a))/(9*(b^4*cos(3*b*log(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 10*(b^2*co
s(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2)*n^2 + cos(3*b*log(c))^2 + sin(3*b*
log(c))^2)
```

**Fricas [A]** time = 0.500971, size = 308, normalized size = 2.07

$$\frac{6b^2n^2x \cos(bn \log(x) + b \log(c) + a) + (b^2n^2 + 1)x \cos(bn \log(x) + b \log(c) + a)^3 + 3(2b^3n^3x + (b^3n^3 + bn)x \cos(bn \log(x) + b \log(c) + a)) \sin(bn \log(x) + b \log(c) + a)}{9b^4n^4 + 10b^2n^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] (6*b^2*n^2*x*cos(b*n*log(x) + b*log(c) + a) + (b^2*n^2 + 1)*x*cos(b*n*log(x)
) + b*log(c) + a)^3 + 3*(2*b^3*n^3*x + (b^3*n^3 + b*n)*x*cos(b*n*log(x) + b
*log(c) + a)^2)*sin(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4 + 10*b^2*n^2 + 1
)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**3,x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.99 \quad \int \frac{\cos^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=42

$$\frac{\sin(a+b \log(cx^n))}{bn} - \frac{\sin^3(a+b \log(cx^n))}{3bn}$$

[Out] Sin[a + b\*Log[c\*x^n]]/(b\*n) - Sin[a + b\*Log[c\*x^n]]^3/(3\*b\*n)

**Rubi [A]** time = 0.032968, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2633}

$$\frac{\sin(a+b \log(cx^n))}{bn} - \frac{\sin^3(a+b \log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^3/x,x]

[Out] Sin[a + b\*Log[c\*x^n]]/(b\*n) - Sin[a + b\*Log[c\*x^n]]^3/(3\*b\*n)

**Rule 2633**

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{\cos^3(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cos^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\text{Subst}\left(\int (1-x^2) dx, x, -\sin(a+b \log(cx^n))\right)}{bn} \\ &= \frac{\sin(a+b \log(cx^n))}{bn} - \frac{\sin^3(a+b \log(cx^n))}{3bn} \end{aligned}$$

**Mathematica [A]** time = 0.0561742, size = 42, normalized size = 1.

$$\frac{\sin(a+b \log(cx^n))}{bn} - \frac{\sin^3(a+b \log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^3/x,x]

[Out] Sin[a + b\*Log[c\*x^n]]/(b\*n) - Sin[a + b\*Log[c\*x^n]]^3/(3\*b\*n)

**Maple [A]** time = 0.033, size = 35, normalized size = 0.8

$$\frac{(2 + (\cos(a + b \ln(cx^n)))^2) \sin(a + b \ln(cx^n))}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*ln(c*x^n))^3/x,x)
```

```
[Out] 1/3/n/b*(2+cos(a+b*ln(c*x^n))^2)*sin(a+b*ln(c*x^n))
```

**Maxima [B]** time = 1.11751, size = 313, normalized size = 7.45

$$\frac{(\cos(3b \log(c)) \sin(6b \log(c)) - \cos(6b \log(c)) \sin(3b \log(c)) + \sin(3b \log(c))) \cos(3b \log(x^n) + 3a) + 9(\cos(3b \log(c)) \sin(6b \log(c)) - \cos(6b \log(c)) \sin(3b \log(c)) + \sin(3b \log(c))) \cos(3b \log(x^n) + 3a) + 9(\cos(3b \log(c)) \sin(6b \log(c)) - \cos(6b \log(c)) \sin(3b \log(c)) + \sin(3b \log(c))) \cos(3b \log(x^n) + 3a)}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*((cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(3*b*log(c)) +
sin(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) + 9*(cos(3*b*log(c))*sin(4*b*log(c))
) - cos(4*b*log(c))*sin(3*b*log(c)) + cos(2*b*log(c))*sin(3*b*log(c)) - co
s(3*b*log(c))*sin(2*b*log(c)))*cos(b*log(x^n) + a) + (cos(6*b*log(c))*cos(3
*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c))*sin(3*b*log
(x^n) + 3*a) + 9*(cos(4*b*log(c))*cos(3*b*log(c)) + cos(3*b*log(c))*cos(2*b
*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(2*b*log(c)
))*sin(b*log(x^n) + a))/(b*n)
```

**Fricas [A]** time = 0.489189, size = 109, normalized size = 2.6

$$\frac{(\cos(bn \log(x) + b \log(c) + a)^2 + 2) \sin(bn \log(x) + b \log(c) + a)}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*n*log(x) + b*log(c) + a)^2 + 2)*sin(b*n*log(x) + b*log(c) + a)/(
b*n)
```

**Sympy [A]** time = 27.5633, size = 82, normalized size = 1.95

$$\begin{cases} \log(x) \cos^3(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cos^3(a + b \log(c)) & \text{for } n = 0 \\ \frac{2 \sin^3(a + bn \log(x) + b \log(c))}{3bn} + \frac{\sin(a + bn \log(x) + b \log(c)) \cos^2(a + bn \log(x) + b \log(c))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*cos(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos
(a + b*log(c))**3, Eq(n, 0)), (2*sin(a + b*n*log(x) + b*log(c))**3/(3*b*n)
+ sin(a + b*n*log(x) + b*log(c))*cos(a + b*n*log(x) + b*log(c))**2/(b*n), T
rue))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^3/x,x, algorithm="giac")

[Out] integrate(cos(b\*log(c\*x^n) + a)^3/x, x)



$$3.100 \quad \int \frac{\cos^3(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=158

$$\frac{6b^3n^3 \sin(a+b \log(cx^n))}{x(9b^4n^4+10b^2n^2+1)} - \frac{\cos^3(a+b \log(cx^n))}{x(9b^2n^2+1)} - \frac{6b^2n^2 \cos(a+b \log(cx^n))}{x(9b^4n^4+10b^2n^2+1)} + \frac{3bn \sin(a+b \log(cx^n)) \cos^2(a+b \log(cx^n))}{x(9b^2n^2+1)}$$

[Out]  $(-6*b^2*n^2*Cos[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - Cos[a + b*Log[c*x^n]]^3/((1 + 9*b^2*n^2)*x) + (6*b^3*n^3*Sin[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) + (3*b*n*Cos[a + b*Log[c*x^n]]^2*Sin[a + b*Log[c*x^n]])/((1 + 9*b^2*n^2)*x)$

**Rubi [A]** time = 0.0482041, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4488, 4486}

$$\frac{6b^3n^3 \sin(a+b \log(cx^n))}{x(9b^4n^4+10b^2n^2+1)} - \frac{\cos^3(a+b \log(cx^n))}{x(9b^2n^2+1)} - \frac{6b^2n^2 \cos(a+b \log(cx^n))}{x(9b^4n^4+10b^2n^2+1)} + \frac{3bn \sin(a+b \log(cx^n)) \cos^2(a+b \log(cx^n))}{x(9b^2n^2+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^3/x^2, x]

[Out]  $(-6*b^2*n^2*Cos[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) - Cos[a + b*Log[c*x^n]]^3/((1 + 9*b^2*n^2)*x) + (6*b^3*n^3*Sin[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x) + (3*b*n*Cos[a + b*Log[c*x^n]]^2*Sin[a + b*Log[c*x^n]])/((1 + 9*b^2*n^2)*x)$

#### Rule 4488

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p - 1))/(b^2\*d^2\*n^2\*p^2 + (m + 1)^2), Int[(e\*x)^m\*Cos[d\*(a + b\*Log[c\*x^n])]]^(p - 2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]\*Cos[d\*(a + b\*Log[c\*x^n])]]^(p - 1)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m + 1)^2, 0]

#### Rule 4486

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]\*(e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])]]/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] + Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])]]/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^3(a+b \log(cx^n))}{x^2} dx &= -\frac{\cos^3(a+b \log(cx^n))}{(1+9b^2n^2)x} + \frac{3bn \cos^2(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{(1+9b^2n^2)x} + \frac{(6b^2n^2) \int \frac{\cos^3(a+b \log(cx^n))}{x^2} dx}{1+9b^2n^2} \\ &= -\frac{6b^2n^2 \cos(a+b \log(cx^n))}{(1+10b^2n^2+9b^4n^4)x} - \frac{\cos^3(a+b \log(cx^n))}{(1+9b^2n^2)x} + \frac{6b^3n^3 \sin(a+b \log(cx^n))}{(1+10b^2n^2+9b^4n^4)x} + \frac{3bn \sin(a+b \log(cx^n)) \cos^2(a+b \log(cx^n))}{(1+9b^2n^2)x} \end{aligned}$$

**Mathematica [A]** time = 0.45156, size = 122, normalized size = 0.77

$$\frac{3(9b^2n^2 + 1)\cos(a + b\log(cx^n)) + (b^2n^2 + 1)\cos(3(a + b\log(cx^n))) - 6bn\sin(a + b\log(cx^n))((b^2n^2 + 1)\cos(2(a + b\log(cx^n))))}{4x(9b^4n^4 + 10b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^3/x^2,x]

[Out] -(3\*(1 + 9\*b^2\*n^2)\*Cos[a + b\*Log[c\*x^n]] + (1 + b^2\*n^2)\*Cos[3\*(a + b\*Log[c\*x^n])]) - 6\*b\*n\*(1 + 5\*b^2\*n^2 + (1 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])])\*Sin[a + b\*Log[c\*x^n]]/(4\*(1 + 10\*b^2\*n^2 + 9\*b^4\*n^4)\*x)

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{(\cos(a + b \ln(cx^n)))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^3/x^2,x)

[Out] int(cos(a+b\*ln(c\*x^n))^3/x^2,x)

**Maxima [B]** time = 1.23463, size = 1342, normalized size = 8.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^3/x^2,x, algorithm="maxima")

[Out] 1/8\*((3\*(b^3\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*sin(3\*b\*log(c)))\*n^3 - (b^2\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c)))\*n^2 + 3\*(b\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*sin(3\*b\*log(c)))\*n - cos(6\*b\*log(c))\*cos(3\*b\*log(c)) - sin(6\*b\*log(c))\*sin(3\*b\*log(c)) - cos(3\*b\*log(c))\*cos(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b^3\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 - 9\*(b^2\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^2\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(4\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(3\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(2\*b\*log(c))\*sin(3\*b\*log(c)) - b\*cos(3\*b\*log(c))\*sin(2\*b\*log(c)))\*n - cos(4\*b\*log(c))\*cos(3\*b\*log(c)) - cos(3\*b\*log(c))\*cos(2\*b\*log(c)) - sin(4\*b\*log(c))\*sin(3\*b\*log(c)) - sin(3\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(b\*log(x^n) + a) + (3\*(b^3\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c)))\*n^3 + (b^2\*cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + b^2\*sin(3\*b\*log(c)))\*n^2 + 3\*(b\*cos(6\*b\*log(c))\*cos(3\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(3\*b\*log(c)) + b\*cos(3\*b\*log(c)))\*n + cos(3\*b\*log(c))\*sin(6\*b\*log(c)) - cos(6\*b\*log(c))\*sin(3\*b\*log(c)) + sin(3\*b\*log(c))\*sin(3\*b\*log(x^n) + 3\*a) + 3\*(9\*(b^3\*cos(4\*b\*log(c))\*cos(3\*b\*log(c)) + b^3\*cos(3\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*

$$\begin{aligned} & \sin(3b \log(c)) + b^3 \sin(3b \log(c)) \sin(2b \log(c)) \cdot n^3 + 9(b^2 \cos(3b \log(c)) \sin(4b \log(c)) - b^2 \cos(4b \log(c)) \sin(3b \log(c)) + b^2 \cos(2b \log(c)) \sin(3b \log(c)) - b^2 \cos(3b \log(c)) \sin(2b \log(c))) \cdot n^2 + (b \cos(4b \log(c)) \cos(3b \log(c)) + b \cos(3b \log(c)) \cos(2b \log(c)) + b \sin(4b \log(c)) \sin(3b \log(c)) + b \sin(3b \log(c)) \sin(2b \log(c))) \cdot n + \cos(3b \log(c)) \sin(4b \log(c)) - \cos(4b \log(c)) \sin(3b \log(c)) + \cos(2b \log(c)) \sin(3b \log(c)) - \cos(3b \log(c)) \sin(2b \log(c)) \cdot \sin(b \log(x^n) + a) / \\ & ((9(b^4 \cos(3b \log(c))^2 + b^4 \sin(3b \log(c))^2) \cdot n^4 + 10(b^2 \cos(3b \log(c))^2 + b^2 \sin(3b \log(c))^2) \cdot n^2 + \cos(3b \log(c))^2 + \sin(3b \log(c))^2) \cdot x) \end{aligned}$$

**Fricas [A]** time = 0.508676, size = 304, normalized size = 1.92

$$\frac{6b^2n^2 \cos(bn \log(x) + b \log(c) + a) + (b^2n^2 + 1) \cos(bn \log(x) + b \log(c) + a)^3 - 3(2b^3n^3 + (b^3n^3 + bn) \cos(bn \log(x) + b \log(c) + a)) \sin(bn \log(x) + b \log(c) + a)}{(9b^4n^4 + 10b^2n^2 + 1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^3/x^2,x, algorithm="fricas")

[Out]  $-(6b^2n^2 \cos(bn \log(x) + b \log(c) + a) + (b^2n^2 + 1) \cos(bn \log(x) + b \log(c) + a)^3 - 3(2b^3n^3 + (b^3n^3 + bn) \cos(bn \log(x) + b \log(c) + a)) \sin(bn \log(x) + b \log(c) + a)) / ((9b^4n^4 + 10b^2n^2 + 1)x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*ln(c\*x\*\*n))\*\*3/x\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^3/x^2,x, algorithm="giac")

[Out] integrate(cos(b\*log(c\*x^n) + a)^3/x^2, x)

### 3.101 $\int \cos^4(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=191

$$\frac{12b^2n^2x \cos^2(a + b \log(cx^n))}{64b^4n^4 + 20b^2n^2 + 1} + \frac{x \cos^4(a + b \log(cx^n))}{16b^2n^2 + 1} + \frac{4bnx \sin(a + b \log(cx^n)) \cos^3(a + b \log(cx^n))}{16b^2n^2 + 1} + \frac{24b^3n^3x \sin(a + b \log(cx^n)) \cos^2(a + b \log(cx^n))}{64b^4n^4 + 20b^2n^2 + 1}$$

[Out]  $(24*b^4*n^4*x)/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^2*x*\text{Cos}[a + b*\text{Log}[c*x^n]]^2)/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (x*\text{Cos}[a + b*\text{Log}[c*x^n]]^4)/(1 + 16*b^2*n^2) + (24*b^3*n^3*x*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (4*b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]]^3*\text{Sin}[a + b*\text{Log}[c*x^n]])/(1 + 16*b^2*n^2)$

**Rubi [A]** time = 0.0449368, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {4478, 8}

$$\frac{12b^2n^2x \cos^2(a + b \log(cx^n))}{64b^4n^4 + 20b^2n^2 + 1} + \frac{x \cos^4(a + b \log(cx^n))}{16b^2n^2 + 1} + \frac{4bnx \sin(a + b \log(cx^n)) \cos^3(a + b \log(cx^n))}{16b^2n^2 + 1} + \frac{24b^3n^3x \sin(a + b \log(cx^n)) \cos^2(a + b \log(cx^n))}{64b^4n^4 + 20b^2n^2 + 1}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[a + b*\text{Log}[c*x^n]]^4, x]$

[Out]  $(24*b^4*n^4*x)/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^2*x*\text{Cos}[a + b*\text{Log}[c*x^n]]^2)/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (x*\text{Cos}[a + b*\text{Log}[c*x^n]]^4)/(1 + 16*b^2*n^2) + (24*b^3*n^3*x*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Sin}[a + b*\text{Log}[c*x^n]])/(1 + 20*b^2*n^2 + 64*b^4*n^4) + (4*b*n*x*\text{Cos}[a + b*\text{Log}[c*x^n]]^3*\text{Sin}[a + b*\text{Log}[c*x^n]])/(1 + 16*b^2*n^2)$

#### Rule 4478

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(x*\text{Cos}[d*(a + b*\text{Log}[c*x^n])]^p)/(b^2*d^2*n^2*p^2 + 1), x] + (\text{Dist}[(b^2*d^2*n^2*p*(p - 1))/(b^2*d^2*n^2*p^2 + 1), \text{Int}[\text{Cos}[d*(a + b*\text{Log}[c*x^n])]^{(p - 2)}, x], x] + \text{Simp}[(b*d*n*p*x*\text{Cos}[d*(a + b*\text{Log}[c*x^n])]^{(p - 1)}*\text{Sin}[d*(a + b*\text{Log}[c*x^n])])/(b^2*d^2*n^2*p^2 + 1), x]) /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{I GtQ}[p, 1] \&\& \text{NeQ}[b^2*d^2*n^2*p^2 + 1, 0]$

#### Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

#### Rubi steps

$$\begin{aligned} \int \cos^4(a + b \log(cx^n)) dx &= \frac{x \cos^4(a + b \log(cx^n))}{1 + 16b^2n^2} + \frac{4bnx \cos^3(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 16b^2n^2} + \frac{(12b^2n^2) \int \cos^2(a + b \log(cx^n)) dx}{1 + 20b^2n^2 + 64b^4n^4} \\ &= \frac{12b^2n^2x \cos^2(a + b \log(cx^n))}{1 + 20b^2n^2 + 64b^4n^4} + \frac{x \cos^4(a + b \log(cx^n))}{1 + 16b^2n^2} + \frac{24b^3n^3x \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{1 + 20b^2n^2 + 64b^4n^4} \\ &= \frac{24b^4n^4x}{1 + 20b^2n^2 + 64b^4n^4} + \frac{12b^2n^2x \cos^2(a + b \log(cx^n))}{1 + 20b^2n^2 + 64b^4n^4} + \frac{x \cos^4(a + b \log(cx^n))}{1 + 16b^2n^2} + \frac{24b^3n^3x \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{1 + 20b^2n^2 + 64b^4n^4} \end{aligned}$$

**Mathematica [A]** time = 0.434809, size = 167, normalized size = 0.87

$$\frac{x(128b^3n^3 \sin(2(a + b \log(cx^n))) + 16b^3n^3 \sin(4(a + b \log(cx^n))) + (64b^2n^2 + 4) \cos(2(a + b \log(cx^n))) + (4b^2n^2 + 1) \cos(4(a + b \log(cx^n))))}{8(64b^4n^4 + 20b^2n^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^4,x]

[Out] (x\*(3 + 60\*b^2\*n^2 + 192\*b^4\*n^4 + (4 + 64\*b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])]) + (1 + 4\*b^2\*n^2)\*Cos[4\*(a + b\*Log[c\*x^n])] + 8\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])] + 128\*b^3\*n^3\*Sin[2\*(a + b\*Log[c\*x^n])] + 4\*b\*n\*Sin[4\*(a + b\*Log[c\*x^n])] + 16\*b^3\*n^3\*Sin[4\*(a + b\*Log[c\*x^n])])/(8\*(1 + 20\*b^2\*n^2 + 64\*b^4\*n^4))

**Maple [F]** time = 0.09, size = 0, normalized size = 0.

$$\int (\cos(a + b \ln(cx^n)))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^4,x)

[Out] int(cos(a+b\*ln(c\*x^n))^4,x)

**Maxima [B]** time = 1.27288, size = 1455, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^4,x, algorithm="maxima")

[Out] 1/16\*((16\*(b^3\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^3\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*sin(4\*b\*log(c)))\*n^3 + 4\*(b^2\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c)))\*n^2 + 4\*(b\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*sin(4\*b\*log(c)))\*n + cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + cos(4\*b\*log(c)))\*x\*cos(4\*b\*log(x^n) + 4\*a) + 4\*(32\*(b^3\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b^3\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b^3\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 + 16\*(b^2\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b^2\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + 2\*(b\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n + cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*x\*cos(2\*b\*log(x^n) + 2\*a) + (16\*(b^3\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b^3\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*cos(4\*b\*log(c)))\*n^3 - 4\*(b^2\*cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - b^2\*cos(8\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*sin(4\*b\*log(c)))\*n^2 + 4\*(b\*cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(8\*b\*log(c))\*sin(4\*b\*log(c)) + b\*cos(4\*b\*log(c)))\*n - cos(4\*b\*log(c))\*sin(8\*b\*log(c)) + cos(8\*b\*log(c))\*sin(4\*b\*log(c)) - sin(4\*b\*log(c)))\*x\*sin(4\*b\*log(x^n) + 4\*a) + 4\*(32\*(b^3\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b^3\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b^3\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^3\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^3 - 16\*(b^2\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)) + b^2\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + 2\*(b\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b\*cos(4\*b\*log(c))

```
*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*b*log(c)) - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) + 6*(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*x)/(64*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 20*(b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + cos(4*b*log(c))^2 + sin(4*b*log(c))^2)
```

**Fricas [A]** time = 0.514719, size = 381, normalized size = 1.99

$$\frac{24b^4n^4x + 12b^2n^2x \cos(bn \log(x) + b \log(c) + a)^2 + (4b^2n^2 + 1)x \cos(bn \log(x) + b \log(c) + a)^4 + 4(6b^3n^3x \cos(bn \log(x) + b \log(c) + a) + (4b^3n^3 + b^n)x \cos(bn \log(x) + b \log(c) + a)^3) \sin(bn \log(x) + b \log(c) + a)}{64b^4n^4 + 20b^2n^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] (24*b^4*n^4*x + 12*b^2*n^2*x*cos(b*n*log(x) + b*log(c) + a)^2 + (4*b^2*n^2 + 1)*x*cos(b*n*log(x) + b*log(c) + a)^4 + 4*(6*b^3*n^3*x*cos(b*n*log(x) + b*log(c) + a) + (4*b^3*n^3 + b^n)*x*cos(b*n*log(x) + b*log(c) + a)^3)*sin(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 + 20*b^2*n^2 + 1)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.102 \quad \int \frac{\cos^4(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=73

$$\frac{\sin(a+b \log(cx^n)) \cos^3(a+b \log(cx^n))}{4bn} + \frac{3 \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{8bn} + \frac{3 \log(x)}{8}$$

[Out] (3\*Log[x])/8 + (3\*Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(8\*b\*n) + (Cos[a + b\*Log[c\*x^n]]^3\*Sin[a + b\*Log[c\*x^n]])/(4\*b\*n)

**Rubi [A]** time = 0.0441463, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2635, 8}

$$\frac{\sin(a+b \log(cx^n)) \cos^3(a+b \log(cx^n))}{4bn} + \frac{3 \sin(a+b \log(cx^n)) \cos(a+b \log(cx^n))}{8bn} + \frac{3 \log(x)}{8}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^4/x,x]

[Out] (3\*Log[x])/8 + (3\*Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]])/(8\*b\*n) + (Cos[a + b\*Log[c\*x^n]]^3\*Sin[a + b\*Log[c\*x^n]])/(4\*b\*n)

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x] \* (b\*SIN[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^4(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cos^4(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\cos^3(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{4bn} + \frac{3 \text{Subst}\left(\int \cos^2(a+bx) dx, x, \log(cx^n)\right)}{4n} \\ &= \frac{3 \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{8bn} + \frac{\cos^3(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{4bn} \\ &= \frac{3 \log(x)}{8} + \frac{3 \cos(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{8bn} + \frac{\cos^3(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{4bn} \end{aligned}$$

**Mathematica [A]** time = 0.10127, size = 51, normalized size = 0.7

$$\frac{12(a+b \log(cx^n)) + 8 \sin(2(a+b \log(cx^n))) + \sin(4(a+b \log(cx^n)))}{32bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^4/x,x]

[Out] (12\*(a + b\*Log[c\*x^n]) + 8\*Sin[2\*(a + b\*Log[c\*x^n])] + Sin[4\*(a + b\*Log[c\*x^n])])/(32\*b\*n)

**Maple [A]** time = 0.027, size = 84, normalized size = 1.2

$$\frac{(\cos(a + b \ln(cx^n)))^3 \sin(a + b \ln(cx^n))}{4bn} + \frac{3 \cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{8bn} + \frac{3 \ln(cx^n)}{8n} + \frac{3a}{8bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^4/x,x)

[Out] 1/4\*cos(a+b\*ln(c\*x^n))^3\*sin(a+b\*ln(c\*x^n))/b/n+3/8\*cos(a+b\*ln(c\*x^n))\*sin(a+b\*ln(c\*x^n))/b/n+3/8/n\*ln(c\*x^n)+3/8/b/n\*a

**Maxima [A]** time = 1.14422, size = 126, normalized size = 1.73

$$\frac{12bn \log(x) + \cos(4b \log(x^n) + 4a) \sin(4b \log(c)) + 8 \cos(2b \log(x^n) + 2a) \sin(2b \log(c)) + \cos(4b \log(c)) \sin(4b \log(x^n) + 4a) + 8 \cos(2b \log(c)) \sin(2b \log(x^n) + 2a)}{32bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^4/x,x, algorithm="maxima")

[Out] 1/32\*(12\*b\*n\*log(x) + cos(4\*b\*log(x^n) + 4\*a)\*sin(4\*b\*log(c)) + 8\*cos(2\*b\*log(x^n) + 2\*a)\*sin(2\*b\*log(c)) + cos(4\*b\*log(c))\*sin(4\*b\*log(x^n) + 4\*a) + 8\*cos(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a))/(b\*n)

**Fricas [A]** time = 0.500121, size = 177, normalized size = 2.42

$$\frac{3bn \log(x) + (2 \cos(bn \log(x) + b \log(c) + a)^3 + 3 \cos(bn \log(x) + b \log(c) + a)) \sin(bn \log(x) + b \log(c) + a)}{8bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^4/x,x, algorithm="fricas")

[Out] 1/8\*(3\*b\*n\*log(x) + (2\*cos(b\*n\*log(x) + b\*log(c) + a)^3 + 3\*cos(b\*n\*log(x) + b\*log(c) + a))\*sin(b\*n\*log(x) + b\*log(c) + a))/(b\*n)

**Sympy [A]** time = 121.549, size = 110, normalized size = 1.51

$$\frac{\begin{cases} \log(x) \cos(2a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cos(2a + 2b \log(c)) & \text{for } n = 0 \\ \frac{\sin(2a + 2bn \log(x) + 2b \log(c))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\begin{cases} \log(x) \cos(4a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cos(4a + 4b \log(c)) & \text{for } n = 0 \\ \frac{\sin(4a + 4bn \log(x) + 4b \log(c))}{4bn} & \text{otherwise} \end{cases}}{8}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos(
2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*n*log(x) + 2*b*log(c))/(2*b*n
, True))/2 + Piecewise((log(x)*cos(4*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))),
(log(x)*cos(4*a + 4*b*log(c)), Eq(n, 0)), (sin(4*a + 4*b*n*log(x) + 4*b*lo
g(c))/(4*b*n), True))/8 + 3*log(x)/8
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^4/x, x)
```

### 3.103 $\int \cos(\log(6 + 3x)) dx$

**Optimal.** Leaf size=29

$$\frac{1}{2}(x+2)\sin(\log(3(x+2))) + \frac{1}{2}(x+2)\cos(\log(3(x+2)))$$

[Out]  $((2+x)*\text{Cos}[\text{Log}[3*(2+x)]])/2 + ((2+x)*\text{Sin}[\text{Log}[3*(2+x)]])/2$

**Rubi [A]** time = 0.0137795, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4476}

$$\frac{1}{2}(x+2)\sin(\log(3(x+2))) + \frac{1}{2}(x+2)\cos(\log(3(x+2)))$$

Antiderivative was successfully verified.

[In] Int[Cos[Log[6 + 3\*x]], x]

[Out]  $((2+x)*\text{Cos}[\text{Log}[3*(2+x)]])/2 + ((2+x)*\text{Sin}[\text{Log}[3*(2+x)]])/2$

#### Rule 4476

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)], x\_Symbol] :> Simp[(x\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] + Simp[(b\*d\*n\*x\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b^2\*d^2\*n^2 + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \cos(\log(6 + 3x)) dx &= \frac{1}{3} \text{Subst}\left(\int \cos(\log(x)) dx, x, 6 + 3x\right) \\ &= \frac{1}{2}(2+x)\cos(\log(3(2+x))) + \frac{1}{2}(2+x)\sin(\log(3(2+x))) \end{aligned}$$

**Mathematica [A]** time = 0.0119223, size = 22, normalized size = 0.76

$$\frac{1}{2}(x+2)(\sin(\log(3(x+2))) + \cos(\log(3(x+2))))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Log[6 + 3\*x]], x]

[Out]  $((2+x)*(Cos[Log[3*(2+x)]] + Sin[Log[3*(2+x)]])/2$

**Maple [C]** time = 0.037, size = 34, normalized size = 1.2

$$\left(\frac{1}{4} - \frac{i}{4}\right)(2+x)(6+3x)^i + \frac{\left(\frac{1}{4} + \frac{i}{4}\right)(2+x)}{(6+3x)^i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(ln(6+3*x)),x)`

[Out]  $(1/4-1/4*I)*(2+x)*(6+3*x)^I+(1/4+1/4*I)*(2+x)/((6+3*x)^I)$

**Maxima [A]** time = 0.968133, size = 27, normalized size = 0.93

$$\frac{1}{2}(x+2)(\cos(\log(3x+6)) + \sin(\log(3x+6)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(log(6+3*x)),x, algorithm="maxima")`

[Out]  $1/2*(x+2)*(\cos(\log(3*x+6)) + \sin(\log(3*x+6)))$

**Fricas [A]** time = 0.469818, size = 85, normalized size = 2.93

$$\frac{1}{2}(x+2)\cos(\log(3x+6)) + \frac{1}{2}(x+2)\sin(\log(3x+6))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(log(6+3*x)),x, algorithm="fricas")`

[Out]  $1/2*(x+2)*\cos(\log(3*x+6)) + 1/2*(x+2)*\sin(\log(3*x+6))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos(\log(3x+6)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(ln(6+3*x)),x)`

[Out] `Integral(cos(log(3*x+6)), x)`

**Giac [A]** time = 1.14413, size = 34, normalized size = 1.17

$$\frac{1}{2}(x+2)\cos(\log(3x+6)) + \frac{1}{2}(x+2)\sin(\log(3x+6))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(log(6+3*x)),x, algorithm="giac")`

[Out]  $1/2*(x+2)*\cos(\log(3*x+6)) + 1/2*(x+2)*\sin(\log(3*x+6))$

$$3.104 \quad \int x^m \cos \left( a + \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=101

$$\frac{x^{m+1} e^{\frac{a(m+1)}{n \sqrt{\frac{(m+1)^2}{n^2}}}} (cx^n)^{\frac{m+1}{n}}}{4(m+1)} + \frac{1}{2} x^{m+1} \log(x) e^{\frac{am \sqrt{\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{-\frac{m+1}{n}}$$

[Out] (E^((a\*(1+m))/(Sqrt[-((1+m)^2/n^2)]\*n))\*x^(1+m)\*(c\*x^n)^((1+m)/n))/(4\*(1+m)) + (E^((a\*Sqrt[-((1+m)^2/n^2)]\*n)/(1+m))\*x^(1+m)\*Log[x])/(2\*(c\*x^n)^((1+m)/n))

**Rubi [A]** time = 0.145609, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {4494, 4490}

$$\frac{x^{m+1} e^{\frac{a(m+1)}{n \sqrt{\frac{(m+1)^2}{n^2}}}} (cx^n)^{\frac{m+1}{n}}}{4(m+1)} + \frac{1}{2} x^{m+1} \log(x) e^{\frac{am \sqrt{\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{-\frac{m+1}{n}}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cos[a + Sqrt[-((1+m)^2/n^2)]\*Log[c\*x^n]], x]

[Out] (E^((a\*(1+m))/(Sqrt[-((1+m)^2/n^2)]\*n))\*x^(1+m)\*(c\*x^n)^((1+m)/n))/(4\*(1+m)) + (E^((a\*Sqrt[-((1+m)^2/n^2)]\*n)/(1+m))\*x^(1+m)\*Log[x])/(2\*(c\*x^n)^((1+m)/n))

#### Rule 4494

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[x^((m+1)/n-1)\*Cos[d\*(a+b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4490

Int[Cos[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/2^p, Int[ExpandIntegrand[(e\*x)^(m\*(E^((a\*b\*d^2\*p)/(m+1)))/x^((m+1)/p) + x^((m+1)/p)/E^((a\*b\*d^2\*p)/(m+1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m+1)^2, 0]

#### Rubi steps

$$\int x^m \cos\left(a + \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n)\right) dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \cos\left(a + \sqrt{-\frac{(1+m)^2}{n^2}} \log(x)\right) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \left(\frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}n}}}{x} + e^{\sqrt{-\frac{(1+m)^2}{n^2}}n} x^{-1+\frac{2(1+m)}{n}}\right) dx, x, cx^n\right)}{2n}$$

$$= \frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}}n}} x^{1+m} (cx^n)^{\frac{1+m}{n}}}{4(1+m)} + \frac{1}{2} e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}n}{1+m}} x^{1+m} (cx^n)^{-\frac{1+m}{n}} \log(x)$$

**Mathematica [F]** time = 0.252802, size = 0, normalized size = 0.

$$\int x^m \cos\left(a + \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n)\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m\*Cos[a + Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n]], x]

[Out] Integrate[x^m\*Cos[a + Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n]], x]

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int x^m \cos\left(a + \ln(cx^n) \sqrt{-\frac{(1+m)^2}{n^2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cos(a+ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)), x)

[Out] int(x^m\*cos(a+ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)), x)

**Maxima [A]** time = 1.23863, size = 111, normalized size = 1.1

$$\frac{c^{\frac{2m}{n} + \frac{2}{n}} x \cos(a) e^{\left(m \log(x) + \frac{m \log(x^n)}{n} + \frac{\log(x^n)}{n}\right)} + 2(m \cos(a) + \cos(a)) \log(x)}{4\left(c^{\frac{m}{n} + \frac{1}{n}} m + c^{\frac{m}{n} + \frac{1}{n}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)), x, algorithm="maxima")

[Out] 1/4\*(c^(2\*m/n + 2/n)\*x\*cos(a)\*e^(m\*log(x) + m\*log(x^n)/n + log(x^n)/n) + 2\*(m\*cos(a) + cos(a))\*log(x))/(c^(m/n + 1/n)\*m + c^(m/n + 1/n))

---

**Fricas [C]** time = 0.48708, size = 150, normalized size = 1.49

$$\frac{\left(x^2 x^{2m} + 2(m+1)e^{\left(\frac{2(ian-(m+1)\log(c))}{n}\right)} \log(x)\right) e^{\left(\frac{ian-(m+1)\log(c)}{n}\right)}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)),x, algorithm="fricas")

[Out] 1/4\*(x^2\*x^(2\*m) + 2\*(m + 1)\*e^(2\*(I\*a\*n - (m + 1)\*log(c))/n)\*log(x))\*e^(-(I\*a\*n - (m + 1)\*log(c))/n)/(m + 1)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \cos\left(a + \sqrt{-\frac{m^2}{n^2} - \frac{2m}{n^2} - \frac{1}{n^2}} \log(cx^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cos(a+ln(c\*x\*\*n)\*(-(1+m)\*\*2/n\*\*2)\*\*(1/2)),x)

[Out] Integral(x\*\*m\*cos(a + sqrt(-m\*\*2/n\*\*2 - 2\*m/n\*\*2 - 1/n\*\*2)\*log(c\*x\*\*n)), x)

---

**Giac [C]** time = 1.85404, size = 360, normalized size = 3.56

$$\frac{mn^2 xx^m e^{\left(i a - \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)} + mn^2 xx^m e^{\left(-i a + \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)} + n^2 xx^m e^{\left(i a - \frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)} + n xx^m |mn + n|}{2(m^2 n^2 + 2mn^2 - (mn + n)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2)),x, algorithm="giac")

[Out] 1/2\*(m\*n^2\*x\*x^m\*e^(I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + m\*n^2\*x\*x^m\*e^(-I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + n^2\*x\*x^m\*e^(I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + n\*x\*x^m\*abs(m\*n + n)\*e^(I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + n^2\*x\*x^m\*e^(-I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) - n\*x\*x^m\*abs(m\*n + n)\*e^(-I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2))/(m^2\*n^2 + 2\*m\*n^2 - (m\*n + n)^2 + n^2)

$$3.105 \quad \int \cos \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=62

$$\frac{1}{4} x e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{2} x e^{a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n}$$

[Out] (x\*(c\*x^n)^n^(-1))/(4\*E^(a\*Sqrt[-n^(-2)]\*n)) + (E^(a\*Sqrt[-n^(-2)]\*n)\*x\*Log[x])/(2\*(c\*x^n)^n^(-1))

**Rubi [A]** time = 0.0448417, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {4484, 4490}

$$\frac{1}{4} x e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{2} x e^{a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

[Out] (x\*(c\*x^n)^n^(-1))/(4\*E^(a\*Sqrt[-n^(-2)]\*n)) + (E^(a\*Sqrt[-n^(-2)]\*n)\*x\*Log[x])/(2\*(c\*x^n)^n^(-1))

**Rule 4484**

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

**Rule 4490**

Int[Cos[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/2^p, Int[ExpandIntegrand[(e\*x)^m\*(E^((a\*b\*d^2\*p)/(m + 1)))/x^((m + 1)/p) + x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

**Rubi steps**

$$\begin{aligned} \int \cos \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x (cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1+\frac{1}{n}} \cos \left( a + \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= \frac{(x (cx^n)^{-1/n}) \text{Subst} \left( \int \left( \frac{e^{a \sqrt{-\frac{1}{n^2}} n}}{x} + e^{-a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{2}{n}} \right) dx, x, cx^n \right)}{2n} \\ &= \frac{1}{4} e^{-a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{\frac{1}{n}} + \frac{1}{2} e^{a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{-1/n} \log(x) \end{aligned}$$

**Mathematica [F]** time = 0.0857273, size = 0, normalized size = 0.

$$\int \cos \left( a + \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[a + Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

[Out] Integrate[Cos[a + Sqrt[-n^(-2)]\*Log[c\*x^n]], x]

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int \cos\left(a + \ln(cx^n) \sqrt{-n^{-2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+ln(c\*x^n)\*(-1/n^2)^(1/2)), x)

[Out] int(cos(a+ln(c\*x^n)\*(-1/n^2)^(1/2)), x)

**Maxima [A]** time = 1.11668, size = 39, normalized size = 0.63

$$\frac{c^{\frac{2}{n}} x^2 \cos(a) + 2 \cos(a) \log(x)}{4 c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+log(c\*x^n)\*(-1/n^2)^(1/2)), x, algorithm="maxima")

[Out] 1/4\*(c^(2/n)\*x^2\*cos(a) + 2\*cos(a)\*log(x))/c^(1/n)

**Fricas [C]** time = 0.472266, size = 96, normalized size = 1.55

$$\frac{1}{4} \left( x^2 + 2 e^{\left(\frac{2(ian - \log(c))}{n}\right)} \log(x) \right) e^{\left(-\frac{ian - \log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+log(c\*x^n)\*(-1/n^2)^(1/2)), x, algorithm="fricas")

[Out] 1/4\*(x^2 + 2\*e^(2\*(I\*a\*n - log(c))/n)\*log(x))\*e^(-(I\*a\*n - log(c))/n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos\left(a + \sqrt{-\frac{1}{n^2}} \log(cx^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2)), x)



```
[Out] Integral(cos(a + sqrt(-1/n**2)*log(c*x**n)), x)
```

---

**Giac [A]** time = 1.28404, size = 1, normalized size = 0.02

+∞

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+log(c*x^n)*(-1/n^2)^(1/2)),x, algorithm="giac")
```

```
[Out] +Infinity
```

$$3.106 \quad \int x^m \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=117

$$\frac{x^{m+1} e^{-\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{\frac{m+1}{n}}}{8(m+1)} + \frac{1}{4} x^{m+1} \log(x) e^{\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{-\frac{m+1}{n}} + \frac{x^{m+1}}{2(m+1)}$$

[Out]  $x^{(1+m)/(2*(1+m))} + (x^{(1+m)}*(c*x^n)^{((1+m)/n)})/(8*E^{((2*a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))*(1+m)}) + (E^{((2*a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))*x^{(1+m)*Log[x]}})/(4*(c*x^n)^{((1+m)/n)})$

**Rubi [A]** time = 0.117212, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$ , Rules used = {4494, 4490}

$$\frac{x^{m+1} e^{-\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{\frac{m+1}{n}}}{8(m+1)} + \frac{1}{4} x^{m+1} \log(x) e^{\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}} (cx^n)^{-\frac{m+1}{n}} + \frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{m+1} \text{Cos}[a + (\text{Sqrt}[-((1+m)^2/n^2]]) * \text{Log}[c*x^n])/2]^2, x]$

[Out]  $x^{(1+m)/(2*(1+m))} + (x^{(1+m)}*(c*x^n)^{((1+m)/n)})/(8*E^{((2*a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))*(1+m)}) + (E^{((2*a*Sqrt[-((1+m)^2/n^2)]*n)/(1+m))*x^{(1+m)*Log[x]}})/(4*(c*x^n)^{((1+m)/n)})$

#### Rule 4494

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]]*(b_.)*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)*\text{Cos}[d*(a+b*\text{Log}[x])]}]^p, x], x, c*x^n], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\text{NeQ}[c, 1] \mid \mid \text{NeQ}[n, 1])$

#### Rule 4490

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[x]* (b_.)]*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/2^p, \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(E^{((a*b*d^2*p)/(m+1))}/x^{((m+1)/p)} + x^{((m+1)/p})/E^{((a*b*d^2*p)/(m+1))}]^p, x], x] /;$   $\text{FreeQ}\{a, b, d, e, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[b^2*d^2*p^2 + (m+1)^2, 0]$

#### Rubi steps

$$\int x^m \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx = \frac{\left( x^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{n}} \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(x) \right) dx, x \right)}{n}$$

$$= \frac{\left( x^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int \left( \frac{e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{n}}}{x} + 2x^{-1+\frac{1+m}{n}} + e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{n}} x^{-\frac{1+m}{n}} \right) dx, x \right)}{4n}$$

$$= \frac{x^{1+m}}{2(1+m)} + \frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{n}} x^{1+m} (cx^n)^{\frac{1+m}{n}}}{8(1+m)} + \frac{1}{4} e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}}{n}} x^{1+m} (cx^n)^{-\frac{1+m}{n}}$$

**Mathematica [F]** time = 0.340159, size = 0, normalized size = 0.

$$\int x^m \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^m\*Cos[a + (Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2]^2,x]

[Out] Integrate[x^m\*Cos[a + (Sqrt[-((1 + m)^2/n^2)]\*Log[c\*x^n])/2]^2, x]

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int x^m \left( \cos \left( a + \frac{\ln(cx^n)}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cos(a+1/2\*ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^2,x)

[Out] int(x^m\*cos(a+1/2\*ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^2,x)

**Maxima [A]** time = 1.2687, size = 232, normalized size = 1.98

$$\frac{4 \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{m}{n} + \frac{1}{n}} x x^m + c^{\frac{2m}{n} + \frac{2}{n}} x \cos(2a) e^{\left( m \log(x) + \frac{m \log(x^n)}{n} + \frac{\log(x^n)}{n} \right)} + 2 \left( \cos(2a)^3 + \cos(2a) \sin(2a)^2 \right)}{8 \left( \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{m}{n} + \frac{1}{n}} m + \left( \cos(2a)^2 + \sin(2a)^2 \right) c^{\frac{m}{n} + \frac{1}{n}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+1/2\*log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="maxima")

[Out] 1/8\*(4\*(cos(2\*a)^2 + sin(2\*a)^2)\*c^(m/n + 1/n)\*x\*x^m + c^(2\*m/n + 2/n)\*x\*cos(2\*a)\*e^(m\*log(x) + m\*log(x^n)/n + log(x^n)/n) + 2\*(cos(2\*a)^3 + cos(2\*a)\*

$$\frac{\sin(2a)^2 + (\cos(2a)^3 + \cos(2a)\sin(2a)^2)m \log(x)}{(\cos(2a)^2 + \sin(2a)^2)c^{(m/n + 1/n)m} + (\cos(2a)^2 + \sin(2a)^2)c^{(m/n + 1/n)}}$$

**Fricas [C]** time = 0.50012, size = 315, normalized size = 2.69

$$\frac{\left(2(m+1)e^{\left(-\frac{2((m+1)n \log(x) - 2ian + (m+1) \log(c))}{n}\right)} \log(x) + 4e^{\left(-\frac{(m+1)n \log(x) - 2ian + (m+1) \log(c)}{n}\right)} + 1\right) e^{\left(\frac{2((m+1)n \log(x) - 2ian + (m+1) \log(c))}{n} + \frac{2ian - (m+1) \log(c)}{n}\right)}}{8(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+1/2\*log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="fricas")

[Out] 1/8\*(2\*(m+1)\*e^(-2\*((m+1)\*n\*log(x) - 2\*I\*a\*n + (m+1)\*log(c))/n)\*log(x) + 4\*e^(-((m+1)\*n\*log(x) - 2\*I\*a\*n + (m+1)\*log(c))/n) + 1)\*e^(2\*((m+1)\*n\*log(x) - 2\*I\*a\*n + (m+1)\*log(c))/n + (2\*I\*a\*n - (m+1)\*log(c))/n)/(m+1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \cos^2 \left( a + \frac{\sqrt{-\frac{m^2}{n^2} - \frac{2m}{n^2} - \frac{1}{n^2}} \log(cx^n)}{2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cos(a+1/2\*ln(c\*x\*\*n)\*(-(1+m)\*\*2/n\*\*2)\*\*(1/2))\*\*2,x)

[Out] Integral(x\*\*m\*cos(a + sqrt(-m\*\*2/n\*\*2 - 2\*m/n\*\*2 - 1/n\*\*2)\*log(c\*x\*\*n)/2)\*\*2, x)

**Giac [C]** time = 2.71286, size = 672, normalized size = 5.74

$$\frac{m^2 n^2 x x^m e^{\left(2i a - \frac{n|mn+n| \log(x) + |mn+n| \log(c)}{n^2}\right)} + m^2 n^2 x x^m e^{\left(-2i a + \frac{n|mn+n| \log(x) + |mn+n| \log(c)}{n^2}\right)} + 2 m^2 n^2 x x^m + 2 m n^2 x x^m e^{\left(2i a - \frac{n|mn+n| \log(x) + |mn+n| \log(c)}{n^2}\right)}}{n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+1/2\*log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^2,x, algorithm="giac")

[Out] 1/4\*(m^2\*n^2\*x\*x^m\*e^(2\*I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + m^2\*n^2\*x\*x^m\*e^(-2\*I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 2\*m^2\*n^2\*x\*x^m + 2\*m\*n^2\*x\*x^m\*e^(2\*I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + m\*n\*x\*x^m\*abs(m\*n + n)\*e^(2\*I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 2\*m\*n^2\*x\*x^m\*e^(-2\*I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) - m\*n\*x\*x^m\*abs(m\*n + n)\*e^(-2\*I\*a + (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 4\*m\*n^2\*x\*x^m + n^2\*x\*x^m\*e^(2\*I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + n\*x\*x^m\*abs(m\*n + n)\*e^(2\*I\*a - (n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2)

$$\begin{aligned}
& (c)/n^2) + n^2*x*x^m*e^{(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log \\
& (c))/n^2) - n*x*x^m*abs(m*n + n)*e^{(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m \\
& *n + n)*log(c))/n^2) - 2*(m*n + n)^2*x*x^m + 2*n^2*x*x^m)/(m^3*n^2 + 3*m^2* \\
& n^2 - (m*n + n)^2*m + 3*m*n^2 - (m*n + n)^2 + n^2)
\end{aligned}$$

$$3.107 \quad \int \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=68

$$\frac{1}{8} x e^{-2a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{4} x e^{2a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n} + \frac{x}{2}$$

[Out]  $x/2 + (x*(c*x^n)^n)^{-1}/(8*E^{(2*a*Sqrt[-n^(-2)]*n)}) + (E^{(2*a*Sqrt[-n^(-2)]*n)}*x*\text{Log}[x])/(4*(c*x^n)^n)^{-1}$

**Rubi [A]** time = 0.0557935, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {4484, 4490}

$$\frac{1}{8} x e^{-2a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{4} x e^{2a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[a + (\text{Sqrt}[-n^(-2)])*\text{Log}[c*x^n])/2]^2, x]$

[Out]  $x/2 + (x*(c*x^n)^n)^{-1}/(8*E^{(2*a*Sqrt[-n^(-2)]*n)}) + (E^{(2*a*Sqrt[-n^(-2)]*n)}*x*\text{Log}[x])/(4*(c*x^n)^n)^{-1}$

#### Rule 4484

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[(c_.)*(x_.)^{n_.}]]*(b_.)*(d_.)]^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[x/(n*(c*x^n)^{1/n}), \text{Subst}[\text{Int}[x^{1/n - 1}*\text{Cos}[d*(a + b*\text{Log}[x])]^p, x], x, c*x^n], x] /;$  FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4490

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[x_]* (b_.)]*(d_.)]^{(p_.)*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/2^p, \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(E^{(a*b*d^2*p)/(m+1)})/x^{(m+1)/p} + x^{(m+1)/p}/E^{(a*b*d^2*p)/(m+1)}]^p, x], x] /;$  FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m+1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x(cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1+\frac{1}{n}} \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= \frac{(x(cx^n)^{-1/n}) \text{Subst} \left( \int \left( \frac{e^{2a \sqrt{-\frac{1}{n^2}} n}}{x} + 2x^{-1+\frac{1}{n}} + e^{-2a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{2}{n}} \right) dx, x, cx^n \right)}{4n} \\ &= \frac{x}{2} + \frac{1}{8} e^{-2a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{\frac{1}{n}} + \frac{1}{4} e^{2a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{-1/n} \log(x) \end{aligned}$$

**Mathematica [F]** time = 0.100558, size = 0, normalized size = 0.

$$\int \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[a + (Sqrt[-n^(-2)])\*Log[c\*x^n])/2]^2, x]

[Out] Integrate[Cos[a + (Sqrt[-n^(-2)])\*Log[c\*x^n])/2]^2, x]

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \left( \cos \left( a + \frac{\ln(cx^n)}{2} \sqrt{-n^{-2}} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+1/2\*ln(c\*x^n)\*(-1/n^2)^(1/2))^2, x)

[Out] int(cos(a+1/2\*ln(c\*x^n)\*(-1/n^2)^(1/2))^2, x)

**Maxima [A]** time = 1.29058, size = 55, normalized size = 0.81

$$\frac{c^{\frac{2}{n}} x^2 \cos(2a) + 4 c^{\left(\frac{1}{n}\right)} x + 2 \cos(2a) \log(x)}{8 c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+1/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2, x, algorithm="maxima")

[Out] 1/8\*(c^(2/n)\*x^2\*cos(2\*a) + 4\*c^(1/n)\*x + 2\*cos(2\*a)\*log(x))/c^(1/n)

**Fricas [C]** time = 0.471724, size = 143, normalized size = 2.1

$$\frac{1}{8} \left( x^2 + 4 x e^{\left(\frac{2i a n - \log(c)}{n}\right)} + 2 e^{\left(\frac{2(2i a n - \log(c))}{n}\right)} \log(x) \right) e^{\left(-\frac{2i a n - \log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+1/2\*log(c\*x^n)\*(-1/n^2)^(1/2))^2, x, algorithm="fricas")

[Out] 1/8\*(x^2 + 4\*x\*e^((2\*I\*a\*n - log(c))/n) + 2\*e^(2\*(2\*I\*a\*n - log(c))/n)\*log(x))\*e^(-(2\*I\*a\*n - log(c))/n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos^2 \left( a + \frac{\sqrt{-\frac{1}{n^2}} \log(cx^n)}{2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/2*ln(c*x**n)*(-1/n**2)**(1/2))**2,x)
```

```
[Out] Integral(cos(a + sqrt(-1/n**2)*log(c*x**n)/2)**2, x)
```

---

**Giac [A]** time = 1.61904, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+1/2*log(c*x^n)*(-1/n^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] +Infinity
```



$$3.108 \quad \int x^m \cos^3 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=226

$$\frac{4n\sqrt{-\frac{(m+1)^2}{n^2}}x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log(cx^n)\right)}{5(m+1)^2} - \frac{4x^{m+1}\cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log(cx^n)\right)}{5(m+1)} + \frac{8x^{m+1}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log(cx^n)\right)}{5(m+1)}$$

[Out] (8\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2])/(5\*(1+m)) - (4\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2]^3)/(5\*(1+m)) + (4\*Sqrt[-((1+m)^2/n^2)]\*n\*x^(1+m)\*Sin[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2])/(5\*(1+m)^2) - (6\*Sqrt[-((1+m)^2/n^2)]\*n\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2]^2\*Sin[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2])/(5\*(1+m)^2)

**Rubi [A]** time = 0.0817879, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$ , Rules used = {4488, 4486}

$$\frac{4n\sqrt{-\frac{(m+1)^2}{n^2}}x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log(cx^n)\right)}{5(m+1)^2} - \frac{4x^{m+1}\cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log(cx^n)\right)}{5(m+1)} + \frac{8x^{m+1}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log(cx^n)\right)}{5(m+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cos[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2]^3,x]

[Out] (8\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2])/(5\*(1+m)) - (4\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2]^3)/(5\*(1+m)) + (4\*Sqrt[-((1+m)^2/n^2)]\*n\*x^(1+m)\*Sin[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2])/(5\*(1+m)^2) - (6\*Sqrt[-((1+m)^2/n^2)]\*n\*x^(1+m)\*Cos[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2]^2\*Sin[a+(Sqrt[-((1+m)^2/n^2])\*Log[c\*x^n])/2])/(5\*(1+m)^2)

#### Rule 4488

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((m+1)\*(e\*x)^(m+1)\*Cos[d\*(a+b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p-1))/(b^2\*d^2\*n^2\*p^2 + (m+1)^2), Int[(e\*x)^m\*Cos[d\*(a+b\*Log[c\*x^n])]^(p-2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m+1)\*Sin[d\*(a+b\*Log[c\*x^n])]\*Cos[d\*(a+b\*Log[c\*x^n])]^(p-1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m+1)^2, 0]

#### Rule 4486

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((m+1)\*(e\*x)^(m+1)\*Cos[d\*(a+b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m+1)^2), x] + Simp[(b\*d\*n\*(e\*x)^(m+1)\*Sin[d\*(a+b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m+1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2 + (m+1)^2, 0]

#### Rubi steps

$$\int x^m \cos^3 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right) dx = -\frac{4x^{1+m} \cos^3 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right)}{5(1+m)} - \frac{6\sqrt{-\frac{(1+m)^2}{n^2}} nx^{1+m} \cos^2 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right)}{5(1+m)}$$

$$= \frac{8x^{1+m} \cos \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right)}{5(1+m)} - \frac{4x^{1+m} \cos^3 \left( a + \frac{1}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \log(cx^n) \right)}{5(1+m)}$$

**Mathematica [A]** time = 1.35216, size = 158, normalized size = 0.7

$$\frac{x^{m+1} \left( n \sqrt{-\frac{(m+1)^2}{n^2}} \left( 5 \sin \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right) - 3 \sin \left( 3a + \frac{3}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right) \right) + 10(m+1) \cos \left( a + \frac{1}{2} \sqrt{-\frac{(m+1)^2}{n^2}} \log(cx^n) \right) \right)}{10(m+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^m \* Cos[a + (Sqrt[-((1 + m)^2/n^2)]) \* Log[c\*x^n])/2]^3, x]

[Out] (x^(1 + m) \* (10 \* (1 + m) \* Cos[a + (Sqrt[-((1 + m)^2/n^2)]) \* Log[c\*x^n])/2] - 2 \* (1 + m) \* Cos[3\*a + (3\*Sqrt[-((1 + m)^2/n^2)]) \* Log[c\*x^n])/2] + Sqrt[-((1 + m)^2/n^2)] \* n \* (5 \* Sin[a + (Sqrt[-((1 + m)^2/n^2)]) \* Log[c\*x^n])/2] - 3 \* Sin[3\*a + (3\*Sqrt[-((1 + m)^2/n^2)]) \* Log[c\*x^n])/2])) / (10 \* (1 + m)^2)

**Maple [F]** time = 0.094, size = 0, normalized size = 0.

$$\int x^m \left( \cos \left( a + \frac{\ln(cx^n)}{2} \sqrt{-\frac{(1+m)^2}{n^2}} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cos(a+1/2\*ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^3,x)

[Out] int(x^m\*cos(a+1/2\*ln(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^3,x)

**Maxima [A]** time = 1.32917, size = 263, normalized size = 1.16

$$\frac{\left( c^{\frac{3m}{n} + \frac{3}{n}} x \cos(3a) e^{\left( m \log(x) + \frac{3m \log(x^n)}{n} + \frac{3 \log(x^n)}{n} \right)} + 5 c^{\frac{2m}{n} + \frac{2}{n}} x \cos(a) e^{\left( m \log(x) + \frac{2m \log(x^n)}{n} + \frac{2 \log(x^n)}{n} \right)} + 15 c^{\frac{m}{n} + \frac{1}{n}} x \cos(a) e^{\left( m \log(x) + \frac{m \log(x^n)}{n} + \frac{\log(x^n)}{n} \right)} \right)}{20 \left( c^{\frac{3m}{2n} + \frac{3}{2n}} m + c^{\frac{3m}{2n} + \frac{3}{2n}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+1/2\*log(c\*x^n)\*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="maxima")

[Out] 1/20\*(c^(3\*m/n + 3/n)\*x\*cos(3\*a)\*e^(m\*log(x) + 3\*m\*log(x^n)/n + 3\*log(x^n)/n) + 5\*c^(2\*m/n + 2/n)\*x\*cos(a)\*e^(m\*log(x) + 2\*m\*log(x^n)/n + 2\*log(x^n)/n) + 15\*c^(m/n + 1/n)\*x\*cos(a)\*e^(m\*log(x) + m\*log(x^n)/n + log(x^n)/n) - 5\*x\*x^m\*cos(3\*a)\*e^(-3/2\*m\*log(x^n)/n - 3/2\*log(x^n)/n)/(c^(3/2\*m/n + 3/2/n))

$$*m + c^{(3/2*m/n + 3/2/n)}$$

**Fricas [C]** time = 0.500286, size = 379, normalized size = 1.68

$$\frac{\left(5 e^{\left(-\frac{(m+1)n \log(x)-2i a n+(m+1) \log(c)}{n}\right)}+15 e^{\left(-\frac{2((m+1)n \log(x)-2i a n+(m+1) \log(c))}{n}\right)}-5 e^{\left(-\frac{3((m+1)n \log(x)-2i a n+(m+1) \log(c))}{n}\right)}+1\right) e^{\left(\frac{5((m+1)n \log(x)-2i a n}{2 n}\right)}}{20(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+1/2\*log(c\*x^n))\*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="fricas")

[Out] 1/20\*(5\*e^(-((m + 1)\*n\*log(x) - 2\*I\*a\*n + (m + 1)\*log(c))/n) + 15\*e^(-2\*((m + 1)\*n\*log(x) - 2\*I\*a\*n + (m + 1)\*log(c))/n) - 5\*e^(-3\*((m + 1)\*n\*log(x) - 2\*I\*a\*n + (m + 1)\*log(c))/n) + 1)\*e^(5/2\*((m + 1)\*n\*log(x) - 2\*I\*a\*n + (m + 1)\*log(c))/n + (2\*I\*a\*n - (m + 1)\*log(c))/n)/(m + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \cos^3 \left( a + \frac{\sqrt{-\frac{m^2}{n^2} - \frac{2m}{n^2} - \frac{1}{n^2}} \log(cx^n)}{2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cos(a+1/2\*ln(c\*x\*\*n))\*(-(1+m)\*\*2/n\*\*2)\*\*(1/2))\*\*3,x)

[Out] Integral(x\*\*m\*cos(a + sqrt(-m\*\*2/n\*\*2 - 2\*m/n\*\*2 - 1/n\*\*2)\*log(c\*x\*\*n)/2)\*\*3, x)

**Giac [C]** time = 4.3396, size = 2525, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+1/2\*log(c\*x^n))\*(-(1+m)^2/n^2)^(1/2))^3,x, algorithm="giac")

[Out] 1/4\*(8\*m^3\*n^4\*x\*x^m\*e^(3\*I\*a - 3/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 24\*m^3\*n^4\*x\*x^m\*e^(I\*a - 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 24\*m^3\*n^4\*x\*x^m\*e^(-I\*a + 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 8\*m^3\*n^4\*x\*x^m\*e^(-3\*I\*a + 3/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 24\*m^2\*n^4\*x\*x^m\*e^(3\*I\*a - 3/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 12\*m^2\*n^3\*x\*x^m\*abs(m\*n + n)\*e^(3\*I\*a - 3/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 72\*m^2\*n^4\*x\*x^m\*e^(I\*a - 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 12\*m^2\*n^3\*x\*x^m\*abs(m\*n + n)\*e^(I\*a - 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 72\*m^2\*n^4\*x\*x^m\*e^(-I\*a + 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) - 12\*m^2\*n^3\*x\*x^m\*abs(m\*n + n)\*e^(-I\*a + 1/2\*(n\*abs(m\*n + n)\*log(x) + abs(m\*n + n)\*log(c))/n^2) + 24\*m^2\*n^4\*x\*x^m

$$\begin{aligned}
& *e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 12*m^{\wedge}2*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 2*(m*n + n)^{\wedge}2*m*n^{\wedge}2*x*x^{\wedge}m*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 24*m*n^{\wedge}4*x*x^{\wedge}m*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} + 24*m*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2)} - 54*(m*n + n)^{\wedge}2*m*n^{\wedge}2*x*x^{\wedge}m*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 72*m*n^{\wedge}4*x*x^{\wedge}m*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 24*m*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 54*(m*n + n)^{\wedge}2*m*n^{\wedge}2*x*x^{\wedge}m*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 72*m*n^{\wedge}4*x*x^{\wedge}m*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 24*m*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 2*(m*n + n)^{\wedge}2*m*n^{\wedge}2*x*x^{\wedge}m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 24*m*n^{\wedge}4*x*x^{\wedge}m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 24*m*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 2*(m*n + n)^{\wedge}2*n^{\wedge}2*x*x^{\wedge}m*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 8*n^{\wedge}4*x*x^{\wedge}m*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 3*(m*n + n)^{\wedge}2*n*x*x^{\wedge}m*abs(m*n + n)*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 12*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 54*(m*n + n)^{\wedge}2*n^{\wedge}2*x*x^{\wedge}m*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 24*n^{\wedge}4*x*x^{\wedge}m*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 27*(m*n + n)^{\wedge}2*n*x*x^{\wedge}m*abs(m*n + n)*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 12*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 54*(m*n + n)^{\wedge}2*n^{\wedge}2*x*x^{\wedge}m*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 24*n^{\wedge}4*x*x^{\wedge}m*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 27*(m*n + n)^{\wedge}2*n*x*x^{\wedge}m*abs(m*n + n)*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 12*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 2*(m*n + n)^{\wedge}2*n^{\wedge}2*x*x^{\wedge}m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 8*n^{\wedge}4*x*x^{\wedge}m*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} + 3*(m*n + n)^{\wedge}2*n*x*x^{\wedge}m*abs(m*n + n)*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)} - 12*n^{\wedge}3*x*x^{\wedge}m*abs(m*n + n)*e^{(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)))/n^2)})/(16*m^{\wedge}4*n^{\wedge}4 + 64*m^{\wedge}3*n^{\wedge}4 - 40*(m*n + n)^{\wedge}2*m^{\wedge}2*n^{\wedge}2 + 96*m^{\wedge}2*n^{\wedge}4 - 80*(m*n + n)^{\wedge}2*m*n^{\wedge}2 + 64*m*n^{\wedge}4 + 9*(m*n + n)^{\wedge}4 - 40*(m*n + n)^{\wedge}2*n^{\wedge}2 + 16*n^{\wedge}4)
\end{aligned}$$

$$3.109 \quad \int \cos^3 \left( a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

**Optimal.** Leaf size=128

$$\frac{9}{16} x e^{a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{1}{3}/n} + \frac{9}{32} x e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{3}/n} + \frac{1}{16} x e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{8} x e^{3a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n}$$

[Out] (9\*E^(a\*Sqrt[-n^(-2)]\*n)\*x)/(16\*(c\*x^n)^(1/(3\*n))) + (9\*x\*(c\*x^n)^(1/(3\*n)))/(32\*E^(a\*Sqrt[-n^(-2)]\*n)) + (x\*(c\*x^n)^n^(-1))/(16\*E^(3\*a\*Sqrt[-n^(-2)]\*n)) + (E^(3\*a\*Sqrt[-n^(-2)]\*n)\*x\*Log[x])/(8\*(c\*x^n)^n^(-1))

**Rubi [A]** time = 0.0958812, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {4484, 4490}

$$\frac{9}{16} x e^{a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{-\frac{1}{3}/n} + \frac{9}{32} x e^{-a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{3}/n} + \frac{1}{16} x e^{-3a \sqrt{-\frac{1}{n^2}} n} (cx^n)^{\frac{1}{n}} + \frac{1}{8} x e^{3a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + (Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3, x]

[Out] (9\*E^(a\*Sqrt[-n^(-2)]\*n)\*x)/(16\*(c\*x^n)^(1/(3\*n))) + (9\*x\*(c\*x^n)^(1/(3\*n)))/(32\*E^(a\*Sqrt[-n^(-2)]\*n)) + (x\*(c\*x^n)^n^(-1))/(16\*E^(3\*a\*Sqrt[-n^(-2)]\*n)) + (E^(3\*a\*Sqrt[-n^(-2)]\*n)\*x\*Log[x])/(8\*(c\*x^n)^n^(-1))

#### Rule 4484

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4490

Int[Cos[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/2^p, Int[ExpandIntegrand[(e\*x)^(m\*(E^((a\*b\*d^2\*p)/(m + 1)))/x^((m + 1)/p) + x^((m + 1)/p)/E^((a\*b\*d^2\*p)/(m + 1))]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2\*d^2\*p^2 + (m + 1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int \cos^3 \left( a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx &= \frac{(x(cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1+\frac{1}{n}} \cos^3 \left( a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \log(x) \right) dx, x, cx^n \right)}{n} \\ &= \frac{(x(cx^n)^{-1/n}) \text{Subst} \left( \int \left( \frac{e^{3a \sqrt{-\frac{1}{n^2}} n}}{x} + 3e^{a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{2}{3n}} + 3e^{-a \sqrt{-\frac{1}{n^2}} n} x^{-1+\frac{4}{3n}} + e^{-3a \sqrt{-\frac{1}{n^2}} n} \right) dx, x, cx^n \right)}{8n} \\ &= \frac{9}{16} e^{a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{-\frac{1}{3}/n} + \frac{9}{32} e^{-a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{\frac{1}{3}/n} + \frac{1}{16} e^{-3a \sqrt{-\frac{1}{n^2}} n} x (cx^n)^{\frac{1}{n}} + \frac{1}{8} e^{3a \sqrt{-\frac{1}{n^2}} n} \log(x) (cx^n)^{-1/n} \end{aligned}$$

**Mathematica [F]** time = 0.138508, size = 0, normalized size = 0.

$$\int \cos^3 \left( a + \frac{1}{3} \sqrt{-\frac{1}{n^2}} \log(cx^n) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[a + (Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3,x]

[Out] Integrate[Cos[a + (Sqrt[-n^(-2)]\*Log[c\*x^n])/3]^3, x]

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int \left( \cos \left( a + \frac{\ln(cx^n)}{3} \sqrt{-n^{-2}} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+1/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3,x)

[Out] int(cos(a+1/3\*ln(c\*x^n)\*(-1/n^2)^(1/2))^3,x)

**Maxima [A]** time = 1.14431, size = 143, normalized size = 1.12

$$\frac{9c^{\frac{5}{3n}}x(x^n)^{\frac{2}{3n}}\cos(a) + 4c^{\frac{1}{3n}}(x^n)^{\frac{1}{3n}}\cos(3a)\log(x) + 18c^{\left(\frac{1}{n}\right)}x\cos(a) + 2c^{\frac{7}{3n}}\cos(3a)e^{\left(\frac{\log(x^n)}{3n} + 2\log(x)\right)}}{32c^{\frac{4}{3n}}(x^n)^{\frac{1}{3n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+1/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3,x, algorithm="maxima")

[Out] 1/32\*(9\*c^(5/3/n)\*x\*(x^n)^(2/3/n)\*cos(a) + 4\*c^(1/3/n)\*(x^n)^(1/3/n)\*cos(3\*a)\*log(x) + 18\*c^(1/n)\*x\*cos(a) + 2\*c^(7/3/n)\*cos(3\*a)\*e^(1/3\*log(x^n)/n + 2\*log(x)))/(c^(4/3/n)\*(x^n)^(1/3/n))

**Fricas [C]** time = 0.478143, size = 227, normalized size = 1.77

$$\frac{1}{32} \left( 9x^{\frac{4}{3}} e^{\left(\frac{2(3ian-\log(c))}{3n}\right)} + 2x^2 + 12e^{\left(\frac{2(3ian-\log(c))}{n}\right)} \log\left(x^{\frac{1}{3}}\right) + 18x^{\frac{2}{3}} e^{\left(\frac{4(3ian-\log(c))}{3n}\right)} \right) e^{\left(-\frac{3ian-\log(c)}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+1/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3,x, algorithm="fricas")

[Out] 1/32\*(9\*x^(4/3)\*e^(2/3\*(3\*I\*a\*n - log(c))/n) + 2\*x^2 + 12\*e^(2\*(3\*I\*a\*n - log(c))/n)\*log(x^(1/3)) + 18\*x^(2/3)\*e^(4/3\*(3\*I\*a\*n - log(c))/n))\*e^(-(3\*I\*a\*n - log(c))/n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos^3 \left( a + \frac{\sqrt{-\frac{1}{n^2}} \log(cx^n)}{3} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+1/3\*ln(c\*x\*\*n)\*(-1/n\*\*2)\*\*(1/2))\*\*3, x)

[Out] Integral(cos(a + sqrt(-1/n\*\*2)\*log(c\*x\*\*n)/3)\*\*3, x)

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+1/3\*log(c\*x^n)\*(-1/n^2)^(1/2))^3, x, algorithm="giac")

[Out] Exception raised: NotImplementedError

### 3.110 $\int \sqrt{\cos(a + b \log(cx^n))} dx$

**Optimal.** Leaf size=110

$$\frac{{}_2F_1\left(-\frac{1}{2}, -\frac{bn+2i}{4bn}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right) \sqrt{\cos(a + b \log(cx^n))}}{(2 - ibn) \sqrt{1 + e^{2ia}(cx^n)^{2ib}}}$$

[Out] (2\*x\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*Hypergeometric2F1[-1/2, -(2\*I + b\*n)/(4\*b\*n), (3 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/((2 - I\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

**Rubi [A]** time = 0.0715192, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4484, 4492, 364}

$$\frac{{}_2F_1\left(-\frac{1}{2}, -\frac{bn+2i}{4bn}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right) \sqrt{\cos(a + b \log(cx^n))}}{(2 - ibn) \sqrt{1 + e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*Hypergeometric2F1[-1/2, -(2\*I + b\*n)/(4\*b\*n), (3 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/((2 - I\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

#### Rule 4484

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\int \sqrt{\cos(a + b \log(cx^n))} dx = \frac{(x (cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1+\frac{1}{n}} \sqrt{\cos(a + b \log(x))} dx, x, cx^n \right)}{n}$$

$$= \frac{\left( x (cx^n)^{\frac{ib}{2} - \frac{1}{n}} \sqrt{\cos(a + b \log(cx^n))} \right) \text{Subst} \left( \int x^{-1-\frac{ib}{2} + \frac{1}{n}} \sqrt{1 + e^{2ia} x^{2ib}} dx, x, cx^n \right)}{n \sqrt{1 + e^{2ia} (cx^n)^{2ib}}}$$

$$= \frac{2x \sqrt{\cos(a + b \log(cx^n))} {}_2F_1 \left( -\frac{1}{2}, -\frac{2i+bn}{4bn}; \frac{1}{4} \left( 3 - \frac{2i}{bn} \right); -e^{2ia} (cx^n)^{2ib} \right)}{(2 - ibn) \sqrt{1 + e^{2ia} (cx^n)^{2ib}}}$$

**Mathematica [B]** time = 3.4273, size = 377, normalized size = 3.43

$$\frac{2x \sqrt{\cos(a + b \log(cx^n))} \cos(a + b \log(cx^n) - bn \log(x))}{bn \sin(a + b \log(cx^n) - bn \log(x)) - 2 \cos(a + b \log(cx^n) - bn \log(x))} + \frac{2e^{ia} b n x (cx^n)^{ib} \sqrt{2 + 2e^{2ia} (cx^n)^{2ib}}}{(bn + 2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Cos[a + b\*Log[c\*x^n]]], x]

[Out] (2\*b\*E^(I\*a)\*n\*x\*(c\*x^n)^(I\*b)\*Sqrt[2 + 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*((2\*I + b\*n)\*x^((2\*I)\*b\*n)\*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), -E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)] + (-2\*I + 3\*b\*n)\*Hypergeometric2F1[1/2, -(2\*I + b\*n)/(4\*b\*n), 3/4 - (I/2)/(b\*n), -E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]))/((2\*I + b\*n)\*(-2\*I + 3\*b\*n)\*Sqrt[1/(E^(I\*a)\*(c\*x^n)^(I\*b)) + E^(I\*a)\*(c\*x^n)^(I\*b)]\*((-2 + I\*b\*n)\*x^((2\*I)\*b\*n) - I\*E^((2\*I)\*a)\*(-2\*I + b\*n)\*(c\*x^n)^((2\*I)\*b)) - (2\*x\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]])/(-2\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]] + b\*n\*Sin[a - b\*n\*Log[x] + b\*Log[c\*x^n]]))

**Maple [F]** time = 0.205, size = 0, normalized size = 0.

$$\int \sqrt{\cos(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(cos(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(cos(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*ln(c\*x\*\*n))\*\*(1/2),x)

[Out] Integral(sqrt(cos(a + b\*log(c\*x\*\*n))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(b\*log(c\*x^n) + a)), x)

$$3.111 \quad \int \frac{\sqrt{\cos(a+b \log(cx^n))}}{x} dx$$

**Optimal.** Leaf size=24

$$\frac{2E\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{bn}$$

[Out] (2\*EllipticE[(a + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Rubi [A]** time = 0.0269538, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2639}

$$\frac{2E\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[a + b\*Log[c\*x^n]]]/x,x]

[Out] (2\*EllipticE[(a + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Rule 2639**

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt{\cos(a+b \log(cx^n))}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{\cos(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2E\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0932007, size = 24, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[a + b\*Log[c\*x^n]]]/x,x]

[Out] (2\*EllipticE[(a + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Maple [B]** time = 2.044, size = 181, normalized size = 7.5

$$2 \frac{\sqrt{\left(2 \left(\cos\left(\frac{a}{2} + \frac{1}{2} b \ln\left(cx^n\right)\right)\right)^2 - 1\right) \left(\sin\left(\frac{a}{2} + \frac{1}{2} b \ln\left(cx^n\right)\right)\right)^2 \sqrt{\left(\sin\left(\frac{a}{2} + \frac{1}{2} b \ln\left(cx^n\right)\right)\right)^2} \sqrt{-2 \left(\cos\left(\frac{a}{2} + \frac{1}{2} b \ln\left(cx^n\right)\right)\right)^2}}{n \sqrt{-2 \left(\sin\left(\frac{a}{2} + \frac{1}{2} b \ln\left(cx^n\right)\right)\right)^4 + \left(\sin\left(\frac{a}{2} + \frac{1}{2} b \ln\left(cx^n\right)\right)\right)^2 \sin\left(\frac{a}{2} + \frac{1}{2} b \ln\left(cx^n\right)\right) \sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a+b*ln(c*x^n))^(1/2)/x,x)`

[Out] 
$$\frac{2}{n} \left( (2 \cos(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)) - 1) \sin(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)) \right)^{\frac{1}{2}} \left( \sin(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)) \right)^{\frac{1}{2}} \left( -2 \cos(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)) + 1 \right)^{\frac{1}{2}} \text{EllipticE}(\cos(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)), 2^{\frac{1}{2}}) / (-2 \sin(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)))^4 + \sin(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)) \right)^{\frac{1}{2}} / \sin(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)) / (2 \cos(\frac{1}{2}a + \frac{1}{2}b \ln(cx^n)) - 1)^{\frac{1}{2}} / b$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt(cos(b*log(c*x^n) + a))/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(b \log(cx^n) + a)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")`

[Out] `integral(sqrt(cos(b*log(c*x^n) + a))/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(a + b \log(cx^n))}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a+b*ln(c*x**n))**(1/2)/x,x)`

[Out] `Integral(sqrt(cos(a + b*log(c*x**n)))/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*log(c*x^n) + a))/x, x)
```

### 3.112 $\int \cos^{\frac{3}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=109

$$\frac{2x \operatorname{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right), \frac{1}{4}\left(1 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right) \cos^{\frac{3}{2}}(a + b \log(cx^n))}{(2 - 3ibn)(1 + e^{2ia}(cx^n)^{2ib})^{3/2}}$$

[Out] (2\*x\*Cos[a + b\*Log[c\*x^n]]^(3/2)\*Hypergeometric2F1[-3/2, (-3 - (2\*I)/(b\*n))/4, (1 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 - (3\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(3/2))

**Rubi [A]** time = 0.0678703, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4484, 4492, 364}

$$\frac{2x {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right); \frac{1}{4}\left(1 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right) \cos^{\frac{3}{2}}(a + b \log(cx^n))}{(2 - 3ibn)(1 + e^{2ia}(cx^n)^{2ib})^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x\*Cos[a + b\*Log[c\*x^n]]^(3/2)\*Hypergeometric2F1[-3/2, (-3 - (2\*I)/(b\*n))/4, (1 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 - (3\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(3/2))

#### Rule 4484

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \cos^{\frac{3}{2}}(a + b \log(cx^n)) dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \cos^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{\frac{3ib}{2}-\frac{1}{n}} \cos^{\frac{3}{2}}(a + b \log(cx^n))\right) \operatorname{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1}{n}} (1 + e^{2ia}x^{2ib})^{3/2} dx, x, cx^n\right)}{n(1 + e^{2ia}(cx^n)^{2ib})^{3/2}}$$

$$= \frac{2x \cos^{\frac{3}{2}}(a + b \log(cx^n)) {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right); \frac{1}{4}\left(1 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2 - 3ibn)(1 + e^{2ia}(cx^n)^{2ib})^{3/2}}$$

**Mathematica [A]** time = 1.66796, size = 163, normalized size = 1.5

$$\frac{x \left( (bn - 2i) (3bn \sin(2(a + b \log(cx^n))) + 4 \cos^2(a + b \log(cx^n))) - 6ib^2n^2 (1 + e^{2ia}(cx^n)^{2ib}) \operatorname{Hypergeometric2F1}\left(\dots\right) \right)}{(bn - 2i) (9b^2n^2 + 4) \sqrt{\cos(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (x\*((-6\*I)\*b^2\*n^2\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, 3/4 - (I/2)/(b\*n), 5/4 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + (-2\*I + b\*n)\*(4\*Cos[a + b\*Log[c\*x^n]]^2 + 3\*b\*n\*Sin[2\*(a + b\*Log[c\*x^n])])))/((-2\*I + b\*n)\*(4 + 9\*b^2\*n^2)\*Sqrt[Cos[a + b\*Log[c\*x^n]])]

**Maple [F]** time = 0.162, size = 0, normalized size = 0.

$$\int (\cos(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(cos(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^(3/2), x)
```



$$3.113 \quad \int \frac{\cos^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=63

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(a+b \log(cx^n)), 2\right)}{3bn} + \frac{2 \sin(a+b \log(cx^n)) \sqrt{\cos(a+b \log(cx^n))}}{3bn}$$

[Out] (2\*EllipticF[(a + b\*Log[c\*x^n])/2, 2])/(3\*b\*n) + (2\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*Sin[a + b\*Log[c\*x^n]])/(3\*b\*n)

**Rubi [A]** time = 0.0426202, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2635, 2641}

$$\frac{2F\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{3bn} + \frac{2 \sin(a+b \log(cx^n)) \sqrt{\cos(a+b \log(cx^n))}}{3bn}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^(3/2)/x,x]

[Out] (2\*EllipticF[(a + b\*Log[c\*x^n])/2, 2])/(3\*b\*n) + (2\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*Sin[a + b\*Log[c\*x^n]])/(3\*b\*n)

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x] \* (b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^3(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cos^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2\sqrt{\cos(a+b \log(cx^n))} \sin(a+b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos(a+bx)}} dx, x, \log(cx^n)\right)}{3n} \\ &= \frac{2F\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{3bn} + \frac{2\sqrt{\cos(a+b \log(cx^n))} \sin(a+b \log(cx^n))}{3bn} \end{aligned}$$

**Mathematica [A]** time = 0.112796, size = 54, normalized size = 0.86

$$\frac{2\left(\text{EllipticF}\left(\frac{1}{2}(a+b \log(cx^n)), 2\right) + \sin(a+b \log(cx^n)) \sqrt{\cos(a+b \log(cx^n))}\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^(3/2)/x,x]

[Out] (2\*(EllipticF[(a + b\*Log[c\*x^n])/2, 2] + Sqrt[Cos[a + b\*Log[c\*x^n]]]\*Sin[a + b\*Log[c\*x^n]]))/(3\*b\*n)

**Maple [B]** time = 2.182, size = 247, normalized size = 3.9

$$-\frac{2}{3bn} \sqrt{\left(2 \left(\cos\left(\frac{a}{2} + \frac{1}{2} b \ln(cx^n)\right)\right)^2 - 1\right) \left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right)\right)^2} \left(4 \cos\left(\frac{a}{2} + \frac{1}{2} b \ln(cx^n)\right) \left(\sin\left(\frac{a}{2} + \frac{1}{2} b \ln(cx^n)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^(3/2)/x,x)

[Out] -2/3/n\*((2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(4\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*EllipticF(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n)))/(-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(3/2)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="fricas")

[Out] integral(cos(b\*log(c\*x^n) + a)^(3/2)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*ln(c\*x\*\*n))\*\*(3/2)/x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="giac")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(3/2)/x, x)

### 3.114 $\int \cos^{\frac{5}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=110

$$\frac{2x \operatorname{Hypergeometric2F1}\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right), -\frac{bn+2i}{4bn}, -e^{2ia}(cx^n)^{2ib}\right) \cos^{\frac{5}{2}}(a + b \log(cx^n))}{(2 - 5ibn)(1 + e^{2ia}(cx^n)^{2ib})^{5/2}}$$

[Out] (2\*x\*Cos[a + b\*Log[c\*x^n]]^(5/2)\*Hypergeometric2F1[-5/2, (-5 - (2\*I)/(b\*n))/4, -(2\*I + b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))])/((2 - (5\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(5/2))

**Rubi [A]** time = 0.0718108, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4484, 4492, 364}

$$\frac{2x {}_2F_1\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right); -\frac{bn+2i}{4bn}; -e^{2ia}(cx^n)^{2ib}\right) \cos^{\frac{5}{2}}(a + b \log(cx^n))}{(2 - 5ibn)(1 + e^{2ia}(cx^n)^{2ib})^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x\*Cos[a + b\*Log[c\*x^n]]^(5/2)\*Hypergeometric2F1[-5/2, (-5 - (2\*I)/(b\*n))/4, -(2\*I + b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))])/((2 - (5\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(5/2))

#### Rule 4484

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \cos^{\frac{5}{2}}(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \cos^{\frac{5}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{\left(x(cx^n)^{\frac{5ib}{2}-\frac{1}{n}} \cos^{\frac{5}{2}}(a + b \log(cx^n))\right) \operatorname{Subst}\left(\int x^{-1-\frac{5ib}{2}+\frac{1}{n}} (1 + e^{2ia}x^{2ib})^{5/2} dx, x, cx^n\right)}{n(1 + e^{2ia}(cx^n)^{2ib})^{5/2}} \\ &= \frac{2x \cos^{\frac{5}{2}}(a + b \log(cx^n)) {}_2F_1\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right); -\frac{2i+bn}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(2 - 5ibn)(1 + e^{2ia}(cx^n)^{2ib})^{5/2}} \end{aligned}$$

**Mathematica [B]** time = 7.1712, size = 696, normalized size = 6.33

$$\frac{30b^3n^3x^{1-ibn}e^{i(a+b(\log(cx^n)-n\log(x)))}\sqrt{2+2x^{2ibn}e^{2i(a+b(\log(cx^n)-n\log(x)))}}\left((bn+2i)x^{2ibn}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4} - \frac{i}{2bn}, \frac{7}{4}\right)\right)}{(2-5ibn)(bn+2i)(3bn-2i)(5bn-2i)\left((bn-2i)e^{2i(a+b(\log(cx^n)-n\log(x)))}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (30\*b^3\*E^(I\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* n^3 \* x^(1 - I\*b\*n) \* Sqrt[2 + 2\*E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n)] \* ((2\*I + b\*n) \* x^((2\*I)\*b\*n) \* Hypergeometric2F1[1/2, 3/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n))] + (-2\*I + 3\*b\*n) \* Hypergeometric2F1[1/2, -(2\*I + b\*n)/(4\*b\*n), 3/4 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n))]) / ((2 - (5\*I)\*b\*n) \* (2\*I + b\*n) \* (-2\*I + 3\*b\*n) \* (-2\*I + 5\*b\*n) \* (-2\*I - b\*n + E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* (-2\*I + b\*n)) \* Sqrt[(1 + E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n)) / (E^(I\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^(I\*b\*n))]) + Sqrt[Cos[a + b\*n\*Log[x] + b\*(-(n\*Log[x]) + Log[c\*x^n])] \* ((-2\*x\*(2\*Cos[a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) + 15\*b^2\*n^2\*Cos[a + b\*(-(n\*Log[x]) + Log[c\*x^n])) - b\*n\*Sin[a + b\*(-(n\*Log[x]) + Log[c\*x^n])])]) / ((-2\*I + 5\*b\*n)\*(2\*I + 5\*b\*n)\*(-2\*Cos[a + b\*(-(n\*Log[x]) + Log[c\*x^n])) + b\*n\*Sin[a + b\*(-(n\*Log[x]) + Log[c\*x^n])])]) + (x\*Sin[2\*b\*n\*Log[x]]\*(5\*b\*n\*Cos[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] - 2\*Sin[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))]) / ((-2\*I + 5\*b\*n)\*(2\*I + 5\*b\*n)) + (x\*Cos[2\*b\*n\*Log[x]]\*(2\*Cos[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 5\*b\*n\*Sin[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))]) / ((-2\*I + 5\*b\*n)\*(2\*I + 5\*b\*n)))

**Maple [F]** time = 0.151, size = 0, normalized size = 0.

$$\int (\cos(a + b \ln(cx^n)))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^(5/2), x)

[Out] int(cos(a+b\*ln(c\*x^n))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos(b \log(cx^n) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(5/2), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos(b \log(cx^n) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(5/2),x, algorithm="giac")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(5/2), x)

$$3.115 \quad \int \frac{\cos^{\frac{5}{2}}(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=63

$$\frac{6E\left(\frac{1}{2}(a+b \log(cx^n))\right) \Big| 2}{5bn} + \frac{2 \sin(a+b \log(cx^n)) \cos^{\frac{3}{2}}(a+b \log(cx^n))}{5bn}$$

[Out] (6\*EllipticE[(a + b\*Log[c\*x^n])/2, 2])/(5\*b\*n) + (2\*Cos[a + b\*Log[c\*x^n]]^(3/2)\*Sin[a + b\*Log[c\*x^n]])/(5\*b\*n)

**Rubi [A]** time = 0.0423989, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2635, 2639}

$$\frac{6E\left(\frac{1}{2}(a+b \log(cx^n))\right) \Big| 2}{5bn} + \frac{2 \sin(a+b \log(cx^n)) \cos^{\frac{3}{2}}(a+b \log(cx^n))}{5bn}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^(5/2)/x, x]

[Out] (6\*EllipticE[(a + b\*Log[c\*x^n])/2, 2])/(5\*b\*n) + (2\*Cos[a + b\*Log[c\*x^n]]^(3/2)\*Sin[a + b\*Log[c\*x^n]])/(5\*b\*n)

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x] \* (b\*SIN[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cos^{\frac{5}{2}}(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cos^{\frac{5}{2}}(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2 \cos^{\frac{3}{2}}(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{5bn} + \frac{3 \text{Subst}\left(\int \sqrt{\cos(a+bx)} dx, x, \log(cx^n)\right)}{5n} \\ &= \frac{6E\left(\frac{1}{2}(a+b \log(cx^n))\right) \Big| 2}{5bn} + \frac{2 \cos^{\frac{3}{2}}(a+b \log(cx^n)) \sin(a+b \log(cx^n))}{5bn} \end{aligned}$$

**Mathematica [A]** time = 0.132875, size = 58, normalized size = 0.92

$$\frac{6E\left(\frac{1}{2}(a+b \log(cx^n))\right) \Big| 2 + \sin(2(a+b \log(cx^n))) \sqrt{\cos(a+b \log(cx^n))}}{5bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^(5/2)/x,x]

[Out] (6\*EllipticE[(a + b\*Log[c\*x^n])/2, 2] + Sqrt[Cos[a + b\*Log[c\*x^n]]]\*Sin[2\*(a + b\*Log[c\*x^n])])/(5\*b\*n)

**Maple [B]** time = 2.823, size = 280, normalized size = 4.4

$$-\frac{2}{5bn} \sqrt{\left(2 \left(\cos\left(\frac{a}{2} + \frac{1}{2}b \ln(cx^n)\right)\right)^2 - 1\right) \left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right)\right)^2} \left(-8 \cos\left(\frac{a}{2} + \frac{1}{2}b \ln(cx^n)\right) \sin\left(\frac{a}{2} + \frac{1}{2}b \ln(cx^n)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^(5/2)/x,x)

[Out] 
$$-2/5/n * ((2 * \cos(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 2 - 1) * \sin(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 2) ^ (1/2) * (-8 * \cos(1/2 * a + 1/2 * b * \ln(c * x^n)) * \sin(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 6 + 8 * \cos(1/2 * a + 1/2 * b * \ln(c * x^n)) * \sin(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 4 - 3 * (\sin(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 2) ^ (1/2) * (2 * \sin(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * a + 1/2 * b * \ln(c * x^n)), 2 ^ (1/2)) - 2 * \sin(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 2 * \cos(1/2 * a + 1/2 * b * \ln(c * x^n))) / (-2 * \sin(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 4 + \sin(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 2) ^ (1/2) / \sin(1/2 * a + 1/2 * b * \ln(c * x^n)) / (2 * \cos(1/2 * a + 1/2 * b * \ln(c * x^n)) ^ 2 - 1) ^ (1/2) / b$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="maxima")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(5/2)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="fricas")

[Out] integral(cos(b\*log(c\*x^n) + a)^(5/2)/x, x)



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*ln(c\*x\*\*n))\*\*(5/2)/x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(b \log(cx^n) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="giac")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(5/2)/x, x)

$$3.116 \quad \int \frac{1}{\sqrt{\cos(a+b \log(cx^n))}} dx$$

**Optimal.** Leaf size=109

$$\frac{2x\sqrt{1 + e^{2ia} (cx^n)^{2ib}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right), \frac{1}{4}\left(5 - \frac{2i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{(2 + ibn)\sqrt{\cos(a + b \log(cx^n))}}$$

[Out] (2\*x\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, (1 - (2\*I)/(b\*n))/4, (5 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + I\*b\*n)\*Sqrt[Cos[a + b\*Log[c\*x^n]]])

**Rubi [A]** time = 0.0667546, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4484, 4492, 364}

$$\frac{2x\sqrt{1 + e^{2ia} (cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right); \frac{1}{4}\left(5 - \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(2 + ibn)\sqrt{\cos(a + b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Cos[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, (1 - (2\*I)/(b\*n))/4, (5 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + I\*b\*n)\*Sqrt[Cos[a + b\*Log[c\*x^n]]])

#### Rule 4484

Int[Cos[(a\_.) + Log[c\_.\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{\sqrt{\cos(a + b \log(cx^n))}} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sqrt{\cos(a+b \log(x))}} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{ib}{2}-\frac{1}{n}} \sqrt{1 + e^{2ia}(cx^n)^{2ib}}\right) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1}{n}}}{\sqrt{1+e^{2ia}x^{2ib}}} dx, x, cx^n\right)}{n\sqrt{\cos(a + b \log(cx^n))}}$$

$$= \frac{2x\sqrt{1 + e^{2ia}(cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right); \frac{1}{4}\left(5 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2 + ibn)\sqrt{\cos(a + b \log(cx^n))}}$$

**Mathematica [A]** time = 0.380223, size = 99, normalized size = 0.91

$$\frac{2ix(1 + e^{2i(a+b \log(cx^n))}) \operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right)}{(bn - 2i)\sqrt{\cos(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[Cos[a + b\*Log[c\*x^n]]], x]

[Out]  $((-2I)*(1 + E^{((2I)*(a + b*Log[c*x^n])}))*x*\operatorname{Hypergeometric2F1}[1, 3/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^{((2I)*(a + b*Log[c*x^n])}))/((-2I + b*n)*\operatorname{Sqrt}[\operatorname{Cos}[a + b*Log[c*x^n]]])$

**Maple [F]** time = 0.16, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(a + b \ln(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(1/cos(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(1/sqrt(cos(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(a + b*log(c*x**n))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(b*log(c*x^n) + a)), x)
```

$$3.117 \quad \int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}} dx$$

**Optimal.** Leaf size=24

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)), 2\right)}{bn}$$

[Out] (2\*EllipticF[(a + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Rubi [A]** time = 0.0279455, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2641}

$$\frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[Cos[a + b\*Log[c\*x^n]]]), x]

[Out] (2\*EllipticF[(a + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Rule 2641**

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2F\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0853128, size = 24, normalized size = 1.

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)), 2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[Cos[a + b\*Log[c\*x^n]]]), x]

[Out] (2\*EllipticF[(a + b\*Log[c\*x^n])/2, 2])/(b\*n)

**Maple [C]** time = 0.037, size = 26, normalized size = 1.1

$$2 \frac{\text{InverseJacobiAM}\left(a/2 + 1/2 b \ln(cx^n), \sqrt{2}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/cos(a+b*ln(c*x^n))^(1/2),x)`

[Out] `2/b/n*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cos(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(x*sqrt(cos(b*log(c*x^n) + a))), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x\sqrt{\cos(b \log(cx^n) + a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")`

[Out] `integral(1/(x*sqrt(cos(b*log(c*x^n) + a))), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cos(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/cos(a+b*ln(c*x**n))**(1/2),x)`

[Out] `Integral(1/(x*sqrt(cos(a + b*log(c*x**n))))), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\cos(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(x*sqrt(cos(b*log(c*x^n) + a))), x)`

$$3.118 \quad \int \frac{1}{\cos^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=109

$$\frac{2x \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{3/2} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1}{4} \left(3 - \frac{2i}{bn}\right), \frac{1}{4} \left(7 - \frac{2i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 3ibn) \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

[Out] (2\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(3/2)\*Hypergeometric2F1[3/2, (3 - (2\*I)/(b\*n))/4, (7 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + (3\*I)\*b\*n)\*Cos[a + b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.0698297, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4484, 4492, 364}

$$\frac{2x \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4} \left(3 - \frac{2i}{bn}\right); \frac{1}{4} \left(7 - \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 3ibn) \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^(-3/2), x]

[Out] (2\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(3/2)\*Hypergeometric2F1[3/2, (3 - (2\*I)/(b\*n))/4, (7 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + (3\*I)\*b\*n)\*Cos[a + b\*Log[c\*x^n]]^(3/2))

#### Rule 4484

Int[Cos[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)]\*(d\_)^(p\_), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_) + Log[x\_]\*(b\_)]\*(d\_)^(p\_)\*((e\_)\*(x\_))^(m\_), x\_Symbol] := Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(a + b \log(cx^n))} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cos^{\frac{3}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{3ib}{2}-\frac{1}{n}} (1 + e^{2ia}(cx^n)^{2ib})^{3/2}\right) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^{3/2}} dx, x, cx^n\right)}{n \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x(1 + e^{2ia}(cx^n)^{2ib})^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right); \frac{1}{4}\left(7 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2 + 3ibn) \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

**Mathematica [B]** time = 3.61347, size = 431, normalized size = 3.95

$$x \left( (3bn - 2i)x^{-ibn} \left( 2x^{ibn} \sqrt{e^{-ia}(cx^n)^{-ib} + e^{ia}(cx^n)^{ib}} (bn \cos(bn \log(x)) - 2 \sin(bn \log(x))) - (bn - 2i) \sqrt{2 + 2e^{2ia}(cx^n)^{2ib}} \operatorname{Hy} \right. \right. \\ \left. \left. bn(3bn - 2i) \sqrt{e^{-ia}(cx^n)^{-ib} + e^{ia}(cx^n)^{ib}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^(-3/2), x]

[Out] (x\*((-(4 + b^2\*n^2)\*x^(I\*b\*n)\*Sqrt[2 + 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]) + ((-2\*I + 3\*b\*n)\*(-(2\*I + b\*n)\*Sqrt[2 + 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*Hypergeometric2F1[1/2, -(2\*I + b\*n)/(4\*b\*n), 3/4 - (I/2)/(b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]) + 2\*x^(I\*b\*n)\*Sqrt[1/(E^(I\*a)\*(c\*x^n)^(I\*b)) + E^(I\*a)\*(c\*x^n)^(I\*b)]\*(b\*n\*Cos[b\*n\*Log[x]] - 2\*Sin[b\*n\*Log[x]]))/x^(I\*b\*n))/((b\*n\*(-2\*I + 3\*b\*n)\*Sqrt[1/(E^(I\*a)\*(c\*x^n)^(I\*b)) + E^(I\*a)\*(c\*x^n)^(I\*b)]\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*(-2\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]] + b\*n\*Sin[a - b\*n\*Log[x] + b\*Log[c\*x^n]]))

**Maple [F]** time = 0.153, size = 0, normalized size = 0.

$$\int (\cos(a + b \ln(cx^n)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(1/cos(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/cos(a+b\*log(c\*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(-3/2), x)

**Fricas [F-2]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Integral(cos(a + b\*log(c\*x\*\*n))\*\*(-3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(-3/2), x)

$$3.119 \quad \int \frac{1}{x \cos^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=59

$$\frac{2 \sin(a+b \log(cx^n))}{bn \sqrt{\cos(a+b \log(cx^n))}} - \frac{2E\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{bn}$$

[Out]  $(-2*\text{EllipticE}[(a + b*\text{Log}[c*x^n])/2, 2])/(b*n) + (2*\text{Sin}[a + b*\text{Log}[c*x^n]])/(b*n*\text{Sqrt}[\text{Cos}[a + b*\text{Log}[c*x^n]])]$

**Rubi [A]** time = 0.0410599, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2636, 2639}

$$\frac{2 \sin(a+b \log(cx^n))}{bn \sqrt{\cos(a+b \log(cx^n))}} - \frac{2E\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{Cos}[a + b*\text{Log}[c*x^n]]^{(3/2)}), x]$

[Out]  $(-2*\text{EllipticE}[(a + b*\text{Log}[c*x^n])/2, 2])/(b*n) + (2*\text{Sin}[a + b*\text{Log}[c*x^n]])/(b*n*\text{Sqrt}[\text{Cos}[a + b*\text{Log}[c*x^n]])]$

#### Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n + 1)})/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n + 2)}, x], x] /;$  FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x\_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$  FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x \cos^{\frac{3}{2}}(a+b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2 \sin(a+b \log(cx^n))}{bn \sqrt{\cos(a+b \log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\cos(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{2E\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{bn} + \frac{2 \sin(a+b \log(cx^n))}{bn \sqrt{\cos(a+b \log(cx^n))}} \end{aligned}$$

**Mathematica [A]** time = 0.152792, size = 54, normalized size = 0.92

$$\frac{2\left(\frac{\sin(a+b \log(cx^n))}{\sqrt{\cos(a+b \log(cx^n))}} - E\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Cos[a + b\*Log[c\*x^n]]^(3/2)),x]

[Out] (2\*(-EllipticE[(a + b\*Log[c\*x^n])/2, 2] + Sin[a + b\*Log[c\*x^n]]/Sqrt[Cos[a + b\*Log[c\*x^n]]]))/(b\*n)

**Maple [A]** time = 2.377, size = 139, normalized size = 2.4

$$-2 \frac{\sqrt{(\sin(a/2 + 1/2 b \ln(cx^n)))^2} \sqrt{2 (\sin(a/2 + 1/2 b \ln(cx^n)))^2 - 1} \text{EllipticE}(\cos(a/2 + 1/2 b \ln(cx^n)), \sqrt{2}) - 2 (\sin(a/2 + 1/2 b \ln(cx^n)))}{n \sin(a/2 + 1/2 b \ln(cx^n)) \sqrt{2 (\cos(a/2 + 1/2 b \ln(cx^n)))^2 - 1} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/cos(a+b\*ln(c\*x^n))^(3/2),x)

[Out] -2/n\*((sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*EllipticE(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n)))/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \cos(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cos(a+b\*log(c\*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x\*cos(b\*log(c\*x^n) + a)^(3/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \cos(b \log(cx^n) + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cos(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] integral(1/(x\*cos(b\*log(c\*x^n) + a)^(3/2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cos(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \cos(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cos(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] integrate(1/(x\*cos(b\*log(c\*x^n) + a)^(3/2)), x)

$$3.120 \quad \int \frac{1}{\cos^2(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=109

$$\frac{2x \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{5/2} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{1}{4} \left(5 - \frac{2i}{bn}\right), \frac{1}{4} \left(9 - \frac{2i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 5ibn) \cos^2(a + b \log(cx^n))}$$

[Out] (2\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(5/2)\*Hypergeometric2F1[5/2, (5 - (2\*I)/(b\*n))/4, (9 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + (5\*I)\*b\*n)\*Cos[a + b\*Log[c\*x^n]]^(5/2))

**Rubi [A]** time = 0.0728147, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4484, 4492, 364}

$$\frac{2x \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4} \left(5 - \frac{2i}{bn}\right); \frac{1}{4} \left(9 - \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 5ibn) \cos^2(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^(-5/2), x]

[Out] (2\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(5/2)\*Hypergeometric2F1[5/2, (5 - (2\*I)/(b\*n))/4, (9 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + (5\*I)\*b\*n)\*Cos[a + b\*Log[c\*x^n]]^(5/2))

#### Rule 4484

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_)\*((e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(a + b \log(cx^n))} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\cos^{\frac{5}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{5ib}{2}-\frac{1}{n}}(1 + e^{2ia}(cx^n)^{2ib})^{5/2}\right) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^{5/2}} dx, x, cx^n\right)}{n \cos^{\frac{5}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x(1 + e^{2ia}(cx^n)^{2ib})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right); \frac{1}{4}\left(9 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2 + 5ibn) \cos^{\frac{5}{2}}(a + b \log(cx^n))}$$

**Mathematica [A]** time = 1.10207, size = 147, normalized size = 1.35

$$\frac{2x \left( (2 - ibn) (1 + e^{2ia} (cx^n)^{2ib}) \cos(a + b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right) + bn \sin(a + b \log(cx^n)) \right)}{3b^2 n^2 \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^(-5/2), x]

[Out] (2\*x\*(-2\*Cos[a + b\*Log[c\*x^n]] + (2 - I\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Cos[a + b\*Log[c\*x^n]]\*Hypergeometric2F1[1, 3/4 - (I/2)/(b\*n), 5/4 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + b\*n\*Sin[a + b\*Log[c\*x^n]]))/(3\*b^2\*n^2\*Cos[a + b\*Log[c\*x^n]]^(3/2))

**Maple [F]** time = 0.163, size = 0, normalized size = 0.

$$\int (\cos(a + b \ln(cx^n)))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(a+b\*ln(c\*x^n))^(5/2), x)

[Out] int(1/cos(a+b\*ln(c\*x^n))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a+b\*log(c\*x^n))^(5/2), x, algorithm="maxima")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(-5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a+b\*log(c\*x^n))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a+b\*log(c\*x^n))^(5/2),x, algorithm="giac")

[Out] integrate(cos(b\*log(c\*x^n) + a)^(-5/2), x)

$$3.121 \quad \int \frac{1}{x \cos^{\frac{5}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=63

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(a+b \log(cx^n)), 2\right)}{3bn} + \frac{2 \sin(a+b \log(cx^n))}{3bn \cos^{\frac{3}{2}}(a+b \log(cx^n))}$$

[Out] (2\*EllipticF[(a + b\*Log[c\*x^n])/2, 2])/(3\*b\*n) + (2\*Sin[a + b\*Log[c\*x^n]])/(3\*b\*n\*Cos[a + b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.0435381, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2636, 2641}

$$\frac{2F\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{3bn} + \frac{2 \sin(a+b \log(cx^n))}{3bn \cos^{\frac{3}{2}}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Cos[a + b\*Log[c\*x^n]]^(5/2)),x]

[Out] (2\*EllipticF[(a + b\*Log[c\*x^n])/2, 2])/(3\*b\*n) + (2\*Sin[a + b\*Log[c\*x^n]])/(3\*b\*n\*Cos[a + b\*Log[c\*x^n]]^(3/2))

#### Rule 2636

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n + 1))/(b\*d\*(n + 1)), x] + Dist[(n + 2)/(b^2\*(n + 1)), Int[(b\*Sin[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x \cos^{\frac{5}{2}}(a+b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2 \sin(a+b \log(cx^n))}{3bn \cos^{\frac{3}{2}}(a+b \log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cos(a+bx)}} dx, x, \log(cx^n)\right)}{3n} \\ &= \frac{2F\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{3bn} + \frac{2 \sin(a+b \log(cx^n))}{3bn \cos^{\frac{3}{2}}(a+b \log(cx^n))} \end{aligned}$$



**Mathematica [A]** time = 0.145647, size = 54, normalized size = 0.86

$$\frac{2 \left( \text{EllipticF} \left( \frac{1}{2} (a + b \log(cx^n)), 2 \right) + \frac{\sin(a + b \log(cx^n))}{\cos^2(a + b \log(cx^n))} \right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Cos[a + b\*Log[c\*x^n]]^(5/2)), x]

[Out] (2\*(EllipticF[(a + b\*Log[c\*x^n])/2, 2] + Sin[a + b\*Log[c\*x^n]]/Cos[a + b\*Log[c\*x^n]]^(3/2)))/(3\*b\*n)

**Maple [B]** time = 2.319, size = 291, normalized size = 4.6

$$-\frac{2}{3bn} \left( -2 \sqrt{(\sin(a/2 + 1/2 b \ln(cx^n)))^2} \sqrt{2 (\sin(a/2 + 1/2 b \ln(cx^n)))^2 - 1} \text{EllipticF} \left( \cos(a/2 + 1/2 b \ln(cx^n)), \sqrt{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/cos(a+b\*ln(c\*x^n))^(5/2), x)

[Out] -2/3/n\*(-2\*(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*EllipticF(cos(1/2\*a+1/2\*b\*ln(c\*x^n)), 2^(1/2))\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2+(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*EllipticF(cos(1/2\*a+1/2\*b\*ln(c\*x^n)), 2^(1/2))-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))\*((2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)/(-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(3/2)/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \cos(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cos(a+b\*log(c\*x^n))^(5/2), x, algorithm="maxima")

[Out] integrate(1/(x\*cos(b\*log(c\*x^n) + a)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{x \cos(b \log(cx^n) + a)^{\frac{5}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/(x*cos(b*log(c*x^n) + a)^(5/2)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \cos(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*cos(b*log(c*x^n) + a)^(5/2)), x)
```

$$3.122 \quad \int \frac{1}{\cos^{\frac{3}{2}}(a-2i \log(cx))} dx$$

**Optimal.** Leaf size=48

$$-\frac{e^{-2ia} (1 + e^{2ia} c^4 x^4)}{2c^4 x^3 \cos^{\frac{3}{2}}(a - 2i \log(cx))}$$

[Out]  $-(1 + c^4 E^{((2*I)*a)*x^4}) / (2*c^4 * E^{((2*I)*a)*x^3} * \text{Cos}[a - (2*I)*\text{Log}[c*x]]^{\frac{3}{2}})$

**Rubi [A]** time = 0.0361138, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4484, 4482, 261}

$$-\frac{e^{-2ia} (1 + e^{2ia} c^4 x^4)}{2c^4 x^3 \cos^{\frac{3}{2}}(a - 2i \log(cx))}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cos}[a - (2*I)*\text{Log}[c*x]]^{(-3/2)}, x]$

[Out]  $-(1 + c^4 * E^{((2*I)*a)*x^4}) / (2*c^4 * E^{((2*I)*a)*x^3} * \text{Cos}[a - (2*I)*\text{Log}[c*x]]^{\frac{3}{2}})$

#### Rule 4484

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[x/(n*(c*x^n)^{(1/n})], \text{Subst}[\text{Int}[x^{(1/n - 1)}*\text{Cos}[d*(a + b*\text{Log}[x])]^p, x], x, c*x^n], x] /;$   $\text{FreeQ}\{a, b, c, d, n, p\}, x] \&\& (\text{NeQ}[c, 1] \mid \mid \text{NeQ}[n, 1])$

#### Rule 4482

$\text{Int}[\text{Cos}[(a_.) + \text{Log}[x_]*](b_.)]*(d_.)]^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[(\text{Cos}[d*(a + b*\text{Log}[x])]^p * x^{(I*b*d*p)}) / (1 + E^{(2*I*a*d)*x^{(2*I*b*d)}})^p, \text{Int}[(1 + E^{(2*I*a*d)*x^{(2*I*b*d)}})^p / x^{(I*b*d*p)}, x], x] /;$   $\text{FreeQ}\{a, b, d, p\}, x] \&\& !\text{IntegerQ}[p]$

#### Rule 261

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p + 1)} / (b*n*(p + 1)), x] /;$   $\text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{EqQ}[m, n - 1] \&\& \text{NeQ}[p, -1]$

#### Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(a - 2i \log(cx))} dx = \frac{\text{Subst}\left(\int \frac{1}{\cos^{\frac{3}{2}}(a - 2i \log(x))} dx, x, cx\right)}{c}$$

$$= \frac{(1 + c^4 e^{2ia} x^4)^{3/2} \text{Subst}\left(\int \frac{x^3}{(1 + e^{2ia} x^4)^{3/2}} dx, x, cx\right)}{c^4 x^3 \cos^{\frac{3}{2}}(a - 2i \log(cx))}$$

$$= \frac{e^{-2ia} (1 + c^4 e^{2ia} x^4)}{2c^4 x^3 \cos^{\frac{3}{2}}(a - 2i \log(cx))}$$

**Mathematica [A]** time = 0.115488, size = 82, normalized size = 1.71

$$-\frac{x(\cos(a) - i \sin(a)) \sqrt{\frac{2 \cos(a)(c^4 x^4 + 1) + 2i \sin(a)(c^4 x^4 - 1)}{c^2 x^2}}}{\cos(a)(c^4 x^4 + 1) + i \sin(a)(c^4 x^4 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a - (2\*I)\*Log[c\*x]]^(-3/2), x]

[Out] -((x\*(Cos[a] - I\*Sin[a])\*Sqrt[(2\*(1 + c^4\*x^4)\*Cos[a] + (2\*I)\*(-1 + c^4\*x^4)\*Sin[a])/(c^2\*x^2)])/((1 + c^4\*x^4)\*Cos[a] + I\*(-1 + c^4\*x^4)\*Sin[a]))

**Maple [F]** time = 0.343, size = 0, normalized size = 0.

$$\int (\cos(a - 2i \ln(cx)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(a-2\*I\*ln(c\*x))^(3/2), x)

[Out] int(1/cos(a-2\*I\*ln(c\*x))^(3/2), x)

**Maxima [B]** time = 1.80503, size = 252, normalized size = 5.25

$$\frac{\left(\left(\sqrt{2} \cos\left(\frac{3}{2}a\right) + i \sqrt{2} \sin\left(\frac{3}{2}a\right)\right)c^4 x^4 + \sqrt{2} \cos\left(\frac{1}{2}a\right) - i \sqrt{2} \sin\left(\frac{1}{2}a\right)\right) \cos\left(\frac{3}{2} \arctan\left(c^4 x^4 \sin(2a), c^4 x^4 \cos(2a) + 1\right)\right)}{\left(\cos(2a)^2 + \sin(2a)^2\right)c^8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a-2\*I\*log(c\*x))^(3/2), x, algorithm="maxima")

[Out] -(((sqrt(2)\*cos(3/2\*a) + I\*sqrt(2)\*sin(3/2\*a))\*c^4\*x^4 + sqrt(2)\*cos(1/2\*a) - I\*sqrt(2)\*sin(1/2\*a))\*cos(3/2\*arctan2(c^4\*x^4\*sin(2\*a), c^4\*x^4\*cos(2\*a) + 1)) + ((-I\*sqrt(2)\*cos(3/2\*a) + sqrt(2)\*sin(3/2\*a))\*c^4\*x^4 - I\*sqrt(2)\*cos(1/2\*a) - sqrt(2)\*sin(1/2\*a))\*sin(3/2\*arctan2(c^4\*x^4\*sin(2\*a), c^4\*x^4\*cos(2\*a) + 1))/(((cos(2\*a)^2 + sin(2\*a)^2)\*c^8\*x^8 + 2\*c^4\*x^4\*cos(2\*a) +

$1)^{(3/4)} * c)$

**Fricas [A]** time = 0.456619, size = 103, normalized size = 2.15

$$\frac{2\sqrt{\frac{1}{2}}x e^{\left(-\frac{3}{2}i a - 3 \log(cx)\right)}}{\sqrt{e^{(-2i a - 4 \log(cx))} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a-2\*I\*log(c\*x))^(3/2), x, algorithm="fricas")

[Out] -2\*sqrt(1/2)\*x\*e^(-3/2\*I\*a - 3\*log(c\*x))/sqrt(e^(-2\*I\*a - 4\*log(c\*x)) + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{3}{2}}(a - 2i \log(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a-2\*I\*ln(c\*x))\*\*(3/2), x)

[Out] Integral(cos(a - 2\*I\*log(c\*x))\*\*(-3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(a - 2i \log(cx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(a-2\*I\*log(c\*x))^(3/2), x, algorithm="giac")

[Out] integrate(cos(a - 2\*I\*log(c\*x))^(3/2), x)

### 3.123 $\int x^m \cos^4(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=266

$$\frac{(m+1)x^{m+1} \cos^4(a + b \log(cx^n))}{16b^2n^2 + (m+1)^2} + \frac{12b^2(m+1)n^2x^{m+1} \cos^2(a + b \log(cx^n))}{(4b^2n^2 + (m+1)^2)(16b^2n^2 + (m+1)^2)} + \frac{4bnx^{m+1} \sin(a + b \log(cx^n)) \cos^3(a + b \log(cx^n))}{16b^2n^2 + (m+1)^2}$$

[Out] (24\*b^4\*n^4\*x^(1+m))/((1+m)\*((1+m)^2+4\*b^2\*n^2)\*((1+m)^2+16\*b^2\*n^2)) + (12\*b^2\*(1+m)\*n^2\*x^(1+m)\*Cos[a+b\*Log[c\*x^n]]^2)/(((1+m)^2+4\*b^2\*n^2)\*((1+m)^2+16\*b^2\*n^2)) + ((1+m)\*x^(1+m)\*Cos[a+b\*Log[c\*x^n]]^4)/((1+m)^2+16\*b^2\*n^2) + (24\*b^3\*n^3\*x^(1+m)\*Cos[a+b\*Log[c\*x^n]]\*Sin[a+b\*Log[c\*x^n]])/(((1+m)^2+4\*b^2\*n^2)\*((1+m)^2+16\*b^2\*n^2)) + (4\*b\*n\*x^(1+m)\*Cos[a+b\*Log[c\*x^n]]^3\*Sin[a+b\*Log[c\*x^n]])/((1+m)^2+16\*b^2\*n^2)

**Rubi [A]** time = 0.124769, antiderivative size = 260, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4488, 30}

$$\frac{(m+1)x^{m+1} \cos^4(a + b \log(cx^n))}{16b^2n^2 + (m+1)^2} + \frac{12b^2(m+1)n^2x^{m+1} \cos^2(a + b \log(cx^n))}{20b^2(m+1)^2n^2 + 64b^4n^4 + (m+1)^4} + \frac{4bnx^{m+1} \sin(a + b \log(cx^n)) \cos^3(a + b \log(cx^n))}{16b^2n^2 + (m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cos[a + b\*Log[c\*x^n]]^4, x]

[Out] (24\*b^4\*n^4\*x^(1+m))/((1+m)\*((1+m)^2+4\*b^2\*n^2)\*((1+m)^2+16\*b^2\*n^2)) + (12\*b^2\*(1+m)\*n^2\*x^(1+m)\*Cos[a+b\*Log[c\*x^n]]^2)/((1+m)^4+20\*b^2\*(1+m)^2\*n^2+64\*b^4\*n^4) + ((1+m)\*x^(1+m)\*Cos[a+b\*Log[c\*x^n]]^4)/((1+m)^2+16\*b^2\*n^2) + (24\*b^3\*n^3\*x^(1+m)\*Cos[a+b\*Log[c\*x^n]]\*Sin[a+b\*Log[c\*x^n]])/((1+m)^4+20\*b^2\*(1+m)^2\*n^2+64\*b^4\*n^4) + (4\*b\*n\*x^(1+m)\*Cos[a+b\*Log[c\*x^n]]^3\*Sin[a+b\*Log[c\*x^n]])/((1+m)^2+16\*b^2\*n^2)

#### Rule 4488

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((m+1)\*(e\*x)^(m+1)\*Cos[d\*(a+b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2+e\*(m+1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p-1))/(b^2\*d^2\*n^2\*p^2+(m+1)^2), Int[(e\*x)^m\*Cos[d\*(a+b\*Log[c\*x^n])]^(p-2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m+1)\*Sin[d\*(a+b\*Log[c\*x^n]])\*Cos[d\*(a+b\*Log[c\*x^n])]^(p-1)/(b^2\*d^2\*e\*n^2\*p^2+e\*(m+1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2+(m+1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^m \cos^4(a + b \log(cx^n)) dx &= \frac{(1+m)x^{1+m} \cos^4(a + b \log(cx^n))}{(1+m)^2 + 16b^2n^2} + \frac{4bnx^{1+m} \cos^3(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{(1+m)^2 + 16b^2n^2} \\ &= \frac{12b^2(1+m)n^2x^{1+m} \cos^2(a + b \log(cx^n))}{(1+m)^4 + 20b^2(1+m)^2n^2 + 64b^4n^4} + \frac{(1+m)x^{1+m} \cos^4(a + b \log(cx^n))}{(1+m)^2 + 16b^2n^2} + \frac{24bnx^{1+m} \cos^3(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{(1+m)^2 + 16b^2n^2} \\ &= \frac{24b^4n^4x^{1+m}}{(1+m)((1+m)^4 + 20b^2(1+m)^2n^2 + 64b^4n^4)} + \frac{12b^2(1+m)n^2x^{1+m} \cos^2(a + b \log(cx^n))}{(1+m)^4 + 20b^2(1+m)^2n^2 + 64b^4n^4} \end{aligned}$$

**Mathematica [A]** time = 4.03467, size = 312, normalized size = 1.17

$$\frac{1}{8}x^{m+1} \left( -\frac{4 \sin(2bn \log(x)) ((m+1) \sin(2(a + b \log(cx^n) - bn \log(x))) - 2bn \cos(2(a + b \log(cx^n) - bn \log(x))))}{4b^2n^2 + m^2 + 2m + 1} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m \* Cos[a + b \* Log[c \* x^n]]^4, x]

[Out] (x^(1+m) \* (3/(1+m) - (4 \* Sin[2 \* b \* n \* Log[x]] \* (-2 \* b \* n \* Cos[2 \* (a - b \* n \* Log[x] + b \* Log[c \* x^n])]) + (1+m) \* Sin[2 \* (a - b \* n \* Log[x] + b \* Log[c \* x^n])])) / (1 + 2 \* m + m^2 + 4 \* b^2 \* n^2) + (4 \* Cos[2 \* b \* n \* Log[x]] \* ((1+m) \* Cos[2 \* (a - b \* n \* Log[x] + b \* Log[c \* x^n])]) + 2 \* b \* n \* Sin[2 \* (a - b \* n \* Log[x] + b \* Log[c \* x^n])])) / (1 + 2 \* m + m^2 + 4 \* b^2 \* n^2) - (Sin[4 \* b \* n \* Log[x]] \* (-4 \* b \* n \* Cos[4 \* (a - b \* n \* Log[x] + b \* Log[c \* x^n])]) + (1+m) \* Sin[4 \* (a - b \* n \* Log[x] + b \* Log[c \* x^n])])) / (1 + 2 \* m + m^2 + 16 \* b^2 \* n^2) + (Cos[4 \* b \* n \* Log[x]] \* ((1+m) \* Cos[4 \* (a - b \* n \* Log[x] + b \* Log[c \* x^n])]) + 4 \* b \* n \* Sin[4 \* (a - b \* n \* Log[x] + b \* Log[c \* x^n])])) / (1 + 2 \* m + m^2 + 16 \* b^2 \* n^2)) / 8

**Maple [F]** time = 0.096, size = 0, normalized size = 0.

$$\int x^m (\cos(a + b \ln(cx^n)))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m \* cos(a + b \* ln(c \* x^n))^4, x)

[Out] int(x^m \* cos(a + b \* ln(c \* x^n))^4, x)

**Maxima [B]** time = 2.06513, size = 4775, normalized size = 17.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m \* cos(a + b \* log(c \* x^n))^4, x, algorithm="maxima")

[Out] 1/16 \* (((cos(8 \* b \* log(c)) \* cos(4 \* b \* log(c)) + sin(8 \* b \* log(c)) \* sin(4 \* b \* log(c)) + cos(4 \* b \* log(c))) \* m^4 + 4 \* (cos(8 \* b \* log(c)) \* cos(4 \* b \* log(c)) + sin(8 \* b \* log(c)) \* sin(4 \* b \* log(c)) + cos(4 \* b \* log(c))) \* m^3 + 16 \* (b^3 \* cos(4 \* b \* log(c)) \* sin(8 \* b \* log(c)) - b^3 \* cos(8 \* b \* log(c)) \* sin(4 \* b \* log(c)) + b^3 \* sin(4 \* b \* log(c)) + (b^3 \* cos(4 \* b \* log(c)) \* sin(8 \* b \* log(c)) - b^3 \* cos(8 \* b \* log(c)) \* sin(4 \* b \* log(c)) + b^3

```

*sin(4*b*log(c))*m)*n^3 + 6*(cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log
(c))*sin(4*b*log(c)) + cos(4*b*log(c)))m^2 + 4*(b^2*cos(8*b*log(c))*cos(4*
b*log(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + (b^2*cos(8*b*log(c))*cos(
4*b*log(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))m^
2 + b^2*cos(4*b*log(c)) + 2*(b^2*cos(8*b*log(c))*cos(4*b*log(c)) + b^2*sin(
8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))m)*n^2 + 4*(cos(8*b*log(
c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c)))m
+ 4*((b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c))
+ b*sin(4*b*log(c)))m^3 + 3*(b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*
b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c)))m^2 + b*cos(4*b*log(c))*sin(
8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + 3*(b*cos(4*b*log(c))*sin(
8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c)))m + b*
sin(4*b*log(c))*n + cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(
4*b*log(c)) + cos(4*b*log(c))*x*x^m*cos(4*b*log(x^n) + 4*a) + 4*((cos(6*b*
log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))
*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))m^4 + 4*(cos(6*b*log(c)
)*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(4
*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))m^3 + 32*(b^3*cos(4*b*log(c))
*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log(c)
)*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)) + (b^3*cos(4*b*log(
c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log
(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))m)*n^3 + 6*(cos
(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c)) + sin(6*b*lo
g(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))m^2 + 16*(b^2*cos(
6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin
(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)) + (b^2*c
os(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*
sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))m^2
+ 2*(b^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(
c)) + b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log
(c)))m)*n^2 + 4*(cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b
*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)
))m + 2*((b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*lo
g(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)
))m^3 + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*
log(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log
(c)))m^2 + b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*lo
g(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(
c)) + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(
c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)
))m)*n + cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log(c))
+ sin(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c))*x*x^m*
cos(2*b*log(x^n) + 2*a) - ((cos(4*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c)
))*sin(4*b*log(c)) + sin(4*b*log(c)))m^4 + 4*(cos(4*b*log(c))*sin(8*b*log(
c)) - cos(8*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c)))m^3 - 16*(b^3*cos(
8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*cos
(4*b*log(c)) + (b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*log(c))*s
in(4*b*log(c)) + b^3*cos(4*b*log(c)))m)*n^3 + 6*(cos(4*b*log(c))*sin(8*b*lo
g(c)) - cos(8*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c)))m^2 + 4*(b^2*co
s(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + (b^2*
cos(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2
*sin(4*b*log(c)))m^2 + b^2*sin(4*b*log(c)) + 2*(b^2*cos(4*b*log(c))*sin(8*
b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c)))m)*n
^2 + 4*(cos(4*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)) +
sin(4*b*log(c)))m - 4*((b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log
(c))*sin(4*b*log(c)) + b*cos(4*b*log(c)))m^3 + 3*(b*cos(8*b*log(c))*cos(4*
b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c)) + b*cos(4*b*log(c)))m^2 + b*
cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c)) + 3*(b*
cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c)) + b*cos

```



$$\begin{aligned}
& (4*b*\log(c))*m + b*\cos(4*b*\log(c))*n + \cos(4*b*\log(c))*\sin(8*b*\log(c)) - \\
& \cos(8*b*\log(c))*\sin(4*b*\log(c)) + \sin(4*b*\log(c))*x*x^m*\sin(4*b*\log(x^n) + \\
& 4*a) - 4*((\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c) \\
& )) + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*m^4 \\
& + 4*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)) + \cos \\
& (2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*m^3 - 32* \\
& (b^3*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^3*\cos(4*b*\log(c))*\cos(2*b*\log(c)) \\
& + b^3*\sin(6*b*\log(c))*\sin(4*b*\log(c)) + b^3*\sin(4*b*\log(c))*\sin(2*b*\log(c)) \\
& + (b^3*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^3*\cos(4*b*\log(c))*\cos(2*b*\log(c) \\
& )) + b^3*\sin(6*b*\log(c))*\sin(4*b*\log(c)) + b^3*\sin(4*b*\log(c))*\sin(2*b*\log(c) \\
& ))*m)*n^3 + 6*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b* \\
& \log(c)) + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c) \\
& ))*m^2 + 16*(b^2*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^2*\cos(6*b*\log(c))*\sin(4 \\
& *b*\log(c)) + b^2*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^2*\cos(4*b*\log(c))*\sin( \\
& 2*b*\log(c)) + (b^2*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^2*\cos(6*b*\log(c))*\sin \\
& (4*b*\log(c)) + b^2*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^2*\cos(4*b*\log(c))*\sin \\
& (2*b*\log(c)))*m^2 + 2*(b^2*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^2*\cos(6*b* \\
& \log(c))*\sin(4*b*\log(c)) + b^2*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^2*\cos(4*b \\
& *\log(c))*\sin(2*b*\log(c)))*m)*n^2 + 4*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos \\
& (6*b*\log(c))*\sin(4*b*\log(c)) + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log \\
& (c))*\sin(2*b*\log(c)))*m - 2*((b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b*\cos(4* \\
& b*\log(c))*\cos(2*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(4*b*\log(c)) + b*\sin(4*b* \\
& \log(c))*\sin(2*b*\log(c)))*m^3 + 3*(b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b*\cos( \\
& 4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(4*b*\log(c)) + b*\sin(4*b \\
& *\log(c))*\sin(2*b*\log(c)))*m^2 + b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b*\cos(4 \\
& *b*\log(c))*\cos(2*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(4*b*\log(c)) + b*\sin(4*b* \\
& \log(c))*\sin(2*b*\log(c)) + 3*(b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b*\cos(4*b* \\
& \log(c))*\cos(2*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(4*b*\log(c)) + b*\sin(4*b*\log \\
& (c))*\sin(2*b*\log(c)))*m)*n + \cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c) \\
& )*\sin(4*b*\log(c)) + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin \\
& (2*b*\log(c))*x*x^m*\sin(2*b*\log(x^n) + 2*a) + 6*((\cos(4*b*\log(c))^2 + \sin(4 \\
& *b*\log(c))^2)*m^4 + 64*(b^4*\cos(4*b*\log(c))^2 + b^4*\sin(4*b*\log(c))^2)*n^4 \\
& + 4*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*m^3 + 6*(\cos(4*b*\log(c))^2 + \sin \\
& (4*b*\log(c))^2)*m^2 + 20*(b^2*\cos(4*b*\log(c))^2 + b^2*\sin(4*b*\log(c))^2 + \\
& (b^2*\cos(4*b*\log(c))^2 + b^2*\sin(4*b*\log(c))^2)*m^2 + 2*(b^2*\cos(4*b*\log(c) \\
& )^2 + b^2*\sin(4*b*\log(c))^2)*m)*n^2 + 4*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c) \\
& )^2)*m + \cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*x*x^m)/((\cos(4*b*\log(c))^2 \\
& + \sin(4*b*\log(c))^2)*m^5 + 5*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*m^4 + \\
& 64*(b^4*\cos(4*b*\log(c))^2 + b^4*\sin(4*b*\log(c))^2 + (b^4*\cos(4*b*\log(c))^2 \\
& + b^4*\sin(4*b*\log(c))^2)*m)*n^4 + 10*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2 \\
& )*m^3 + 10*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*m^2 + 20*((b^2*\cos(4*b* \\
& \log(c))^2 + b^2*\sin(4*b*\log(c))^2)*m^3 + b^2*\cos(4*b*\log(c))^2 + b^2*\sin(4*b \\
& *\log(c))^2 + 3*(b^2*\cos(4*b*\log(c))^2 + b^2*\sin(4*b*\log(c))^2)*m^2 + 3*(b^2 \\
& *\cos(4*b*\log(c))^2 + b^2*\sin(4*b*\log(c))^2)*m)*n^2 + 5*(\cos(4*b*\log(c))^2 + \\
& \sin(4*b*\log(c))^2)*m + \cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)
\end{aligned}$$

**Fricas [A]** time = 0.562355, size = 667, normalized size = 2.51

$$\frac{4 \left( (b^3 m + b^3) n^3 x \cos(b n \log(x) + b \log(c) + a) + \left( 4 (b^3 m + b^3) n^3 + (b m^3 + 3 b m^2 + 3 b m + b) n \right) x \cos(b n \log(x) + \right.}{
}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*log(c\*x^n))^4,x, algorithm="fricas")

[Out] (4\*(6\*(b^3\*m + b^3)\*n^3\*x\*cos(b\*n\*log(x) + b\*log(c) + a) + (4\*(b^3\*m + b^3)\*n^3 + (b\*m^3 + 3\*b\*m^2 + 3\*b\*m + b)\*n)\*x\*cos(b\*n\*log(x) + b\*log(c) + a)^3)

$$\frac{x^m \sin(bn \log(x) + b \log(c) + a) + (24b^4 n^4 x + 12(b^2 m^2 + 2b^2 m + b^2) n^2 x \cos(bn \log(x) + b \log(c) + a)^2 + (m^4 + 4m^3 + 4(b^2 m^2 + 2b^2 m + b^2) n^2 + 6m^2 + 4m + 1) x \cos(bn \log(x) + b \log(c) + a)^4) x^m}{(m^5 + 64(b^4 m + b^4) n^4 + 5m^4 + 10m^3 + 20(b^2 m^3 + 3b^2 m^2 + 3b^2 m + b^2) n^2 + 10m^2 + 5m + 1)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cos(a+b\*ln(c\*x\*\*n))\*\*4,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*log(c\*x^n))^4,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

### 3.124 $\int x^m \cos^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=201

$$\frac{6b^3n^3x^{m+1} \sin(a + b \log(cx^n))}{(b^2n^2 + (m+1)^2)(9b^2n^2 + (m+1)^2)} + \frac{(m+1)x^{m+1} \cos^3(a + b \log(cx^n))}{9b^2n^2 + (m+1)^2} + \frac{6b^2(m+1)n^2x^{m+1} \cos(a + b \log(cx^n))}{(b^2n^2 + (m+1)^2)(9b^2n^2 + (m+1)^2)}$$

[Out]  $(6*b^2*(1+m)*n^2*x^(1+m)*Cos[a + b*Log[c*x^n]])/(((1+m)^2 + b^2*n^2)*((1+m)^2 + 9*b^2*n^2)) + ((1+m)*x^(1+m)*Cos[a + b*Log[c*x^n]]^3)/((1+m)^2 + 9*b^2*n^2) + (6*b^3*n^3*x^(1+m)*Sin[a + b*Log[c*x^n]])/(((1+m)^2 + b^2*n^2)*((1+m)^2 + 9*b^2*n^2)) + (3*b*n*x^(1+m)*Cos[a + b*Log[c*x^n]]^2*Sin[a + b*Log[c*x^n]])/((1+m)^2 + 9*b^2*n^2)$

**Rubi [A]** time = 0.0781204, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4488, 4486}

$$\frac{6b^3n^3x^{m+1} \sin(a + b \log(cx^n))}{(b^2n^2 + (m+1)^2)(9b^2n^2 + (m+1)^2)} + \frac{(m+1)x^{m+1} \cos^3(a + b \log(cx^n))}{9b^2n^2 + (m+1)^2} + \frac{6b^2(m+1)n^2x^{m+1} \cos(a + b \log(cx^n))}{(b^2n^2 + (m+1)^2)(9b^2n^2 + (m+1)^2)}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cos[a + b\*Log[c\*x^n]]^3,x]

[Out]  $(6*b^2*(1+m)*n^2*x^(1+m)*Cos[a + b*Log[c*x^n]])/(((1+m)^2 + b^2*n^2)*((1+m)^2 + 9*b^2*n^2)) + ((1+m)*x^(1+m)*Cos[a + b*Log[c*x^n]]^3)/((1+m)^2 + 9*b^2*n^2) + (6*b^3*n^3*x^(1+m)*Sin[a + b*Log[c*x^n]])/(((1+m)^2 + b^2*n^2)*((1+m)^2 + 9*b^2*n^2)) + (3*b*n*x^(1+m)*Cos[a + b*Log[c*x^n]]^2*Sin[a + b*Log[c*x^n]])/((1+m)^2 + 9*b^2*n^2)$

#### Rule 4488

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((m+1)\*(e\*x)^(m+1)\*Cos[d\*(a + b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p-1))/(b^2\*d^2\*n^2\*p^2 + (m+1)^2), Int[(e\*x)^(m)\*Cos[d\*(a + b\*Log[c\*x^n])]^(p-2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m+1)\*Sin[d\*(a + b\*Log[c\*x^n])]\*Cos[d\*(a + b\*Log[c\*x^n])]^(p-1)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m+1)^2, 0]

#### Rule 4486

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((m+1)\*(e\*x)^(m+1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m+1)^2), x] + Simp[(b\*d\*n\*(e\*x)^(m+1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m+1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b^2\*d^2\*n^2 + (m+1)^2, 0]

#### Rubi steps

$$\begin{aligned} \int x^m \cos^3(a + b \log(cx^n)) dx &= \frac{(1+m)x^{1+m} \cos^3(a + b \log(cx^n))}{(1+m)^2 + 9b^2n^2} + \frac{3bnx^{1+m} \cos^2(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{(1+m)^2 + 9b^2n^2} \\ &= \frac{6b^2(1+m)n^2x^{1+m} \cos(a + b \log(cx^n))}{((1+m)^2 + b^2n^2)((1+m)^2 + 9b^2n^2)} + \frac{(1+m)x^{1+m} \cos^3(a + b \log(cx^n))}{(1+m)^2 + 9b^2n^2} + \frac{6bnx^{1+m} \cos^2(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{(1+m)^2 + 9b^2n^2} \end{aligned}$$

**Mathematica [A]** time = 1.88161, size = 292, normalized size = 1.45

$$\frac{1}{4}x^{m+1} \left( -\frac{3 \sin(bn \log(x)) ((m+1) \sin(a + b \log(cx^n) - bn \log(x)) - bn \cos(a + b \log(cx^n) - bn \log(x)))}{b^2n^2 + m^2 + 2m + 1} + \frac{3 \cos(bn \log(x)) ((m+1) \cos(a + b \log(cx^n) - bn \log(x)) + bn \sin(a + b \log(cx^n) - bn \log(x)))}{b^2n^2 + m^2 + 2m + 1} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m * Cos[a + b * Log[c * x^n]]^3, x]
```

```
[Out] (x^(1 + m) * ((-3 * Sin[b * n * Log[x]] * (-b * n * Cos[a - b * n * Log[x] + b * Log[c * x^n]]) + (1 + m) * Sin[a - b * n * Log[x] + b * Log[c * x^n]])) / (1 + 2 * m + m^2 + b^2 * n^2) + (3 * Cos[b * n * Log[x]] * ((1 + m) * Cos[a - b * n * Log[x] + b * Log[c * x^n]] + b * n * Sin[a - b * n * Log[x] + b * Log[c * x^n]])) / (1 + 2 * m + m^2 + b^2 * n^2) - (Sin[3 * b * n * Log[x]] * (-3 * b * n * Cos[3 * (a - b * n * Log[x] + b * Log[c * x^n])] + (1 + m) * Sin[3 * (a - b * n * Log[x] + b * Log[c * x^n])]) / (1 + 2 * m + m^2 + 9 * b^2 * n^2) + (Cos[3 * b * n * Log[x]] * ((1 + m) * Cos[3 * (a - b * n * Log[x] + b * Log[c * x^n])] + 3 * b * n * Sin[3 * (a - b * n * Log[x] + b * Log[c * x^n])])) / (1 + 2 * m + m^2 + 9 * b^2 * n^2)) / 4
```

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int x^m (\cos(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m * cos(a + b * ln(c * x^n))^3, x)
```

```
[Out] int(x^m * cos(a + b * ln(c * x^n))^3, x)
```

**Maxima [B]** time = 1.5955, size = 3175, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m * cos(a + b * log(c * x^n))^3, x, algorithm="maxima")
```

```
[Out] 1/8 * (((cos(6 * b * log(c)) * cos(3 * b * log(c)) + sin(6 * b * log(c)) * sin(3 * b * log(c)) + cos(3 * b * log(c))) * m^3 + 3 * (b^3 * cos(3 * b * log(c)) * sin(6 * b * log(c)) - b^3 * cos(6 * b * log(c)) * sin(3 * b * log(c)) + b^3 * sin(3 * b * log(c))) * n^3 + 3 * (cos(6 * b * log(c)) * cos(3 * b * log(c)) + sin(6 * b * log(c)) * sin(3 * b * log(c)) + cos(3 * b * log(c))) * m^2 + (b^2 * cos(6 * b * log(c)) * cos(3 * b * log(c)) + b^2 * sin(6 * b * log(c)) * sin(3 * b * log(c)) + b^2 * cos(3 * b * log(c)) + (b^2 * cos(6 * b * log(c)) * cos(3 * b * log(c)) + b^2 * sin(6 * b * log(c)) * sin(3 * b * log(c)) + b^2 * cos(3 * b * log(c))) * m) * n^2 + 3 * (cos(6 * b * log(c)) * cos(3 * b * log(c)) + sin(6 * b * log(c)) * sin(3 * b * log(c)) + cos(3 * b * log(c))) * m + 3 * ((b * cos(3 * b * log(c)) * sin(6 * b * log(c)) - b * cos(6 * b * log(c)) * sin(3 * b * log(c)) + b * sin(3 * b * log(c))) * m^2 + b * cos(3 * b * log(c)) * sin(6 * b * log(c)) - b * cos(6 * b * log(c)) * sin(3 * b * log(c)) + 2 * (b * cos(3 * b * log(c)) * sin(6 * b * log(c)) - b * cos(6 * b * log(c)) * sin(3 * b * log(c)) + b * sin(3 * b * log(c))) * m + b * sin(3 * b * log(c))) * n + cos(6 * b * log(c)) * cos(3 * b * log(c)) + sin(6 * b * log(c)) * sin(3 * b * log(c)) + cos(3 * b * log(c))) * x * x^m * cos(3 * b * log(x^n) + 3 * a) + 3 * ((cos(4 * b * log(c)) * cos(3 * b * log(c)) + cos(3 * b * log(c)) * cos(2 * b * log(c)) + sin(4 * b * log(c)) * sin(3 * b * log(c)) + sin(3 * b * log(c)) * sin(2 * b * log(c))) * m^3 + 9 * (b^3 * cos(3 * b * log(c)) * sin(4 * b * log(c)) - b^3 * cos(4 * b * log(c)) * sin(3 * b * log(c)) + b^3 * cos(2 * b * log(c)) * sin(3 * b * log(c)) - b^3 * co
```

$$\begin{aligned}
& s(3*b*log(c))*sin(2*b*log(c))*n^3 + 3*(cos(4*b*log(c))*cos(3*b*log(c)) + c \\
& os(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b* \\
& log(c))*sin(2*b*log(c))*m^2 + 9*(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2 \\
& *cos(3*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^ \\
& 2*sin(3*b*log(c))*sin(2*b*log(c)) + (b^2*cos(4*b*log(c))*cos(3*b*log(c)) + \\
& b^2*cos(3*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + \\
& b^2*sin(3*b*log(c))*sin(2*b*log(c)))*m)*n^2 + 3*(cos(4*b*log(c))*cos(3*b*log \\
& (c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) \\
& + sin(3*b*log(c))*sin(2*b*log(c))*m + ((b*cos(3*b*log(c))*sin(4*b*log(c)) \\
& - b*cos(4*b*log(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b \\
& *cos(3*b*log(c))*sin(2*b*log(c)))*m^2 + b*cos(3*b*log(c))*sin(4*b*log(c)) - \\
& b*cos(4*b*log(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b* \\
& cos(3*b*log(c))*sin(2*b*log(c)) + 2*(b*cos(3*b*log(c))*sin(4*b*log(c)) - b* \\
& cos(4*b*log(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos \\
& (3*b*log(c))*sin(2*b*log(c)))*m)*n + cos(4*b*log(c))*cos(3*b*log(c)) + cos( \\
& 3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*log \\
& (c))*sin(2*b*log(c))*x*x^m*cos(b*log(x^n) + a) - ((cos(3*b*log(c))*sin(6*b \\
& *log(c)) - cos(6*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c)))*m^3 - 3*(b^3* \\
& cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*log(c)) + b^3 \\
& *cos(3*b*log(c)))*n^3 + 3*(cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c) \\
& )*sin(3*b*log(c)) + sin(3*b*log(c)))*m^2 + (b^2*cos(3*b*log(c))*sin(6*b*log \\
& (c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)) + (b^2*cos \\
& (3*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*si \\
& n(3*b*log(c)))*m)*n^2 + 3*(cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c) \\
& )*sin(3*b*log(c)) + sin(3*b*log(c)))*m - 3*((b*cos(6*b*log(c))*cos(3*b*log( \\
& c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b*log(c)))*m^2 + b*cos(6* \\
& b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + 2*(b*cos(6* \\
& b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b*log \\
& (c)))*m + b*cos(3*b*log(c))*n + cos(3*b*log(c))*sin(6*b*log(c)) - cos(6* \\
& b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*x*x^m*sin(3*b*log(x^n) + 3*a) \\
& - 3*((cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)) + c \\
& os(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*m^3 - 9*( \\
& b^3*cos(4*b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + \\
& b^3*sin(4*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c))) \\
& *n^3 + 3*(cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)) \\
& + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*m^2 + \\
& 9*(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c) \\
& )) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log( \\
& c)) + (b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log \\
& (c)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log \\
& (c)))*m)*n^2 + 3*(cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3 \\
& *b*log(c)) + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log( \\
& c)))*m - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log \\
& (c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c) \\
& ))*m^2 + b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log \\
& (c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c) \\
& ) + 2*(b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log(c) \\
& ) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))* \\
& m)*n + cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)) + \\
& cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c))*x*x^m*si \\
& n(b*log(x^n) + a)/((cos(3*b*log(c))^2 + sin(3*b*log(c))^2)*m^4 + 9*(b^4*co \\
& s(3*b*log(c))^2 + b^4*sin(3*b*log(c))^2)*n^4 + 4*(cos(3*b*log(c))^2 + sin(3 \\
& *b*log(c))^2)*m^3 + 6*(cos(3*b*log(c))^2 + sin(3*b*log(c))^2)*m^2 + 10*(b^2 \\
& *cos(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2 + (b^2*cos(3*b*log(c))^2 + b^2*si \\
& n(3*b*log(c))^2)*m^2 + 2*(b^2*cos(3*b*log(c))^2 + b^2*sin(3*b*log(c))^2)*m \\
& )*n^2 + 4*(cos(3*b*log(c))^2 + sin(3*b*log(c))^2)*m + cos(3*b*log(c))^2 + s \\
& in(3*b*log(c))^2)
\end{aligned}$$

**Fricas [A]** time = 0.53167, size = 467, normalized size = 2.32

$$\frac{3(2b^3n^3x + (b^3n^3 + (bm^2 + 2bm + b)n)x \cos(bn \log(x) + b \log(c) + a)^2)x^m \sin(bn \log(x) + b \log(c) + a) + (6(b^2m + 9b^4n^4 + m^4 + 4m^3 + 10(b^2m^2 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] (3\*(2\*b^3\*n^3\*x + (b^3\*n^3 + (b\*m^2 + 2\*b\*m + b)\*n)\*x\*cos(b\*n\*log(x) + b\*log(c) + a)^2)\*x^m\*sin(b\*n\*log(x) + b\*log(c) + a) + (6\*(b^2\*m + b^2)\*n^2\*x\*cos(b\*n\*log(x) + b\*log(c) + a) + (m^3 + (b^2\*m + b^2)\*n^2 + 3\*m^2 + 3\*m + 1)\*x\*cos(b\*n\*log(x) + b\*log(c) + a)^3)\*x^m)/(9\*b^4\*n^4 + m^4 + 4\*m^3 + 10\*(b^2\*m^2 + 2\*b^2\*m + b^2)\*n^2 + 6\*m^2 + 4\*m + 1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cos(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

### 3.125 $\int x^m \cos^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=120

$$\frac{(m+1)x^{m+1} \cos^2(a + b \log(cx^n))}{4b^2n^2 + (m+1)^2} + \frac{2bnx^{m+1} \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + (m+1)^2} + \frac{2b^2n^2x^{m+1}}{(m+1)(4b^2n^2 + (m+1)^2)}$$

[Out]  $(2*b^2*n^2*x^(1+m))/((1+m)*((1+m)^2+4*b^2*n^2)) + ((1+m)*x^(1+m))*Cos[a+b*Log[c*x^n]]^2/((1+m)^2+4*b^2*n^2) + (2*b*n*x^(1+m))*Cos[a+b*Log[c*x^n]]*Sin[a+b*Log[c*x^n]]/((1+m)^2+4*b^2*n^2)$

**Rubi [A]** time = 0.0318313, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {4488, 30}

$$\frac{(m+1)x^{m+1} \cos^2(a + b \log(cx^n))}{4b^2n^2 + (m+1)^2} + \frac{2bnx^{m+1} \sin(a + b \log(cx^n)) \cos(a + b \log(cx^n))}{4b^2n^2 + (m+1)^2} + \frac{2b^2n^2x^{m+1}}{(m+1)(4b^2n^2 + (m+1)^2)}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cos[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(2*b^2*n^2*x^(1+m))/((1+m)*((1+m)^2+4*b^2*n^2)) + ((1+m)*x^(1+m))*Cos[a+b*Log[c*x^n]]^2/((1+m)^2+4*b^2*n^2) + (2*b*n*x^(1+m))*Cos[a+b*Log[c*x^n]]*Sin[a+b*Log[c*x^n]]/((1+m)^2+4*b^2*n^2)$

#### Rule 4488

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((m+1)\*(e\*x)^(m+1)\*Cos[d\*(a+b\*Log[c\*x^n])]^p)/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x] + (Dist[(b^2\*d^2\*n^2\*p\*(p-1))/(b^2\*d^2\*n^2\*p^2 + (m+1)^2), Int[(e\*x)^m\*Cos[d\*(a+b\*Log[c\*x^n])]^(p-2), x], x] + Simp[(b\*d\*n\*p\*(e\*x)^(m+1)\*Sin[d\*(a+b\*Log[c\*x^n])]\*Cos[d\*(a+b\*Log[c\*x^n])]^(p-1))/(b^2\*d^2\*e\*n^2\*p^2 + e\*(m+1)^2), x]) /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2\*d^2\*n^2\*p^2 + (m+1)^2, 0]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^m \cos^2(a + b \log(cx^n)) dx &= \frac{(1+m)x^{1+m} \cos^2(a + b \log(cx^n))}{(1+m)^2 + 4b^2n^2} + \frac{2bnx^{1+m} \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{(1+m)^2 + 4b^2n^2} \\ &= \frac{2b^2n^2x^{1+m}}{(1+m)((1+m)^2 + 4b^2n^2)} + \frac{(1+m)x^{1+m} \cos^2(a + b \log(cx^n))}{(1+m)^2 + 4b^2n^2} + \frac{2bnx^{1+m} \cos(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{(1+m)^2 + 4b^2n^2} \end{aligned}$$

**Mathematica [C]** time = 0.319739, size = 91, normalized size = 0.76

$$\frac{x^{m+1} (2b(m+1)n \sin(2(a + b \log(cx^n))) + (m+1)^2 \cos(2(a + b \log(cx^n))) + 4b^2n^2 + m^2 + 2m + 1)}{2(m+1)(-2ibn + m + 1)(2ibn + m + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*cos[a + b\*Log[c\*x^n]]^2,x]

[Out] (x^(1 + m)\*(1 + 2\*m + m^2 + 4\*b^2\*n^2 + (1 + m)^2\*cos[2\*(a + b\*Log[c\*x^n])] + 2\*b\*(1 + m)\*n\*sin[2\*(a + b\*Log[c\*x^n])]))/(2\*(1 + m)\*(1 + m - (2\*I)\*b\*n) \* (1 + m + (2\*I)\*b\*n))

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int x^m (\cos(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cos(a+b\*ln(c\*x^n))^2,x)

[Out] int(x^m\*cos(a+b\*ln(c\*x^n))^2,x)

**Maxima [B]** time = 1.25935, size = 872, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 1/4\*(((cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)) + cos(2\*b\*log(c)))^2\*m^2 + 2\*(cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)) + cos(2\*b\*log(c)))^2\*m + 2\*(b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)) + (b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)) + b\*sin(2\*b\*log(c)))^2\*m + b\*sin(2\*b\*log(c)))^2\*n + cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)) + cos(2\*b\*log(c)))^2\*x\*x^m\*cos(2\*b\*log(x^n) + 2\*a) - ((cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c)) + sin(2\*b\*log(c)))^2\*m^2 + 2\*(cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c)) + sin(2\*b\*log(c)))^2\*m - 2\*(b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)) + (b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)) + b\*cos(2\*b\*log(c)))^2\*m + b\*cos(2\*b\*log(c)))^2\*n + cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c)) + sin(2\*b\*log(c)))^2\*x\*x^m\*sin(2\*b\*log(x^n) + 2\*a) + 2\*((cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*m^2 + 4\*(b^2\*cos(2\*b\*log(c))^2 + b^2\*sin(2\*b\*log(c))^2)\*n^2 + 2\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*m + cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*x\*x^m)/((cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*m^3 + 3\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*m^2 + 4\*(b^2\*cos(2\*b\*log(c))^2 + b^2\*sin(2\*b\*log(c))^2 + (b^2\*cos(2\*b\*log(c))^2 + b^2\*sin(2\*b\*log(c))^2)\*m)\*n^2 + 3\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*m + cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)

**Fricas [A]** time = 0.501269, size = 275, normalized size = 2.29

$$\frac{2(bm + b)nxx^m \cos(bn \log(x) + b \log(c) + a) \sin(bn \log(x) + b \log(c) + a) + (2b^2n^2x + (m^2 + 2m + 1)x \cos(bn \log(x) + b \log(c) + a))}{m^3 + 4(b^2m + b^2)n^2 + 3m^2 + 3m + 1}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] (2*(b*m + b)*n*x*x^m*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c) + a) + (2*b^2*n^2*x + (m^2 + 2*m + 1)*x*cos(b*n*log(x) + b*log(c) + a)^2)*x^m)/(m^3 + 4*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*ln(c*x**n))**2,x)
```

```
[Out] Exception raised: TypeError
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.126 $\int x^m \cos(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=70

$$\frac{bnx^{m+1} \sin(a + b \log(cx^n))}{b^2n^2 + (m+1)^2} + \frac{(m+1)x^{m+1} \cos(a + b \log(cx^n))}{b^2n^2 + (m+1)^2}$$

[Out]  $((1 + m)*x^{(1 + m)}*\text{Cos}[a + b*\text{Log}[c*x^n]])/((1 + m)^2 + b^2*n^2) + (b*n*x^{(1 + m)}*\text{Sin}[a + b*\text{Log}[c*x^n]])/((1 + m)^2 + b^2*n^2)$

**Rubi [A]** time = 0.0164579, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {4486}

$$\frac{bnx^{m+1} \sin(a + b \log(cx^n))}{b^2n^2 + (m+1)^2} + \frac{(m+1)x^{m+1} \cos(a + b \log(cx^n))}{b^2n^2 + (m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cos[a + b\*Log[c\*x^n]],x]

[Out]  $((1 + m)*x^{(1 + m)}*\text{Cos}[a + b*\text{Log}[c*x^n]])/((1 + m)^2 + b^2*n^2) + (b*n*x^{(1 + m)}*\text{Sin}[a + b*\text{Log}[c*x^n]])/((1 + m)^2 + b^2*n^2)$

#### Rule 4486

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((m + 1)\*(e\*x)^(m + 1)\*Cos[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] + Simp[(b\*d\*n\*(e\*x)^(m + 1)\*Sin[d\*(a + b\*Log[c\*x^n])])/(b^2\*d^2\*e\*n^2 + e\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, e, m, n}, x] & & NeQ[b^2\*d^2\*n^2 + (m + 1)^2, 0]

#### Rubi steps

$$\int x^m \cos(a + b \log(cx^n)) dx = \frac{(1 + m)x^{1+m} \cos(a + b \log(cx^n))}{(1 + m)^2 + b^2n^2} + \frac{bnx^{1+m} \sin(a + b \log(cx^n))}{(1 + m)^2 + b^2n^2}$$

**Mathematica [A]** time = 0.129747, size = 53, normalized size = 0.76

$$\frac{x^{m+1} ((m+1) \cos(a + b \log(cx^n)) + bn \sin(a + b \log(cx^n)))}{b^2n^2 + m^2 + 2m + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Cos[a + b\*Log[c\*x^n]],x]

[Out]  $(x^{(1 + m)}*((1 + m)*\text{Cos}[a + b*\text{Log}[c*x^n]] + b*n*\text{Sin}[a + b*\text{Log}[c*x^n]]))/((1 + 2*m + m^2 + b^2*n^2))$

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int x^m \cos(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*cos(a+b*ln(c*x^n)),x)`

[Out] `int(x^m*cos(a+b*ln(c*x^n)),x)`

**Maxima [B]** time = 1.1769, size = 423, normalized size = 6.04

$$\frac{((\cos(2b \log(c)) \cos(b \log(c)) + \sin(2b \log(c)) \sin(b \log(c)) + \cos(b \log(c)))^m + (b \cos(b \log(c)) \sin(2b \log(c)) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*cos(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out] 
$$\frac{1}{2} \left( (\cos(2b \log(c)) \cos(b \log(c)) + \sin(2b \log(c)) \sin(b \log(c)) + \cos(b \log(c)))^m + (b \cos(b \log(c)) \sin(2b \log(c)) - b \cos(2b \log(c)) \sin(b \log(c)) + b \sin(b \log(c)))^n + \cos(2b \log(c)) \cos(b \log(c)) + \sin(2b \log(c)) \sin(b \log(c)) + \cos(b \log(c)) \right) x^m \cos(b \log(x^n) + a) - ((\cos(b \log(c)) \sin(2b \log(c)) - \cos(2b \log(c)) \sin(b \log(c)) + \sin(b \log(c)))^m - (b \cos(2b \log(c)) \cos(b \log(c)) + b \sin(2b \log(c)) \sin(b \log(c)) + b \cos(b \log(c)))^n + \cos(b \log(c)) \sin(2b \log(c)) - \cos(2b \log(c)) \sin(b \log(c)) + \sin(b \log(c))) x^m \sin(b \log(x^n) + a) / ((\cos(b \log(c))^2 + \sin(b \log(c))^2)^m + (b^2 \cos(b \log(c))^2 + b^2 \sin(b \log(c))^2)^n + 2(\cos(b \log(c))^2 + \sin(b \log(c))^2)^m + \cos(b \log(c))^2 + \sin(b \log(c))^2)$$

**Fricas [A]** time = 0.484151, size = 158, normalized size = 2.26

$$\frac{bnx^m \sin(bn \log(x) + b \log(c) + a) + (m+1)xx^m \cos(bn \log(x) + b \log(c) + a)}{b^2n^2 + m^2 + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*cos(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out] 
$$(b \cdot n \cdot x^m \cdot \sin(b \cdot n \cdot \log(x) + b \cdot \log(c) + a) + (m + 1) \cdot x^m \cdot \cos(b \cdot n \cdot \log(x) + b \cdot \log(c) + a)) / (b^2 \cdot n^2 + m^2 + 2 \cdot m + 1)$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*cos(a+b*ln(c*x**n)),x)`

[Out] Exception raised: TypeError

**Giac [B]** time = 1.69173, size = 6969, normalized size = 99.56

result too large to display







$$\begin{aligned} & \operatorname{gn}(x) - 1/4*\pi*m)^2*\tan(1/2*a)^2 + 2*m*\tan(1/2*b*n*\log(\operatorname{abs}(x)) + 1/2*b*\log(\operatorname{abs}(c)))^2 \\ & + 2*m*\tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)^2 + 2*m*\tan(1/2*a)^2 + m^2 \\ & + \tan(1/2*b*n*\log(\operatorname{abs}(x)) + 1/2*b*\log(\operatorname{abs}(c)))^2 + \tan(1/4*\pi*m*\operatorname{sgn}(x) - 1/4*\pi*m)^2 \\ & + \tan(1/2*a)^2 + 2*m + 1) \end{aligned}$$

### 3.127 $\int x^m \cos^{\frac{3}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \cos^{\frac{3}{2}}(a + b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(-\frac{3}{2}, -\frac{3bn+2im+2i}{4bn}, -\frac{-bn+2im+2i}{4bn}, -e^{2ia}(cx^n)^{2ib}\right)}{(-3ibn + 2m + 2)(1 + e^{2ia}(cx^n)^{2ib})^{3/2}}$$

[Out] (2\*x^(1 + m)\*Cos[a + b\*Log[c\*x^n]]^(3/2)\*Hypergeometric2F1[-3/2, -(2\*I + (2\*I)\*m + 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m - (3\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2))

**Rubi [A]** time = 0.10096, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4494, 4492, 364}

$$\frac{2x^{m+1} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-\frac{2i(m+1)}{bn} - 3\right); -\frac{2im-bn+2i}{4bn}; -e^{2ia}(cx^n)^{2ib}\right) \cos^{\frac{3}{2}}(a + b \log(cx^n))}{(-3ibn + 2m + 2)(1 + e^{2ia}(cx^n)^{2ib})^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cos[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^(1 + m)\*Cos[a + b\*Log[c\*x^n]]^(3/2)\*Hypergeometric2F1[-3/2, (-3 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m - (3\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2))

#### Rule 4494

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\begin{aligned} \int x^m \cos^{\frac{3}{2}}(a + b \log(cx^n)) dx &= \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \cos^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{\left(x^{1+m} (cx^n)^{\frac{3ib}{2}-\frac{1+m}{n}} \cos^{\frac{3}{2}}(a + b \log(cx^n))\right) \text{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1+m}{n}} (1 + e^{2ia} x^{2ib})^{3/2} dx, x, cx^n\right)}{n (1 + e^{2ia} (cx^n)^{2ib})^{3/2}} \\ &= \frac{2x^{1+m} \cos^{\frac{3}{2}}(a + b \log(cx^n)) {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 2m - 3ibn) (1 + e^{2ia} (cx^n)^{2ib})^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 2.00886, size = 204, normalized size = 1.57

$$\frac{x^{m+1} \left(6b^2 n^2 (1 + e^{2ia} (cx^n)^{2ib}) \text{Hypergeometric2F1}\left(1, -\frac{-3bn+2im+2i}{4bn}, -\frac{-5bn+2im+2i}{4bn}, -e^{2i(a+b \log(cx^n))}\right) + (ibn + 2m + 2)\right)}{(ibn + 2m + 2)(-3ibn + 2m + 2)(3ibn + 2m + 2)\sqrt{\cos(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m \* Cos[a + b \* Log[c \* x^n]]^(3/2), x]

[Out] (x^(1 + m) \* (6 \* b^2 \* n^2 \* (1 + E^((2 \* I) \* a) \* (c \* x^n)^((2 \* I) \* b))) \* Hypergeometric2F1[1, -(2 \* I + (2 \* I) \* m - 3 \* b \* n) / (4 \* b \* n), -(2 \* I + (2 \* I) \* m - 5 \* b \* n) / (4 \* b \* n), -E^((2 \* I) \* (a + b \* Log[c \* x^n]))] + (2 + 2 \* m + I \* b \* n) \* (4 \* (1 + m) \* Cos[a + b \* Log[c \* x^n]]^2 + 3 \* b \* n \* Sin[2 \* (a + b \* Log[c \* x^n])])) / ((2 + 2 \* m + I \* b \* n) \* (2 + 2 \* m - (3 \* I) \* b \* n) \* (2 + 2 \* m + (3 \* I) \* b \* n) \* Sqrt[Cos[a + b \* Log[c \* x^n]]])

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int x^m (\cos(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m \* cos(a + b \* ln(c \* x^n))^(3/2), x)

[Out] int(x^m \* cos(a + b \* ln(c \* x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \cos(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m \* cos(a + b \* log(c \* x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(x^m \* cos(b \* log(c \* x^n) + a)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cos(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \cos(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] integrate(x^m\*cos(b\*log(c\*x^n) + a)^(3/2), x)

### 3.128 $\int x^m \sqrt{\cos(a + b \log(cx^n))} dx$

**Optimal.** Leaf size=129

$$\frac{2x^{m+1} \sqrt{\cos(a + b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{bn+2im+2i}{4bn}, -\frac{-3bn+2im+2i}{4bn}, -e^{2ia}(cx^n)^{2ib}\right)}{(-ibn + 2m + 2) \sqrt{1 + e^{2ia}(cx^n)^{2ib}}}$$

[Out] (2\*x^(1 + m)\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*Hypergeometric2F1[-1/2, -(2\*I + (2\*I)\*m + b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/((2 + 2\*m - I\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

**Rubi [A]** time = 0.0990102, antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4494, 4492, 364}

$$\frac{2x^{m+1} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-\frac{2i(m+1)}{bn} - 1\right); -\frac{2im-3bn+2i}{4bn}; -e^{2ia}(cx^n)^{2ib}\right) \sqrt{\cos(a + b \log(cx^n))}}{(-ibn + 2m + 2) \sqrt{1 + e^{2ia}(cx^n)^{2ib}}}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sqrt[Cos[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x^(1 + m)\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*Hypergeometric2F1[-1/2, (-1 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/((2 + 2\*m - I\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])

#### Rule 4494

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \sqrt{\cos(a + b \log(cx^n))} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sqrt{\cos(a + b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{\frac{ib}{2}-\frac{1+m}{n}} \sqrt{\cos(a + b \log(cx^n))}\right) \text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1+m}{n}} \sqrt{1 + e^{2ia} x^{2ib}} dx, x, cx^n\right)}{n\sqrt{1 + e^{2ia} (cx^n)^{2ib}}}$$

$$= \frac{2x^{1+m} \sqrt{\cos(a + b \log(cx^n))} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \left(-1 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-3bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 2m - ibn)\sqrt{1 + e^{2ia} (cx^n)^{2ib}}}$$

**Mathematica [B]** time = 5.23837, size = 436, normalized size = 3.38

$$\frac{2x^{m+1} \sqrt{\cos(a + b \log(cx^n))} \cos(a + b \log(cx^n) - bn \log(x))}{2(m+1) \cos(a + b \log(cx^n) - bn \log(x)) - bn \sin(a + b \log(cx^n) - bn \log(x))} - \frac{2e^{ia} b n x^{m+1} (cx^n)^{ib} \sqrt{2 + 2e^{2ia} (cx^n)^{2ib}}}{(2 + 2m - ibn)\sqrt{1 + e^{2ia} (cx^n)^{2ib}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Sqrt[Cos[a + b\*Log[c\*x^n]]],x]

[Out]  $(-2*b*E^{(I*a)*n}*x^{(1+m)}*(c*x^n)^{(I*b)}*\text{Sqrt}[2 + 2*E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}]*((2*I + (2*I)*m + b*n)*x^{((2*I)*b*n)}*\text{Hypergeometric2F1}[1/2, ((-I/2)*(1+m + ((3*I)/2)*b*n))/(b*n), -(2*I + (2*I)*m - 7*b*n)/(4*b*n), -E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}]) + (-2*I - (2*I)*m + 3*b*n)*\text{Hypergeometric2F1}[1/2, -(2*I + (2*I)*m + b*n)/(4*b*n), -(2*I + (2*I)*m - 3*b*n)/(4*b*n), -E^{((2*I)*a)*(c*x^n)^{(2*I)*b}})])/((2 + 2*m - I*b*n)*(2 + 2*m + (3*I)*b*n)*\text{Sqrt}[1/(E^{(I*a)*(c*x^n)^{(I*b)}} + E^{(I*a)*(c*x^n)^{(I*b)})]*((2 + 2*m - I*b*n)*x^{((2*I)*b*n)} + E^{((2*I)*a)*(2 + 2*m + I*b*n)*(c*x^n)^{(2*I)*b}})] + (2*x^{(1+m)}*\text{Sqrt}[Cos[a + b*Log[c*x^n]])]*Cos[a - b*n*Log[x] + b*Log[c*x^n]])/(2*(1+m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]] - b*n*Sin[a - b*n*Log[x] + b*Log[c*x^n]])$

**Maple [F]** time = 0.2, size = 0, normalized size = 0.

$$\int x^m \sqrt{\cos(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cos(a+b\*ln(c\*x^n))^(1/2),x)

[Out] int(x^m\*cos(a+b\*ln(c\*x^n))^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\cos(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cos(a+b\*log(c\*x^n))^(1/2),x, algorithm="maxima")

[Out] integrate(x<sup>m</sup>\*sqrt(cos(b\*log(c\*x<sup>n</sup>) + a)), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*cos(a+b\*log(c\*x<sup>n</sup>))<sup>(1/2)</sup>,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\cos(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cos(a+b\*ln(c\*x\*\*n))\*\*(1/2),x)

[Out] Integral(x\*\*m\*sqrt(cos(a + b\*log(c\*x\*\*n))), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\cos(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*cos(a+b\*log(c\*x<sup>n</sup>))<sup>(1/2)</sup>,x, algorithm="giac")

[Out] integrate(x<sup>m</sup>\*sqrt(cos(b\*log(c\*x<sup>n</sup>) + a)), x)

$$3.129 \quad \int \frac{x^m}{\sqrt{\cos(a+b \log(cx^n))}} dx$$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \sqrt{1 + e^{2ia} (cx^n)^{2ib}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{-bn+2im+2i}{4bn}, -\frac{-5bn+2im+2i}{4bn}, -e^{2ia} (cx^n)^{2ib}\right)}{(ibn + 2m + 2) \sqrt{\cos(a + b \log(cx^n))}}$$

[Out] (2\*x^(1 + m)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m + I\*b\*n)\*Sqrt[Cos[a + b\*Log[c\*x^n]]])

**Rubi [A]** time = 0.0928662, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4494, 4492, 364}

$$\frac{2x^{m+1} \sqrt{1 + e^{2ia} (cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, -\frac{2im-bn+2i}{4bn}; -\frac{2im-5bn+2i}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(ibn + 2m + 2) \sqrt{\cos(a + b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[x^m/Sqrt[Cos[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x^(1 + m)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m + I\*b\*n)\*Sqrt[Cos[a + b\*Log[c\*x^n]]])

#### Rule 4494

Int[Cos[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^(m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p)/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))]^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{x^m}{\sqrt{\cos(a + b \log(cx^n))}} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sqrt{\cos(a+b \log(x))}} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{ib}{2}-\frac{1+m}{n}} \sqrt{1 + e^{2ia} (cx^n)^{2ib}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1+m}{n}}}{\sqrt{1+e^{2ia}x^{2ib}}} dx, x, cx^n\right)}{n\sqrt{\cos(a + b \log(cx^n))}}$$

$$= \frac{2x^{1+m} \sqrt{1 + e^{2ia} (cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, -\frac{2i+2im-bn}{4bn}; -\frac{2i+2im-5bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 2m + ibn)\sqrt{\cos(a + b \log(cx^n))}}$$

**Mathematica [A]** time = 0.584231, size = 119, normalized size = 0.92

$$\frac{2x^{m+1} \left(1 + e^{2i(a+b \log(cx^n))}\right) \text{Hypergeometric2F1}\left(1, -\frac{-3bn+2im+2i}{4bn}, -\frac{-5bn+2im+2i}{4bn}, -e^{2i(a+b \log(cx^n))}\right)}{(ibn + 2m + 2)\sqrt{\cos(a + b \log(cx^n))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m/Sqrt[Cos[a + b\*Log[c\*x^n]]], x]

[Out] (2\*(1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))))\*x^(1 + m)\*Hypergeometric2F1[1, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n])))]/((2 + 2\*m + I\*b\*n)\*Sqrt[Cos[a + b\*Log[c\*x^n]])]

**Maple [F]** time = 0.178, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{\cos(a + b \ln(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/cos(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(x^m/cos(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\cos(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/cos(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(x^m/sqrt(cos(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\cos(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/cos(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x**m/sqrt(cos(a + b*log(c*x**n))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\cos(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(cos(b*log(c*x^n) + a)), x)
```



$$3.130 \quad \int \frac{x^m}{\cos^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{3/2} \text{Hypergeometric2F1}\left(\frac{3}{2}, -\frac{-3bn+2im+2i}{4bn}, -\frac{-7bn+2im+2i}{4bn}, -e^{2ia} (cx^n)^{2ib}\right)}{(3ibn + 2m + 2) \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

[Out] (2\*x^(1 + m)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Hypergeometric2F1[3/2, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m + (3\*I)\*b\*n)\*Cos[a + b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.0986783, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4494, 4492, 364}

$$\frac{2x^{m+1} \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4} \left(3 - \frac{2i(m+1)}{bn}\right); -\frac{2im-7bn+2i}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(3ibn + 2m + 2) \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x^m/Cos[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^(1 + m)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Hypergeometric2F1[3/2, (3 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m + (3\*I)\*b\*n)\*Cos[a + b\*Log[c\*x^n]]^(3/2))

#### Rule 4494

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a + b \log(cx^n))} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\cos^{\frac{3}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{3ib}{2}-\frac{1+m}{n}} \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{3/2}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1+m}{n}}}{(1+e^{2ia}x^{2ib})^{3/2}} dx, x, cx^n\right)}{n \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x^{1+m} \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4} \left(3 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-7bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 2m + 3ibn) \cos^{\frac{3}{2}}(a + b \log(cx^n))}$$

**Mathematica [B]** time = 5.05932, size = 487, normalized size = 3.75

$$x^{-ibn+m+1} \left( (b^2n^2 + 4m^2 + 8m + 4) x^{2ibn} \sqrt{2 + 2e^{2ia} (cx^n)^{2ib}} \sqrt{\cos(a + b \log(cx^n))} \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{i\left(\frac{3ibn}{2} + m + 1\right)}{2bn}, \dots\right) \right)$$

$bn(3bn)$

Warning: Unable to verify antiderivative.

[In] Integrate[x<sup>m</sup>/Cos[a + b\*Log[c\*x<sup>n</sup>]]<sup>(3/2)</sup>, x]

[Out] -((x<sup>(1 + m - I\*b\*n)</sup>\*(4 + 8\*m + 4\*m<sup>2</sup> + b<sup>2</sup>\*n<sup>2</sup>)\*x<sup>((2\*I)\*b\*n)</sup>\*Sqrt[2 + 2\*E<sup>((2\*I)\*a)</sup>\*(c\*x<sup>n</sup>)<sup>((2\*I)\*b)</sup>]\*Sqrt[Cos[a + b\*Log[c\*x<sup>n</sup>]]]\*Hypergeometric2F1[1/2, ((-I/2)\*(1 + m + ((3\*I)/2)\*b\*n))/(b\*n), -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), -(E<sup>((2\*I)\*a)</sup>\*(c\*x<sup>n</sup>)<sup>((2\*I)\*b)</sup>)] + (-2\*I - (2\*I)\*m + 3\*b\*n)\*((-2\*I - (2\*I)\*m + b\*n)\*Sqrt[2 + 2\*E<sup>((2\*I)\*a)</sup>\*(c\*x<sup>n</sup>)<sup>((2\*I)\*b)</sup>]\*Sqrt[Cos[a + b\*Log[c\*x<sup>n</sup>]]]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m + b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(E<sup>((2\*I)\*a)</sup>\*(c\*x<sup>n</sup>)<sup>((2\*I)\*b)</sup>)] - 2\*x<sup>(I\*b\*n)</sup>\*Sqrt[1/(E<sup>(I\*a)</sup>\*(c\*x<sup>n</sup>)<sup>(I\*b)</sup>) + E<sup>(I\*a)</sup>\*(c\*x<sup>n</sup>)<sup>(I\*b)</sup>]\*(b\*n\*Cos[b\*n\*Log[x]] - 2\*(1 + m)\*Sin[b\*n\*Log[x]])))/(b\*n\*(-2\*I - (2\*I)\*m + 3\*b\*n)\*Sqrt[1/(E<sup>(I\*a)</sup>\*(c\*x<sup>n</sup>)<sup>(I\*b)</sup>) + E<sup>(I\*a)</sup>\*(c\*x<sup>n</sup>)<sup>(I\*b)</sup>]\*Sqrt[Cos[a + b\*Log[c\*x<sup>n</sup>]]]\*(-2\*(1 + m)\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x<sup>n</sup>]] + b\*n\*Sin[a - b\*n\*Log[x] + b\*Log[c\*x<sup>n</sup>]])

**Maple [F]** time = 0.188, size = 0, normalized size = 0.

$$\int x^m (\cos(a + b \ln(cx^n)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>m</sup>/cos(a+b\*ln(c\*x<sup>n</sup>))<sup>(3/2)</sup>, x)

[Out] int(x<sup>m</sup>/cos(a+b\*ln(c\*x<sup>n</sup>))<sup>(3/2)</sup>, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\cos(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/cos(b*log(c*x^n) + a)^(3/2), x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/cos(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\cos(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/cos(b*log(c*x^n) + a)^(3/2), x)
```

$$3.131 \quad \int \frac{x^m}{\cos^2(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{2}, -\frac{-5bn+2im+2i}{4bn}, -\frac{-9bn+2im+2i}{4bn}, -e^{2ia} (cx^n)^{2ib}\right)}{(5ibn + 2m + 2) \cos^2(a + b \log(cx^n))}$$

[Out] (2\*x^(1 + m)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Hypergeometric2F1[5/2, -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 9\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m + (5\*I)\*b\*n)\*Cos[a + b\*Log[c\*x^n]]^(5/2))

**Rubi [A]** time = 0.0996792, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4494, 4492, 364}

$$\frac{2x^{m+1} \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4} \left(5 - \frac{2i(m+1)}{bn}\right); -\frac{2im-9bn+2i}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(5ibn + 2m + 2) \cos^2(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x^m/Cos[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x^(1 + m)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Hypergeometric2F1[5/2, (5 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 9\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m + (5\*I)\*b\*n)\*Cos[a + b\*Log[c\*x^n]]^(5/2))

#### Rule 4494

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a + b \log(cx^n))} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\cos^{\frac{5}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{5ib}{2}-\frac{1+m}{n}} (1 + e^{2ia} (cx^n)^{2ib})^{5/2}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1+m}{n}}}{(1+e^{2ia}x^{2ib})^{5/2}} dx, x, cx^n\right)}{n \cos^{\frac{5}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x^{1+m} (1 + e^{2ia} (cx^n)^{2ib})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4} \left(5 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-9bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 2m + 5ibn) \cos^{\frac{5}{2}}(a + b \log(cx^n))}$$

**Mathematica [A]** time = 2.21002, size = 205, normalized size = 1.58

$$2x^{m+1} \left(-ibn + 2m + 2\right) \left(1 + e^{2ia} (cx^n)^{2ib}\right) \cos(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, -\frac{-3bn+2im+2i}{4bn}, -\frac{-5bn+2im+2i}{4bn}, -\right)$$

3b<sup>2</sup>

Warning: Unable to verify antiderivative.

[In] Integrate[x<sup>m</sup>/Cos[a + b\*Log[c\*x<sup>n</sup>]]<sup>(5/2)</sup>, x]

[Out] (2\*x<sup>(1 + m)</sup>\*((2 + 2\*m - I\*b\*n)\*(1 + E<sup>((2\*I)\*a)</sup>\*(c\*x<sup>n</sup>)<sup>((2\*I)\*b)</sup>))\*Cos[a + b\*Log[c\*x<sup>n</sup>]]\*Hypergeometric2F1[1, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -E<sup>((2\*I)\*a + b\*Log[c\*x<sup>n</sup>])</sup>]] + b\*n\*Sec[a - b\*n\*Log[x] + b\*Log[c\*x<sup>n</sup>]]\*Sin[b\*n\*Log[x]] + Cos[a + b\*Log[c\*x<sup>n</sup>]]\*(-2\*(1 + m) + b\*n\*Tan[a - b\*n\*Log[x] + b\*Log[c\*x<sup>n</sup>]]))/(3\*b<sup>2</sup>\*n<sup>2</sup>\*Cos[a + b\*Log[c\*x<sup>n</sup>]]<sup>(3/2)</sup>)

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int x^m (\cos(a + b \ln(cx^n)))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>m</sup>/cos(a+b\*ln(c\*x<sup>n</sup>))<sup>(5/2)</sup>, x)

[Out] int(x<sup>m</sup>/cos(a+b\*ln(c\*x<sup>n</sup>))<sup>(5/2)</sup>, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\cos(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/cos(a+b\*log(c\*x<sup>n</sup>))<sup>(5/2)</sup>, x, algorithm="maxima")

[Out] integrate(x<sup>m</sup>/cos(b\*log(c\*x<sup>n</sup>) + a)<sup>(5/2)</sup>, x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/cos(a+b\*log(c\*x<sup>n</sup>))<sup>(5/2)</sup>,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/cos(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\cos(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/cos(a+b\*log(c\*x<sup>n</sup>))<sup>(5/2)</sup>,x, algorithm="giac")

[Out] integrate(x<sup>m</sup>/cos(b\*log(c\*x<sup>n</sup>) + a)<sup>(5/2)</sup>, x)

### 3.132 $\int (ex)^m \cos^p(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=144

$$\frac{(ex)^{m+1} \left(1 + e^{2iad} (cx^n)^{2ibd}\right)^{-p} \cos^p(d(a + b \log(cx^n))) \operatorname{Hypergeometric2F1}\left(-p, -\frac{bdnp+im+i}{2bdn}, \frac{1}{2}\left(-\frac{i(m+1)}{bdn} - p + 2\right), -e^{2iad} (cx^n)^{2ibd}\right)}{e(-ibdnp + m + 1)}$$

[Out]  $((e*x)^{(1+m)}*\operatorname{Cos}[d*(a + b*\operatorname{Log}[c*x^n])]^p*\operatorname{Hypergeometric2F1}[-p, -(I + I*m + b*d*n*p)/(2*b*d*n), (2 - (I*(1 + m))/(b*d*n) - p)/2, -(E^{((2*I)*a*d)}*(c*x^n)^{(2*I)*b*d})])/(e*(1 + m - I*b*d*n*p)*(1 + E^{((2*I)*a*d)}*(c*x^n)^{(2*I)*b*d}))^p$

**Rubi [A]** time = 0.103792, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4494, 4492, 364}

$$\frac{(ex)^{m+1} \left(1 + e^{2iad} (cx^n)^{2ibd}\right)^{-p} {}_2F_1\left(-p, -\frac{im+bdnp+i}{2bdn}; \frac{1}{2}\left(-\frac{i(m+1)}{bdn} - p + 2\right); -e^{2iad} (cx^n)^{2ibd}\right) \cos^p(d(a + b \log(cx^n)))}{e(-ibdnp + m + 1)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^m*\operatorname{Cos}[d*(a + b*\operatorname{Log}[c*x^n])]^p, x]$

[Out]  $((e*x)^{(1+m)}*\operatorname{Cos}[d*(a + b*\operatorname{Log}[c*x^n])]^p*\operatorname{Hypergeometric2F1}[-p, -(I + I*m + b*d*n*p)/(2*b*d*n), (2 - (I*(1 + m))/(b*d*n) - p)/2, -(E^{((2*I)*a*d)}*(c*x^n)^{(2*I)*b*d})])/(e*(1 + m - I*b*d*n*p)*(1 + E^{((2*I)*a*d)}*(c*x^n)^{(2*I)*b*d}))^p$

#### Rule 4494

$\operatorname{Int}[\operatorname{Cos}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]]*(b_.)*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] :> \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[x^{((m+1)/n-1)}*\operatorname{Cos}[d*(a + b*\operatorname{Log}[x])]^p, x], x, c*x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\operatorname{NeQ}[c, 1] \parallel \operatorname{NeQ}[n, 1])$

#### Rule 4492

$\operatorname{Int}[\operatorname{Cos}[(a_.) + \operatorname{Log}[x_]*](b_.)*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] :> \operatorname{Dist}[(\operatorname{Cos}[d*(a + b*\operatorname{Log}[x])]^p*x^{(I*b*d*p)})/(1 + E^{(2*I*a*d)}*x^{(2*I*b*d)})^p, \operatorname{Int}[(e*x)^m*(1 + E^{(2*I*a*d)}*x^{(2*I*b*d)})^p/x^{(I*b*d*p)}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, m, p\}, x] \&\& !\operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] :> \operatorname{Simp}[(a^p*(c*x)^{(m+1)}*\operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -((b*x^n)/a)])/(c*(m+1)), x] /; \operatorname{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \parallel \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\begin{aligned} \int (ex)^m \cos^p(d(a + b \log(cx^n))) dx &= \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \cos^p(d(a + b \log(x))) dx, x, cx^n\right)}{en} \\ &= \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}+ibdp} \left(1 + e^{2iad} (cx^n)^{2ibd}\right)^{-p} \cos^p(d(a + b \log(cx^n)))\right) \text{Subst}\left(\int x \right)}{en} \\ &= \frac{(ex)^{1+m} \left(1 + e^{2iad} (cx^n)^{2ibd}\right)^{-p} \cos^p(d(a + b \log(cx^n))) {}_2F_1\left(-p, -\frac{i+im+bdnp}{2bdn}; \frac{1}{2}\left(2 - \right)\right)}{e(1 + m - ibdnp)} \end{aligned}$$

**Mathematica [A]** time = 0.985341, size = 123, normalized size = 0.85

$$\frac{x(ex)^m \left(1 + e^{2id(a+b \log(cx^n))}\right) \cos^p(d(a + b \log(cx^n))) \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-\frac{i(m+1)}{bdn} + p + 2\right), -\frac{i(m+1)}{2bdn} - \frac{p}{2} + 1, -e^{2id(a+b \log(cx^n))}\right)}{-ibdnp + m + 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m \* Cos[d\*(a + b\*Log[c\*x^n])]^p, x]

[Out] ((1 + E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))) \* x \* (e\*x)^m \* Cos[d\*(a + b\*Log[c\*x^n])]^p \* Hypergeometric2F1[1, (2 - (I\*(1 + m))/(b\*d\*n) + p)/2, 1 - ((I/2)\*(1 + m))/(b\*d\*n) - p/2, -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) / (1 + m - I\*b\*d\*n\*p)

**Maple [F]** time = 0.185, size = 0, normalized size = 0.

$$\int (ex)^m (\cos(d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m \* cos(d\*(a+b\*ln(c\*x^n)))^p, x)

[Out] int((e\*x)^m \* cos(d\*(a+b\*ln(c\*x^n)))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cos((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m \* cos(d\*(a+b\*log(c\*x^n)))^p, x, algorithm="maxima")

[Out] integrate((e\*x)^m \* cos((b\*log(c\*x^n) + a)\*d)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \cos(bd \log(cx^n) + ad)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*cos(b*d*log(c*x^n) + a*d)^p, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cos(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.133 $\int x \cos^p (a + b \log (cx^n)) dx$

**Optimal.** Leaf size=114

$$\frac{x^2 (1 + e^{2ia} (cx^n)^{2ib})^{-p} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}\left(-p - \frac{2i}{bn}\right), -p, \frac{1}{2}\left(-\frac{2i}{bn} - p + 2\right), -e^{2ia} (cx^n)^{2ib}\right) \cos^p (a + b \log (cx^n))}{2 - ibnp}$$

[Out] (x^2\*Cos[a + b\*Log[c\*x^n]]^p\*Hypergeometric2F1[((-2\*I)/(b\*n) - p)/2, -p, (2 - (2\*I)/(b\*n) - p)/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/((2 - I\*b\*n\*p)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p

**Rubi [A]** time = 0.0768798, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4494, 4492, 364}

$$\frac{x^2 (1 + e^{2ia} (cx^n)^{2ib})^{-p} {}_2F_1\left(\frac{1}{2}\left(-p - \frac{2i}{bn}\right), -p; \frac{1}{2}\left(-p - \frac{2i}{bn} + 2\right); -e^{2ia} (cx^n)^{2ib}\right) \cos^p (a + b \log (cx^n))}{2 - ibnp}$$

Antiderivative was successfully verified.

[In] Int[x\*Cos[a + b\*Log[c\*x^n]]^p,x]

[Out] (x^2\*Cos[a + b\*Log[c\*x^n]]^p\*Hypergeometric2F1[((-2\*I)/(b\*n) - p)/2, -p, (2 - (2\*I)/(b\*n) - p)/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/((2 - I\*b\*n\*p)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p

#### Rule 4494

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x \cos^p(a + b \log(cx^n)) dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int x^{-1+\frac{2}{n}} \cos^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(x^2 (cx^n)^{-\frac{2}{n}+ibp} (1 + e^{2ia} (cx^n)^{2ib})^{-p} \cos^p(a + b \log(cx^n))) \text{Subst}\left(\int x^{-1+\frac{2}{n}-ibp} (1 + e^{2ia} (cx^n)^{2ib})^{-p} \cos^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{x^2 (1 + e^{2ia} (cx^n)^{2ib})^{-p} \cos^p(a + b \log(cx^n)) {}_2F_1\left(\frac{1}{2}\left(-\frac{2i}{bn} - p\right), -p; \frac{1}{2}\left(2 - \frac{2i}{bn} - p\right); -e^{2ia} (cx^n)^{2ib}\right)}{2 - ibnp} \end{aligned}$$

**Mathematica [A]** time = 0.633202, size = 102, normalized size = 0.89

$$\frac{ix^2 (1 + e^{2i(a+b \log(cx^n))}) \cos^p(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, -\frac{i}{bn} + \frac{p}{2} + 1, -\frac{i}{bn} - \frac{p}{2} + 1, -e^{2i(a+b \log(cx^n))}\right)}{bnp + 2i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Cos[a + b\*Log[c\*x^n]]^p,x]

[Out] (I\*(1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x^2 \* Cos[a + b\*Log[c\*x^n]]^p \* Hypergeometric2F1[1, 1 - I/(b\*n) + p/2, 1 - I/(b\*n) - p/2, -E^((2\*I)\*(a + b\*Log[c\*x^n]))]) / (2\*I + b\*n\*p)

**Maple [F]** time = 0.127, size = 0, normalized size = 0.

$$\int x (\cos(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cos(a+b\*ln(c\*x^n))^p,x)

[Out] int(x\*cos(a+b\*ln(c\*x^n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \cos(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cos(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] integrate(x\*cos(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \cos(b \log(cx^n) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(x*cos(b*log(c*x^n) + a)^p, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \cos^p(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(x*cos(a + b*log(c*x**n))**p, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \cos(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*log(c*x^n) + a)^p, x)
```

### 3.134 $\int \cos^p(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=112

$$\frac{x \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{-p} \operatorname{Hypergeometric2F1}\left(-p, -\frac{bnp+i}{2bn}, \frac{1}{2}\left(-\frac{i}{bn} - p + 2\right), -e^{2ia} (cx^n)^{2ib}\right) \cos^p(a + b \log(cx^n))}{1 - ibnp}$$

[Out] (x\*Cos[a + b\*Log[c\*x^n]]^p\*Hypergeometric2F1[-p, -(I + b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) - p)/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((1 - I\*b\*n\*p)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p)

**Rubi [A]** time = 0.0706092, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4484, 4492, 364}

$$\frac{x \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{-p} {}_2F_1\left(-p, -\frac{bnp+i}{2bn}; \frac{1}{2}\left(-p - \frac{i}{bn} + 2\right); -e^{2ia} (cx^n)^{2ib}\right) \cos^p(a + b \log(cx^n))}{1 - ibnp}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b\*Log[c\*x^n]]^p, x]

[Out] (x\*Cos[a + b\*Log[c\*x^n]]^p\*Hypergeometric2F1[-p, -(I + b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) - p)/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((1 - I\*b\*n\*p)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p)

#### Rule 4484

Int[Cos[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Cos[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4492

Int[Cos[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := > Dist[(Cos[d\*(a + b\*Log[x])]^p\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, Int[((e\*x)^m\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \cos^p(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int x^{-1+\frac{1}{n}} \cos^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{\left(x(cx^n)^{-\frac{1}{n}+ibp} (1 + e^{2ia} (cx^n)^{2ib})^{-p} \cos^p(a + b \log(cx^n))\right) \text{Subst}\left(\int x^{-1+\frac{1}{n}-ibp} (1 + e^{2ia} x^{2ib})^p\right)}{n} \\ &= \frac{x(1 + e^{2ia} (cx^n)^{2ib})^{-p} \cos^p(a + b \log(cx^n)) {}_2F_1\left(-p, -\frac{i+bnp}{2bn}; \frac{1}{2}\left(2 - \frac{i}{bn} - p\right); -e^{2ia} (cx^n)^{2ib}\right)}{1 - ibnp} \end{aligned}$$

**Mathematica [A]** time = 0.556962, size = 102, normalized size = 0.91

$$\frac{ix(1 + e^{2i(a+b \log(cx^n))}) \cos^p(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-\frac{i}{bn} + p + 2\right), -\frac{i}{2bn} - \frac{p}{2} + 1, -e^{2i(a+b \log(cx^n))}\right)}{bnp + i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[a + b\*Log[c\*x^n]]^p, x]

[Out] (I\*(1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))) \* x \* Cos[a + b\*Log[c\*x^n]]^p \* Hypergeometric2F1[1, (2 - I/(b\*n) + p)/2, 1 - (I/2)/(b\*n) - p/2, -E^((2\*I)\*(a + b\*Log[c\*x^n]))]) / (I + b\*n\*p)

**Maple [F]** time = 0.113, size = 0, normalized size = 0.

$$\int (\cos(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a+b\*ln(c\*x^n))^p, x)

[Out] int(cos(a+b\*ln(c\*x^n))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a+b\*log(c\*x^n))^p, x, algorithm="maxima")

[Out] integrate(cos(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\cos(b \log(cx^n) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(cos(b*log(c*x^n) + a)^p, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos^p(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(cos(a + b*log(c*x**n))**p, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \cos(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(cos(b*log(c*x^n) + a)^p, x)
```

### 3.135 $\int x^3 \tan(a + i \log(x)) dx$

**Optimal.** Leaf size=47

$$-ie^{2ia}x^2 + ie^{4ia} \log(x^2 + e^{2ia}) + \frac{ix^4}{4}$$

[Out]  $(-I)*E^{((2*I)*a)}*x^2 + (I/4)*x^4 + I*E^{((4*I)*a)}*Log[E^{((2*I)*a)} + x^2]$

**Rubi [F]** time = 0.0300232, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \tan(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*Tan[a + I\*Log[x]], x]

[Out] Defer[Int][x^3\*Tan[a + I\*Log[x]], x]

Rubi steps

$$\int x^3 \tan(a + i \log(x)) dx = \int x^3 \tan(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.0323966, size = 132, normalized size = 2.81

$$x^2 \sin(2a) - ix^2 \cos(2a) + \frac{1}{2}i \cos(4a) \log(2x^2 \cos(2a) + x^4 + 1) - \frac{1}{2} \sin(4a) \log(2x^2 \cos(2a) + x^4 + 1) + \cos(4a) \tan^{-1} \left( \frac{x^2 \cos(2a) + x^4 + 1}{2x^2 \cos(2a) + x^4 + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Tan[a + I\*Log[x]], x]

[Out]  $(I/4)*x^4 - I*x^2*\text{Cos}[2*a] + \text{ArcTan}[\frac{(1 + x^2)*\text{Cos}[a]}{\text{Sin}[a] - x^2*\text{Sin}[a]}] * \text{Cos}[4*a] + (I/2)*\text{Cos}[4*a]*\text{Log}[1 + x^4 + 2*x^2*\text{Cos}[2*a]] + x^2*\text{Sin}[2*a] + I*\text{ArcTan}[\frac{(1 + x^2)*\text{Cos}[a]}{\text{Sin}[a] - x^2*\text{Sin}[a]}] * \text{Sin}[4*a] - (\text{Log}[1 + x^4 + 2*x^2*\text{Cos}[2*a]] * \text{Sin}[4*a])/2$

**Maple [A]** time = 0.07, size = 49, normalized size = 1.

$$-\frac{i}{4}x^4 - i \left( x^2 (e^{ia})^2 - \frac{x^4}{2} - (e^{ia})^4 \ln \left( (e^{ia})^2 + x^2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*tan(a+I\*ln(x)), x)

[Out]  $-1/4*I*x^4 - I*(x^2*\exp(I*a)^2 - 1/2*x^4 - \exp(I*a)^4*\ln(\exp(I*a)^2 + x^2))$



---

**Maxima [B]** time = 1.05784, size = 122, normalized size = 2.6

$$\frac{1}{4}ix^4 + x^2(-i \cos(2a) + \sin(2a)) - \frac{1}{4}(4 \cos(4a) + 4i \sin(4a)) \arctan(\sin(2a), x^2 + \cos(2a)) + \frac{1}{2}(i \cos(4a) - \sin(4a)) \log(x^2 + \cos(2a) + \cos(2a)^2 + \sin(2a)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*tan(a+I\*log(x)),x, algorithm="maxima")

[Out] 1/4\*I\*x^4 + x^2\*(-I\*cos(2\*a) + sin(2\*a)) - 1/4\*(4\*cos(4\*a) + 4\*I\*sin(4\*a))\*arctan2(sin(2\*a), x^2 + cos(2\*a)) + 1/2\*(I\*cos(4\*a) - sin(4\*a))\*log(x^4 + 2\*x^2\*cos(2\*a) + cos(2\*a)^2 + sin(2\*a)^2)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-ix^3e^{(2ia-2\log(x))} + ix^3}{e^{(2ia-2\log(x))} + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*tan(a+I\*log(x)),x, algorithm="fricas")

[Out] integral((-I\*x^3\*e^(2\*I\*a - 2\*log(x)) + I\*x^3)/(e^(2\*I\*a - 2\*log(x)) + 1), x)

---

**Sympy [A]** time = 0.683258, size = 37, normalized size = 0.79

$$\frac{ix^4}{4} - ix^2e^{2ia} + ie^{4ia} \log(x^2 + e^{2ia})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*tan(a+I\*ln(x)),x)

[Out] I\*x\*\*4/4 - I\*x\*\*2\*exp(2\*I\*a) + I\*exp(4\*I\*a)\*log(x\*\*2 + exp(2\*I\*a))

---

**Giac [A]** time = 1.1721, size = 46, normalized size = 0.98

$$\frac{1}{4}ix^4 - ix^2e^{(2ia)} + ie^{(4ia)} \log(ix^2 + ie^{(2ia)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*tan(a+I\*log(x)),x, algorithm="giac")

[Out] 1/4\*I\*x^4 - I\*x^2\*e^(2\*I\*a) + I\*e^(4\*I\*a)\*log(I\*x^2 + I\*e^(2\*I\*a))

### 3.136 $\int x^2 \tan(a + i \log(x)) dx$

**Optimal.** Leaf size=43

$$-2ie^{2ia}x + 2ie^{3ia} \tan^{-1}(e^{-ia}x) + \frac{ix^3}{3}$$

[Out]  $(-2*I)*E^{((2*I)*a)*x} + (I/3)*x^3 + (2*I)*E^{((3*I)*a)*ArcTan[x/E^{(I*a)}]}$

**Rubi [F]** time = 0.022892, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \tan(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Tan[a + I\*Log[x]],x]

[Out] Defer[Int][x^2\*Tan[a + I\*Log[x]], x]

Rubi steps

$$\int x^2 \tan(a + i \log(x)) dx = \int x^2 \tan(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.0177957, size = 66, normalized size = 1.53

$$2x \sin(2a) - 2ix \cos(2a) + 2i \cos(3a) \tan^{-1}(x \cos(a) - ix \sin(a)) - 2 \sin(3a) \tan^{-1}(x \cos(a) - ix \sin(a)) + \frac{ix^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Tan[a + I\*Log[x]],x]

[Out]  $(I/3)*x^3 - (2*I)*x*\text{Cos}[2*a] + (2*I)*\text{ArcTan}[x*\text{Cos}[a] - I*x*\text{Sin}[a]]*\text{Cos}[3*a] + 2*x*\text{Sin}[2*a] - 2*\text{ArcTan}[x*\text{Cos}[a] - I*x*\text{Sin}[a]]*\text{Sin}[3*a]$

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int x^2 \tan(a + i \ln(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*tan(a+I\*ln(x)),x)

[Out] int(x^2\*tan(a+I\*ln(x)),x)

**Maxima [B]** time = 1.63307, size = 204, normalized size = 4.74

$$\frac{1}{3}ix^3 - 2x(i \cos(2a) - \sin(2a)) - (i \cos(3a) - \sin(3a)) \arctan\left(\frac{2x \cos(a)}{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}, \frac{x^2 - \cos(a)^2 - \sin(a)^2}{x^2 + \cos(a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*tan(a+I\*log(x)),x, algorithm="maxima")

[Out] 1/3\*I\*x^3 - 2\*x\*(I\*cos(2\*a) - sin(2\*a)) - (I\*cos(3\*a) - sin(3\*a))\*arctan2(2\*x\*cos(a)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2)) + 1/6\*(3\*cos(3\*a) + 3\*I\*sin(3\*a))\*log((x^2 + cos(a)^2 + 2\*x\*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-ix^2e^{(2ia-2\log(x))} + ix^2}{e^{(2ia-2\log(x))} + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*tan(a+I\*log(x)),x, algorithm="fricas")

[Out] integral((-I\*x^2\*e^(2\*I\*a - 2\*log(x)) + I\*x^2)/(e^(2\*I\*a - 2\*log(x)) + 1), x)

**Sympy [A]** time = 0.69732, size = 44, normalized size = 1.02

$$\frac{ix^3}{3} - 2ixe^{2ia} + (\log(x - ie^{ia}) - \log(x + ie^{ia}))e^{3ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*tan(a+I\*ln(x)),x)

[Out] I\*x\*\*3/3 - 2\*I\*x\*exp(2\*I\*a) + (log(x - I\*exp(I\*a)) - log(x + I\*exp(I\*a)))\*exp(3\*I\*a)

**Giac [A]** time = 1.16951, size = 35, normalized size = 0.81

$$\frac{1}{3}ix^3 + 2i \arctan(xe^{-ia})e^{(3ia)} - 2ixe^{(2ia)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*tan(a+I\*log(x)),x, algorithm="giac")

[Out] 1/3\*I\*x^3 + 2\*I\*arctan(x\*e^(-I\*a))\*e^(3\*I\*a) - 2\*I\*x\*e^(2\*I\*a)

### 3.137 $\int x \tan(a + i \log(x)) dx$

**Optimal.** Leaf size=33

$$\frac{ix^2}{2} - ie^{2ia} \log(x^2 + e^{2ia})$$

[Out] (I/2)\*x^2 - I\*E^((2\*I)\*a)\*Log[E^((2\*I)\*a) + x^2]

**Rubi [F]** time = 0.017491, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \tan(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x\*Tan[a + I\*Log[x]],x]

[Out] Defer[Int][x\*Tan[a + I\*Log[x]], x]

Rubi steps

$$\int x \tan(a + i \log(x)) dx = \int x \tan(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.0224594, size = 114, normalized size = 3.45

$$-\frac{1}{2}i \cos(2a) \log(2x^2 \cos(2a) + x^4 + 1) + \frac{1}{2} \sin(2a) \log(2x^2 \cos(2a) + x^4 + 1) - \cos(2a) \tan^{-1}\left(\frac{(x^2 + 1) \cos(a)}{\sin(a) - x^2 \sin(a)}\right) - i \sin(2a)$$

Antiderivative was successfully verified.

[In] Integrate[x\*Tan[a + I\*Log[x]],x]

[Out] (I/2)\*x^2 - ArcTan[((1 + x^2)\*Cos[a])/(Sin[a] - x^2\*Sin[a])]\*Cos[2\*a] - (I/2)\*Cos[2\*a]\*Log[1 + x^4 + 2\*x^2\*Cos[2\*a]] - I\*ArcTan[((1 + x^2)\*Cos[a])/(Sin[a] - x^2\*Sin[a])]\*Sin[2\*a] + (Log[1 + x^4 + 2\*x^2\*Cos[2\*a]]\*Sin[2\*a])/2

**Maple [A]** time = 0.062, size = 37, normalized size = 1.1

$$-\frac{i}{2}x^2 - i\left(-x^2 + (e^{ia})^2 \ln\left((e^{ia})^2 + x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*tan(a+I\*ln(x)),x)

[Out] -1/2\*I\*x^2-I\*(-x^2+exp(I\*a)^2\*ln(exp(I\*a)^2+x^2))

---

**Maxima [B]** time = 1.05837, size = 99, normalized size = 3.

$$\frac{1}{2}ix^2 + \frac{1}{2}(2\cos(2a) + 2i\sin(2a))\arctan(\sin(2a), x^2 + \cos(2a)) + \frac{1}{2}(-i\cos(2a) + \sin(2a))\log(x^4 + 2x^2\cos(2a) + \cos(2a)^2 + \sin(2a)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*tan(a+I\*log(x)),x, algorithm="maxima")

[Out] 1/2\*I\*x^2 + 1/2\*(2\*cos(2\*a) + 2\*I\*sin(2\*a))\*arctan2(sin(2\*a), x^2 + cos(2\*a)) + 1/2\*(-I\*cos(2\*a) + sin(2\*a))\*log(x^4 + 2\*x^2\*cos(2\*a) + cos(2\*a)^2 + sin(2\*a)^2)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-ix e^{(2ia-2)\log(x)} + ix}{e^{(2ia-2)\log(x)} + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*tan(a+I\*log(x)),x, algorithm="fricas")

[Out] integral((-I\*x\*e^(2\*I\*a - 2\*log(x)) + I\*x)/(e^(2\*I\*a - 2\*log(x)) + 1), x)

---

**Sympy [A]** time = 0.518396, size = 26, normalized size = 0.79

$$\frac{ix^2}{2} - ie^{2ia}\log(x^2 + e^{2ia})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*tan(a+I\*ln(x)),x)

[Out] I\*x\*\*2/2 - I\*exp(2\*I\*a)\*log(x\*\*2 + exp(2\*I\*a))

---

**Giac [A]** time = 1.19309, size = 34, normalized size = 1.03

$$\frac{1}{2}ix^2 - ie^{(2ia)}\log(-ix^2 - ie^{(2ia)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*tan(a+I\*log(x)),x, algorithm="giac")

[Out] 1/2\*I\*x^2 - I\*e^(2\*I\*a)\*log(-I\*x^2 - I\*e^(2\*I\*a))

### 3.138 $\int \tan(a + i \log(x)) dx$

**Optimal.** Leaf size=27

$$ix - 2ie^{ia} \tan^{-1}(e^{-ia}x)$$

[Out]  $I*x - (2*I)*E^{(I*a)}*ArcTan[x/E^{(I*a)}]$

**Rubi [F]** time = 0.0074411, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \tan(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In]  $Int[Tan[a + I*Log[x]], x]$

[Out]  $Defer[Int][Tan[a + I*Log[x]], x]$

Rubi steps

$$\int \tan(a + i \log(x)) dx = \int \tan(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.0071506, size = 42, normalized size = 1.56

$$-2i \cos(a) \tan^{-1}(x \cos(a) - ix \sin(a)) + 2 \sin(a) \tan^{-1}(x \cos(a) - ix \sin(a)) + ix$$

Antiderivative was successfully verified.

[In]  $Integrate[Tan[a + I*Log[x]], x]$

[Out]  $I*x - (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[a] + 2*ArcTan[x*Cos[a] - I*x*Sin[a]]*Sin[a]$

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \tan(a + i \ln(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $int(\tan(a+I*\ln(x)), x)$

[Out]  $int(\tan(a+I*\ln(x)), x)$

**Maxima [B]** time = 1.70817, size = 165, normalized size = 6.11

$$(i \cos(a) - \sin(a)) \arctan\left(\frac{2x \cos(a)}{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}, \frac{x^2 - \cos(a)^2 - \sin(a)^2}{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}\right) - \frac{1}{2}(\cos(a) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x)),x, algorithm="maxima")

[Out] (I\*cos(a) - sin(a))\*arctan2(2\*x\*cos(a)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2)) - 1/2\*(cos(a) + I\*sin(a))\*log((x^2 + cos(a)^2 + 2\*x\*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2)) + I\*x

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i e^{(2i a - 2 \log(x))} + i}{e^{(2i a - 2 \log(x))} + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x)),x, algorithm="fricas")

[Out] integral((-I\*e^(2\*I\*a - 2\*log(x)) + I)/(e^(2\*I\*a - 2\*log(x)) + 1), x)

**Sympy [A]** time = 0.50162, size = 27, normalized size = 1.

$$ix + (-\log(x - ie^{ia}) + \log(x + ie^{ia}))e^{ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*ln(x)),x)

[Out] I\*x + (-log(x - I\*exp(I\*a)) + log(x + I\*exp(I\*a)))\*exp(I\*a)

**Giac [A]** time = 1.14532, size = 41, normalized size = 1.52

$$\frac{2 \arctan\left(\frac{ix}{\sqrt{-e^{(2ia)}}}\right) e^{(2ia)}}{\sqrt{-e^{(2ia)}}} + ix$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x)),x, algorithm="giac")

[Out] 2\*arctan(I\*x/sqrt(-e^(2\*I\*a)))\*e^(2\*I\*a)/sqrt(-e^(2\*I\*a)) + I\*x

$$3.139 \quad \int \frac{\tan(a+i \log(x))}{x} dx$$

**Optimal.** Leaf size=14

$$i \log(\cos(a + i \log(x)))$$

[Out] I\*Log[Cos[a + I\*Log[x]]]

**Rubi [A]** time = 0.0127726, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3475}

$$i \log(\cos(a + i \log(x)))$$

Antiderivative was successfully verified.

[In] Int[Tan[a + I\*Log[x]]/x,x]

[Out] I\*Log[Cos[a + I\*Log[x]]]

**Rule 3475**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\tan(a + i \log(x))}{x} dx &= \text{Subst}\left(\int \tan(a + ix) dx, x, \log(x)\right) \\ &= i \log(\cos(a + i \log(x))) \end{aligned}$$

**Mathematica [A]** time = 0.021204, size = 14, normalized size = 1.

$$i \log(\cos(a + i \log(x)))$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I\*Log[x]]/x,x]

[Out] I\*Log[Cos[a + I\*Log[x]]]

**Maple [A]** time = 0.013, size = 17, normalized size = 1.2

$$-\frac{i}{2} \ln(1 + (\tan(a + i \ln(x)))^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+I\*ln(x))/x,x)



[Out]  $-1/2*I*\ln(1+\tan(a+I*\ln(x))^2)$

**Maxima [A]** time = 0.997935, size = 14, normalized size = 1.

$$-i \log(\sec(a + i \log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))/x,x, algorithm="maxima")`

[Out]  $-I*\log(\sec(a + I*\log(x)))$

**Fricas [A]** time = 0.485468, size = 59, normalized size = 4.21

$$i \log(x) + i \log(e^{(2i a - 2 \log(x))} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))/x,x, algorithm="fricas")`

[Out]  $I*\log(x) + I*\log(e^{(2*I*a - 2*\log(x))} + 1)$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*ln(x))/x,x)`

[Out] Exception raised: PolynomialError

**Giac [A]** time = 1.14129, size = 14, normalized size = 1.

$$i \log(\cos(a + i \log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))/x,x, algorithm="giac")`

[Out]  $I*\log(\cos(a + I*\log(x)))$

$$3.140 \quad \int \frac{\tan(a+i \log(x))}{x^2} dx$$

**Optimal.** Leaf size=29

$$2ie^{-ia} \tan^{-1}(e^{-ia}x) + \frac{i}{x}$$

[Out] I/x + ((2\*I)\*ArcTan[x/E^(I\*a)])/E^(I\*a)

**Rubi [F]** time = 0.0277522, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan(a + i \log(x))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + I\*Log[x]]/x^2, x]

[Out] Defer[Int][Tan[a + I\*Log[x]]/x^2, x]

Rubi steps

$$\int \frac{\tan(a + i \log(x))}{x^2} dx = \int \frac{\tan(a + i \log(x))}{x^2} dx$$

**Mathematica [A]** time = 0.0238917, size = 44, normalized size = 1.52

$$2i \cos(a) \tan^{-1}(x \cos(a) - ix \sin(a)) + 2 \sin(a) \tan^{-1}(x \cos(a) - ix \sin(a)) + \frac{i}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I\*Log[x]]/x^2, x]

[Out] I/x + (2\*I)\*ArcTan[x\*Cos[a] - I\*x\*Sin[a]]\*Cos[a] + 2\*ArcTan[x\*Cos[a] - I\*x\*Sin[a]]\*Sin[a]

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{\tan(a + i \ln(x))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+I\*ln(x))/x^2, x)

[Out] int(tan(a+I\*ln(x))/x^2, x)

**Maxima [B]** time = 1.63297, size = 171, normalized size = 5.9

$$\frac{2x(-i \cos(a) - \sin(a)) \arctan\left(\frac{2x \cos(a)}{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}, \frac{x^2 - \cos(a)^2 - \sin(a)^2}{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}\right) + x(\cos(a) - i \sin(a)) \log\left(\frac{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))/x^2,x, algorithm="maxima")

[Out] 1/2\*(2\*x\*(-I\*cos(a) - sin(a))\*arctan2(2\*x\*cos(a)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2)) + x\*(cos(a) - I\*sin(a))\*log((x^2 + cos(a)^2 + 2\*x\*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2\*x\*sin(a) + sin(a)^2)) + 2\*I)/x

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i e^{(2i a - 2 \log(x))} + i}{x^2 e^{(2i a - 2 \log(x))} + x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))/x^2,x, algorithm="fricas")

[Out] integral((-I\*e^(2\*I\*a - 2\*log(x)) + I)/(x^2\*e^(2\*I\*a - 2\*log(x)) + x^2), x)

**Sympy [A]** time = 0.456843, size = 27, normalized size = 0.93

$$\left(\log(x - i e^{ia}) - \log(x + i e^{ia})\right) e^{-ia} + \frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*ln(x))/x\*\*2,x)

[Out] (log(x - I\*exp(I\*a)) - log(x + I\*exp(I\*a)))\*exp(-I\*a) + I/x

**Giac [A]** time = 1.17649, size = 38, normalized size = 1.31

$$-\frac{2 \arctan\left(\frac{ix}{\sqrt{-e^{(2ia)}}}\right)}{\sqrt{-e^{(2ia)}}} + \frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))/x^2,x, algorithm="giac")

[Out] -2\*arctan(I\*x/sqrt(-e^(2\*I\*a)))/sqrt(-e^(2\*I\*a)) + I/x

$$3.141 \quad \int \frac{\tan(a+i \log(x))}{x^3} dx$$

**Optimal.** Leaf size=35

$$\frac{i}{2x^2} - ie^{-2ia} \log\left(1 + \frac{e^{2ia}}{x^2}\right)$$

[Out] (I/2)/x^2 - (I\*Log[1 + E^((2\*I)\*a)/x^2])/E^((2\*I)\*a)

**Rubi [F]** time = 0.0267757, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan(a + i \log(x))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + I\*Log[x]]/x^3, x]

[Out] Defer[Int][Tan[a + I\*Log[x]]/x^3, x]

Rubi steps

$$\int \frac{\tan(a + i \log(x))}{x^3} dx = \int \frac{\tan(a + i \log(x))}{x^3} dx$$

**Mathematica [B]** time = 0.0318199, size = 132, normalized size = 3.77

$$-\frac{1}{2}i \cos(2a) \log(2x^2 \cos(2a) + x^4 + 1) - \frac{1}{2} \sin(2a) \log(2x^2 \cos(2a) + x^4 + 1) + \cos(2a) \left( -\tan^{-1} \left( \frac{(x^2 + 1) \cos(a)}{\sin(a) - x^2 \sin(a)} \right) \right) +$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I\*Log[x]]/x^3, x]

[Out] (I/2)/x^2 - ArcTan[((1 + x^2)\*Cos[a])/(Sin[a] - x^2\*Sin[a])]\*Cos[2\*a] + (2\*I)\*Cos[2\*a]\*Log[x] - (I/2)\*Cos[2\*a]\*Log[1 + x^4 + 2\*x^2\*Cos[2\*a]] + I\*ArcTan[((1 + x^2)\*Cos[a])/(Sin[a] - x^2\*Sin[a])]\*Sin[2\*a] + 2\*Log[x]\*Sin[2\*a] - (Log[1 + x^4 + 2\*x^2\*Cos[2\*a]]\*Sin[2\*a])/2

**Maple [A]** time = 0.057, size = 43, normalized size = 1.2

$$\frac{i}{x^2} - i \left( \frac{\ln\left(\frac{(e^{ia})^2 + x^2}{(e^{ia})^2}\right)}{(e^{ia})^2} - 2 \frac{\ln(x)}{(e^{ia})^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+I\*ln(x))/x^3, x)

[Out]  $1/2*I/x^2 - I*(1/\exp(I*a)^2*\ln(\exp(I*a)^2+x^2) - 2/\exp(I*a)^2*\ln(x))$

**Maxima [B]** time = 1.08791, size = 130, normalized size = 3.71

$$\frac{x^2(i \cos(2a) + \sin(2a)) \log(x^4 + 2x^2 \cos(2a) + \cos(2a)^2 + \sin(2a)^2) - ((2 \cos(2a) - 2i \sin(2a)) \arctan(\sin(2a) / x^2))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))/x^3,x, algorithm="maxima")`

[Out]  $-1/2*(x^2*(I*\cos(2*a) + \sin(2*a))*\log(x^4 + 2*x^2*\cos(2*a) + \cos(2*a)^2 + \sin(2*a)^2) - ((2*\cos(2*a) - 2*I*\sin(2*a))*\arctan2(\sin(2*a), x^2 + \cos(2*a)) + 4*(I*\cos(2*a) + \sin(2*a))*\log(x))*x^2 - I)/x^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i e^{(2i a - 2 \log(x))} + i}{x^3 e^{(2i a - 2 \log(x))} + x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))/x^3,x, algorithm="fricas")`

[Out] `integral((-I*e^(2*I*a - 2*log(x)) + I)/(x^3*e^(2*I*a - 2*log(x)) + x^3), x)`

**Sympy [A]** time = 0.696203, size = 39, normalized size = 1.11

$$2ie^{-2ia} \log(x) - ie^{-2ia} \log(x^2 + e^{2ia}) + \frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*ln(x))/x**3,x)`

[Out] `2*I*exp(-2*I*a)*log(x) - I*exp(-2*I*a)*log(x**2 + exp(2*I*a)) + I/(2*x**2)`

**Giac [A]** time = 1.13705, size = 45, normalized size = 1.29

$$-i e^{(-2ia)} \log(-i x^2 - i e^{(2ia)}) + 2i e^{(-2ia)} \log(x) + \frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))/x^3,x, algorithm="giac")`

[Out] `-I*e^(-2*I*a)*log(-I*x^2 - I*e^(2*I*a)) + 2*I*e^(-2*I*a)*log(x) + 1/2*I/x^2`

$$3.142 \quad \int \frac{\tan(a+i \log(x))}{x^4} dx$$

**Optimal.** Leaf size=45

$$-\frac{2ie^{-2ia}}{x} - 2ie^{-3ia} \tan^{-1}(e^{-ia}x) + \frac{i}{3x^3}$$

[Out] (I/3)/x^3 - (2\*I)/(E^((2\*I)\*a)\*x) - ((2\*I)\*ArcTan[x/E^(I\*a)])/E^((3\*I)\*a)

**Rubi [F]** time = 0.0270119, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan(a + i \log(x))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + I\*Log[x]]/x^4, x]

[Out] Defer[Int][Tan[a + I\*Log[x]]/x^4, x]

Rubi steps

$$\int \frac{\tan(a + i \log(x))}{x^4} dx = \int \frac{\tan(a + i \log(x))}{x^4} dx$$

**Mathematica [A]** time = 0.0249357, size = 70, normalized size = 1.56

$$-\frac{2 \sin(2a)}{x} - \frac{2i \cos(2a)}{x} - 2i \cos(3a) \tan^{-1}(x \cos(a) - ix \sin(a)) - 2 \sin(3a) \tan^{-1}(x \cos(a) - ix \sin(a)) + \frac{i}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I\*Log[x]]/x^4, x]

[Out] (I/3)/x^3 - ((2\*I)\*Cos[2\*a])/x - (2\*I)\*ArcTan[x\*Cos[a] - I\*x\*Sin[a]]\*Cos[3\*a] - (2\*Sin[2\*a])/x - 2\*ArcTan[x\*Cos[a] - I\*x\*Sin[a]]\*Sin[3\*a]

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{\tan(a + i \ln(x))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+I\*ln(x))/x^4, x)

[Out] int(tan(a+I\*ln(x))/x^4, x)

**Maxima [B]** time = 1.67508, size = 212, normalized size = 4.71

$$\frac{6x^3(-i \cos(3a) - \sin(3a)) \arctan\left(\frac{2x \cos(a)}{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}, \frac{x^2 - \cos(a)^2 - \sin(a)^2}{x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2}\right) + x^3(3 \cos(3a) - 3i \sin(3a))}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))/x^4,x, algorithm="maxima")

[Out]  $-1/6*(6*x^3*(-I*\cos(3*a) - \sin(3*a))*\arctan2(2*x*\cos(a)/(x^2 + \cos(a)^2 - 2*x*\sin(a) + \sin(a)^2), (x^2 - \cos(a)^2 - \sin(a)^2)/(x^2 + \cos(a)^2 - 2*x*\sin(a) + \sin(a)^2)) + x^3*(3*\cos(3*a) - 3*I*\sin(3*a))*\log((x^2 + \cos(a)^2 + 2*x*\sin(a) + \sin(a)^2)/(x^2 + \cos(a)^2 - 2*x*\sin(a) + \sin(a)^2)) + 12*x^2*(I*\cos(2*a) + \sin(2*a)) - 2*I)/x^3$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-i e^{(2i a - 2 \log(x))} + i}{x^4 e^{(2i a - 2 \log(x))} + x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))/x^4,x, algorithm="fricas")

[Out] integral((-I\*e^(2\*I\*a - 2\*log(x)) + I)/(x^4\*e^(2\*I\*a - 2\*log(x)) + x^4), x)

**Sympy [A]** time = 1.25665, size = 53, normalized size = 1.18

$$\left(-\log(x - i e^{ia}) + \log(x + i e^{ia})\right) e^{-3ia} - \frac{(6ix^2 - ie^{2ia})e^{-2ia}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*ln(x))/x\*\*4,x)

[Out]  $(-\log(x - I*\exp(I*a)) + \log(x + I*\exp(I*a)))*\exp(-3*I*a) - (6*I*x**2 - I*\exp(2*I*a))*\exp(-2*I*a)/(3*x**3)$

**Giac [A]** time = 1.14238, size = 38, normalized size = 0.84

$$-2i \arctan\left(xe^{(-ia)}\right) e^{(-3ia)} - \frac{2i e^{(-2ia)}}{x} + \frac{i}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))/x^4,x, algorithm="giac")

[Out]  $-2*I*\arctan(x*e^{(-I*a)})*e^{(-3*I*a)} - 2*I*e^{(-2*I*a)}/x + 1/3*I/x^3$

### 3.143 $\int x^3 \tan^2(a + i \log(x)) dx$

**Optimal.** Leaf size=63

$$2e^{2ia}x^2 - \frac{2e^{6ia}}{x^2 + e^{2ia}} - 4e^{4ia} \log(x^2 + e^{2ia}) - \frac{x^4}{4}$$

[Out]  $2 * E^{((2 * I) * a)} * x^2 - x^4 / 4 - (2 * E^{((6 * I) * a)}) / (E^{((2 * I) * a)} + x^2) - 4 * E^{((4 * I) * a)} * \text{Log}[E^{((2 * I) * a)} + x^2]$

**Rubi [F]** time = 0.0699981, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \tan^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*Tan[a + I\*Log[x]]^2,x]

[Out] Defer[Int][x^3\*Tan[a + I\*Log[x]]^2, x]

Rubi steps

$$\int x^3 \tan^2(a + i \log(x)) dx = \int x^3 \tan^2(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.167011, size = 155, normalized size = 2.46

$$2ix^2 \sin(2a) + 2x^2 \cos(2a) - 2 \cos(4a) \log(2x^2 \cos(2a) + x^4 + 1) - \frac{2(\cos(5a) + i \sin(5a))}{(x^2 + 1) \cos(a) - i(x^2 - 1) \sin(a)} - 2i \sin(4a) \log(2x^2 \cos(2a) + x^4 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Tan[a + I\*Log[x]]^2,x]

[Out]  $-x^4/4 + 2*x^2*\text{Cos}[2*a] - (4*I)*\text{ArcTan}[\frac{(1 + x^2)*\text{Cot}[a]}{(-1 + x^2)}]*\text{Cos}[4*a] - 2*\text{Cos}[4*a]*\text{Log}[1 + x^4 + 2*x^2*\text{Cos}[2*a]] + (2*I)*x^2*\text{Sin}[2*a] + 4*\text{ArcTan}[\frac{(1 + x^2)*\text{Cot}[a]}{(-1 + x^2)}]*\text{Sin}[4*a] - (2*I)*\text{Log}[1 + x^4 + 2*x^2*\text{Cos}[2*a]]*\text{Sin}[4*a] - (2*(\text{Cos}[5*a] + I*\text{Sin}[5*a]))/((1 + x^2)*\text{Cos}[a] - I*(-1 + x^2)*\text{Sin}[a])$

**Maple [A]** time = 0.078, size = 62, normalized size = 1.

$$-\frac{9x^4}{4} + 2 \frac{x^4}{(e^{i(a+i \ln(x))})^2 + 1} + 4x^2 (e^{ia})^2 - 4 (e^{ia})^4 \ln((e^{ia})^2 + x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*tan(a+I\*ln(x))^2,x)



[Out]  $-9/4*x^4+2*x^4/(\exp(I*(a+I*\ln(x)))^2+1)+4*x^2*\exp(I*a)^2-4*\exp(I*a)^4*\ln(\exp(I*a)^2+x^2)$

**Maxima [B]** time = 1.15875, size = 312, normalized size = 4.95

$$x^6 - x^4(7 \cos(2a) + 7i \sin(2a)) - (16(-i \cos(4a) + \sin(4a)) \arctan(\sin(2a), x^2 + \cos(2a)) + 8 \cos(4a) + 8i \sin(4a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*tan(a+I\*log(x))^2,x, algorithm="maxima")

[Out]  $-(x^6 - x^4(7*\cos(2*a) + 7*I*\sin(2*a)) - (16*(-I*\cos(4*a) + \sin(4*a))*\arctan2(\sin(2*a), x^2 + \cos(2*a)) + 8*\cos(4*a) + 8*I*\sin(4*a))*x^2 - (16*(-I*\cos(2*a) + \sin(2*a))*\cos(4*a) + (16*\cos(2*a) + 16*I*\sin(2*a))*\sin(4*a))*\arctan2(\sin(2*a), x^2 + \cos(2*a)) + (x^2*(8*\cos(4*a) + 8*I*\sin(4*a)) + (8*\cos(2*a) + 8*I*\sin(2*a))*\cos(4*a) - 8*(-I*\cos(2*a) + \sin(2*a))*\sin(4*a))*\log(x^4 + 2*x^2*\cos(2*a) + \cos(2*a)^2 + \sin(2*a)^2) + 8*\cos(6*a) + 8*I*\sin(6*a))/(4*x^2 + 4*\cos(2*a) + 4*I*\sin(2*a))$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x^4 + (e^{(2ia-2 \log(x))} + 1) \operatorname{integral}\left(-\frac{x^3 e^{(2ia-2 \log(x))} + 9x^3}{e^{(2ia-2 \log(x))} + 1}, x\right)}{e^{(2ia-2 \log(x))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*tan(a+I\*log(x))^2,x, algorithm="fricas")

[Out]  $(2*x^4 + (e^{(2*I*a - 2*\log(x))} + 1)*\operatorname{integral}(-x^3*e^{(2*I*a - 2*\log(x))} + 9*x^3)/(e^{(2*I*a - 2*\log(x))} + 1), x))/(e^{(2*I*a - 2*\log(x))} + 1)$

**Sympy [A]** time = 0.672236, size = 54, normalized size = 0.86

$$-\frac{x^4}{4} + 2x^2 e^{2ia} - 4e^{4ia} \log(x^2 + e^{2ia}) - \frac{2e^{6ia}}{x^2 + e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*tan(a+I\*ln(x))\*\*2,x)

[Out]  $-x**4/4 + 2*x**2*\exp(2*I*a) - 4*\exp(4*I*a)*\log(x**2 + \exp(2*I*a)) - 2*\exp(6*I*a)/(x**2 + \exp(2*I*a))$

**Giac [B]** time = 1.2509, size = 352, normalized size = 5.59

$$-\frac{x^6}{4\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{3x^4 e^{(2ia)}}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} - \frac{4x^2 e^{(4ia)} \log(-x^2 - e^{(2ia)})}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} + \frac{17x^2 e^{(4ia)}}{4\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} - \frac{8e^{(6ia)} \log(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)})}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*tan(a+I\*log(x))^2,x, algorithm="giac")

[Out] 
$$-1/4*x^6/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) + 3/2*x^4*e^{(2*I*a)}/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) - 4*x^2*e^{(4*I*a)}*\log(-x^2 - e^{(2*I*a)})/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) + 17/4*x^2*e^{(4*I*a)}/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) - 8*e^{(6*I*a)}*\log(-x^2 - e^{(2*I*a)})/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) + e^{(6*I*a)}/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) - 4*e^{(8*I*a)}*\log(-x^2 - e^{(2*I*a)})/((x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)})*x^2) - 3/2*e^{(8*I*a)}/((x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)})*x^2)$$

### 3.144 $\int x^2 \tan^2(a + i \log(x)) dx$

**Optimal.** Leaf size=62

$$-\frac{2e^{2ia}x^3}{x^2 + e^{2ia}} + 6e^{2ia}x - 6e^{3ia} \tan^{-1}(e^{-ia}x) - \frac{x^3}{3}$$

[Out]  $6E^{((2I)*a)*x} - x^3/3 - (2E^{((2I)*a)*x^3})/(E^{((2I)*a)} + x^2) - 6E^{((3I)*a)*ArcTan[x/E^{(I*a)}]}$

**Rubi [F]** time = 0.0498105, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \tan^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Tan[a + I\*Log[x]]^2,x]

[Out] Defer[Int][x^2\*Tan[a + I\*Log[x]]^2, x]

Rubi steps

$$\int x^2 \tan^2(a + i \log(x)) dx = \int x^2 \tan^2(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.119622, size = 100, normalized size = 1.61

$$\frac{2x(\cos(3a) + i \sin(3a))}{(x^2 + 1) \cos(a) - i(x^2 - 1) \sin(a)} + 4ix \sin(2a) + 4x \cos(2a) - 6 \cos(3a) \tan^{-1}(x(\cos(a) - i \sin(a))) - 6i \sin(3a) \tan^{-1}(x(\cos(a) - i \sin(a)))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Tan[a + I\*Log[x]]^2,x]

[Out]  $-x^3/3 + 4*x*\text{Cos}[2*a] - 6*\text{ArcTan}[x*(\text{Cos}[a] - I*\text{Sin}[a])]*\text{Cos}[3*a] + (4*I)*x*\text{Sin}[2*a] + (2*x*(\text{Cos}[3*a] + I*\text{Sin}[3*a]))/((1 + x^2)*\text{Cos}[a] - I*(-1 + x^2)*\text{Sin}[a]) - (6*I)*\text{ArcTan}[x*(\text{Cos}[a] - I*\text{Sin}[a])]*\text{Sin}[3*a]$

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int x^2 (\tan(a + i \ln(x)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*tan(a+I\*ln(x))^2,x)

[Out] int(x^2\*tan(a+I\*ln(x))^2,x)

---

**Maxima [B]** time = 1.64488, size = 363, normalized size = 5.85

$$2x^5 - x^3(22 \cos(2a) + 22i \sin(2a)) - x(36 \cos(4a) + 36i \sin(4a)) - (x^2(18 \cos(3a) + 18i \sin(3a)) + (18 \cos(2a) + 18i \sin(2a)) \cos(3a) - 18(-I \cos(2a) + \sin(2a)) \sin(3a)) \arctan2(2x \cos(a)/(x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2), (x^2 - \cos(a)^2 - \sin(a)^2)/(x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2)) + (9x^2(-I \cos(3a) + \sin(3a)) + 9(-I \cos(2a) + \sin(2a)) \cos(3a) + (9 \cos(2a) + 9I \sin(2a)) \sin(3a)) \log((x^2 + \cos(a)^2 + 2x \sin(a) + \sin(a)^2)/(x^2 + \cos(a)^2 - 2x \sin(a) + \sin(a)^2)))/(6x^2 + 6 \cos(2a) + 6I \sin(2a))$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*tan(a+I\*log(x))^2,x, algorithm="maxima")

[Out] 
$$\frac{-2x^5 - x^3(22\cos(2a) + 22I\sin(2a)) - x(36\cos(4a) + 36I\sin(4a)) - (x^2(18\cos(3a) + 18I\sin(3a)) + (18\cos(2a) + 18I\sin(2a))\cos(3a) - 18(-I\cos(2a) + \sin(2a))\sin(3a))\arctan2(2x\cos(a)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2), (x^2 - \cos(a)^2 - \sin(a)^2)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2)) + (9x^2(-I\cos(3a) + \sin(3a)) + 9(-I\cos(2a) + \sin(2a))\cos(3a) + (9\cos(2a) + 9I\sin(2a))\sin(3a))\log((x^2 + \cos(a)^2 + 2x\sin(a) + \sin(a)^2)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2))}{6x^2 + 6\cos(2a) + 6I\sin(2a)}$$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x^3 + (e^{(2ia-2\log(x))} + 1)\operatorname{integral}\left(-\frac{x^2 e^{(2ia-2\log(x))+7x^2}}{e^{(2ia-2\log(x))+1}}, x\right)}{e^{(2ia-2\log(x))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*tan(a+I\*log(x))^2,x, algorithm="fricas")

[Out] 
$$\frac{(2x^3 + (e^{(2Ia - 2\log(x))} + 1)\operatorname{integral}(-(x^2 e^{(2Ia - 2\log(x))} + 7x^2)/(e^{(2Ia - 2\log(x))} + 1), x))/(e^{(2Ia - 2\log(x))} + 1)}$$

---

**Sympy [A]** time = 0.655233, size = 66, normalized size = 1.06

$$-\frac{x^3}{3} + 4xe^{2ia} + \frac{2xe^{4ia}}{x^2 + e^{2ia}} - 3(-i \log(x - ie^{ia}) + i \log(x + ie^{ia}))e^{3ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*tan(a+I\*ln(x))\*\*2,x)

[Out] 
$$-x^{**3}/3 + 4*x*\exp(2*I*a) + 2*x*\exp(4*I*a)/(x^{**2} + \exp(2*I*a)) - 3*(-I*\log(x - I*\exp(I*a)) + I*\log(x + I*\exp(I*a)))*\exp(3*I*a)$$

---

**Giac [B]** time = 1.24315, size = 190, normalized size = 3.06

$$-\frac{x^5}{3\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{10x^3 e^{(2ia)}}{3\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} - 6 \arctan\left(xe^{(-ia)}\right)e^{(3ia)} + \frac{35xe^{(4ia)}}{3\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{2xe^{(4ia)}}{x^2 + e^{(2ia)}} + \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*tan(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/3*x^5/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 10/3*x^3*e^(2*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) - 6*arctan(x*e^(-I*a))*e^(3*I*a) + 35/3*x*e^(4*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 2*x*e^(4*I*a)/(x^2 + e^(2*I*a)) + 8*e^(6*I*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x)
```

### 3.145 $\int x \tan^2(a + i \log(x)) dx$

**Optimal.** Leaf size=51

$$\frac{2e^{4ia}}{x^2 + e^{2ia}} + 2e^{2ia} \log(x^2 + e^{2ia}) - \frac{x^2}{2}$$

[Out]  $-x^2/2 + (2E^{((4*I)*a)})/(E^{((2*I)*a)} + x^2) + 2E^{((2*I)*a)}*Log[E^{((2*I)*a)} + x^2]$

**Rubi [F]** time = 0.0318918, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \tan^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x\*Tan[a + I\*Log[x]]^2, x]

[Out] Defer[Int][x\*Tan[a + I\*Log[x]]^2, x]

Rubi steps

$$\int x \tan^2(a + i \log(x)) dx = \int x \tan^2(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.113743, size = 135, normalized size = 2.65

$$\cos(2a) \log(2x^2 \cos(2a) + x^4 + 1) + \frac{2 \cos(3a) + 2i \sin(3a)}{(x^2 + 1) \cos(a) - i(x^2 - 1) \sin(a)} + i \sin(2a) \log(2x^2 \cos(2a) + x^4 + 1) + 2i \cos(2a)$$

Antiderivative was successfully verified.

[In] Integrate[x\*Tan[a + I\*Log[x]]^2, x]

[Out]  $-x^2/2 + (2*I)*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Cos[2*a] + Cos[2*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] - 2*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Sin[2*a] + I*Log[1 + x^4 + 2*x^2*Cos[2*a]]*Sin[2*a] + (2*Cos[3*a] + (2*I)*Sin[3*a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a])$

**Maple [A]** time = 0.072, size = 50, normalized size = 1.

$$-\frac{5x^2}{2} + 2 \frac{x^2}{(e^{i(a+i \ln(x))})^2 + 1} + 2 (e^{ia})^2 \ln((e^{ia})^2 + x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*tan(a+I\*ln(x))^2, x)

[Out]  $-5/2*x^2+2*x^2/(exp(I*(a+I*ln(x)))^2+1)+2*exp(I*a)^2*ln(exp(I*a)^2+x^2)$

**Maxima [B]** time = 1.02712, size = 261, normalized size = 5.12

$$\frac{x^4 + (4(-i \cos(2a) + \sin(2a)) \arctan(\sin(2a), x^2 + \cos(2a)) + \cos(2a) + i \sin(2a))x^2 - (4i \cos(2a)^2 - 8 \cos(2a))}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(a+I*log(x))^2,x, algorithm="maxima")`

[Out]  $-(x^4 + (4*(-I*\cos(2*a) + \sin(2*a))*\arctan2(\sin(2*a), x^2 + \cos(2*a)) + \cos(2*a) + I*\sin(2*a))*x^2 - (4*I*\cos(2*a)^2 - 8*\cos(2*a)*\sin(2*a) - 4*I*\sin(2*a)^2)*\arctan2(\sin(2*a), x^2 + \cos(2*a)) - (x^2*(2*\cos(2*a) + 2*I*\sin(2*a)) + 2*\cos(2*a)^2 + 4*I*\cos(2*a)*\sin(2*a) - 2*\sin(2*a)^2)*\log(x^4 + 2*x^2*\cos(2*a) + \cos(2*a)^2 + \sin(2*a)^2) - 4*\cos(4*a) - 4*I*\sin(4*a))/(2*x^2 + 2*\cos(2*a) + 2*I*\sin(2*a))$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x^2 + (e^{(2ia-2\log(x))} + 1)\operatorname{integral}\left(-\frac{xe^{(2ia-2\log(x))+5x}}{e^{(2ia-2\log(x))+1}}, x\right)}{e^{(2ia-2\log(x))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(a+I*log(x))^2,x, algorithm="fricas")`

[Out]  $(2*x^2 + (e^{(2*I*a - 2*\log(x))} + 1)*\operatorname{integral}(-(x*e^{(2*I*a - 2*\log(x))} + 5*x)/(e^{(2*I*a - 2*\log(x))} + 1), x))/(e^{(2*I*a - 2*\log(x))} + 1)$

**Sympy [A]** time = 0.742233, size = 42, normalized size = 0.82

$$-\frac{x^2}{2} + 2e^{2ia} \log(x^2 + e^{2ia}) + \frac{2e^{4ia}}{x^2 + e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(a+I*ln(x))**2,x)`

[Out]  $-x**2/2 + 2*exp(2*I*a)*log(x**2 + exp(2*I*a)) + 2*exp(4*I*a)/(x**2 + exp(2*I*a))$

**Giac [B]** time = 1.28563, size = 298, normalized size = 5.84

$$\frac{x^4}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{2x^2e^{(2ia)}\log(x^2 + e^{(2ia)})}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} - \frac{5x^2e^{(2ia)}}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{4e^{(4ia)}\log(x^2 + e^{(2ia)})}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} - \frac{3e^{(4ia)}}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*tan(a+I\*log(x))^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*x^4/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) + 2*x^2*e^{(2*I*a)}*\log(x^2 + e^{(2*I*a)})/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) - 5/2*x^2*e^{(2*I*a)}/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) \\ & + 4*e^{(4*I*a)}*\log(x^2 + e^{(2*I*a)})/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) - 3/2*e^{(4*I*a)}/(x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)}) + \\ & 2*e^{(6*I*a)}*\log(x^2 + e^{(2*I*a)})/((x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)})*x^2) \\ & + 1/2*e^{(6*I*a)}/((x^2 + e^{(4*I*a)}/x^2 + 2*e^{(2*I*a)})*x^2) \end{aligned}$$



### 3.146 $\int \tan^2(a + i \log(x)) dx$

**Optimal.** Leaf size=46

$$-\frac{2e^{2ia}x}{x^2 + e^{2ia}} + 2e^{ia} \tan^{-1}(e^{-ia}x) - x$$

[Out]  $-x - (2E^{((2*I)*a)*x})/(E^{((2*I)*a)} + x^2) + 2E^{(I*a)}*ArcTan[x/E^{(I*a)}]$

**Rubi [F]** time = 0.0104115, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \tan^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + I\*Log[x]]^2,x]

[Out] Defer[Int][Tan[a + I\*Log[x]]^2, x]

Rubi steps

$$\int \tan^2(a + i \log(x)) dx = \int \tan^2(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.0887918, size = 70, normalized size = 1.52

$$\frac{-x(x^2 + 3)\cos(a) + ix(x^2 - 3)\sin(a)}{(x^2 + 1)\cos(a) - i(x^2 - 1)\sin(a)} + 2(\cos(a) + i\sin(a))\tan^{-1}(x(\cos(a) - i\sin(a)))$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I\*Log[x]]^2,x]

[Out]  $2*ArcTan[x*(Cos[a] - I*Sin[a])]*(Cos[a] + I*Sin[a]) + (-x*(3 + x^2)*Cos[a] + I*x*(-3 + x^2)*Sin[a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a])$

**Maple [F]** time = 0.061, size = 0, normalized size = 0.

$$\int (\tan(a + i \ln(x)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+I\*ln(x))^2,x)

[Out] int(tan(a+I\*ln(x))^2,x)

**Maxima [B]** time = 1.59874, size = 305, normalized size = 6.63

$$2x^3 + x(6 \cos(2a) + 6i \sin(2a)) + (x^2(2 \cos(a) + 2i \sin(a)) + (2 \cos(a) + 2i \sin(a)) \cos(2a) - 2(-i \cos(a) + \sin(a)))$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))^2,x, algorithm="maxima")

[Out]  $-(2x^3 + x(6\cos(2a) + 6I\sin(2a))) + (x^2(2\cos(a) + 2I\sin(a)) + (2\cos(a) + 2I\sin(a))\cos(2a) - 2(-I\cos(a) + \sin(a))\sin(2a))\arctan(2x\cos(a)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2), (x^2 - \cos(a)^2 - \sin(a)^2)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2)) + (x^2(I\cos(a) - \sin(a)) + (I\cos(a) - \sin(a))\cos(2a) - (\cos(a) + I\sin(a))\sin(2a))\log((x^2 + \cos(a)^2 + 2x\sin(a) + \sin(a)^2)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2)))/(2x^2 + 2\cos(2a) + 2I\sin(2a))$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(e^{(2ia-2\log(x))} + 1)\operatorname{integral}\left(-\frac{e^{(2ia-2\log(x))+3}}{e^{(2ia-2\log(x))+1}}, x\right) + 2x}{e^{(2ia-2\log(x))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))^2,x, algorithm="fricas")

[Out]  $((e^{(2I*a - 2*\log(x))} + 1)*\operatorname{integral}(-(e^{(2I*a - 2*\log(x))} + 3)/(e^{(2I*a - 2*\log(x))} + 1), x) + 2*x)/(e^{(2I*a - 2*\log(x))} + 1)$

---

**Sympy [A]** time = 0.60995, size = 51, normalized size = 1.11

$$-x - \frac{2xe^{2ia}}{x^2 + e^{2ia}} - (i \log(x - ie^{ia}) - i \log(x + ie^{ia}))e^{ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*ln(x))\*\*2,x)

[Out]  $-x - 2*x*\exp(2*I*a)/(x**2 + \exp(2*I*a)) - (I*\log(x - I*\exp(I*a)) - I*\log(x + I*\exp(I*a)))*\exp(I*a)$

---

**Giac [B]** time = 1.18675, size = 154, normalized size = 3.35

$$-\frac{x^3}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} + 2\left(\arctan\left(xe^{(-ia)}\right)e^{(-ia)} - \frac{x}{x^2 + e^{(2ia)}}\right)e^{(2ia)} - \frac{6xe^{(2ia)}}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} - \frac{5e^{(4ia)}}{\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))^2,x, algorithm="giac")

```
[Out] -x^3/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 2*(arctan(x*e^(-I*a))*e^(-I*a) -  
x/(x^2 + e^(2*I*a)))*e^(2*I*a) - 6*x*e^(2*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(  
2*I*a)) - 5*e^(4*I*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x)
```

$$3.147 \quad \int \frac{\tan^2(a+i \log(x))}{x} dx$$

**Optimal.** Leaf size=18

$$-\log(x) - i \tan(a + i \log(x))$$

[Out] -Log[x] - I\*Tan[a + I\*Log[x]]

**Rubi [A]** time = 0.0249887, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3473, 8}

$$-\log(x) - i \tan(a + i \log(x))$$

Antiderivative was successfully verified.

[In] Int[Tan[a + I\*Log[x]]^2/x, x]

[Out] -Log[x] - I\*Tan[a + I\*Log[x]]

**Rule 3473**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

**Rule 8**

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\tan^2(a + i \log(x))}{x} dx &= \text{Subst} \left( \int \tan^2(a + ix) dx, x, \log(x) \right) \\ &= -i \tan(a + i \log(x)) - \text{Subst} \left( \int 1 dx, x, \log(x) \right) \\ &= -\log(x) - i \tan(a + i \log(x)) \end{aligned}$$

**Mathematica [A]** time = 0.0385149, size = 28, normalized size = 1.56

$$i \tan^{-1}(\tan(a + i \log(x))) - i \tan(a + i \log(x))$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I\*Log[x]]^2/x, x]

[Out] I\*ArcTan[Tan[a + I\*Log[x]]] - I\*Tan[a + I\*Log[x]]

**Maple [A]** time = 0.014, size = 23, normalized size = 1.3

$$-i \tan(a + i \ln(x)) + i(a + i \ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(a+I*ln(x))^2/x,x)`

[Out] `-I*tan(a+I*ln(x))+I*(a+I*ln(x))`

**Maxima [A]** time = 1.50866, size = 23, normalized size = 1.28

$$i a - \log(x) - i \tan(a + i \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))^2/x,x, algorithm="maxima")`

[Out] `I*a - log(x) - I*tan(a + I*log(x))`

**Fricas [B]** time = 0.463124, size = 97, normalized size = 5.39

$$-\frac{e^{(2i a - 2 \log(x))} \log(x) + \log(x) - 2}{e^{(2i a - 2 \log(x))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))^2/x,x, algorithm="fricas")`

[Out] `-(e^(2*I*a - 2*log(x))*log(x) + log(x) - 2)/(e^(2*I*a - 2*log(x)) + 1)`

**Sympy [A]** time = 0.529706, size = 22, normalized size = 1.22

$$-\log(x) - \frac{2e^{2ia}}{x^2 + e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*ln(x))**2/x,x)`

[Out] `-log(x) - 2*exp(2*I*a)/(x**2 + exp(2*I*a))`

**Giac [B]** time = 1.16253, size = 130, normalized size = 7.22

$$\frac{\frac{2 e^{(2i a) \log(x)}}{x^2} - \frac{e^{(2i a) \log\left(\frac{e^{(2i a)}}{x^2} + 1\right)}}{x^2} + \frac{e^{(2i a) \log\left(-\frac{e^{(2i a)}}{x^2} - 1\right)}}{x^2} + 2 \log(x) - \log\left(\frac{e^{(2i a)}}{x^2} + 1\right) + \log\left(-\frac{e^{(2i a)}}{x^2} - 1\right) - 4}{2\left(\frac{e^{(2i a)}}{x^2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+I*log(x))^2/x,x, algorithm="giac")`

```
[Out] -1/2*(2*e^(2*I*a)*log(x)/x^2 - e^(2*I*a)*log(e^(2*I*a)/x^2 + 1)/x^2 + e^(2*I*a)*log(-e^(2*I*a)/x^2 - 1)/x^2 + 2*log(x) - log(e^(2*I*a)/x^2 + 1) + log(-e^(2*I*a)/x^2 - 1) - 4)/(e^(2*I*a)/x^2 + 1)
```

$$3.148 \quad \int \frac{\tan^2(a+i \log(x))}{x^2} dx$$

**Optimal.** Leaf size=60

$$\frac{3x}{x^2 + e^{2ia}} + \frac{e^{2ia}}{x(x^2 + e^{2ia})} + 2e^{-ia} \tan^{-1}(e^{-ia}x)$$

[Out]  $E^{\left(\left(2I\right)a\right)} / \left(x \cdot \left(E^{\left(\left(2I\right)a\right)} + x^2\right)\right) + \left(3x\right) / \left(E^{\left(\left(2I\right)a\right)} + x^2\right) + \left(2 \cdot \text{ArcTan}\left[x / E^{\left(Ia\right)}\right]\right) / E^{\left(Ia\right)}$

**Rubi [F]** time = 0.0491912, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan^2(a + i \log(x))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + I\*Log[x]]^2/x^2, x]

[Out] Defer[Int][Tan[a + I\*Log[x]]^2/x^2, x]

Rubi steps

$$\int \frac{\tan^2(a + i \log(x))}{x^2} dx = \int \frac{\tan^2(a + i \log(x))}{x^2} dx$$

**Mathematica [A]** time = 0.108788, size = 72, normalized size = 1.2

$$\frac{2x(\cos(a) - i \sin(a))}{(x^2 + 1) \cos(a) - i(x^2 - 1) \sin(a)} + 2 \cos(a) \tan^{-1}(x(\cos(a) - i \sin(a))) - 2i \sin(a) \tan^{-1}(x(\cos(a) - i \sin(a))) + \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I\*Log[x]]^2/x^2, x]

[Out]  $x^{(-1)} + 2 \cdot \text{ArcTan}\left[x \cdot \left(\text{Cos}[a] - I \cdot \text{Sin}[a]\right)\right] \cdot \text{Cos}[a] - \left(2I\right) \cdot \text{ArcTan}\left[x \cdot \left(\text{Cos}[a] - I \cdot \text{Sin}[a]\right)\right] \cdot \text{Sin}[a] + \left(2x \cdot \left(\text{Cos}[a] - I \cdot \text{Sin}[a]\right)\right) / \left(\left(1 + x^2\right) \cdot \text{Cos}[a] - I \cdot \left(-1 + x^2\right) \cdot \text{Sin}[a]\right)$

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int \frac{(\tan(a + i \ln(x)))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+I\*ln(x))^2/x^2, x)

[Out]  $\int (\tan(a + I \ln(x)))^2 / x^2, x$

**Maxima [B]** time = 1.6189, size = 312, normalized size = 5.2

$$6x^2 - \left( x^3(2 \cos(a) - 2i \sin(a)) + ((2 \cos(a) - 2i \sin(a)) \cos(2a) + 2(i \cos(a) + \sin(a)) \sin(2a))x \right) \arctan\left(\frac{2}{x^2 + \cos(a)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\tan(a + I \log(x))^2 / x^2, x, \text{algorithm}="maxima")$

[Out]  $(6x^2 - (x^3(2\cos(a) - 2I\sin(a)) + ((2\cos(a) - 2I\sin(a))\cos(2a) + 2(I\cos(a) + \sin(a))\sin(2a))x) \arctan(2x\cos(a)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2), (x^2 - \cos(a)^2 - \sin(a)^2)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2)) + (x^3(-I\cos(a) - \sin(a)) + ((-I\cos(a) - \sin(a))\cos(2a) + (\cos(a) - I\sin(a))\sin(2a))x) \log((x^2 + \cos(a)^2 + 2x\sin(a) + \sin(a)^2)/(x^2 + \cos(a)^2 - 2x\sin(a) + \sin(a)^2)) + 2\cos(2a) + 2I\sin(2a))/(2x^3 + x(2\cos(2a) + 2I\sin(2a)))$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(xe^{(2ia-2 \log(x))} + x) \text{integral}\left(-\frac{e^{(2ia-2 \log(x))-1}}{x^2 e^{(2ia-2 \log(x))+x^2}}, x\right) + 2}{xe^{(2ia-2 \log(x))} + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\tan(a + I \log(x))^2 / x^2, x, \text{algorithm}="fricas")$

[Out]  $((x \cdot e^{(2Ia - 2 \log(x))} + x) \cdot \text{integral}(- (e^{(2Ia - 2 \log(x))} - 1) / (x^2 \cdot e^{(2Ia - 2 \log(x))} + x^2), x) + 2) / (x \cdot e^{(2Ia - 2 \log(x))} + x))$

**Sympy [A]** time = 0.672558, size = 51, normalized size = 0.85

$$\frac{3x^2 + e^{2ia}}{x^3 + xe^{2ia}} - (i \log(x - ie^{ia}) - i \log(x + ie^{ia})) e^{-ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\tan(a + I \ln(x))^{**2} / x^{**2}, x)$

[Out]  $(3x^{**2} + \exp(2Ia)) / (x^{**3} + x \cdot \exp(2Ia)) - (I \cdot \log(x - I \cdot \exp(Ia)) - I \cdot \log(x + I \cdot \exp(Ia))) \cdot \exp(-Ia)$

**Giac [A]** time = 1.21748, size = 99, normalized size = 1.65

$$2 \left( \arctan(xe^{-ia}) e^{-3ia} + \frac{xe^{-2ia}}{x^2 + e^{2ia}} \right) e^{2ia} + \frac{5}{x \left( \frac{e^{2ia}}{x^2} + 1 \right)} + \frac{e^{2ia}}{x^3 \left( \frac{e^{2ia}}{x^2} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(tan(a+I*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] 2*(arctan(x*e^(-I*a))*e^(-3*I*a) + x*e^(-2*I*a)/(x^2 + e^(2*I*a)))*e^(2*I*a) + 5/(x*(e^(2*I*a)/x^2 + 1)) + e^(2*I*a)/(x^3*(e^(2*I*a)/x^2 + 1))
```

$$3.149 \quad \int \frac{\tan^2(a+i \log(x))}{x^3} dx$$

**Optimal.** Leaf size=55

$$-\frac{2e^{-2ia}}{1 + \frac{e^{2ia}}{x^2}} - 2e^{-2ia} \log\left(1 + \frac{e^{2ia}}{x^2}\right) + \frac{1}{2x^2}$$

[Out]  $-2/(E^((2*I)*a)*(1 + E^((2*I)*a)/x^2)) + 1/(2*x^2) - (2*Log[1 + E^((2*I)*a)/x^2])/E^((2*I)*a)$

**Rubi [F]** time = 0.052695, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan^2(a + i \log(x))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + I\*Log[x]]^2/x^3, x]

[Out] Defer[Int][Tan[a + I\*Log[x]]^2/x^3, x]

Rubi steps

$$\int \frac{\tan^2(a + i \log(x))}{x^3} dx = \int \frac{\tan^2(a + i \log(x))}{x^3} dx$$

**Mathematica [B]** time = 0.170479, size = 150, normalized size = 2.73

$$-\cos(2a) \log(2x^2 \cos(2a) + x^4 + 1) + \frac{2 \cos(a) - 2i \sin(a)}{(x^2 + 1) \cos(a) - i(x^2 - 1) \sin(a)} + i \sin(2a) \log(2x^2 \cos(2a) + x^4 + 1) - 2i \cos(2a)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + I\*Log[x]]^2/x^3, x]

[Out]  $1/(2*x^2) - (2*I)*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Cos[2*a] + 4*Cos[2*a]*Log[x] - Cos[2*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + (2*Cos[a] - (2*I)*Sin[a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a]) - 2*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Sin[2*a] - (4*I)*Log[x]*Sin[2*a] + I*Log[1 + x^4 + 2*x^2*Cos[2*a]]*Sin[2*a]$

**Maple [A]** time = 0.065, size = 61, normalized size = 1.1

$$\frac{1}{2x^2} + 2 \frac{1}{x^2 \left( (e^{i(a+i \ln(x))})^2 + 1 \right)} + 4 \frac{\ln(x)}{(e^{ia})^2} - 2 \frac{\ln \left( (e^{ia})^2 + x^2 \right)}{(e^{ia})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+I\*ln(x))^2/x^3,x)

[Out] 1/2/x^2+2/x^2/(exp(I\*(a+I\*ln(x)))^2+1)+4/exp(I\*a)^2\*ln(x)-2/exp(I\*a)^2\*ln(exp(I\*a)^2+x^2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))^2/x^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(x^2 e^{(2i a - 2 \log(x))} + x^2) \operatorname{integral}\left(-\frac{e^{(2i a - 2 \log(x)) - 3}}{x^3 e^{(2i a - 2 \log(x)) + x^3}}, x\right) + 2}{x^2 e^{(2i a - 2 \log(x))} + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))^2/x^3,x, algorithm="fricas")

[Out] ((x^2\*e^(2\*I\*a - 2\*log(x)) + x^2)\*integral(-(e^(2\*I\*a - 2\*log(x)) - 3)/(x^3 \*e^(2\*I\*a - 2\*log(x)) + x^3), x) + 2)/(x^2\*e^(2\*I\*a - 2\*log(x)) + x^2)

**Sympy [A]** time = 1.0463, size = 60, normalized size = 1.09

$$\frac{5x^2 + e^{2ia}}{2x^4 + 2x^2 e^{2ia}} + 4e^{-2ia} \log(x) - 2e^{-2ia} \log(x^2 + e^{2ia})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*ln(x))\*\*2/x\*\*3,x)

[Out] (5\*x\*\*2 + exp(2\*I\*a))/(2\*x\*\*4 + 2\*x\*\*2\*exp(2\*I\*a)) + 4\*exp(-2\*I\*a)\*log(x) - 2\*exp(-2\*I\*a)\*log(x\*\*2 + exp(2\*I\*a))

**Giac [B]** time = 1.21621, size = 240, normalized size = 4.36

$$-\frac{2 \log(-x^2 - e^{(2i a)})}{\frac{e^{(4i a)}}{x^2} + e^{(2i a)}} + \frac{4 \log(x)}{\frac{e^{(4i a)}}{x^2} + e^{(2i a)}} - \frac{2}{\frac{e^{(4i a)}}{x^2} + e^{(2i a)}} - \frac{2 e^{(2i a)} \log(-x^2 - e^{(2i a)})}{x^2 \left(\frac{e^{(4i a)}}{x^2} + e^{(2i a)}\right)} + \frac{4 e^{(2i a)} \log(x)}{x^2 \left(\frac{e^{(4i a)}}{x^2} + e^{(2i a)}\right)} + \frac{e^{(2i a)}}{2 x^2 \left(\frac{e^{(4i a)}}{x^2} + e^{(2i a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+I\*log(x))^2/x^3,x, algorithm="giac")

```
[Out] -2*log(-x^2 - e^(2*I*a))/(e^(4*I*a)/x^2 + e^(2*I*a)) + 4*log(x)/(e^(4*I*a)/  
x^2 + e^(2*I*a)) - 2/(e^(4*I*a)/x^2 + e^(2*I*a)) - 2*e^(2*I*a)*log(-x^2 - e  
^(2*I*a))/(x^2*(e^(4*I*a)/x^2 + e^(2*I*a))) + 4*e^(2*I*a)*log(x)/(x^2*(e^(4  
*I*a)/x^2 + e^(2*I*a))) + 1/2*e^(2*I*a)/(x^2*(e^(4*I*a)/x^2 + e^(2*I*a))) +  
1/2*e^(4*I*a)/(x^4*(e^(4*I*a)/x^2 + e^(2*I*a)))
```

### 3.150 $\int (ex)^m \tan(a + i \log(x)) dx$

**Optimal.** Leaf size=71

$$\frac{2i(ex)^{m+1} \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m-1), \frac{1-m}{2}, -\frac{e^{2ia}}{x^2}\right)}{e(m+1)} - \frac{i(ex)^{m+1}}{e(m+1)}$$

[Out]  $((-I)*(e*x)^{(1+m)})/(e*(1+m)) + ((2*I)*(e*x)^{(1+m)}*\text{Hypergeometric2F1}[1, (-1-m)/2, (1-m)/2, -(E^((2*I)*a)/x^2)])/(e*(1+m))$

**Rubi [F]** time = 0.0436427, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \tan(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Tan[a + I\*Log[x]], x]

[Out] Defer[Int][(e\*x)^m\*Tan[a + I\*Log[x]], x]

Rubi steps

$$\int (ex)^m \tan(a + i \log(x)) dx = \int (ex)^m \tan(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.184527, size = 124, normalized size = 1.75

$$\frac{x(\cos(a) - i \sin(a))(ex)^m \left( (m+1)x^2(\sin(a) + i \cos(a)) \text{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, -x^2(\cos(2a) - i \sin(2a))\right) + \right)}{(m+1)(m+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Tan[a + I\*Log[x]], x]

[Out]  $(x*(e*x)^m*(\text{Cos}[a] - I*\text{Sin}[a]))*((3+m)*\text{Hypergeometric2F1}[1, (1+m)/2, (3+m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))])*((-I)*\text{Cos}[a] + \text{Sin}[a]) + (1+m)*x^2*\text{Hypergeometric2F1}[1, (3+m)/2, (5+m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))])*(I*\text{Cos}[a] + \text{Sin}[a]))/((1+m)*(3+m))$

**Maple [F]** time = 0.201, size = 0, normalized size = 0.

$$\int (ex)^m \tan(a + i \ln(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*tan(a+I\*ln(x)), x)

[Out] `int((e*x)^m*tan(a+I*ln(x)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(a + i \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="maxima")`

[Out] `integrate((e*x)^m*tan(a + I*log(x)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m (-i e^{2ia-2 \log(x)} + i)}{e^{2ia-2 \log(x)} + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="fricas")`

[Out] `integral((e*x)^m*(-I*e^(2*I*a - 2*log(x)) + I)/(e^(2*I*a - 2*log(x)) + 1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(a + i \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*tan(a+I*ln(x)),x)`

[Out] `Integral((e*x)**m*tan(a + I*log(x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(a + i \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="giac")`

[Out] `integrate((e*x)^m*tan(a + I*log(x)), x)`

### 3.151 $\int (ex)^m \tan^2(a + i \log(x)) dx$

**Optimal.** Leaf size=77

$$-2x(ex)^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m-1), \frac{1-m}{2}, -\frac{e^{2ia}}{x^2}\right) + \frac{2x(ex)^m}{1 + \frac{e^{2ia}}{x^2}} - \frac{x(ex)^m}{m+1}$$

[Out]  $-\left(\frac{x*(e*x)^m}{1+m}\right) + \left(\frac{2*x*(e*x)^m}{1+E^{\left(\frac{2*I}{x^2}\right)*a}}\right) - \frac{2*x*(e*x)^m}{1+\frac{e^{2ia}}{x^2}} - \frac{x(ex)^m}{m+1}$   
 $\text{Hypergeometric2F1}\left[1, \frac{-1-m}{2}, \frac{1-m}{2}, -\left(\frac{E^{\left(\frac{2*I}{x^2}\right)*a}}{x^2}\right)\right]$

**Rubi [F]** time = 0.0868889, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \tan^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[(e*x)^m * \text{Tan}[a + I * \text{Log}[x]]^2, x]$

[Out]  $\text{Defer}[\text{Int}][(e*x)^m * \text{Tan}[a + I * \text{Log}[x]]^2, x]$

Rubi steps

$$\int (ex)^m \tan^2(a + i \log(x)) dx = \int (ex)^m \tan^2(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.41403, size = 172, normalized size = 2.23

$$x(ex)^m \left( -\frac{x^4(\cos(a)-i\sin(a))^2 \text{Hypergeometric2F1}\left(2, \frac{m+5}{2}, \frac{m+7}{2}, -x^2(\cos(2a)-i\sin(2a))\right)}{m+5} + \frac{2x^2 \text{Hypergeometric2F1}\left(2, \frac{m+3}{2}, \frac{m+5}{2}, -x^2(\cos(2a)-i\sin(2a))\right)}{m+3} \right) / (\cos(a) + i\sin(a))^2$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(e*x)^m * \text{Tan}[a + I * \text{Log}[x]]^2, x]$

[Out]  $(x*(e*x)^m * ((2*x^2 * \text{Hypergeometric2F1}[2, (3+m)/2, (5+m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))]) / (3+m) - (x^4 * \text{Hypergeometric2F1}[2, (5+m)/2, (7+m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))]) * (\text{Cos}[a] - I*\text{Sin}[a])^2 / (5+m) - (\text{Hypergeometric2F1}[2, (1+m)/2, (3+m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))]) * (\text{Cos}[2*a] + I*\text{Sin}[2*a]) / (1+m))) / (\text{Cos}[a] + I*\text{Sin}[a])^2$

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int (ex)^m (\tan(a + i \ln(x)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*tan(a+I*ln(x))^2,x)`

[Out] `int((e*x)^m*tan(a+I*ln(x))^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(a + i \log(x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*tan(a + I*log(x))^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2 (ex)^m x + \left( e^{(2ia-2 \log(x))} + 1 \right) \operatorname{integral} \left( -\frac{(ex)^m (2m + e^{(2ia-2 \log(x))+3})}{e^{(2ia-2 \log(x))+1}}, x \right)}{e^{(2ia-2 \log(x))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="fricas")`

[Out] `(2*(e*x)^m*x + (e^(2*I*a - 2*log(x)) + 1)*integral(-(e*x)^m*(2*m + e^(2*I*a - 2*log(x)) + 3)/(e^(2*I*a - 2*log(x)) + 1), x))/(e^(2*I*a - 2*log(x)) + 1)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan^2(a + i \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*tan(a+I*ln(x))**2,x)`

[Out] `Integral((e*x)**m*tan(a + I*log(x))**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(a + i \log(x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="giac")`

[Out] `integrate((e*x)^m*tan(a + I*log(x))^2, x)`



### 3.152 $\int (ex)^m \tan^3(a + i \log(x)) dx$

**Optimal.** Leaf size=184

$$\frac{i(m^2 + 2m + 3)x(ex)^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m-1), \frac{1-m}{2}, -\frac{e^{2ia}}{x^2}\right)}{m+1} + \frac{ie^{-2ia}x\left(\frac{e^{4ia(1-m)}}{x^2} + e^{2ia(m+3)}\right)(ex)^m}{2\left(1 + \frac{e^{2ia}}{x^2}\right)} + \dots$$

[Out]  $((-I/2)*(1 - m)*m*x*(e*x)^m)/(1 + m) + ((I/2)*(1 - E^((2*I)*a)/x^2)^2*x*(e*x)^m)/(1 + E^((2*I)*a)/x^2)^2 + ((I/2)*(E^((2*I)*a)*(3 + m) + (E^((4*I)*a)*(1 - m))/x^2)*x*(e*x)^m)/(E^((2*I)*a)*(1 + E^((2*I)*a)/x^2)) - (I*(3 + 2*m + m^2)*x*(e*x)^m*Hypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, -(E^((2*I)*a)/x^2)])/(1 + m)$

**Rubi [F]** time = 0.0906369, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \tan^3(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Tan[a + I\*Log[x]]^3, x]

[Out] Defer[Int][(e\*x)^m\*Tan[a + I\*Log[x]]^3, x]

Rubi steps

$$\int (ex)^m \tan^3(a + i \log(x)) dx = \int (ex)^m \tan^3(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.929593, size = 255, normalized size = 1.39

$$x(ex)^m \left( -\frac{ix^4(\cos(2a)-i\sin(2a))((m+5)x^2(\cos(a)-i\sin(a))\text{Hypergeometric2F1}\left(3, \frac{m+7}{2}, \frac{m+9}{2}, -x^2(\cos(2a)-i\sin(2a))\right)-3(m+7)(\cos(a)+i\sin(a))\text{Hypergeom}}{(m+5)(m+7)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Tan[a + I\*Log[x]]^3, x]

[Out]  $(x*(e*x)^m*((I*\text{Hypergeometric2F1}[3, (1 + m)/2, (3 + m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))])*(\text{Cos}[a] + I*\text{Sin}[a])^3)/(1 + m) + (3*x^2*\text{Hypergeometric2F1}[3, (3 + m)/2, (5 + m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))])*((-I)*\text{Cos}[a] + \text{Sin}[a]))/(3 + m) - (I*x^4*((5 + m)*x^2*\text{Hypergeometric2F1}[3, (7 + m)/2, (9 + m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))])*(\text{Cos}[a] - I*\text{Sin}[a]) - 3*(7 + m)*\text{Hypergeometric2F1}[3, (5 + m)/2, (7 + m)/2, -(x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))])*(\text{Cos}[a] + I*\text{Sin}[a]))*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))/((5 + m)*(7 + m)))/(\text{Cos}[a] + I*\text{Sin}[a])^3$

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int (ex)^m (\tan(a + i \ln(x)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*tan(a+I\*ln(x))^3,x)

[Out] int((e\*x)^m\*tan(a+I\*ln(x))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(a + i \log(x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*tan(a+I\*log(x))^3,x, algorithm="maxima")

[Out] integrate((e\*x)^m\*tan(a + I\*log(x))^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{((im - i)xe^{(2ia-2 \log(x))} + (im + i)x)(ex)^m + (e^{(4ia-4 \log(x))} + 2e^{(2ia-2 \log(x))} + 1)\text{integral}\left(\frac{(-im^2 - 2im + ie^{(2ia-2 \log(x)) - 2i})(ex)^m}{e^{(2ia-2 \log(x))} + 1}, x\right)}{e^{(4ia-4 \log(x))} + 2e^{(2ia-2 \log(x))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*tan(a+I\*log(x))^3,x, algorithm="fricas")

[Out] (((I\*m - I)\*x\*e^(2\*I\*a - 2\*log(x)) + (I\*m + I)\*x)\*(e\*x)^m + (e^(4\*I\*a - 4\*log(x)) + 2\*e^(2\*I\*a - 2\*log(x)) + 1)\*integral((-I\*m^2 - 2\*I\*m + I\*e^(2\*I\*a - 2\*log(x)) - 2\*I)\*(e\*x)^m/(e^(2\*I\*a - 2\*log(x)) + 1), x))/(e^(4\*I\*a - 4\*log(x)) + 2\*e^(2\*I\*a - 2\*log(x)) + 1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*tan(a+I\*ln(x))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(a + i \log(x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(a+I*log(x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tan(a + I*log(x))^3, x)
```

### 3.153 $\int \tan^p(a + b \log(x)) dx$

**Optimal.** Leaf size=142

$$x(1 - e^{2ia}x^{2ib})^{-p} \left( \frac{i(1 - e^{2ia}x^{2ib})}{1 + e^{2ia}x^{2ib}} \right)^p (1 + e^{2ia}x^{2ib})^p F_1 \left( -\frac{i}{2b}; -p, p; 1 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right)$$

[Out]  $(x*((I*(1 - E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2*I)*b)))^p*(1 + E^((2*I)*a)*x^((2*I)*b))^p*AppellF1[(-I/2)/b, -p, p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))]/(1 - E^((2*I)*a)*x^((2*I)*b))^p$

**Rubi [F]** time = 0.0242006, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \tan^p(a + b \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + b\*Log[x]]^p,x]

[Out] Defer[Int][Tan[a + b\*Log[x]]^p, x]

Rubi steps

$$\int \tan^p(a + b \log(x)) dx = \int \tan^p(a + b \log(x)) dx$$

**Mathematica [B]** time = 0.665116, size = 330, normalized size = 2.32

$$\frac{(2b - i)x \left( -\frac{i(-1 + e^{2ia}x^{2ib})}{1 + e^{2ia}x^{2ib}} \right)^p F_1 \left( -\frac{i}{2b}; -p, p; 1 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right)}{-2e^{2ia}bpx^{2ib}F_1 \left( 1 - \frac{i}{2b}; 1 - p, p; 2 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right) - 2e^{2ia}bpx^{2ib}F_1 \left( 1 - \frac{i}{2b}; -p, p + 1; 2 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right) + (2b - i)x \left( -\frac{i(-1 + e^{2ia}x^{2ib})}{1 + e^{2ia}x^{2ib}} \right)^p F_1 \left( -\frac{i}{2b}; -p, p; 1 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[a + b\*Log[x]]^p,x]

[Out]  $((-I + 2*b)*x*((-I)*(-1 + E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2*I)*b)))^p*AppellF1[(-I/2)/b, -p, p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))]/(-2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 - (I/2)/b, 1 - p, p, 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))] - 2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 - (I/2)/b, -p, 1 + p, 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))] + (-I + 2*b)*AppellF1[(-I/2)/b, -p, p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))]$

**Maple [F]** time = 0.374, size = 0, normalized size = 0.

$$\int (\tan(a + b \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(a+b*ln(x))^p,x)`

[Out] `int(tan(a+b*ln(x))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+b*log(x))^p,x, algorithm="maxima")`

[Out] `integrate(tan(b*log(x) + a)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\tan(b \log(x) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+b*log(x))^p,x, algorithm="fricas")`

[Out] `integral(tan(b*log(x) + a)^p, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan^p(a + b \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+b*ln(x))**p,x)`

[Out] `Integral(tan(a + b*log(x))**p, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+b*log(x))^p,x, algorithm="giac")`

[Out] Timed out

### 3.154 $\int (ex)^m \tan^p(a + b \log(x)) dx$

**Optimal.** Leaf size=162

$$\frac{(ex)^{m+1} (1 - e^{2ia} x^{2ib})^{-p} \left( \frac{i(1 - e^{2ia} x^{2ib})}{1 + e^{2ia} x^{2ib}} \right)^p (1 + e^{2ia} x^{2ib})^p F_1 \left( -\frac{i(m+1)}{2b}; -p, p; 1 - \frac{i(m+1)}{2b}; e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right)}{e(m+1)}$$

[Out]  $((e*x)^{(1+m)}*((I*(1 - E^{((2*I)*a)}*x^{((2*I)*b)})))/(1 + E^{((2*I)*a)}*x^{((2*I)*b)})^p*(1 + E^{((2*I)*a)}*x^{((2*I)*b)})^p*AppellF1[((-I/2)*(1+m))/b, -p, p, 1 - ((I/2)*(1+m))/b, E^{((2*I)*a)}*x^{((2*I)*b)}, -(E^{((2*I)*a)}*x^{((2*I)*b)})]/(e*(1+m)*(1 - E^{((2*I)*a)}*x^{((2*I)*b)})^p)$

**Rubi [F]** time = 0.129776, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \tan^p(a + b \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Tan[a + b\*Log[x]]^p,x]

[Out] Defer[Int][(e\*x)^m\*Tan[a + b\*Log[x]]^p, x]

Rubi steps

$$\int (ex)^m \tan^p(a + b \log(x)) dx = \int (ex)^m \tan^p(a + b \log(x)) dx$$

**Mathematica [A]** time = 0.648355, size = 157, normalized size = 0.97

$$\frac{x(ex)^m (1 - e^{2ia} x^{2ib})^{-p} \left( -\frac{i(-1 + e^{2ia} x^{2ib})}{1 + e^{2ia} x^{2ib}} \right)^p (1 + e^{2ia} x^{2ib})^p F_1 \left( -\frac{i(m+1)}{2b}; -p, p; 1 - \frac{i(m+1)}{2b}; e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Tan[a + b\*Log[x]]^p,x]

[Out]  $(x*(e*x)^m*(((I)*(-1 + E^{((2*I)*a)}*x^{((2*I)*b)})))/(1 + E^{((2*I)*a)}*x^{((2*I)*b)})^p*(1 + E^{((2*I)*a)}*x^{((2*I)*b)})^p*AppellF1[((-I/2)*(1+m))/b, -p, p, 1 - ((I/2)*(1+m))/b, E^{((2*I)*a)}*x^{((2*I)*b)}, -(E^{((2*I)*a)}*x^{((2*I)*b)})]/((1+m)*(1 - E^{((2*I)*a)}*x^{((2*I)*b)})^p)$

**Maple [F]** time = 0.411, size = 0, normalized size = 0.

$$\int (ex)^m (\tan(a + b \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*tan(a+b*ln(x))^p,x)`

[Out] `int((e*x)^m*tan(a+b*ln(x))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*tan(b*log(x) + a)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \tan(b \log(x) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="fricas")`

[Out] `integral((e*x)^m*tan(b*log(x) + a)^p, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan^p(a + b \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*tan(a+b*ln(x))**p,x)`

[Out] `Integral((e*x)**m*tan(a + b*log(x))**p, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="giac")`

[Out] Timed out

### 3.155 $\int \tan^p(a + \log(x)) dx$

**Optimal.** Leaf size=120

$$x(1 - e^{2ia}x^{2i})^{-p} \left( \frac{i(1 - e^{2ia}x^{2i})}{1 + e^{2ia}x^{2i}} \right)^p (1 + e^{2ia}x^{2i})^p F_1 \left( -\frac{i}{2}; -p, p; 1 - \frac{i}{2}; e^{2ia}x^{2i}, -e^{2ia}x^{2i} \right)$$

[Out] (((I\*(1 - E^((2\*I)\*a)\*x^(2\*I)))/(1 + E^((2\*I)\*a)\*x^(2\*I)))^p\*(1 + E^((2\*I)\*a)\*x^(2\*I))^p\*x\*AppellF1[-I/2, -p, p, 1 - I/2, E^((2\*I)\*a)\*x^(2\*I), -(E^((2\*I)\*a)\*x^(2\*I))]/(1 - E^((2\*I)\*a)\*x^(2\*I))^p

**Rubi [F]** time = 0.0214072, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \tan^p(a + \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + Log[x]]^p,x]

[Out] Defer[Int][Tan[a + Log[x]]^p, x]

Rubi steps

$$\int \tan^p(a + \log(x)) dx = \int \tan^p(a + \log(x)) dx$$

**Mathematica [A]** time = 0.490736, size = 240, normalized size = 2.

$$\frac{(1 + 2i)x \left( -\frac{i(-1 + e^{2ia}x^{2i})}{1 + e^{2ia}x^{2i}} \right)^p F_1 \left( -\frac{i}{2}; -p, p; 1 - \frac{i}{2}; e^{2ia}x^{2i}, -e^{2ia}x^{2i} \right)}{(1 + 2i)F_1 \left( -\frac{i}{2}; -p, p; 1 - \frac{i}{2}; e^{2ia}x^{2i}, -e^{2ia}x^{2i} \right) - 2ie^{2ia}px^{2i} \left( F_1 \left( 1 - \frac{i}{2}; 1 - p, p; 2 - \frac{i}{2}; e^{2ia}x^{2i}, -e^{2ia}x^{2i} \right) + F_1 \left( 1 - \frac{i}{2}; -p, p + 1; \dots \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[a + Log[x]]^p,x]

[Out] (((1 + 2\*I)\*((( -I)\*(-1 + E^((2\*I)\*a)\*x^(2\*I)))/(1 + E^((2\*I)\*a)\*x^(2\*I)))^p\*x\*AppellF1[-I/2, -p, p, 1 - I/2, E^((2\*I)\*a)\*x^(2\*I), -(E^((2\*I)\*a)\*x^(2\*I))])/((1 + 2\*I)\*AppellF1[-I/2, -p, p, 1 - I/2, E^((2\*I)\*a)\*x^(2\*I), -(E^((2\*I)\*a)\*x^(2\*I))] - (2\*I)\*E^((2\*I)\*a)\*p\*x^(2\*I)\*(AppellF1[1 - I/2, 1 - p, p, 2 - I/2, E^((2\*I)\*a)\*x^(2\*I), -(E^((2\*I)\*a)\*x^(2\*I))] + AppellF1[1 - I/2, -p, 1 + p, 2 - I/2, E^((2\*I)\*a)\*x^(2\*I), -(E^((2\*I)\*a)\*x^(2\*I))]))

**Maple [F]** time = 0.339, size = 0, normalized size = 0.

$$\int (\tan(a + \ln(x)))^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(a+ln(x))^p,x)`

[Out] `int(tan(a+ln(x))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan(a + \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+log(x))^p,x, algorithm="maxima")`

[Out] `integrate(tan(a + log(x))^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\tan(a + \log(x))^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+log(x))^p,x, algorithm="fricas")`

[Out] `integral(tan(a + log(x))^p, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan^p(a + \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+ln(x))**p,x)`

[Out] `Integral(tan(a + log(x))**p, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan(a + \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+log(x))^p,x, algorithm="giac")`

[Out] `integrate(tan(a + log(x))^p, x)`

### 3.156 $\int \tan^p(a + 2 \log(x)) dx$

**Optimal.** Leaf size=120

$$x(1 - e^{2ia}x^{4i})^{-p} \left( \frac{i(1 - e^{2ia}x^{4i})}{1 + e^{2ia}x^{4i}} \right)^p (1 + e^{2ia}x^{4i})^p F_1 \left( -\frac{i}{4}; -p, p; 1 - \frac{i}{4}; e^{2ia}x^{4i}, -e^{2ia}x^{4i} \right)$$

[Out] (((I\*(1 - E^((2\*I)\*a)\*x^(4\*I)))/(1 + E^((2\*I)\*a)\*x^(4\*I)))^p\*(1 + E^((2\*I)\*a)\*x^(4\*I))^p\*x\*AppellF1[-I/4, -p, p, 1 - I/4, E^((2\*I)\*a)\*x^(4\*I), -(E^((2\*I)\*a)\*x^(4\*I))]/(1 - E^((2\*I)\*a)\*x^(4\*I))^p

**Rubi [F]** time = 0.0196184, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \tan^p(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + 2\*Log[x]]^p,x]

[Out] Defer[Int][Tan[a + 2\*Log[x]]^p, x]

Rubi steps

$$\int \tan^p(a + 2 \log(x)) dx = \int \tan^p(a + 2 \log(x)) dx$$

**Mathematica [A]** time = 0.495122, size = 240, normalized size = 2.

$$\frac{(1 + 4i)x \left( -\frac{i(-1 + e^{2ia}x^{4i})}{1 + e^{2ia}x^{4i}} \right)^p F_1 \left( -\frac{i}{4}; -p, p; 1 - \frac{i}{4}; e^{2ia}x^{4i}, -e^{2ia}x^{4i} \right)}{(1 + 4i)F_1 \left( -\frac{i}{4}; -p, p; 1 - \frac{i}{4}; e^{2ia}x^{4i}, -e^{2ia}x^{4i} \right) - 4ie^{2ia}px^{4i} \left( F_1 \left( 1 - \frac{i}{4}; 1 - p, p; 2 - \frac{i}{4}; e^{2ia}x^{4i}, -e^{2ia}x^{4i} \right) + F_1 \left( 1 - \frac{i}{4}; -p, p + 1; \dots \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[a + 2\*Log[x]]^p,x]

[Out] ((1 + 4\*I)\*(((I)\*(-1 + E^((2\*I)\*a)\*x^(4\*I)))/(1 + E^((2\*I)\*a)\*x^(4\*I)))^p\*x\*AppellF1[-I/4, -p, p, 1 - I/4, E^((2\*I)\*a)\*x^(4\*I), -(E^((2\*I)\*a)\*x^(4\*I))]/((1 + 4\*I)\*AppellF1[-I/4, -p, p, 1 - I/4, E^((2\*I)\*a)\*x^(4\*I), -(E^((2\*I)\*a)\*x^(4\*I))] - (4\*I)\*E^((2\*I)\*a)\*p\*x^(4\*I)\*(AppellF1[1 - I/4, 1 - p, p, 2 - I/4, E^((2\*I)\*a)\*x^(4\*I), -(E^((2\*I)\*a)\*x^(4\*I))] + AppellF1[1 - I/4, -p, 1 + p, 2 - I/4, E^((2\*I)\*a)\*x^(4\*I), -(E^((2\*I)\*a)\*x^(4\*I))]))

**Maple [F]** time = 0.322, size = 0, normalized size = 0.

$$\int (\tan(a + 2 \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(a+2*ln(x))^p,x)`

[Out] `int(tan(a+2*ln(x))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan(a + 2 \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+2*log(x))^p,x, algorithm="maxima")`

[Out] `integrate(tan(a + 2*log(x))^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\tan(a + 2 \log(x))^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+2*log(x))^p,x, algorithm="fricas")`

[Out] `integral(tan(a + 2*log(x))^p, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan^p(a + 2 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+2*ln(x))**p,x)`

[Out] `Integral(tan(a + 2*log(x))**p, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+2*log(x))^p,x, algorithm="giac")`

[Out] Timed out

### 3.157 $\int \tan^p(a + 3 \log(x)) dx$

**Optimal.** Leaf size=120

$$x(1 - e^{2ia}x^{6i})^{-p} \left( \frac{i(1 - e^{2ia}x^{6i})}{1 + e^{2ia}x^{6i}} \right)^p (1 + e^{2ia}x^{6i})^p F_1 \left( -\frac{i}{6}; -p, p; 1 - \frac{i}{6}; e^{2ia}x^{6i}, -e^{2ia}x^{6i} \right)$$

[Out] (((I\*(1 - E^((2\*I)\*a)\*x^(6\*I)))/(1 + E^((2\*I)\*a)\*x^(6\*I)))^p\*(1 + E^((2\*I)\*a)\*x^(6\*I))^p\*x\*AppellF1[-I/6, -p, p, 1 - I/6, E^((2\*I)\*a)\*x^(6\*I), -(E^((2\*I)\*a)\*x^(6\*I))]/(1 - E^((2\*I)\*a)\*x^(6\*I))^p

**Rubi [F]** time = 0.0207789, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \tan^p(a + 3 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Tan[a + 3\*Log[x]]^p,x]

[Out] Defer[Int][Tan[a + 3\*Log[x]]^p, x]

Rubi steps

$$\int \tan^p(a + 3 \log(x)) dx = \int \tan^p(a + 3 \log(x)) dx$$

**Mathematica [A]** time = 0.489129, size = 240, normalized size = 2.

$$\frac{(1 + 6i)x \left( -\frac{i(-1 + e^{2ia}x^{6i})}{1 + e^{2ia}x^{6i}} \right)^p F_1 \left( -\frac{i}{6}; -p, p; 1 - \frac{i}{6}; e^{2ia}x^{6i}, -e^{2ia}x^{6i} \right)}{(1 + 6i)F_1 \left( -\frac{i}{6}; -p, p; 1 - \frac{i}{6}; e^{2ia}x^{6i}, -e^{2ia}x^{6i} \right) - 6ie^{2ia}px^{6i} \left( F_1 \left( 1 - \frac{i}{6}; 1 - p, p; 2 - \frac{i}{6}; e^{2ia}x^{6i}, -e^{2ia}x^{6i} \right) + F_1 \left( 1 - \frac{i}{6}; -p, p + 1; \dots \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[a + 3\*Log[x]]^p,x]

[Out] (((1 + 6\*I)\*((-I)\*(-1 + E^((2\*I)\*a)\*x^(6\*I)))/(1 + E^((2\*I)\*a)\*x^(6\*I)))^p\*x\*AppellF1[-I/6, -p, p, 1 - I/6, E^((2\*I)\*a)\*x^(6\*I), -(E^((2\*I)\*a)\*x^(6\*I))]/((1 + 6\*I)\*AppellF1[-I/6, -p, p, 1 - I/6, E^((2\*I)\*a)\*x^(6\*I), -(E^((2\*I)\*a)\*x^(6\*I))] - (6\*I)\*E^((2\*I)\*a)\*p\*x^(6\*I)\*(AppellF1[1 - I/6, 1 - p, p, 2 - I/6, E^((2\*I)\*a)\*x^(6\*I), -(E^((2\*I)\*a)\*x^(6\*I))] + AppellF1[1 - I/6, -p, 1 + p, 2 - I/6, E^((2\*I)\*a)\*x^(6\*I), -(E^((2\*I)\*a)\*x^(6\*I))]))

**Maple [F]** time = 0.318, size = 0, normalized size = 0.

$$\int (\tan(a + 3 \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(a+3*ln(x))^p,x)`

[Out] `int(tan(a+3*ln(x))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan(a + 3 \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+3*log(x))^p,x, algorithm="maxima")`

[Out] `integrate(tan(a + 3*log(x))^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\tan(a + 3 \log(x))^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+3*log(x))^p,x, algorithm="fricas")`

[Out] `integral(tan(a + 3*log(x))^p, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan^p(a + 3 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+3*ln(x))**p,x)`

[Out] `Integral(tan(a + 3*log(x))**p, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(a+3*log(x))^p,x, algorithm="giac")`

[Out] Timed out

### 3.158 $\int x^3 \tan(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=71

$$\frac{1}{2}ix^4 \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, -e^{2iad}(cx^n)^{2ibd}\right) - \frac{ix^4}{4}$$

[Out]  $(-I/4)*x^4 + (I/2)*x^4*\text{Hypergeometric2F1}[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]$

**Rubi [F]** time = 0.0433235, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \tan(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[x^3*\text{Tan}[d*(a + b*\text{Log}[c*x^n])], x]$

[Out]  $\text{Defer}[\text{Int}[x^3*\text{Tan}[d*(a + b*\text{Log}[c*x^n])], x]$

Rubi steps

$$\int x^3 \tan(d(a + b \log(cx^n))) dx = \int x^3 \tan(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 6.50766, size = 146, normalized size = 2.06

$$\frac{x^4 \left( 2ie^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{2i}{bdn}, 2 - \frac{2i}{bdn}, -e^{2id(a+b \log(cx^n))}\right) + (bdn - 2i) \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, -e^{2id(a+b \log(cx^n))}\right) \right)}{-8 - 4ibdn}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^3*\text{Tan}[d*(a + b*\text{Log}[c*x^n])], x]$

[Out]  $(x^4*((2*I)*E^((2*I)*d*(a + b*\text{Log}[c*x^n]))*\text{Hypergeometric2F1}[1, 1 - (2*I)/(b*d*n), 2 - (2*I)/(b*d*n), -E^((2*I)*d*(a + b*\text{Log}[c*x^n]))] + (-2*I + b*d*n)*\text{Hypergeometric2F1}[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), -E^((2*I)*d*(a + b*\text{Log}[c*x^n]))])))/(-8 - (4*I)*b*d*n)$

**Maple [F]** time = 1.566, size = 0, normalized size = 0.

$$\int x^3 \tan(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3*\text{tan}(d*(a+b*\ln(c*x^n))), x)$

[Out] `int(x^3*tan(d*(a+b*ln(c*x^n))),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \tan((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(x^3*tan((b*log(c*x^n) + a)*d), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \tan(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(x^3*tan(b*d*log(c*x^n) + a*d), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*tan(d*(a+b*ln(c*x**n))),x)`

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Timed out

### 3.159 $\int x^2 \tan(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=75

$$\frac{2}{3}ix^3\text{Hypergeometric2F1}\left(1, -\frac{3i}{2bdn}, 1 - \frac{3i}{2bdn}, -e^{2iad}(cx^n)^{2ibd}\right) - \frac{ix^3}{3}$$

[Out]  $(-I/3)*x^3 + ((2*I)/3)*x^3\text{Hypergeometric2F1}[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]$

**Rubi [F]** time = 0.0312679, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \tan(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Tan[d\*(a + b\*Log[c\*x^n])], x]

[Out] Defer[Int][x^2\*Tan[d\*(a + b\*Log[c\*x^n])], x]

Rubi steps

$$\int x^2 \tan(d(a + b \log(cx^n))) dx = \int x^2 \tan(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 6.06496, size = 155, normalized size = 2.07

$$\frac{x^3 \left( 3ie^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{3i}{2bdn}, 2 - \frac{3i}{2bdn}, -e^{2id(a+b \log(cx^n))}\right) + (2bdn - 3i) \text{Hypergeometric2F1}\left(1, \frac{-3i}{2bdn}, 1 - \frac{3i}{2bdn}, -e^{2id(a+b \log(cx^n))}\right) \right)}{-9 - 6ibdn}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Tan[d\*(a + b\*Log[c\*x^n])], x]

[Out]  $(x^3*((3*I)*E^((2*I)*d*(a + b*Log[c*x^n]))*\text{Hypergeometric2F1}[1, 1 - ((3*I)/2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-3*I + 2*b*d*n)*\text{Hypergeometric2F1}[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]))/(-9 - (6*I)*b*d*n)$

**Maple [F]** time = 1.357, size = 0, normalized size = 0.

$$\int x^2 \tan(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*tan(d\*(a+b\*ln(c\*x^n))), x)



[Out] `int(x^2*tan(d*(a+b*ln(c*x^n))),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \tan((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(x^2*tan((b*log(c*x^n) + a)*d), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \tan(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(x^2*tan(b*d*log(c*x^n) + a*d), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \tan(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*tan(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral(x**2*tan(a*d + b*d*log(c*x**n)), x)`

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Timed out

### 3.160 $\int x \tan(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=69

$$ix^2 \text{Hypergeometric2F1}\left(1, -\frac{i}{bdn}, 1 - \frac{i}{bdn}, -e^{2iad}(cx^n)^{2ibd}\right) - \frac{ix^2}{2}$$

[Out]  $(-I/2)*x^2 + I*x^2*\text{Hypergeometric2F1}[1, (-I)/(b*d*n), 1 - I/(b*d*n), -(E^((2*I)*a*d))*(c*x^n)^((2*I)*b*d)]$

**Rubi [F]** time = 0.0248683, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \tan(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[x*\text{Tan}[d*(a + b*\text{Log}[c*x^n])], x]$

[Out]  $\text{Defer}[\text{Int}[x*\text{Tan}[d*(a + b*\text{Log}[c*x^n])], x]$

Rubi steps

$$\int x \tan(d(a + b \log(cx^n))) dx = \int x \tan(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 6.19783, size = 146, normalized size = 2.12

$$\frac{x^2 \left( i e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{i}{bdn}, 2 - \frac{i}{bdn}, -e^{2id(a+b \log(cx^n))}\right) + (bdn - i) \text{Hypergeometric2F1}\left(1, -\frac{i}{bdn}, -2 - 2ibdn \right) \right)}{-2 - 2ibdn}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x*\text{Tan}[d*(a + b*\text{Log}[c*x^n])], x]$

[Out]  $(x^2*(I*E^((2*I)*d*(a + b*\text{Log}[c*x^n]))*\text{Hypergeometric2F1}[1, 1 - I/(b*d*n), 2 - I/(b*d*n), -E^((2*I)*d*(a + b*\text{Log}[c*x^n]))] + (-I + b*d*n)*\text{Hypergeometric2F1}[1, (-I)/(b*d*n), 1 - I/(b*d*n), -E^((2*I)*d*(a + b*\text{Log}[c*x^n]))]))/(-2 - (2*I)*b*d*n)$

**Maple [F]** time = 1.217, size = 0, normalized size = 0.

$$\int x \tan(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x*\text{tan}(d*(a+b*\ln(c*x^n))), x)$

[Out] `int(x*tan(d*(a+b*ln(c*x^n))),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \tan((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(x*tan((b*log(c*x^n) + a)*d), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \tan(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(x*tan(b*d*log(c*x^n) + a*d), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \tan(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral(x*tan(a*d + b*d*log(c*x**n)), x)`

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Timed out

### 3.161 $\int \tan(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=67

$$2ix \operatorname{Hypergeometric2F1}\left(1, -\frac{i}{2bdn}, 1 - \frac{i}{2bdn}, -e^{2iad}(cx^n)^{2ibd}\right) - ix$$

[Out]  $(-I)*x + (2*I)*x*\operatorname{Hypergeometric2F1}[1, (-I/2)/(b*d*n), 1 - (I/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]$

**Rubi [F]** time = 0.0117656, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \tan(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[\operatorname{Tan}[d*(a + b*\operatorname{Log}[c*x^n])], x]$

[Out]  $\operatorname{Defer}[\operatorname{Int}[\operatorname{Tan}[d*(a + b*\operatorname{Log}[c*x^n])], x]$

Rubi steps

$$\int \tan(d(a + b \log(cx^n))) dx = \int \tan(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 10.9698, size = 151, normalized size = 2.25

$$\frac{x \left( (1 + 2ibdn) \operatorname{Hypergeometric2F1}\left(1, -\frac{i}{2bdn}, 1 - \frac{i}{2bdn}, -e^{2id(a+b \log(cx^n))}\right) - e^{2id(a+b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(1, 1 - \frac{i}{2bdn}, 1 - \frac{i}{2bdn}, -e^{2id(a+b \log(cx^n))}\right) \right)}{2bdn - i}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[\operatorname{Tan}[d*(a + b*\operatorname{Log}[c*x^n])], x]$

[Out]  $(x*(-(E^((2*I)*d*(a + b*\operatorname{Log}[c*x^n]))) * \operatorname{Hypergeometric2F1}[1, 1 - (I/2)/(b*d*n), 2 - (I/2)/(b*d*n), -E^((2*I)*d*(a + b*\operatorname{Log}[c*x^n]))]) + (1 + (2*I)*b*d*n) * \operatorname{Hypergeometric2F1}[1, (-I/2)/(b*d*n), 1 - (I/2)/(b*d*n), -E^((2*I)*d*(a + b*\operatorname{Log}[c*x^n]))])))/(-I + 2*b*d*n)$

**Maple [F]** time = 1.083, size = 0, normalized size = 0.

$$\int \tan(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}(\tan(d*(a+b*\ln(c*x^n))), x)$

[Out] `int(tan(d*(a+b*ln(c*x^n))),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(tan((b*log(c*x^n) + a)*d), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\tan(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(tan(b*d*log(c*x^n) + a*d), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral(tan(d*(a + b*log(c*x**n))), x)`

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Timed out

$$3.162 \quad \int \frac{\tan(d(a+b \log(cx^n)))}{x} dx$$

**Optimal.** Leaf size=26

$$-\frac{\log(\cos(ad + bd \log(cx^n)))}{bdn}$$

[Out]  $-(\text{Log}[\text{Cos}[a*d + b*d*\text{Log}[c*x^n]])]/(b*d*n)$

**Rubi [A]** time = 0.0181544, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {3475}

$$-\frac{\log(\cos(ad + bd \log(cx^n)))}{bdn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[d*(a + b*\text{Log}[c*x^n])]]/x, x]$

[Out]  $-(\text{Log}[\text{Cos}[a*d + b*d*\text{Log}[c*x^n]])]/(b*d*n)$

Rule 3475

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{\tan(d(a + b \log(cx^n)))}{x} dx &= \frac{\text{Subst}\left(\int \tan(d(a + bx)) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\log(\cos(ad + bd \log(cx^n)))}{bdn} \end{aligned}$$

**Mathematica [A]** time = 0.0467243, size = 25, normalized size = 0.96

$$-\frac{\log(\cos(d(a + b \log(cx^n))))}{bdn}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Tan}[d*(a + b*\text{Log}[c*x^n])]]/x, x]$

[Out]  $-(\text{Log}[\text{Cos}[d*(a + b*\text{Log}[c*x^n])]])/(b*d*n)$

**Maple [A]** time = 0.014, size = 30, normalized size = 1.2

$$\frac{\ln(1 + (\tan(d(a + b \ln(cx^n))))^2)}{2bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*(a+b*ln(c*x^n)))/x,x)`

[Out]  $1/2/n/d/b*\ln(1+\tan(d*(a+b*\ln(c*x^n)))^2)$

**Maxima [A]** time = 0.990878, size = 32, normalized size = 1.23

$$\frac{\log(\sec((b \log(cx^n) + a)d))}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")`

[Out]  $\log(\sec((b*\log(c*x^n) + a)*d))/(b*d*n)$

**Fricas [A]** time = 0.493028, size = 97, normalized size = 3.73

$$\frac{\log\left(\frac{1}{2} \cos(2 b d n \log(x) + 2 b d \log(c) + 2 a d) + \frac{1}{2}\right)}{2 b d n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")`

[Out]  $-1/2*\log(1/2*\cos(2*b*d*n*\log(x) + 2*b*d*\log(c) + 2*a*d) + 1/2)/(b*d*n)$

**Sympy [A]** time = 10.9052, size = 44, normalized size = 1.69

$$\begin{cases} \log(x) \tan(ad) & \text{for } b = 0 \\ 0 & \text{for } d = 0 \\ \log(x) \tan(ad + bd \log(c)) & \text{for } n = 0 \\ -\frac{\log(\cos(ad + bd \log(cx^n)))}{bdn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*ln(c*x**n)))/x,x)`

[Out] `Piecewise((log(x)*tan(a*d), Eq(b, 0)), (0, Eq(d, 0)), (log(x)*tan(a*d + b*d*log(c)), Eq(n, 0)), (-log(cos(a*d + b*d*log(c*x**n)))/(b*d*n), True))`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")`

[Out] Timed out

$$3.163 \quad \int \frac{\tan(d(a+b \log(cx^n)))}{x^2} dx$$

**Optimal.** Leaf size=71

$$\frac{i}{x} - \frac{{}_2F_1\left(1, \frac{i}{2bdn}, 1 + \frac{i}{2bdn}, -e^{2iad} (cx^n)^{2ibd}\right)}{x}$$

[Out] I/x - ((2\*I)\*Hypergeometric2F1[1, (I/2)/(b\*d\*n), 1 + (I/2)/(b\*d\*n), -(E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))])/x

**Rubi [F]** time = 0.0308557, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan(d(a+b \log(cx^n)))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[d\*(a + b\*Log[c\*x^n])]/x^2,x]

[Out] Defer[Int][Tan[d\*(a + b\*Log[c\*x^n])]/x^2, x]

Rubi steps

$$\int \frac{\tan(d(a+b \log(cx^n)))}{x^2} dx = \int \frac{\tan(d(a+b \log(cx^n)))}{x^2} dx$$

**Mathematica [B]** time = 4.03371, size = 153, normalized size = 2.15

$$\frac{(1 - 2ibd) \text{Hypergeometric2F1}\left(1, \frac{i}{2bdn}, 1 + \frac{i}{2bdn}, -e^{2id(a+b \log(cx^n))}\right) - e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 + \frac{i}{2bdn}, \right)}{x(2bdn + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[d\*(a + b\*Log[c\*x^n])]/x^2,x]

[Out] (- (E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*Hypergeometric2F1[1, 1 + (I/2)/(b\*d\*n), 2 + (I/2)/(b\*d\*n), -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) + (1 - (2\*I)\*b\*d\*n)\*Hypergeometric2F1[1, (I/2)/(b\*d\*n), 1 + (I/2)/(b\*d\*n), -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))])/((I + 2\*b\*d\*n)\*x)

**Maple [F]** time = 1.364, size = 0, normalized size = 0.

$$\int \frac{\tan(d(a+b \ln(cx^n)))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*(a+b\*ln(c\*x^n)))/x^2,x)



[Out] `int(tan(d*(a+b*ln(c*x^n)))/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan((b \log(cx^n) + a)d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")`

[Out] `integrate(tan((b*log(c*x^n) + a)*d)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan(bd \log(cx^n) + ad)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")`

[Out] `integral(tan(b*d*log(c*x^n) + a*d)/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(ad + bd \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*ln(c*x**n)))/x**2,x)`

[Out] `Integral(tan(a*d + b*d*log(c*x**n))/x**2, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")`

[Out] Timed out

$$3.164 \quad \int \frac{\tan(d(a+b \log(cx^n)))}{x^3} dx$$

**Optimal.** Leaf size=69

$$\frac{i}{2x^2} - \frac{i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, -e^{2iad} (cx^n)^{2ibd}\right)}{x^2}$$

[Out] (I/2)/x^2 - (I\*Hypergeometric2F1[1, I/(b\*d\*n), 1 + I/(b\*d\*n), -E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)])/x^2

**Rubi [F]** time = 0.0305873, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan(d(a+b \log(cx^n)))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[d\*(a + b\*Log[c\*x^n])]/x^3, x]

[Out] Defer[Int][Tan[d\*(a + b\*Log[c\*x^n])]/x^3, x]

Rubi steps

$$\int \frac{\tan(d(a+b \log(cx^n)))}{x^3} dx = \int \frac{\tan(d(a+b \log(cx^n)))}{x^3} dx$$

**Mathematica [B]** time = 3.70281, size = 147, normalized size = 2.13

$$\frac{(1 - ibdn) \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, -e^{2id(a+b \log(cx^n))}\right) - e^{2id(a+b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(1, 1 + \frac{i}{bdn}, 2 + \frac{i}{bdn}, -e^{2id(a+b \log(cx^n))}\right)}{2x^2(bdn + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[d\*(a + b\*Log[c\*x^n])]/x^3, x]

[Out] (-E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*Hypergeometric2F1[1, 1 + I/(b\*d\*n), 2 + I/(b\*d\*n), -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) + (1 - I\*b\*d\*n)\*Hypergeometric2F1[1, I/(b\*d\*n), 1 + I/(b\*d\*n), -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))])/(2\*(I + b\*d\*n)\*x^2)

**Maple [F]** time = 1.454, size = 0, normalized size = 0.

$$\int \frac{\tan(d(a+b \ln(cx^n)))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*(a+b\*ln(c\*x^n)))/x^3, x)

[Out] `int(tan(d*(a+b*ln(c*x^n)))/x^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")`

[Out] `integrate(tan((b*log(c*x^n) + a)*d)/x^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan(bd \log(cx^n) + ad)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")`

[Out] `integral(tan(b*d*log(c*x^n) + a*d)/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(ad + bd \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*ln(c*x**n)))/x**3,x)`

[Out] `Integral(tan(a*d + b*d*log(c*x**n))/x**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")`

[Out] `integrate(tan((b*log(c*x^n) + a)*d)/x^3, x)`

### 3.165 $\int x^3 \tan^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=159

$$\frac{2ix^4 \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, -e^{2iad}(cx^n)^{2ibd}\right)}{bdn} + \frac{ix^4(1 - e^{2iad}(cx^n)^{2ibd})}{bdn(1 + e^{2iad}(cx^n)^{2ibd})} + \frac{x^4(-bdn + 4i)}{4bdn}$$

[Out]  $((4*I - b*d*n)*x^4)/(4*b*d*n) + (I*x^4*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^4*\text{Hypergeometric2F1}[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(b*d*n)$

**Rubi [F]** time = 0.0900549, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \tan^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*Tan[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out] Defer[Int][x^3\*Tan[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int x^3 \tan^2(d(a + b \log(cx^n))) dx = \int x^3 \tan^2(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 6.58613, size = 179, normalized size = 1.13

$$\frac{x^4 \left( (bdn - 2i) \left( 4i \text{Hypergeometric2F1} \left( 1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, -e^{2id(a+b \log(cx^n))} \right) - 4 \tan(d(a + b \log(cx^n))) + bdn \right) - 8e^{2id(a+b \log(cx^n))} \right)}{4bdn(bdn - 2i)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Tan[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out]  $-(x^4*(-8*E^((2*I)*d*(a + b*Log[c*x^n]))*\text{Hypergeometric2F1}[1, 1 - (2*I)/(b*d*n), 2 - (2*I)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (-2*I + b*d*n)*(b*d*n + (4*I)*\text{Hypergeometric2F1}[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) - 4*\text{Tan}[d*(a + b*Log[c*x^n])])/(4*b*d*n*(-2*I + b*d*n))$

**Maple [F]** time = 1.464, size = 0, normalized size = 0.

$$\int x^3 (\tan(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*tan(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(x^3*tan(d*(a+b*ln(c*x^n)))^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \tan(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(x^3*tan(b*d*log(c*x^n) + a*d)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*tan(d*(a+b*ln(c*x**n)))**2,x)`

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] Timed out

### 3.166 $\int x^2 \tan^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=163

$$\frac{2ix^3 \text{Hypergeometric2F1}\left(1, -\frac{3i}{2bdn}, 1 - \frac{3i}{2bdn}, -e^{2iad}(cx^n)^{2ibd}\right)}{bdn} + \frac{ix^3(1 - e^{2iad}(cx^n)^{2ibd})}{bdn(1 + e^{2iad}(cx^n)^{2ibd})} + \frac{x^3(-bdn + 3i)}{3bdn}$$

[Out]  $((3I - b*d*n)*x^3)/(3*b*d*n) + (I*x^3*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^3*\text{Hypergeometric2F1}[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(b*d*n)$

**Rubi [F]** time = 0.0688039, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \tan^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Tan[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out] Defer[Int][x^2\*Tan[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int x^2 \tan^2(d(a + b \log(cx^n))) dx = \int x^2 \tan^2(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 6.29162, size = 189, normalized size = 1.16

$$\frac{x^3 \left( (2bdn - 3i) \left( 3i \text{Hypergeometric2F1} \left( 1, -\frac{3i}{2bdn}, 1 - \frac{3i}{2bdn}, -e^{2id(a+b \log(cx^n))} \right) - 3 \tan(d(a + b \log(cx^n))) + bdn \right) - 9e^{2id(a+b \log(cx^n))} \right)}{3bdn(2bdn - 3i)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Tan[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out]  $-(x^3*(-9E^((2*I)*d*(a + b*Log[c*x^n]))*\text{Hypergeometric2F1}[1, 1 - ((3*I)/2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (-3*I + 2*b*d*n)*(b*d*n + (3*I)*\text{Hypergeometric2F1}[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) - 3*\text{Tan}[d*(a + b*Log[c*x^n])])/(3*b*d*n*(-3*I + 2*b*d*n))$

**Maple [F]** time = 1.36, size = 0, normalized size = 0.

$$\int x^2 (\tan(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*tan(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(x^2*tan(d*(a+b*ln(c*x^n)))^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \tan(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(x^2*tan(b*d*log(c*x^n) + a*d)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*tan(d*(a+b*ln(c*x**n)))**2,x)`

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] Timed out

### 3.167 $\int x \tan^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=159

$$\frac{2ix^2 \text{Hypergeometric2F1}\left(1, -\frac{i}{bdn}, 1 - \frac{i}{bdn}, -e^{2iad}(cx^n)^{2ibd}\right)}{bdn} + \frac{ix^2(1 - e^{2iad}(cx^n)^{2ibd})}{bdn(1 + e^{2iad}(cx^n)^{2ibd})} + \frac{x^2(-bdn + 2i)}{2bdn}$$

[Out]  $((2*I - b*d*n)*x^2)/(2*b*d*n) + (I*x^2*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^2*Hypergeometric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(b*d*n)$

**Rubi [F]** time = 0.0438573, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \tan^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x\*Tan[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out] Defer[Int][x\*Tan[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int x \tan^2(d(a + b \log(cx^n))) dx = \int x \tan^2(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 6.5493, size = 179, normalized size = 1.13

$$\frac{x^2 \left( (bdn - i) \left( 2i \text{Hypergeometric2F1} \left( 1, -\frac{i}{bdn}, 1 - \frac{i}{bdn}, -e^{2id(a+b \log(cx^n))} \right) - 2 \tan(d(a + b \log(cx^n))) + bdn \right) - 2e^{2id(a+b \log(cx^n))} \right)}{2bdn(bdn - i)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Tan[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out]  $-(x^2*(-2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b*d*n), 2 - I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (-I + b*d*n)*(b*d*n + (2*I)*Hypergeometric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) - 2*Tan[d*(a + b*Log[c*x^n])])/(2*b*d*n*(-I + b*d*n))$

**Maple [F]** time = 1.244, size = 0, normalized size = 0.

$$\int x (\tan(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x*tan(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(x*tan(d*(a+b*ln(c*x^n)))^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \tan(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(x*tan(b*d*log(c*x^n) + a*d)^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \tan^2(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(d*(a+b*ln(c*x**n)))**2,x)`

[Out] `Integral(x*tan(a*d + b*d*log(c*x**n))**2, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] Timed out

### 3.168 $\int \tan^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=154

$$\frac{{}_2F_1\left(1, -\frac{i}{2bdn}, 1 - \frac{i}{2bdn}, -e^{2iad}(cx^n)^{2ibd}\right)}{bdn} + \frac{ix(1 - e^{2iad}(cx^n)^{2ibd})}{bdn(1 + e^{2iad}(cx^n)^{2ibd})} + \frac{x(-bdn + i)}{bdn}$$

[Out] ((I - b\*d\*n)\*x)/(b\*d\*n) + (I\*x\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))/(b\*d\*n\*(1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))) - ((2\*I)\*x\*Hypergeometric2F1[1, (-I/2)/(b\*d\*n), 1 - (I/2)/(b\*d\*n), -(E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))])/(b\*d\*n)

**Rubi [F]** time = 0.0140475, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ ., Rules used = {}

$$\int \tan^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[Tan[d\*(a + b\*Log[c\*x^n])]^2, x]

[Out] Defer[Int][Tan[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int \tan^2(d(a + b \log(cx^n))) dx = \int \tan^2(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 11.2312, size = 185, normalized size = 1.2

$$\frac{x e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{i}{2bdn}, 2 - \frac{i}{2bdn}, -e^{2id(a+b \log(cx^n))}\right) - x(2bdn - i) \left(i \text{Hypergeometric2F1}\left(1, -\frac{i}{2bdn}, 1 - \frac{i}{2bdn}, -e^{2id(a+b \log(cx^n))}\right)\right)}{bdn(2bdn - i)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[d\*(a + b\*Log[c\*x^n])]^2, x]

[Out] (E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*x\*Hypergeometric2F1[1, 1 - (I/2)/(b\*d\*n), 2 - (I/2)/(b\*d\*n), -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) - (-I + 2\*b\*d\*n)\*x\*(b\*d\*n + I\*Hypergeometric2F1[1, (-I/2)/(b\*d\*n), 1 - (I/2)/(b\*d\*n), -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) - Tan[d\*(a + b\*Log[c\*x^n])])/(b\*d\*n\*(-I + 2\*b\*d\*n))

**Maple [F]** time = 1.099, size = 0, normalized size = 0.

$$\int (\tan(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(tan(d*(a+b*ln(c*x^n)))^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\tan\left(bd \log\left(cx^n\right) + ad\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(tan(b*d*log(c*x^n) + a*d)^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan^2(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*ln(c*x**n)))**2,x)`

[Out] `Integral(tan(d*(a + b*log(c*x**n)))**2, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] Timed out

$$3.169 \quad \int \frac{\tan^2(d(a+b \log(cx^n)))}{x} dx$$

**Optimal.** Leaf size=29

$$\frac{\tan(ad + bd \log(cx^n))}{bdn} - \log(x)$$

[Out] -Log[x] + Tan[a\*d + b\*d\*Log[c\*x^n]]/(b\*d\*n)

**Rubi [A]** time = 0.0289976, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3473, 8}

$$\frac{\tan(ad + bd \log(cx^n))}{bdn} - \log(x)$$

Antiderivative was successfully verified.

[In] Int[Tan[d\*(a + b\*Log[c\*x^n])]^2/x,x]

[Out] -Log[x] + Tan[a\*d + b\*d\*Log[c\*x^n]]/(b\*d\*n)

**Rule 3473**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

**Rule 8**

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\tan^2(d(a+b \log(cx^n)))}{x} dx &= \frac{\text{Subst}\left(\int \tan^2(d(a+bx)) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\tan(ad + bd \log(cx^n))}{bdn} - \frac{\text{Subst}\left(\int 1 dx, x, \log(cx^n)\right)}{n} \\ &= -\log(x) + \frac{\tan(ad + bd \log(cx^n))}{bdn} \end{aligned}$$

**Mathematica [A]** time = 0.0820285, size = 51, normalized size = 1.76

$$\frac{\tan(ad + bd \log(cx^n))}{bdn} - \frac{\tan^{-1}(\tan(ad + bd \log(cx^n)))}{bdn}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[d\*(a + b\*Log[c\*x^n])]^2/x,x]

[Out] -(ArcTan[Tan[a\*d + b\*d\*Log[c\*x^n]]]/(b\*d\*n)) + Tan[a\*d + b\*d\*Log[c\*x^n]]/(b\*d\*n)

---

**Maple [A]** time = 0.019, size = 50, normalized size = 1.7

$$\frac{\tan(d(a + b \ln(cx^n)))}{bdn} - \frac{\arctan(\tan(d(a + b \ln(cx^n))))}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*(a+b\*ln(c\*x^n)))^2/x,x)

[Out] 1/b/d/n\*tan(d\*(a+b\*ln(c\*x^n)))-1/b/d/n\*arctan(tan(d\*(a+b\*ln(c\*x^n))))

---

**Maxima [B]** time = 1.33222, size = 432, normalized size = 14.9

$$\frac{(bd \cos(2bd \log(c))^2 + bd \sin(2bd \log(c))^2)n \cos(2bd \log(x^n) + 2ad)^2 \log(x) + (bd \cos(2bd \log(c))^2 + bd \sin(2bd \log(c))^2)n \sin(2bd \log(x^n) + 2ad)^2 \log(x)}{2bdn \cos(2bd \log(c)) \cos(2bd \log(x^n) + 2ad) - 2bdn \sin(2bd \log(c)) \sin(2bd \log(x^n) + 2ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*(a+b\*log(c\*x^n)))^2/x,x, algorithm="maxima")

[Out] -((b\*d\*cos(2\*b\*d\*log(c))^2 + b\*d\*sin(2\*b\*d\*log(c))^2)\*n\*cos(2\*b\*d\*log(x^n) + 2\*a\*d)^2\*log(x) + (b\*d\*cos(2\*b\*d\*log(c))^2 + b\*d\*sin(2\*b\*d\*log(c))^2)\*n\*log(x)\*sin(2\*b\*d\*log(x^n) + 2\*a\*d)^2 + b\*d\*n\*log(x) + 2\*(b\*d\*n\*cos(2\*b\*d\*log(c))\*log(x) - sin(2\*b\*d\*log(c)))\*cos(2\*b\*d\*log(x^n) + 2\*a\*d) - 2\*(b\*d\*n\*log(x)\*sin(2\*b\*d\*log(c)) + cos(2\*b\*d\*log(c)))\*sin(2\*b\*d\*log(x^n) + 2\*a\*d))/(2\*b\*d\*n\*cos(2\*b\*d\*log(c))\*cos(2\*b\*d\*log(x^n) + 2\*a\*d) - 2\*b\*d\*n\*sin(2\*b\*d\*log(c))\*sin(2\*b\*d\*log(x^n) + 2\*a\*d) + (b\*d\*cos(2\*b\*d\*log(c))^2 + b\*d\*sin(2\*b\*d\*log(c))^2)\*n\*cos(2\*b\*d\*log(x^n) + 2\*a\*d)^2 + (b\*d\*cos(2\*b\*d\*log(c))^2 + b\*d\*sin(2\*b\*d\*log(c))^2)\*n\*sin(2\*b\*d\*log(x^n) + 2\*a\*d)^2 + b\*d\*n)

---

**Fricas [B]** time = 0.478696, size = 242, normalized size = 8.34

$$\frac{bdn \cos(2bdn \log(x) + 2bd \log(c) + 2ad) \log(x) + bdn \log(x) - \sin(2bdn \log(x) + 2bd \log(c) + 2ad)}{bdn \cos(2bdn \log(x) + 2bd \log(c) + 2ad) + bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*(a+b\*log(c\*x^n)))^2/x,x, algorithm="fricas")

[Out] -(b\*d\*n\*cos(2\*b\*d\*n\*log(x) + 2\*b\*d\*log(c) + 2\*a\*d)\*log(x) + b\*d\*n\*log(x) - sin(2\*b\*d\*n\*log(x) + 2\*b\*d\*log(c) + 2\*a\*d))/(b\*d\*n\*cos(2\*b\*d\*n\*log(x) + 2\*b\*d\*log(c) + 2\*a\*d) + b\*d\*n)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(ad + bd \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] Integral(tan(a*d + b*d*log(c*x**n))**2/x, x)
```

---

**Giac [B]** time = 2.28555, size = 135, normalized size = 4.66

$$\frac{\tan(bd \log(c))^2 \tan(ad)^2 + \tan(bd \log(c))^2 + \tan(ad)^2 + 1}{(bdn \tan(bd \log(c)) + bdn \tan(ad))(\tan(bdn \log(x)) \tan(bd \log(c)) + \tan(bdn \log(x)) \tan(ad) + \tan(bd \log(c)) \tan(bdn \log(x)) \tan(ad))} - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

```
[Out] -(tan(b*d*log(c))^2*tan(a*d)^2 + tan(b*d*log(c))^2 + tan(a*d)^2 + 1)/((b*d*
n*tan(b*d*log(c)) + b*d*n*tan(a*d))*(tan(b*d*n*log(x))*tan(b*d*log(c)) + ta
n(b*d*n*log(x))*tan(a*d) + tan(b*d*log(c))*tan(a*d) - 1)) - log(x)
```

$$3.170 \quad \int \frac{\tan^2(d(a+b \log(cx^n)))}{x^2} dx$$

**Optimal.** Leaf size=157

$$\frac{2i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{2bdn}, 1 + \frac{i}{2bdn}, -e^{2iad} (cx^n)^{2ibd}\right)}{bdnx} + \frac{i(1 - e^{2iad} (cx^n)^{2ibd})}{bdnx(1 + e^{2iad} (cx^n)^{2ibd})} + \frac{1 + \frac{i}{bdn}}{x}$$

[Out] (1 + I/(b\*d\*n))/x + (I\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))/(b\*d\*n\*x\*(1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))) - ((2\*I)\*Hypergeometric2F1[1, (I/2)/(b\*d\*n), 1 + (I/2)/(b\*d\*n), -(E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))])/(b\*d\*n\*x)

**Rubi [F]** time = 0.0551413, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan^2(d(a+b \log(cx^n)))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[d\*(a + b\*Log[c\*x^n])]^2/x^2, x]

[Out] Defer[Int][Tan[d\*(a + b\*Log[c\*x^n])]^2/x^2, x]

Rubi steps

$$\int \frac{\tan^2(d(a+b \log(cx^n)))}{x^2} dx = \int \frac{\tan^2(d(a+b \log(cx^n)))}{x^2} dx$$

**Mathematica [A]** time = 4.31487, size = 184, normalized size = 1.17

$$\frac{(2bdn + i) \left( -i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{2bdn}, 1 + \frac{i}{2bdn}, -e^{2id(a+b \log(cx^n))}\right) + \tan(d(a+b \log(cx^n))) + bdn \right) - e^{2id(a+b \log(cx^n))}}{bdnx(2bdn + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[d\*(a + b\*Log[c\*x^n])]^2/x^2, x]

[Out] (-E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*Hypergeometric2F1[1, 1 + (I/2)/(b\*d\*n), 2 + (I/2)/(b\*d\*n), -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) + (I + 2\*b\*d\*n)\*(b\*d\*n - I\*Hypergeometric2F1[1, (I/2)/(b\*d\*n), 1 + (I/2)/(b\*d\*n), -E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) + Tan[d\*(a + b\*Log[c\*x^n])])/(b\*d\*n\*(I + 2\*b\*d\*n)\*x)

**Maple [F]** time = 1.37, size = 0, normalized size = 0.

$$\int \frac{(\tan(d(a+b \ln(cx^n))))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*(a+b*ln(c*x^n)))^2/x^2,x)`

[Out] `int(tan(d*(a+b*ln(c*x^n)))^2/x^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan(bd \log(cx^n) + ad)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")`

[Out] `integral(tan(b*d*log(c*x^n) + a*d)^2/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(ad + bd \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*ln(c*x**n)))**2/x**2,x)`

[Out] `Integral(tan(a*d + b*d*log(c*x**n))**2/x**2, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")`

[Out] Timed out



$$3.171 \quad \int \frac{\tan^2(d(a+b \log(cx^n)))}{x^3} dx$$

**Optimal.** Leaf size=156

$$-\frac{2i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, -e^{2iad} (cx^n)^{2ibd}\right)}{bdnx^2} + \frac{i(1 - e^{2iad} (cx^n)^{2ibd})}{bdnx^2(1 + e^{2iad} (cx^n)^{2ibd})} + \frac{1 + \frac{2i}{bdn}}{2x^2}$$

[Out]  $(1 + (2*I)/(b*d*n))/(2*x^2) + (I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*x^2*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*\operatorname{Hypergeometric2F1}[1, I/(b*d*n), 1 + I/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(b*d*n*x^2)$

**Rubi [F]** time = 0.053629, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tan^2(d(a+b \log(cx^n)))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Tan[d\*(a + b\*Log[c\*x^n])]^2/x^3, x]

[Out] Defer[Int][Tan[d\*(a + b\*Log[c\*x^n])]^2/x^3, x]

Rubi steps

$$\int \frac{\tan^2(d(a+b \log(cx^n)))}{x^3} dx = \int \frac{\tan^2(d(a+b \log(cx^n)))}{x^3} dx$$

**Mathematica [A]** time = 4.19479, size = 179, normalized size = 1.15

$$\frac{(bdn + i) \left( -2i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, -e^{2id(a+b \log(cx^n))}\right) + 2 \tan(d(a+b \log(cx^n))) + bdn \right) - 2e^{2id(a+b \log(cx^n))}}{2bdnx^2(bdn + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[d\*(a + b\*Log[c\*x^n])]^2/x^3, x]

[Out]  $(-2*E^((2*I)*d*(a + b*Log[c*x^n]))*\operatorname{Hypergeometric2F1}[1, 1 + I/(b*d*n), 2 + I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (I + b*d*n)*(b*d*n - (2*I)*\operatorname{Hypergeometric2F1}[1, I/(b*d*n), 1 + I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + 2*\operatorname{Tan}[d*(a + b*Log[c*x^n])])/(2*b*d*n*(I + b*d*n)*x^2)$

**Maple [F]** time = 1.565, size = 0, normalized size = 0.

$$\int \frac{(\tan(d(a+b \ln(cx^n))))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*(a+b*ln(c*x^n)))^2/x^3,x)`

[Out] `int(tan(d*(a+b*ln(c*x^n)))^2/x^3,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan(bd \log(cx^n) + ad)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")`

[Out] `integral(tan(b*d*log(c*x^n) + a*d)^2/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(ad + bd \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*ln(c*x**n)))**2/x**3,x)`

[Out] `Integral(tan(a*d + b*d*log(c*x**n))**2/x**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan((b \log(cx^n) + a)d)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")`

[Out] `integrate(tan((b*log(c*x^n) + a)*d)^2/x^3, x)`

$$3.172 \quad \int \frac{\tan^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=43

$$\frac{\tan^2(a+b \log(cx^n))}{2bn} + \frac{\log(\cos(a+b \log(cx^n)))}{bn}$$

[Out] Log[Cos[a + b\*Log[c\*x^n]]]/(b\*n) + Tan[a + b\*Log[c\*x^n]]^2/(2\*b\*n)

**Rubi [A]** time = 0.0342154, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3473, 3475}

$$\frac{\tan^2(a+b \log(cx^n))}{2bn} + \frac{\log(\cos(a+b \log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Tan[a + b\*Log[c\*x^n]]^3/x,x]

[Out] Log[Cos[a + b\*Log[c\*x^n]]]/(b\*n) + Tan[a + b\*Log[c\*x^n]]^2/(2\*b\*n)

**Rule 3473**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

**Rule 3475**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\tan^3(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \tan^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\tan^2(a+b \log(cx^n))}{2bn} - \frac{\text{Subst}\left(\int \tan(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\log(\cos(a+b \log(cx^n)))}{bn} + \frac{\tan^2(a+b \log(cx^n))}{2bn} \end{aligned}$$

**Mathematica [A]** time = 0.156367, size = 38, normalized size = 0.88

$$\frac{\tan^2(a+b \log(cx^n)) + 2 \log(\cos(a+b \log(cx^n)))}{2bn}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + b\*Log[c\*x^n]]^3/x,x]

[Out] (2\*Log[Cos[a + b\*Log[c\*x^n]]] + Tan[a + b\*Log[c\*x^n]]^2)/(2\*b\*n)

---

**Maple [A]** time = 0.018, size = 47, normalized size = 1.1

$$\frac{(\tan(a + b \ln(cx^n)))^2}{2bn} - \frac{\ln(1 + (\tan(a + b \ln(cx^n)))^2)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+b\*ln(c\*x^n))^3/x,x)

[Out] 1/2\*tan(a+b\*ln(c\*x^n))^2/b/n-1/2/n/b\*ln(1+tan(a+b\*ln(c\*x^n))^2)

---

**Maxima [B]** time = 1.14161, size = 1677, normalized size = 39.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^3/x,x, algorithm="maxima")

[Out] 1/2\*(8\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*cos(2\*b\*log(x^n) + 2\*a)^2 + 8\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*sin(2\*b\*log(x^n) + 2\*a)^2 + 4\*((cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(2\*b\*log(x^n) + 2\*a) + (cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*sin(2\*b\*log(x^n) + 2\*a))\*cos(4\*b\*log(x^n) + 4\*a) + 4\*cos(2\*b\*log(c))\*cos(2\*b\*log(x^n) + 2\*a) + ((cos(4\*b\*log(c))^2 + sin(4\*b\*log(c))^2)\*cos(4\*b\*log(x^n) + 4\*a)^2 + 4\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*cos(2\*b\*log(x^n) + 2\*a)^2 + (cos(4\*b\*log(c))^2 + sin(4\*b\*log(c))^2)\*sin(4\*b\*log(x^n) + 4\*a)^2 + 4\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*sin(2\*b\*log(x^n) + 2\*a)^2 + 2\*(2\*(cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(2\*b\*log(x^n) + 2\*a) + 2\*(cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*sin(2\*b\*log(x^n) + 2\*a) + cos(4\*b\*log(c))\*cos(4\*b\*log(x^n) + 4\*a) + 4\*cos(2\*b\*log(c))\*cos(2\*b\*log(x^n) + 2\*a) - 2\*(2\*(cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(2\*b\*log(x^n) + 2\*a) - 2\*(cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*sin(2\*b\*log(x^n) + 2\*a) + sin(4\*b\*log(c))\*sin(4\*b\*log(x^n) + 4\*a) - 4\*sin(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a) + 1)\*log((cos(2\*a)^2 + sin(2\*a)^2)\*cos(2\*b\*log(c))^2 + (cos(2\*a)^2 + sin(2\*a)^2)\*sin(2\*b\*log(c))^2 + 2\*(cos(2\*b\*log(c))\*cos(2\*a) - sin(2\*b\*log(c))\*sin(2\*a))\*cos(2\*b\*log(x^n)) + cos(2\*b\*log(x^n))^2 - 2\*(cos(2\*a)\*sin(2\*b\*log(c)) + cos(2\*b\*log(c))\*sin(2\*a))\*sin(2\*b\*log(x^n)) + sin(2\*b\*log(x^n))^2) - 4\*((cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(2\*b\*log(x^n) + 2\*a) - (cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*sin(2\*b\*log(x^n) + 2\*a))\*sin(4\*b\*log(x^n) + 4\*a) - 4\*sin(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a)) / ((b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*cos(4\*b\*log(x^n) + 4\*a)^2 + 4\*b\*n\*cos(2\*b\*log(c))\*cos(2\*b\*log(x^n) + 2\*a) + 4\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*cos(2\*b\*log(x^n) + 2\*a)^2 + (b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*sin(4\*b\*log(x^n) + 4\*a)^2 - 4\*b\*n\*sin(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a) + 4\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*sin(2\*b\*log(x^n) + 2\*a)^2 + b\*n + 2\*(b\*n\*cos(4\*b\*log(c)) + 2\*(b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n\*cos(2\*b\*log(x^n) + 2\*a) + 2\*(b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n\*sin(2\*b\*log(x^n) + 2\*a))\*cos(4\*b\*log(x^n) + 4\*a) - 2\*(2\*(b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n\*cos(2\*b\*log(x^n) + 2\*a) + b\*n\*sin(4\*b\*log(c)) - 2\*(b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n\*sin(2\*b\*log(x^n) + 2\*a))\*sin(4\*b\*log(x^n) + 4\*a)

n) + 4\*a))

**Fricas [A]** time = 0.50836, size = 209, normalized size = 4.86

$$\frac{(\cos(2bn \log(x) + 2b \log(c) + 2a) + 1) \log\left(\frac{1}{2} \cos(2bn \log(x) + 2b \log(c) + 2a) + \frac{1}{2}\right) + 2}{2(bn \cos(2bn \log(x) + 2b \log(c) + 2a) + bn)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^3/x,x, algorithm="fricas")

[Out] 1/2\*((cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) + 1)\*log(1/2\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) + 1/2) + 2)/(b\*n\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) + b\*n)

**Sympy [A]** time = 18.4742, size = 70, normalized size = 1.63

$$\begin{cases} \log(x) \tan^3(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \tan^3(a + b \log(c)) & \text{for } n = 0 \\ -\frac{\log(\tan^2(a+bn \log(x)+b \log(c))+1)}{2bn} + \frac{\tan^2(a+bn \log(x)+b \log(c))}{2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*ln(c\*x\*\*n))\*\*3/x,x)

[Out] Piecewise((log(x)\*tan(a)\*\*3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)\*tan(a + b\*log(c))\*\*3, Eq(n, 0)), (-log(tan(a + b\*n\*log(x) + b\*log(c))\*\*2 + 1)/(2\*b\*n) + tan(a + b\*n\*log(x) + b\*log(c))\*\*2/(2\*b\*n), True))

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^3/x,x, algorithm="giac")

[Out] Timed out

$$3.173 \quad \int \frac{\tan^4(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=45

$$\frac{\tan^3(a+b \log(cx^n))}{3bn} - \frac{\tan(a+b \log(cx^n))}{bn} + \log(x)$$

[Out] Log[x] - Tan[a + b\*Log[c\*x^n]]/(b\*n) + Tan[a + b\*Log[c\*x^n]]^3/(3\*b\*n)

**Rubi [A]** time = 0.0375052, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3473, 8}

$$\frac{\tan^3(a+b \log(cx^n))}{3bn} - \frac{\tan(a+b \log(cx^n))}{bn} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[Tan[a + b\*Log[c\*x^n]]^4/x, x]

[Out] Log[x] - Tan[a + b\*Log[c\*x^n]]/(b\*n) + Tan[a + b\*Log[c\*x^n]]^3/(3\*b\*n)

#### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_.)]^(n\_.), x\_Symbol] :> Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\tan^4(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \tan^4(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\tan^3(a+b \log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \tan^2(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\tan(a+b \log(cx^n))}{bn} + \frac{\tan^3(a+b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int 1 dx, x, \log(cx^n)\right)}{n} \\ &= \log(x) - \frac{\tan(a+b \log(cx^n))}{bn} + \frac{\tan^3(a+b \log(cx^n))}{3bn} \end{aligned}$$

**Mathematica [A]** time = 0.100872, size = 62, normalized size = 1.38

$$\frac{\tan^3(a+b \log(cx^n))}{3bn} + \frac{\tan^{-1}(\tan(a+b \log(cx^n)))}{bn} - \frac{\tan(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + b\*Log[c\*x^n]]^4/x, x]

```
[Out] ArcTan[Tan[a + b*Log[c*x^n]]]/(b*n) - Tan[a + b*Log[c*x^n]]/(b*n) + Tan[a +
b*Log[c*x^n]]^3/(3*b*n)
```

**Maple [A]** time = 0.018, size = 61, normalized size = 1.4

$$\frac{(\tan(a + b \ln(cx^n)))^3}{3bn} - \frac{\tan(a + b \ln(cx^n))}{bn} + \frac{\arctan(\tan(a + b \ln(cx^n)))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] 1/3*tan(a+b*ln(c*x^n))^3/b/n-tan(a+b*ln(c*x^n))/b/n+1/n/b*arctan(tan(a+b*ln
(c*x^n)))
```

**Maxima [B]** time = 1.35695, size = 2931, normalized size = 65.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a
)^2*log(x) + 27*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x
^n) + 4*a)^2*log(x) + 27*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(
2*b*log(x^n) + 2*a)^2*log(x) + 3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2
)*n*log(x)*sin(6*b*log(x^n) + 6*a)^2 + 27*(b*cos(4*b*log(c))^2 + b*sin(4*b*
log(c))^2)*n*log(x)*sin(4*b*log(x^n) + 4*a)^2 + 27*(b*cos(2*b*log(c))^2 + b
*sin(2*b*log(c))^2)*n*log(x)*sin(2*b*log(x^n) + 2*a)^2 + 3*b*n*log(x) + 2*(
3*b*n*cos(6*b*log(c))*log(x) + 3*(3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*
sin(6*b*log(c))*sin(4*b*log(c)))*n*log(x) - 2*cos(4*b*log(c))*sin(6*b*log(c
)) + 2*cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 3*(3*(b*c
os(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*log(x
) - 2*cos(2*b*log(c))*sin(6*b*log(c)) + 2*cos(6*b*log(c))*sin(2*b*log(c)))*
cos(2*b*log(x^n) + 2*a) + 3*(3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6
*b*log(c))*sin(4*b*log(c)))*n*log(x) + 2*cos(6*b*log(c))*cos(4*b*log(c)) +
2*sin(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 3*(3*(b*cos(2
*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*log(x) + 2
*cos(6*b*log(c))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*sin(2
*b*log(x^n) + 2*a) - 4*sin(6*b*log(c))*cos(6*b*log(x^n) + 6*a) + 6*(3*b*n*
cos(4*b*log(c))*log(x) + 9*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*l
og(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a)*log(x) + 9*(b*cos(2*b*log
(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*log(x)*sin(2*b*
log(x^n) + 2*a) - 2*sin(4*b*log(c))*cos(4*b*log(x^n) + 4*a) + 6*(3*b*n*cos
(2*b*log(c))*log(x) - 2*sin(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*(3*b*n
*log(x)*sin(6*b*log(c)) + 3*(3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6
*b*log(c))*sin(4*b*log(c)))*n*log(x) + 2*cos(6*b*log(c))*cos(4*b*log(c)) +
2*sin(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 3*(3*(b*cos(2
*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*log(x) + 2
*cos(6*b*log(c))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*cos(2
*b*log(x^n) + 2*a) - 3*(3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*lo
g(c))*sin(4*b*log(c)))*n*log(x) - 2*cos(4*b*log(c))*sin(6*b*log(c)) + 2*cos
(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 3*(3*(b*cos(6*b*log
(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*log(x) - 2*cos(
```





Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Piecewise((log(x)*tan(a)**4, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*tan(a + b*log(c))**4, Eq(n, 0)), (log(x) + tan(a + b*n*log(x) + b*log(c))**3/(3*b*n) - tan(a + b*n*log(x) + b*log(c))/(b*n), True))
```

**Giac [B]** time = 5.26057, size = 1430, normalized size = 31.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -1/3*(3*tan(b*n*log(x))^2*tan(b*log(c))^6*tan(a)^4 + 6*tan(b*n*log(x))^2*tan(b*log(c))^5*tan(a)^5 + 3*tan(b*n*log(x))*tan(b*log(c))^6*tan(a)^5 + 3*tan(b*n*log(x))^2*tan(b*log(c))^4*tan(a)^6 + 3*tan(b*n*log(x))*tan(b*log(c))^5*tan(a)^6 + tan(b*log(c))^6*tan(a)^6 - 12*tan(b*n*log(x))^2*tan(b*log(c))^5*tan(a)^3 - 6*tan(b*n*log(x))*tan(b*log(c))^6*tan(a)^3 - 24*tan(b*n*log(x))^2*tan(b*log(c))^4*tan(a)^4 - 33*tan(b*n*log(x))*tan(b*log(c))^5*tan(a)^4 - 3*tan(b*log(c))^6*tan(a)^4 - 12*tan(b*n*log(x))^2*tan(b*log(c))^3*tan(a)^5 - 33*tan(b*n*log(x))*tan(b*log(c))^4*tan(a)^5 - 12*tan(b*log(c))^5*tan(a)^5 - 6*tan(b*n*log(x))*tan(b*log(c))^3*tan(a)^6 - 3*tan(b*log(c))^4*tan(a)^6 - 3*tan(b*n*log(x))^2*tan(b*log(c))^6 - 18*tan(b*n*log(x))^2*tan(b*log(c))^5*tan(a) - 9*tan(b*n*log(x))*tan(b*log(c))^6*tan(a) - 27*tan(b*n*log(x))^2*tan(b*log(c))^4*tan(a)^2 - 27*tan(b*n*log(x))*tan(b*log(c))^5*tan(a)^2 - 3*tan(b*log(c))^6*tan(a)^2 - 24*tan(b*n*log(x))^2*tan(b*log(c))^3*tan(a)^3 - 6*tan(b*n*log(x))*tan(b*log(c))^4*tan(a)^3 - 27*tan(b*n*log(x))^2*tan(b*log(c))^2*tan(a)^4 - 6*tan(b*n*log(x))*tan(b*log(c))^3*tan(a)^4 + 21*tan(b*log(c))^4*tan(a)^4 - 18*tan(b*n*log(x))^2*tan(b*log(c))*tan(a)^5 - 27*tan(b*n*log(x))*tan(b*log(c))^2*tan(a)^5 - 3*tan(b*n*log(x))^2*tan(a)^6 - 9*tan(b*n*log(x))*tan(b*log(c))*tan(a)^6 - 3*tan(b*log(c))^2*tan(a)^6 + 9*tan(b*n*log(x))*tan(b*log(c))^5 + tan(b*log(c))^6 - 12*tan(b*n*log(x))^2*tan(b*log(c))^3*tan(a) + 27*tan(b*n*log(x))*tan(b*log(c))^4*tan(a) + 12*tan(b*log(c))^5*tan(a) - 24*tan(b*n*log(x))^2*tan(b*log(c))^2*tan(a)^2 + 6*tan(b*n*log(x))*tan(b*log(c))^3*tan(a)^2 + 21*tan(b*log(c))^4*tan(a)^2 - 12*tan(b*n*log(x))^2*tan(b*log(c))*tan(a)^3 + 6*tan(b*n*log(x))*tan(b*log(c))^2*tan(a)^3 + 27*tan(b*n*log(x))*tan(b*log(c))*tan(a)^4 + 21*tan(b*log(c))^2*tan(a)^4 + 9*tan(b*n*log(x))*tan(a)^5 + 12*tan(b*log(c))*tan(a)^5 + tan(a)^6 + 3*tan(b*n*log(x))^2*tan(b*log(c))^2 + 6*tan(b*n*log(x))*tan(b*log(c))^3 - 3*tan(b*log(c))^4 + 6*tan(b*n*log(x))^2*tan(b*log(c))*tan(a) + 33*tan(b*n*log(x))*tan(b*log(c))^2*tan(a) + 3*tan(b*n*log(x))^2*tan(a)^2 + 33*tan(b*n*log(x))*tan(b*log(c))*tan(a)^2 + 21*tan(b*log(c))^2*tan(a)^2 + 6*tan(b*n*log(x))*tan(a)^3 - 3*tan(a)^4 - 3*tan(b*n*log(x))*tan(b*log(c)) - 3*tan(b*log(c))^2 - 3*tan(b*n*log(x))*tan(a) - 12*tan(b*log(c))*tan(a) - 3*tan(a)^2 + 1)/((b*n*tan(b*log(c))^3 + 3*b*n*tan(b*log(c))^2*tan(a) + 3*b*n*tan(b*log(c))*tan(a)^2 + b*n*tan(a)^3)*(tan(b*n*log(x))*tan(b*log(c)) + tan(b*n*log(x))*tan(a) + tan(b*log(c))*tan(a) - 1)^3) + log(x)
```

$$3.174 \quad \int \frac{\tan^5(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=67

$$\frac{\tan^4(a+b \log(cx^n))}{4bn} - \frac{\tan^2(a+b \log(cx^n))}{2bn} - \frac{\log(\cos(a+b \log(cx^n)))}{bn}$$

[Out]  $-(\text{Log}[\text{Cos}[a + b*\text{Log}[c*x^n]])/(b*n)) - \text{Tan}[a + b*\text{Log}[c*x^n]]^2/(2*b*n) + \text{Tan}[a + b*\text{Log}[c*x^n]]^4/(4*b*n)$

**Rubi [A]** time = 0.044337, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3473, 3475}

$$\frac{\tan^4(a+b \log(cx^n))}{4bn} - \frac{\tan^2(a+b \log(cx^n))}{2bn} - \frac{\log(\cos(a+b \log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Tan}[a + b*\text{Log}[c*x^n]]^5/x, x]$

[Out]  $-(\text{Log}[\text{Cos}[a + b*\text{Log}[c*x^n]])/(b*n)) - \text{Tan}[a + b*\text{Log}[c*x^n]]^2/(2*b*n) + \text{Tan}[a + b*\text{Log}[c*x^n]]^4/(4*b*n)$

#### Rule 3473

$\text{Int}[(b_*)*\text{tan}[(c_*) + (d_*)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n-2)}, x], x] /;$   $\text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1]$

#### Rule 3475

$\text{Int}[\text{tan}[(c_*) + (d_*)*(x_)], x\_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /;$   $\text{FreeQ}\{c, d\}, x$

#### Rubi steps

$$\begin{aligned} \int \frac{\tan^5(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \tan^5(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\tan^4(a+b \log(cx^n))}{4bn} - \frac{\text{Subst}\left(\int \tan^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\tan^2(a+b \log(cx^n))}{2bn} + \frac{\tan^4(a+b \log(cx^n))}{4bn} + \frac{\text{Subst}\left(\int \tan(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\log(\cos(a+b \log(cx^n)))}{bn} - \frac{\tan^2(a+b \log(cx^n))}{2bn} + \frac{\tan^4(a+b \log(cx^n))}{4bn} \end{aligned}$$

**Mathematica [A]** time = 0.163187, size = 55, normalized size = 0.82

$$-\frac{\tan^4(a+b \log(cx^n)) + 2 \tan^2(a+b \log(cx^n)) + 4 \log(\cos(a+b \log(cx^n)))}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + b\*Log[c\*x^n]]^5/x,x]

[Out]  $-(4*\text{Log}[\text{Cos}[a + b*\text{Log}[c*x^n]]] + 2*\text{Tan}[a + b*\text{Log}[c*x^n]]^2 - \text{Tan}[a + b*\text{Log}[c*x^n]]^4)/(4*b*n)$

**Maple [A]** time = 0.017, size = 68, normalized size = 1.

$$\frac{(\tan(a + b \ln(cx^n)))^4}{4bn} - \frac{(\tan(a + b \ln(cx^n)))^2}{2bn} + \frac{\ln(1 + (\tan(a + b \ln(cx^n)))^2)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+b\*ln(c\*x^n))^5/x,x)

[Out]  $1/4*\tan(a+b*\ln(c*x^n))^4/b/n-1/2*\tan(a+b*\ln(c*x^n))^2/b/n+1/2/n/b*\ln(1+\tan(a+b*\ln(c*x^n))^2)$

**Maxima [B]** time = 1.5303, size = 6029, normalized size = 89.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^5/x,x, algorithm="maxima")

[Out]  $-1/2*(32*(\cos(6*b*\log(c))^2 + \sin(6*b*\log(c))^2)*\cos(6*b*\log(x^n) + 6*a)^2 + 48*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\cos(4*b*\log(x^n) + 4*a)^2 + 32*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\cos(2*b*\log(x^n) + 2*a)^2 + 32*(\cos(6*b*\log(c))^2 + \sin(6*b*\log(c))^2)*\sin(6*b*\log(x^n) + 6*a)^2 + 48*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\sin(4*b*\log(x^n) + 4*a)^2 + 32*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\sin(2*b*\log(x^n) + 2*a)^2 + 8*((\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) + (\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + (\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + (\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) + (\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + (\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a)*\cos(8*b*\log(x^n) + 8*a) + 8*(10*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 8*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 10*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 8*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \cos(6*b*\log(c))*\cos(6*b*\log(x^n) + 6*a) + 8*(10*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 10*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \cos(4*b*\log(c))*\cos(4*b*\log(x^n) + 4*a) + 8*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + ((\cos(8*b*\log(c))^2 + \sin(8*b*\log(c))^2)*\cos(8*b*\log(x^n) + 8*a)^2 + 16*(\cos(6*b*\log(c))^2 + \sin(6*b*\log(c))^2)*\cos(6*b*\log(x^n) + 6*a)^2 + 36*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\cos(4*b*\log(x^n) + 4*a)^2 + 16*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\cos(2*b*\log(x^n) + 2*a)^2 + (\cos(8*b*\log(c))^2 + \sin(8*b*\log(c))^2)*\sin(8*b*\log(x^n) + 8*a)^2 + 16*(\cos(6*b*\log(c))^2 + \sin(6*b*\log(c))^2)*\sin(6*b*\log(x^n) + 6*a)^2 + 36*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\sin(4*b*\log(x^n) + 4*a)^2 + 16*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\sin(2*b*\log(x^n) + 2*a)^2$

$$\begin{aligned}
&g(c)^2 + \sin(4*b*\log(c))^2*\sin(4*b*\log(x^n) + 4*a)^2 + 16*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\sin(2*b*\log(x^n) + 2*a)^2 + 2*(4*(\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) + 6*(\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 4*(\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 4*(\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) + 6*(\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 4*(\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \cos(8*b*\log(c))*\cos(8*b*\log(x^n) + 8*a) + 8*(6*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 4*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 6*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 4*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \cos(6*b*\log(c))*\cos(6*b*\log(x^n) + 6*a) + 12*(4*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 4*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \cos(4*b*\log(c))*\cos(4*b*\log(x^n) + 4*a) + 8*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) - 2*(4*(\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) + 6*(\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 4*(\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 4*(\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) - 6*(\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - 4*(\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(8*b*\log(c))*\sin(8*b*\log(x^n) + 8*a) - 8*(6*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 4*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 6*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - 4*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(6*b*\log(c))*\sin(6*b*\log(x^n) + 6*a) - 12*(4*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 4*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(4*b*\log(c))*\sin(4*b*\log(x^n) + 4*a) - 8*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 1)*\log((\cos(2*a)^2 + \sin(2*a)^2)*\cos(2*b*\log(c))^2 + (\cos(2*a)^2 + \sin(2*a)^2)*\sin(2*b*\log(c))^2 + 2*(\cos(2*b*\log(c))*\cos(2*a) - \sin(2*b*\log(c))*\sin(2*a))*\cos(2*b*\log(x^n)) + \cos(2*b*\log(x^n))^2 - 2*(\cos(2*a)*\sin(2*b*\log(c)) + \cos(2*b*\log(c))*\sin(2*a))*\sin(2*b*\log(x^n)) + \sin(2*b*\log(x^n))^2) - 8*((\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) + (\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + (\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - (\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) - (\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - (\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a))*\sin(8*b*\log(x^n) + 8*a) - 8*(10*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 8*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 10*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - 8*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(6*b*\log(c))*\sin(6*b*\log(x^n) + 6*a) - 8*(10*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 10*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(4*b*\log(c))*\sin(4*b*\log(x^n) + 4*a) - 8*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a))/((b*\cos(8*b*\log(c)))^2 +
\end{aligned}$$

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b*sin(8*b*log(c))^2)*n*cos(8*b*log(x^n) + 8*a)^2 + 16*(b*cos(6*b*log(c))^2
+ b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a)^2 + 36*(b*cos(4*b*log(c))^
2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 + 8*b*n*cos(2*b*log(c)
)*cos(2*b*log(x^n) + 2*a) + 16*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*
n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(8*b*log(c))^2 + b*sin(8*b*log(c))^2)*n
*sin(8*b*log(x^n) + 8*a)^2 + 16*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)
*n*sin(6*b*log(x^n) + 6*a)^2 + 36*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^
2)*n*sin(4*b*log(x^n) + 4*a)^2 - 8*b*n*sin(2*b*log(c))*sin(2*b*log(x^n) + 2
*a) + 16*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) + 2
*a)^2 + b*n + 2*(b*n*cos(8*b*log(c)) + 4*(b*cos(8*b*log(c))*cos(6*b*log(c))
+ b*sin(8*b*log(c))*sin(6*b*log(c)))*n*cos(6*b*log(x^n) + 6*a) + 6*(b*cos(
8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*
log(x^n) + 4*a) + 4*(b*cos(8*b*log(c))*cos(2*b*log(c)) + b*sin(8*b*log(c))*
sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) + 4*(b*cos(6*b*log(c))*sin(8*b*1
og(c)) - b*cos(8*b*log(c))*sin(6*b*log(c)))*n*sin(6*b*log(x^n) + 6*a) + 6*(
b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)))*n*si
n(4*b*log(x^n) + 4*a) + 4*(b*cos(2*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*lo
g(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a)*cos(8*b*log(x^n) + 8*a) +
8*(b*n*cos(6*b*log(c)) + 6*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*
log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) + 4*(b*cos(6*b*log(c))*c
os(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*
a) + 6*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)
))*n*sin(4*b*log(x^n) + 4*a) + 4*(b*cos(2*b*log(c))*sin(6*b*log(c)) - b*co
s(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a)*cos(6*b*log(x^n)
+ 6*a) + 12*(b*n*cos(4*b*log(c)) + 4*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b
*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) + 4*(b*cos(2*b*
log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(
x^n) + 2*a)*cos(4*b*log(x^n) + 4*a) - 2*(4*(b*cos(6*b*log(c))*sin(8*b*log(
c)) - b*cos(8*b*log(c))*sin(6*b*log(c)))*n*cos(6*b*log(x^n) + 6*a) + 6*(b*c
os(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)))*n*cos(4
*b*log(x^n) + 4*a) + 4*(b*cos(2*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c)
))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) + b*n*sin(8*b*log(c)) - 4*(b*
cos(8*b*log(c))*cos(6*b*log(c)) + b*sin(8*b*log(c))*sin(6*b*log(c)))*n*sin(
6*b*log(x^n) + 6*a) - 6*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(
c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) - 4*(b*cos(8*b*log(c))*cos(2
*b*log(c)) + b*sin(8*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*
sin(8*b*log(x^n) + 8*a) - 8*(6*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6
*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) + 4*(b*cos(2*b*log(c)
))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) +
2*a) + b*n*sin(6*b*log(c)) - 6*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(
6*b*log(c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) - 4*(b*cos(6*b*log(c)
))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n)
+ 2*a))*sin(6*b*log(x^n) + 6*a) - 12*(4*(b*cos(2*b*log(c))*sin(4*b*log(c))
- b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) + b*n*sin(4*
b*log(c)) - 4*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*
b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4*a))

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**Fricas [B]** time = 0.508384, size = 387, normalized size = 5.78

$$\frac{\left(\cos(2bn \log(x) + 2b \log(c) + 2a)^2 + 2 \cos(2bn \log(x) + 2b \log(c) + 2a) + 1\right) \log\left(\frac{1}{2} \cos(2bn \log(x) + 2b \log(c) + 2a)\right)}{2 \left(bn \cos(2bn \log(x) + 2b \log(c) + 2a)^2 + 2bn \cos(2bn \log(x) + 2b \log(c) + 2a) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^5/x,x, algorithm="fricas")

```
[Out] -1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2 + 2*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)*log(1/2*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1/2) + 4*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2 + 2*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + b*n)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.175 $\int (ex)^m \tan(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=101

$$\frac{2i(ex)^{m+1} \text{Hypergeometric2F1}\left(1, -\frac{i(m+1)}{2bdn}, 1 - \frac{i(m+1)}{2bdn}, -e^{2iad}(cx^n)^{2ibd}\right)}{e(m+1)} - \frac{i(ex)^{m+1}}{e(m+1)}$$

[Out]  $((-I)*(e*x)^{(1+m)})/(e*(1+m)) + ((2*I)*(e*x)^{(1+m)}*\text{Hypergeometric2F1}[1, ((-I/2)*(1+m))/(b*d*n), 1 - ((I/2)*(1+m))/(b*d*n), -(E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})])/(e*(1+m))$

**Rubi [F]** time = 0.0478934, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ , Rules used = {}

$$\int (ex)^m \tan(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])], x]

[Out] Defer[Int][(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])], x]

Rubi steps

$$\int (ex)^m \tan(d(a + b \log(cx^n))) dx = \int (ex)^m \tan(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 14.5072, size = 186, normalized size = 1.84

$$\frac{ix(ex)^m \left( \text{Hypergeometric2F1}\left(1, -\frac{i(m+1)}{2bdn}, 1 - \frac{i(m+1)}{2bdn}, -e^{2id(a+b \log(cx^n))}\right) - \frac{(m+1)e^{2iad}(cx^n)^{2ibd} \text{Hypergeometric2F1}\left(1, -\frac{i(2ibd+n+1)}{2bdn}\right)}{2ibd+n+1} \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])], x]

[Out]  $(I*x*(e*x)^m*(\text{Hypergeometric2F1}[1, ((-I/2)*(1+m))/(b*d*n), 1 - ((I/2)*(1+m))/(b*d*n), -E^{((2*I)*d*(a + b*Log[c*x^n])})] - (E^{((2*I)*a*d)}*(1+m)*(c*x^n)^{((2*I)*b*d)}*\text{Hypergeometric2F1}[1, ((-I/2)*(1+m + (2*I)*b*d*n))/(b*d*n), ((-I/2)*(1+m + (4*I)*b*d*n))/(b*d*n), -E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}])/(1+m + (2*I)*b*d*n))/(1+m)$

**Maple [F]** time = 1.721, size = 0, normalized size = 0.

$$\int (ex)^m \tan(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*tan(d*(a+b*ln(c*x^n))),x)`

[Out] `int((e*x)^m*tan(d*(a+b*ln(c*x^n))),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate((e*x)^m*tan((b*log(c*x^n) + a)*d), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \tan(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral((e*x)^m*tan(b*d*log(c*x^n) + a*d), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*tan(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral((e*x)**m*tan(a*d + b*d*log(c*x**n)), x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Timed out



### 3.176 $\int (ex)^m \tan^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=196

$$\frac{2i(ex)^{m+1} \text{Hypergeometric2F1}\left(1, -\frac{i(m+1)}{2bdn}, 1 - \frac{i(m+1)}{2bdn}, -e^{2iad}(cx^n)^{2ibd}\right)}{bden} + \frac{i(ex)^{m+1}(1 - e^{2iad}(cx^n)^{2ibd})}{bden(1 + e^{2iad}(cx^n)^{2ibd})} + \frac{(ex)^{m+1}(-bdn)}{bden}$$

[Out]  $((I*(1 + m) - b*d*n)*(e*x)^{(1 + m)})/(b*d*e*(1 + m)*n) + (I*(e*x)^{(1 + m)}*(1 - E^{((2*I)*a*d)*(c*x^n)^{((2*I)*b*d)}})/(b*d*e*n*(1 + E^{((2*I)*a*d)*(c*x^n)^{((2*I)*b*d)}}) - ((2*I)*(e*x)^{(1 + m)}*\text{Hypergeometric2F1}[1, ((-I/2)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), -E^{((2*I)*a*d)*(c*x^n)^{((2*I)*b*d)}}])/(b*d*e*n)$

**Rubi [F]** time = 0.0821511, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \tan^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out] Defer[Int][(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int (ex)^m \tan^2(d(a + b \log(cx^n))) dx = \int (ex)^m \tan^2(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 17.4527, size = 550, normalized size = 2.81

$$(m + 1)x^{-m}(ex)^m \sec(d(a + b(\log(cx^n) - n \log(x)))) \left( \frac{x^{m+1} \sin(bdn \log(x)) \sec(d(a + b \log(cx^n)))}{m+1} - \frac{i \cos(d(a + b(\log(cx^n) - n \log(x)))) \exp(i d(a + b \log(cx^n)))}{m+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out]  $-((x*(e*x)^m)/(1 + m)) + (x*(e*x)^m*\text{Sec}[d*(a + b*(-(n*\text{Log}[x]) + \text{Log}[c*x^n]))]*\text{Sec}[b*d*n*\text{Log}[x] + d*(a + b*(-(n*\text{Log}[x]) + \text{Log}[c*x^n]))]*\text{Sin}[b*d*n*\text{Log}[x]])/(b*d*n) - ((1 + m)*(e*x)^m*\text{Sec}[d*(a + b*(-(n*\text{Log}[x]) + \text{Log}[c*x^n]))]*((x^{(1 + m)}*\text{Sec}[d*(a + b*\text{Log}[c*x^n]))*\text{Sin}[b*d*n*\text{Log}[x]])/(1 + m) - (I*\text{Cos}[d*(a + b*(-(n*\text{Log}[x]) + \text{Log}[c*x^n]))]*(-E^{((a + 2*a*m + b*(1 + m)*n*\text{Log}[x] + b*(1 + 2*m)*(-(n*\text{Log}[x]) + \text{Log}[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*\text{Hypergeometric2F1}[1, ((-I/2)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), -E^{((2*I)*d*(a + b*\text{Log}[c*x^n]))}) + E^{((a*(1 + 2*m + (2*I)*b*d*n))/(b*n) + (1 + m + (2*I)*b*d*n)*\text{Log}[x] + ((1 + 2*m + (2*I)*b*d*n)*(-(n*\text{Log}[x]) + \text{Log}[c*x^n]))/n)*(1 + m)*\text{Hypergeometric2F1}[1, ((-I/2)*(1 + m + (2*I)*b*d*n))/(b*d*n), ((-I/2)*(1 + m + (4*I)*b*d*n))/(b*d*n), -E^{((2*I)*d*(a + b*\text{Log}[c*x^n]))})] -$

$$\frac{I \cdot E^{((a + 2 \cdot a \cdot m + b \cdot (1 + m) \cdot n \cdot \text{Log}[x] + b \cdot (1 + 2 \cdot m) \cdot (-n \cdot \text{Log}[x]) + \text{Log}[c \cdot x^n]))} / (b \cdot n)) \cdot (1 + m + (2 \cdot I) \cdot b \cdot d \cdot n) \cdot \text{Tan}[d \cdot (a + b \cdot \text{Log}[c \cdot x^n])]} / (E^{(((1 + 2 \cdot m) \cdot (a + b \cdot (-n \cdot \text{Log}[x]) + \text{Log}[c \cdot x^n]))} / (b \cdot n)) \cdot (1 + m) \cdot (1 + m + (2 \cdot I) \cdot b \cdot d \cdot n))} / (b \cdot d \cdot n \cdot x^m)$$

**Maple [F]** time = 1.703, size = 0, normalized size = 0.

$$\int (ex)^m (\tan(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*tan(d\*(a+b\*ln(c\*x^n)))^2,x)

[Out] int((e\*x)^m\*tan(d\*(a+b\*ln(c\*x^n)))^2,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*tan(d\*(a+b\*log(c\*x^n)))^2,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \tan(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*tan(d\*(a+b\*log(c\*x^n)))^2,x, algorithm="fricas")

[Out] integral((e\*x)^m\*tan(b\*d\*log(c\*x^n) + a\*d)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*tan(d\*(a+b\*ln(c\*x\*\*n))))\*\*2,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.177 $\int (ex)^m \tan^3 (d(a + b \log (cx^n))) dx$

**Optimal.** Leaf size=351

$$\frac{i(ex)^{m+1} \left(-2b^2d^2n^2 + m^2 + 2m + 1\right) \text{Hypergeometric2F1} \left(1, -\frac{i(m+1)}{2bdn}, 1 - \frac{i(m+1)}{2bdn}, -e^{2iad} (cx^n)^{2ibd}\right)}{b^2d^2e(m+1)n^2} - \frac{ie^{-2iad}(ex)^{m+1} \left(\frac{e^{2iad}(-)}{2b^2d^2e}\right)}{2b^2d^2e}$$

[Out]  $-\left((I*(1 + m) - b*d*n)*(1 + m + (2*I)*b*d*n)*(e*x)^{(1 + m)} / (2*b^2*d^2*e*(1 + m)*n^2) - ((e*x)^{(1 + m)}*(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^2 / (2*b*d*e*n*(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^2) - ((I/2)*(e*x)^{(1 + m)}*(E^{((2*I)*a*d)}*(1 + m - (2*I)*b*d*n)) / n - (E^{((4*I)*a*d)}*(1 + m + (2*I)*b*d*n)*(c*x^n)^{((2*I)*b*d)} / n) / (b^2*d^2*e*E^{((2*I)*a*d)}*n*(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})) + (I*(1 + 2*m + m^2 - 2*b^2*d^2*n^2)*(e*x)^{(1 + m)}*Hypergeometric2F1[1, ((-I/2)*(1 + m)) / (b*d*n), 1 - ((I/2)*(1 + m)) / (b*d*n), -E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}]) / (b^2*d^2*e*(1 + m)*n^2)$

**Rubi [F]** time = 0.0759238, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \tan^3 (d(a + b \log (cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^3,x]

[Out] Defer[Int][(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^3, x]

Rubi steps

$$\int (ex)^m \tan^3 (d(a + b \log (cx^n))) dx = \int (ex)^m \tan^3 (d(a + b \log (cx^n))) dx$$

**Mathematica [A]** time = 17.7815, size = 642, normalized size = 1.83

$$x^{-m}(ex)^m \left(2b^2d^2n^2 - m^2 - 2m - 1\right) \sec (d(a + b(\log (cx^n) - n \log (x)))) \left(\frac{x^{m+1} \sin (bdn \log (x)) \sec (d(a + b \log (cx^n)))}{m+1} - \frac{i \cos (d(a + b \log (cx^n)))}{m+1}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^3,x]

[Out]  $(x*(e*x)^m*Sec[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2) / (2*b*d*n) - ((1 + m)*x*(e*x)^m*Sec[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sec[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]]) / (2*b^2*d^2*n^2) - ((-1 - 2*m - m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Sec[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*((x^(1 + m)*Sec[d*(a + b*Log[c*x^n]))*Sin[b*d*n*Log[x]]) / (1 + m) - (I*Cos[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(-E^{(a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n])) / (b*n))*(1 + m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-I/2)*(1 + m)) / (b*d*n), 1 - ((I/2)$

$$\begin{aligned} & * (1 + m) / (b*d*n), -E^{((2*I)*d*(a + b*\text{Log}[c*x^n]))}] + E^{((a*(1 + 2*m + (2*I)*b*d*n)) / (b*n) + (1 + m + (2*I)*b*d*n)*\text{Log}[x] + ((1 + 2*m + (2*I)*b*d*n)* \\ & (-n*\text{Log}[x] + \text{Log}[c*x^n])) / n) * (1 + m) * \text{Hypergeometric2F1}[1, ((-I/2)*(1 + m + (2*I)*b*d*n)) / (b*d*n), ((-I/2)*(1 + m + (4*I)*b*d*n)) / (b*d*n), -E^{((2*I)* \\ & d*(a + b*\text{Log}[c*x^n]))}] - I * E^{((a + 2*a*m + b*(1 + m)*n*\text{Log}[x] + b*(1 + 2*m) \\ & * (-n*\text{Log}[x] + \text{Log}[c*x^n])) / (b*n)) * (1 + m + (2*I)*b*d*n) * \text{Tan}[d*(a + b*\text{Log}[ \\ & c*x^n])]})) / (E^{((1 + 2*m)*(a + b*(-n*\text{Log}[x] + \text{Log}[c*x^n])) / (b*n)) * (1 + m) \\ & * (1 + m + (2*I)*b*d*n))} / (2*b^2*d^2*n^2*x^m) - (x*(e*x)^m * \text{Tan}[d*(a + b*(- \\ & n*\text{Log}[x] + \text{Log}[c*x^n]))]) / (1 + m) \end{aligned}$$

**Maple [F]** time = 1.796, size = 0, normalized size = 0.

$$\int (ex)^m (\tan(d(a + b \ln(cx^n))))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*tan(d\*(a+b\*ln(c\*x^n)))^3,x)

[Out] int((e\*x)^m\*tan(d\*(a+b\*ln(c\*x^n)))^3,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*tan(d\*(a+b\*log(c\*x^n)))^3,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \tan(bd \log(cx^n) + ad)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*tan(d\*(a+b\*log(c\*x^n)))^3,x, algorithm="fricas")

[Out] integral((e\*x)^m\*tan(b\*d\*log(c\*x^n) + a\*d)^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*tan(d\*(a+b\*ln(c\*x\*\*n)))\*\*3,x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")`

[Out] Timed out

### 3.178 $\int \tan^p(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=190

$$x(1 - e^{2iad}(cx^n)^{2ibd})^{-p} \left( \frac{i(1 - e^{2iad}(cx^n)^{2ibd})}{1 + e^{2iad}(cx^n)^{2ibd}} \right)^p (1 + e^{2iad}(cx^n)^{2ibd})^p F_1 \left( -\frac{i}{2bdn}; -p, p; 1 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd} \right)$$

[Out]  $(x*((I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*\text{AppellF1}[(-I/2)/(b*d*n), -p, p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]/(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p$

**Rubi [F]** time = 0.0148668, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \tan^p(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[Tan[d\*(a + b\*Log[c\*x^n])]^p, x]

[Out] Defer[Int][Tan[d\*(a + b\*Log[c\*x^n])]^p, x]

Rubi steps

$$\int \tan^p(d(a + b \log(cx^n))) dx = \int \tan^p(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 1.41376, size = 458, normalized size = 2.41

$$\frac{x(2bdn - i) \left( \frac{i(-1 + e^{2iad}(cx^n)^{2ibd})}{1 + e^{2iad}(cx^n)^{2ibd}} \right)^p F_1 \left( -\frac{i}{2bdn}; -p, p; 1 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd} \right) - 2bdnpe^{2iad}(cx^n)^{2ibd} F_1 \left( 1 - \frac{i}{2bdn}; 1 - p, p; 2 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd} \right) - 2bdnpe^{2iad}(cx^n)^{2ibd} F_1 \left( 1 - \frac{i}{2bdn}; -p, p; 1 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd} \right)}{1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Tan[d\*(a + b\*Log[c\*x^n])]^p, x]

[Out]  $((-I + 2*b*d*n)*x*((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*\text{AppellF1}[(-I/2)/(b*d*n), -p, p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]/(-2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*b*d)*\text{AppellF1}[1 - (I/2)/(b*d*n), 1 - p, p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] - 2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*b*d)*\text{AppellF1}[1 - (I/2)/(b*d*n), -p, 1 + p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] + (-I + 2*b*d*n)*\text{AppellF1}[(-I/2)/(b*d*n), -p, p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]$

**Maple [F]** time = 0.177, size = 0, normalized size = 0.

$$\int (\tan(d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d\*(a+b\*ln(c\*x^n)))^p,x)

[Out] int(tan(d\*(a+b\*ln(c\*x^n)))^p,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="maxima")

[Out] integrate(tan((b\*log(c\*x^n) + a)\*d)^p, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\tan(bd \log(cx^n) + ad)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="fricas")

[Out] integral(tan(b\*d\*log(c\*x^n) + a\*d)^p, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \tan^p(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*(a+b\*ln(c\*x\*\*n)))\*\*p,x)

[Out] Integral(tan(d\*(a + b\*log(c\*x\*\*n)))\*\*p, x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="giac")

[Out] Timed out



### 3.179 $\int (ex)^m \tan^p (d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=210

$$\frac{(ex)^{m+1} (1 - e^{2iad} (cx^n)^{2ibd})^{-p} \left( \frac{i(1 - e^{2iad} (cx^n)^{2ibd})}{1 + e^{2iad} (cx^n)^{2ibd}} \right)^p (1 + e^{2iad} (cx^n)^{2ibd})^p F_1 \left( -\frac{i(m+1)}{2bdn}; -p, p; 1 - \frac{i(m+1)}{2bdn}; e^{2iad} (cx^n)^{2ibd}, -e^{2iad} (cx^n)^{2ibd} \right)}{e(m+1)}$$

[Out]  $((e*x)^{(1+m)}*((I*(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}))/(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}))^p*(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p*Appel1F1[((-I/2)*(1+m))/(b*d*n), -p, p, 1 - ((I/2)*(1+m))/(b*d*n), E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}, -(E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})]/(e*(1+m)*(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p)$

**Rubi [F]** time = 0.104173, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \tan^p (d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out] Defer[Int][(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^p, x]

Rubi steps

$$\int (ex)^m \tan^p (d(a + b \log(cx^n))) dx = \int (ex)^m \tan^p (d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 1.13885, size = 205, normalized size = 0.98

$$\frac{x(ex)^m (1 - e^{2iad} (cx^n)^{2ibd})^{-p} \left( \frac{i(-1 + e^{2iad} (cx^n)^{2ibd})}{1 + e^{2iad} (cx^n)^{2ibd}} \right)^p (1 + e^{2iad} (cx^n)^{2ibd})^p F_1 \left( -\frac{i(m+1)}{2bdn}; -p, p; 1 - \frac{i(m+1)}{2bdn}; e^{2iad} (cx^n)^{2ibd}, -e^{2iad} (cx^n)^{2ibd} \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Tan[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out]  $(x*(e*x)^m*((-I)*(-1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}))/(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}))^p*(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p*Appel1F1[((-I/2)*(1+m))/(b*d*n), -p, p, 1 - ((I/2)*(1+m))/(b*d*n), E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}, -(E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})]/((1+m)*(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p)$

**Maple [F]** time = 0.189, size = 0, normalized size = 0.

$$\int (ex)^m (\tan (d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^p,x)`

[Out] `int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \tan((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*tan((b*log(c*x^n) + a)*d)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \tan(bd \log(cx^n) + ad)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")`

[Out] `integral((e*x)^m*tan(b*d*log(c*x^n) + a*d)^p, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*tan(d*(a+b*ln(c*x**n)))**p,x)`

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")`

[Out] Timed out

$$3.180 \quad \int \frac{\tan^{\frac{5}{2}}(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=201

$$\frac{2 \tan^{\frac{3}{2}}(a+b \log(cx^n))}{3bn} + \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} - \frac{\log(\tan(\dots))}{\dots}$$

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + (2*Tan[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

**Rubi [A]** time = 0.139294, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2 \tan^{\frac{3}{2}}(a+b \log(cx^n))}{3bn} + \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} - \frac{\log(\tan(\dots))}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*Log[c*x^n]]^(5/2)/x, x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + (2*Tan[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

#### Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

#### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]
```

#### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

& AtomQ[SplitProduct[SumBaseQ, b]])

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & & EqQ[c\*d^2 - a\*e^2, 0] & & PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] & & (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] & & NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] & & PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & & EqQ[c\*d^2 - a\*e^2, 0] & & NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] & & EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{\tan^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \tan^{\frac{5}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2 \tan^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \sqrt{\tan(a + bx)} dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2 \tan^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(a + b \log(cx^n))\right)}{bn} \\
&= \frac{2 \tan^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} - \frac{2 \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{2 \tan^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{2 \tan^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} \\
&= -\frac{\log\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} + \frac{\log\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\log\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn}
\end{aligned}$$

**Mathematica [C]** time = 0.259088, size = 50, normalized size = 0.25

$$\frac{2 \tan^{\frac{3}{2}}(a + b \log(cx^n)) \left( \text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -\tan^2(a + b \log(cx^n))\right) - 1 \right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + b\*Log[c\*x^n]]^(5/2)/x,x]

[Out] (-2\*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[a + b\*Log[c\*x^n]]^2])\*Tan[a + b\*Log[c\*x^n]]^(3/2))/(3\*b\*n)

**Maple [A]** time = 0.051, size = 161, normalized size = 0.8

$$\frac{2}{3bn} (\tan(a + b \ln(cx^n)))^{\frac{3}{2}} - \frac{\sqrt{2}}{2bn} \arctan\left(1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))}\right) - \frac{\sqrt{2}}{2bn} \arctan\left(-1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+b\*ln(c\*x^n))^(5/2)/x,x)

[Out] 2/3\*tan(a+b\*ln(c\*x^n))^(3/2)/b/n-1/2\*arctan(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)-1/2\*arctan(-1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)-1/4/b/n\*2^(1/2)\*ln((1-2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n)))/(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n))))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(b \log(cx^n) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(tan(b*log(c*x^n) + a)^(5/2)/x, x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.181 \quad \int \frac{\tan^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=199

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} + \frac{\log\left(\tan(a+b \log(cx^n)) - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{2\sqrt{2bn}}$$

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + (2*Sqrt[Tan[a + b*Log[c*x^n]]])/(b*n)
```

**Rubi [A]** time = 0.127709, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} + \frac{\log\left(\tan(a+b \log(cx^n)) - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{2\sqrt{2bn}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*Log[c*x^n]]^(3/2)/x, x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + (2*Sqrt[Tan[a + b*Log[c*x^n]]])/(b*n)
```

#### Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

#### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]
```

#### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

AtomQ[SplitProduct[SumBaseQ, b]])

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps



$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \tan^{\frac{3}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2\sqrt{\tan(a + b \log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tan(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2\sqrt{\tan(a + b \log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \tan(a + b \log(cx^n))\right)}{bn} \\
&= \frac{2\sqrt{\tan(a + b \log(cx^n))}}{bn} - \frac{2 \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{2\sqrt{\tan(a + b \log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{2\sqrt{\tan(a + b \log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2x+x^2}} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2x+x^2}} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} \\
&= \frac{\log\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} - \frac{\log\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\log\left(\tan(a + b \log(cx^n))\right)}{bn}
\end{aligned}$$

**Mathematica [A]** time = 0.242586, size = 175, normalized size = 0.88

$$\frac{2\sqrt{2} \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right) - 2\sqrt{2} \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + 1\right) + \sqrt{2} \log\left(\tan(a + b \log(cx^n))\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + b\*Log[c\*x^n]]^(3/2)/x,x]

[Out] (2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[a + b\*Log[c\*x^n]]]] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[a + b\*Log[c\*x^n]]]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[Tan[a + b\*Log[c\*x^n]]]] + Tan[a + b\*Log[c\*x^n]] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[Tan[a + b\*Log[c\*x^n]]]] + Tan[a + b\*Log[c\*x^n]] + 8\*Sqrt[Tan[a + b\*Log[c\*x^n]]])/(4\*b\*n)

**Maple [A]** time = 0.031, size = 161, normalized size = 0.8

$$2 \frac{\sqrt{\tan(a + b \ln(cx^n))}}{bn} - \frac{\sqrt{2}}{2bn} \arctan\left(1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))}\right) - \frac{\sqrt{2}}{2bn} \arctan\left(-1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))}\right) + \frac{\log(\tan(a + b \ln(cx^n)))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+b\*ln(c\*x^n))^(3/2)/x,x)

[Out] 2\*tan(a+b\*ln(c\*x^n))^(1/2)/b/n-1/2\*arctan(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)-1/2\*arctan(-1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)-1/4/b/n\*2^(1/2)\*ln((1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n)))/(1-2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n))))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate(tan(b\*log(c\*x^n) + a)^(3/2)/x, x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*ln(c\*x\*\*n))\*\*(3/2)/x,x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="giac")

[Out] Timed out

$$3.182 \quad \int \frac{\sqrt{\tan(a+b \log(cx^n))}}{x} dx$$

**Optimal.** Leaf size=176

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} + \frac{\log\left(\tan(a+b \log(cx^n)) - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{2\sqrt{2bn}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]/(Sqrt[2]*b*n)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]/(2*Sqrt[2]*b*n) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]/(2*Sqrt[2]*b*n)]
```

**Rubi [A]** time = 0.12044, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} + \frac{\log\left(\tan(a+b \log(cx^n)) - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{2\sqrt{2bn}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]/(Sqrt[2]*b*n)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]/(2*Sqrt[2]*b*n) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]/(2*Sqrt[2]*b*n)]
```

#### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && IntegerQ[n]
```

#### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

#### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

$/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&$   
 $\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

### Rule 617

$\text{Int}[\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}^{-1}, x\_Symbol] := \text{With}[\{q = 1 - 4*S$   
 $\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b$   
 $], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \|\ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{Free}$   
 $\text{Q}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 204

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] := -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a,$   
 $2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[$   
 $a, 0] \|\ \text{LtQ}[b, 0])$

### Rule 1165

$\text{Int}[\{(d\_)+ (e\_)*(x\_)^2\}/\{(a\_)+ (c\_)*(x\_)^4\}, x\_Symbol] := \text{With}[\{q = \text{Rt}[-$   
 $(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x],$   
 $x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{Fre}$   
 $\text{eQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

### Rule 628

$\text{Int}[\{(d\_)+ (e\_)*(x\_)\}/\{(a\_)+ (b\_)*(x\_)+ (c\_)*(x\_)^2\}, x\_Symbol] := \text{S}$   
 $\text{imp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d,$   
 $e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\tan(a + b \log(cx^n))}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{\tan(a + bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(a + b \log(cx^n))\right)}{bn} \\ &= \frac{2 \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\ &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} \\ &= \frac{\log\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} - \frac{\log\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\ &= -\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\log\left(1 - \tan^2(a + b \log(cx^n))\right)}{2bn} \end{aligned}$$

**Mathematica [C]** time = 0.0969592, size = 48, normalized size = 0.27

$$\frac{2 \tan^{\frac{3}{2}}(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -\tan^2(a + b \log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Tan[a + b\*Log[c\*x^n]]]/x,x]

[Out] (2\*Hypergeometric2F1[3/4, 1, 7/4, -Tan[a + b\*Log[c\*x^n]]^2]\*Tan[a + b\*Log[c\*x^n]]^(3/2))/(3\*b\*n)

**Maple [A]** time = 0.03, size = 140, normalized size = 0.8

$$\frac{\sqrt{2}}{4bn} \ln \left( \left( 1 - \sqrt{2} \sqrt{\tan(a + b \ln(cx^n))} + \tan(a + b \ln(cx^n)) \right) \left( 1 + \sqrt{2} \sqrt{\tan(a + b \ln(cx^n))} + \tan(a + b \ln(cx^n)) \right)^{-1} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a+b\*ln(c\*x^n))^(1/2)/x,x)

[Out] 1/4/b/n\*2^(1/2)\*ln((1-2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n)))/(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n))))+1/2\*arctan(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)+1/2\*arctan(-1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(tan(b\*log(c\*x^n) + a))/x, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*log(c\*x^n))^(1/2)/x,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\tan(a + b \log(cx^n))}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(a+b\*ln(c\*x\*\*n))\*\*(1/2)/x,x)

```
[Out] Integral(sqrt(tan(a + b*log(c*x**n)))/x, x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.183 \quad \int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}} dx$$

**Optimal.** Leaf size=176

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2bn}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))} + 1\right)}{\sqrt{2bn}} - \frac{\log\left(\tan(a+b\log(cx^n)) - \sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{2\sqrt{2bn}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]/(Sqrt[2]*b*n)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]/(2*Sqrt[2]*b*n) + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]/(2*Sqrt[2]*b*n)]
```

**Rubi [A]** time = 0.120569, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2bn}} + \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))} + 1\right)}{\sqrt{2bn}} - \frac{\log\left(\tan(a+b\log(cx^n)) - \sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{2\sqrt{2bn}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Tan[a + b*Log[c*x^n]]]), x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]/(Sqrt[2]*b*n)) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]/(2*Sqrt[2]*b*n) + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]/(2*Sqrt[2]*b*n)]
```

#### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && IntegerQ[n]
```

#### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))
```

#### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

$x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

### Rule 617

$\text{Int}[\frac{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}^{-1}, x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 204

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^2}{(a_.) + (b_.)*(x_.)^2}^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{\tan(a + b \log(cx^n))}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tan(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \tan(a + b \log(cx^n))\right)}{bn} \\ &= \frac{2 \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\ &= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} \\ &= -\frac{\log\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} + \frac{\log\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\ &= -\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\log\left(\tan(a + b \log(cx^n)) - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + 1\right)}{2\sqrt{2}bn} \end{aligned}$$

**Mathematica [A]** time = 0.13451, size = 142, normalized size = 0.81

$$\frac{-2 \tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right) + 2 \tan^{-1}\left(\sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + 1\right) - \log\left(\tan(a + b \log(cx^n)) - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + 1\right)}{2\sqrt{2}bn}$$



Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[Tan[a + b\*Log[c\*x^n]]]),x]

[Out] (-2\*ArcTan[1 - Sqrt[2]\*Sqrt[Tan[a + b\*Log[c\*x^n]]]] + 2\*ArcTan[1 + Sqrt[2]\*Sqrt[Tan[a + b\*Log[c\*x^n]]]] - Log[1 - Sqrt[2]\*Sqrt[Tan[a + b\*Log[c\*x^n]]] + Tan[a + b\*Log[c\*x^n]]] + Log[1 + Sqrt[2]\*Sqrt[Tan[a + b\*Log[c\*x^n]]] + Tan[a + b\*Log[c\*x^n]])/(2\*Sqrt[2]\*b\*n)

**Maple [A]** time = 0.031, size = 140, normalized size = 0.8

$$\frac{\sqrt{2}}{4bn} \ln \left( \left( 1 + \sqrt{2} \sqrt{\tan(a + b \ln(cx^n)) + \tan(a + b \ln(cx^n))} \right) \left( 1 - \sqrt{2} \sqrt{\tan(a + b \ln(cx^n)) + \tan(a + b \ln(cx^n))} \right)^{-1} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/tan(a+b\*ln(c\*x^n))^(1/2),x)

[Out] 1/4/b/n\*2^(1/2)\*ln((1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n)))/(1-2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n))))+1/2\*arctan(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)+1/2\*arctan(-1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt{\tan(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*log(c\*x^n))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(x\*sqrt(tan(b\*log(c\*x^n) + a))), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*log(c\*x^n))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt{\tan(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(tan(a + b*log(c*x**n))))), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/tan(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.184 \quad \int \frac{1}{x \tan^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=199

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} - \frac{\log\left(\tan(a+b \log(cx^n)) - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{2\sqrt{2bn}}$$

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - 2/(b*n*Sqrt[Tan[a + b*Log[c*x^n]]])
```

**Rubi [A]** time = 0.133643, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} - \frac{\log\left(\tan(a+b \log(cx^n)) - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{2\sqrt{2bn}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Tan[a + b*Log[c*x^n]]^(3/2)), x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - 2/(b*n*Sqrt[Tan[a + b*Log[c*x^n]]])
```

#### Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

#### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]
```

#### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

& AtomQ[SplitProduct[SumBaseQ, b]])

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & & EqQ[c\*d^2 - a\*e^2, 0] & & PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] & & (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] & & NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] & & PosQ[a/b] & & (LtQ[a, 0] || LtQ[b, 0])

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] & & EqQ[c\*d^2 - a\*e^2, 0] & & NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] & & EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x \tan^{\frac{3}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\tan^{\frac{3}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2}{bn\sqrt{\tan(a + b \log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\tan(a + bx)} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2}{bn\sqrt{\tan(a + b \log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \tan(a + b \log(cx^n))\right)}{bn} \\
&= -\frac{2}{bn\sqrt{\tan(a + b \log(cx^n))}} - \frac{2 \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{2}{bn\sqrt{\tan(a + b \log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} \\
&= -\frac{\log\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} + \frac{\log\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\log\left(\frac{1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))}{1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))}\right)}{2\sqrt{2}bn}
\end{aligned}$$

**Mathematica [C]** time = 0.118378, size = 46, normalized size = 0.23

$$-\frac{2\text{Hypergeometric2F1}\left(-\frac{1}{4}, 1, \frac{3}{4}, -\tan^2(a + b \log(cx^n))\right)}{bn\sqrt{\tan(a + b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Tan[a + b\*Log[c\*x^n]]^(3/2)), x]

[Out] (-2\*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[a + b\*Log[c\*x^n]]^2])/(b\*n\*Sqrt[Tan[a + b\*Log[c\*x^n]]])

**Maple [A]** time = 0.037, size = 161, normalized size = 0.8

$$-\frac{\sqrt{2}}{2bn} \arctan\left(1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))}\right) - \frac{\sqrt{2}}{2bn} \arctan\left(-1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))}\right) - \frac{\sqrt{2}}{4bn} \ln\left(\frac{1 - \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))} + \tan(a + b \ln(cx^n))}{1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))} + \tan(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/tan(a+b\*ln(c\*x^n))^(3/2), x)

[Out] -1/2\*arctan(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)-1/2\*arctan(-1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)-1/4/b/n\*2^(1/2)\*ln((1-2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n)))/(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n))))-2/b/n/tan(a+b\*ln(c\*x^n))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \tan(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*log(c\*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x\*tan(b\*log(c\*x^n) + a)^(3/2)), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \tan^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Integral(1/(x\*tan(a + b\*log(c\*x\*\*n))\*\*(3/2)), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] Timed out

$$3.185 \quad \int \frac{1}{x \tan^{\frac{5}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=201

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} - \frac{2}{3bn \tan^{\frac{3}{2}}(a+b \log(cx^n))} + \frac{\log(\tan(a+b \log(cx^n)))}{bn}$$

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - 2/(3*b*n*Tan[a + b*Log[c*x^n]]^(3/2))
```

**Rubi [A]** time = 0.128344, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a+b \log(cx^n))}\right)}{\sqrt{2bn}} - \frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(a+b \log(cx^n))} + 1\right)}{\sqrt{2bn}} - \frac{2}{3bn \tan^{\frac{3}{2}}(a+b \log(cx^n))} + \frac{\log(\tan(a+b \log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Tan[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) - ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/(Sqrt[2]*b*n) + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]*b*n) - 2/(3*b*n*Tan[a + b*Log[c*x^n]]^(3/2))
```

#### Rule 3474

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Tan[c + d*x])^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

#### Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]
```

#### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 211

```
Int[((a_.) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps



$$\begin{aligned}
\int \frac{1}{x \tan^{\frac{5}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\tan^{\frac{5}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2}{3bn \tan^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tan(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2}{3bn \tan^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \tan(a + b \log(cx^n))\right)}{bn} \\
&= -\frac{2}{3bn \tan^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{2 \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{2}{3bn \tan^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{2}{3bn \tan^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\tan(a + b \log(cx^n))}\right)}{2bn} \\
&= \frac{\log\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} - \frac{\log\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\log\left(\frac{1 - \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))}{1 + \sqrt{2}\sqrt{\tan(a + b \log(cx^n))} + \tan(a + b \log(cx^n))}\right)}{\sqrt{2}bn}
\end{aligned}$$

**Mathematica [C]** time = 0.208501, size = 48, normalized size = 0.24

$$-\frac{2\text{Hypergeometric2F1}\left(-\frac{3}{4}, 1, \frac{1}{4}, -\tan^2(a + b \log(cx^n))\right)}{3bn \tan^{\frac{3}{2}}(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Tan[a + b\*Log[c\*x^n]]^(5/2)), x]

[Out] (-2\*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[a + b\*Log[c\*x^n]]^2])/(3\*b\*n\*Tan[a + b\*Log[c\*x^n]]^(3/2))

**Maple [A]** time = 0.038, size = 161, normalized size = 0.8

$$-\frac{2}{3bn} (\tan(a + b \ln(cx^n)))^{-\frac{3}{2}} - \frac{\sqrt{2}}{2bn} \arctan\left(1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))}\right) - \frac{\sqrt{2}}{2bn} \arctan\left(-1 + \sqrt{2}\sqrt{\tan(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/tan(a+b\*ln(c\*x^n))^(5/2), x)

[Out] -2/3/b/n/tan(a+b\*ln(c\*x^n))^(3/2)-1/2\*arctan(1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)-1/2\*arctan(-1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)-1/4/b/n\*2^(1/2)\*ln((1+2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n)))/(1-2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n))))

))/(1-2^(1/2)\*tan(a+b\*ln(c\*x^n))^(1/2)+tan(a+b\*ln(c\*x^n)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \tan(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*log(c\*x^n))^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x\*tan(b\*log(c\*x^n) + a)^(5/2)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*log(c\*x^n))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/tan(a+b\*log(c\*x^n))^(5/2),x, algorithm="giac")

[Out] Timed out

### 3.186 $\int x^3 \cot(a + i \log(x)) dx$

**Optimal.** Leaf size=49

$$-ie^{2ia}x^2 - ie^{4ia} \log(-x^2 + e^{2ia}) - \frac{ix^4}{4}$$

[Out]  $(-I)*E^{((2*I)*a)}*x^2 - (I/4)*x^4 - I*E^{((4*I)*a)}*Log[E^{((2*I)*a)} - x^2]$

**Rubi [F]** time = 0.0260149, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \cot(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*Cot[a + I\*Log[x]], x]

[Out] Defer[Int][x^3\*Cot[a + I\*Log[x]], x]

Rubi steps

$$\int x^3 \cot(a + i \log(x)) dx = \int x^3 \cot(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.0337094, size = 137, normalized size = 2.8

$$x^2 \sin(2a) - ix^2 \cos(2a) - \frac{1}{2}i \cos(4a) \log(-2x^2 \cos(2a) + x^4 + 1) + \frac{1}{2} \sin(4a) \log(-2x^2 \cos(2a) + x^4 + 1) - \cos(4a) \tan^{-1}\left(\frac{-1 + x^2 \cos(2a)}{-\sin(2a) - x^2 \sin(2a)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Cot[a + I\*Log[x]], x]

[Out]  $(-I/4)*x^4 - I*x^2*\text{Cos}[2*a] - \text{ArcTan}[((-1 + x^2)*\text{Cos}[a])/(-\text{Sin}[a] - x^2*\text{Sin}[a])]*\text{Cos}[4*a] - (I/2)*\text{Cos}[4*a]*\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]] + x^2*\text{Sin}[2*a] - I*\text{ArcTan}[((-1 + x^2)*\text{Cos}[a])/(-\text{Sin}[a] - x^2*\text{Sin}[a])]*\text{Sin}[4*a] + (\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]]*\text{Sin}[4*a])/2$

**Maple [A]** time = 0.079, size = 65, normalized size = 1.3

$$\frac{i}{4}x^4 + i\left(-\frac{x^4}{2} - x^2(e^{ia})^2 - (e^{ia})^4 \ln(e^{ia} - x) - (e^{ia})^4 \ln(e^{ia} + x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cot(a+I\*ln(x)), x)

[Out]  $\frac{1}{4}i x^4 + I \left( -\frac{1}{2}x^4 - x^2 \exp(Ia)^2 - \exp(Ia)^4 \ln(\exp(Ia) - x) - \exp(Ia)^4 \ln(\exp(Ia) + x) \right)$

**Maxima [B]** time = 1.08604, size = 184, normalized size = 3.76

$$-\frac{1}{4}ix^4 - x^2(i \cos(2a) - \sin(2a)) + \frac{1}{4}(4 \cos(4a) + 4i \sin(4a)) \arctan(\sin(a), x + \cos(a)) - \frac{1}{4}(4 \cos(4a) + 4i \sin(4a)) \arctan(\sin(a), x - \cos(a)) - \frac{1}{2}(I \cos(4a) - \sin(4a)) \log(x^2 + 2x \cos(a) + \cos(a)^2 + \sin(a)^2) - \frac{1}{2}(I \cos(4a) - \sin(4a)) \log(x^2 - 2x \cos(a) + \cos(a)^2 + \sin(a)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cot(a+I\*log(x)),x, algorithm="maxima")

[Out]  $-\frac{1}{4}ix^4 - x^2(I \cos(2a) - \sin(2a)) + \frac{1}{4}(4 \cos(4a) + 4i \sin(4a)) \arctan2(\sin(a), x + \cos(a)) - \frac{1}{4}(4 \cos(4a) + 4i \sin(4a)) \arctan2(\sin(a), x - \cos(a)) - \frac{1}{2}(I \cos(4a) - \sin(4a)) \log(x^2 + 2x \cos(a) + \cos(a)^2 + \sin(a)^2) - \frac{1}{2}(I \cos(4a) - \sin(4a)) \log(x^2 - 2x \cos(a) + \cos(a)^2 + \sin(a)^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ix^3 e^{(2ia-2 \log(x))} + ix^3}{e^{(2ia-2 \log(x))} - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cot(a+I\*log(x)),x, algorithm="fricas")

[Out] integral((I\*x^3\*e^(2\*I\*a - 2\*log(x)) + I\*x^3)/(e^(2\*I\*a - 2\*log(x)) - 1), x)

**Sympy [A]** time = 0.606986, size = 39, normalized size = 0.8

$$-\frac{ix^4}{4} - ix^2 e^{2ia} - i e^{4ia} \log(x^2 - e^{2ia})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cot(a+I\*ln(x)),x)

[Out]  $-I x^{**4}/4 - I x^{**2} \exp(2I a) - I \exp(4I a) \log(x^{**2} - \exp(2I a))$

**Giac [A]** time = 1.55213, size = 68, normalized size = 1.39

$$-\frac{1}{4}ix^4 - ix^2 e^{(2ia)} + \frac{1}{2} \pi e^{(4ia)} - i e^{(4ia)} \log(x + e^{(ia)}) - i e^{(4ia)} \log(-x + e^{(ia)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cot(a+I\*log(x)),x, algorithm="giac")

[Out]  $-\frac{1}{4}ix^4 - I x^2 e^{(2I a)} + \frac{1}{2} \pi e^{(4I a)} - I e^{(4I a)} \log(x + e^{(I a)}) - I e^{(4I a)} \log(-x + e^{(I a)})$

### 3.187 $\int x^2 \cot(a + i \log(x)) dx$

**Optimal.** Leaf size=43

$$-2ie^{2ia}x + 2ie^{3ia} \tanh^{-1}(e^{-ia}x) - \frac{ix^3}{3}$$

[Out]  $(-2*I)*E^{((2*I)*a)*x} - (I/3)*x^3 + (2*I)*E^{((3*I)*a)*ArcTanh[x/E^{(I*a)}]$

**Rubi [F]** time = 0.0223548, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \cot(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Cot[a + I\*Log[x]],x]

[Out] Defer[Int][x^2\*Cot[a + I\*Log[x]], x]

Rubi steps

$$\int x^2 \cot(a + i \log(x)) dx = \int x^2 \cot(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.0181679, size = 66, normalized size = 1.53

$$2x \sin(2a) - 2ix \cos(2a) + 2i \cos(3a) \tanh^{-1}(x \cos(a) - ix \sin(a)) - 2 \sin(3a) \tanh^{-1}(x \cos(a) - ix \sin(a)) - \frac{ix^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cot[a + I\*Log[x]],x]

[Out]  $(-I/3)*x^3 - (2*I)*x*\text{Cos}[2*a] + (2*I)*\text{ArcTanh}[x*\text{Cos}[a] - I*x*\text{Sin}[a]]*\text{Cos}[3*a] + 2*x*\text{Sin}[2*a] - 2*\text{ArcTanh}[x*\text{Cos}[a] - I*x*\text{Sin}[a]]*\text{Sin}[3*a]$

**Maple [A]** time = 0.089, size = 62, normalized size = 1.4

$$\frac{i}{3}x^3 + i \left( -\frac{2x^3}{3} - 2(e^{ia})^2 x + (e^{ia})^3 \ln(e^{ia} + x) - (e^{ia})^3 \ln(e^{ia} - x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cot(a+I\*ln(x)),x)

[Out]  $1/3*I*x^3 + I*(-2/3*x^3 - 2*\exp(I*a)^2*x + \exp(I*a)^3*\ln(\exp(I*a)+x) - \exp(I*a)^3*\ln(\exp(I*a)-x))$

---

**Maxima [B]** time = 1.18713, size = 176, normalized size = 4.09

$$-\frac{1}{3}ix^3 + 2x(-i \cos(2a) + \sin(2a)) - \frac{1}{6}(6 \cos(3a) + 6i \sin(3a)) \arctan(\sin(a), x + \cos(a)) - \frac{1}{6}(6 \cos(3a) + 6i \sin(3a)) \arctan(\sin(a), x - \cos(a)) + \frac{1}{2}(i \cos(3a) - \sin(3a)) \log(x^2 + 2x \cos(a) + \cos(a)^2 + \sin(a)^2) + \frac{1}{2}(-i \cos(3a) + \sin(3a)) \log(x^2 - 2x \cos(a) + \cos(a)^2 + \sin(a)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cot(a+I\*log(x)),x, algorithm="maxima")

[Out] -1/3\*I\*x^3 + 2\*x\*(-I\*cos(2\*a) + sin(2\*a)) - 1/6\*(6\*cos(3\*a) + 6\*I\*sin(3\*a))\*arctan2(sin(a), x + cos(a)) - 1/6\*(6\*cos(3\*a) + 6\*I\*sin(3\*a))\*arctan2(sin(a), x - cos(a)) + 1/2\*(I\*cos(3\*a) - sin(3\*a))\*log(x^2 + 2\*x\*cos(a) + cos(a)^2 + sin(a)^2) + 1/2\*(-I\*cos(3\*a) + sin(3\*a))\*log(x^2 - 2\*x\*cos(a) + cos(a)^2 + sin(a)^2)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ix^2e^{(2ia-2\log(x))} + ix^2}{e^{(2ia-2\log(x))} - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cot(a+I\*log(x)),x, algorithm="fricas")

[Out] integral((I\*x^2\*e^(2\*I\*a - 2\*log(x)) + I\*x^2)/(e^(2\*I\*a - 2\*log(x)) - 1), x)

---

**Sympy [A]** time = 0.497822, size = 46, normalized size = 1.07

$$-\frac{ix^3}{3} - 2ixe^{2ia} - (i \log(x - e^{ia}) - i \log(x + e^{ia})) e^{3ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cot(a+I\*ln(x)),x)

[Out] -I\*x\*\*3/3 - 2\*I\*x\*exp(2\*I\*a) - (I\*log(x - exp(I\*a)) - I\*log(x + exp(I\*a)))\*exp(3\*I\*a)

---

**Giac [A]** time = 1.46495, size = 63, normalized size = 1.47

$$-\frac{1}{3}ix^3 - 2ixe^{(2ia)} + ie^{(3ia)} \log(ix + ie^{(ia)}) - ie^{(3ia)} \log(-ix + ie^{(ia)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cot(a+I\*log(x)),x, algorithm="giac")

[Out] -1/3\*I\*x^3 - 2\*I\*x\*e^(2\*I\*a) + I\*e^(3\*I\*a)\*log(I\*x + I\*e^(I\*a)) - I\*e^(3\*I\*a)\*log(-I\*x + I\*e^(I\*a))

### 3.188 $\int x \cot(a + i \log(x)) dx$

**Optimal.** Leaf size=35

$$-ie^{2ia} \log(-x^2 + e^{2ia}) - \frac{ix^2}{2}$$

[Out]  $(-I/2)*x^2 - I*E^{((2*I)*a)}*Log[E^{((2*I)*a)} - x^2]$

**Rubi [F]** time = 0.014523, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \cot(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x\*Cot[a + I\*Log[x]],x]

[Out] Defer[Int][x\*Cot[a + I\*Log[x]], x]

Rubi steps

$$\int x \cot(a + i \log(x)) dx = \int x \cot(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.0233421, size = 118, normalized size = 3.37

$$-\frac{1}{2}i \cos(2a) \log(-2x^2 \cos(2a) + x^4 + 1) + \frac{1}{2} \sin(2a) \log(-2x^2 \cos(2a) + x^4 + 1) - \cos(2a) \tan^{-1} \left( \frac{(x^2 - 1) \cos(a)}{x^2(-\sin(a)) - \sin(a)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cot[a + I\*Log[x]],x]

[Out]  $(-I/2)*x^2 - \text{ArcTan}[\frac{(-1 + x^2)*\text{Cos}[a]}{(-\text{Sin}[a] - x^2*\text{Sin}[a])}]*\text{Cos}[2*a] - (I/2)*\text{Cos}[2*a]*\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]] - I*\text{ArcTan}[\frac{(-1 + x^2)*\text{Cos}[a]}{(-\text{Sin}[a] - x^2*\text{Sin}[a])}]*\text{Sin}[2*a] + (\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]]*\text{Sin}[2*a])/2]$

**Maple [A]** time = 0.069, size = 53, normalized size = 1.5

$$\frac{i}{2}x^2 + i \left( -x^2 - (e^{ia})^2 \ln(e^{ia} - x) - (e^{ia})^2 \ln(e^{ia} + x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cot(a+I\*ln(x)),x)

[Out]  $1/2*I*x^2 + I*(-x^2 - \exp(I*a)^2*\ln(\exp(I*a) - x) - \exp(I*a)^2*\ln(\exp(I*a) + x))$

---

**Maxima [B]** time = 1.09912, size = 154, normalized size = 4.4

$$-\frac{1}{2}ix^2 + \frac{1}{2}(2\cos(2a) + 2i\sin(2a))\arctan(\sin(a), x + \cos(a)) - \frac{1}{2}(2\cos(2a) + 2i\sin(2a))\arctan(\sin(a), x - \cos(a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cot(a+I\*log(x)),x, algorithm="maxima")

[Out]  $-\frac{1}{2}I*x^2 + \frac{1}{2}*(2*\cos(2*a) + 2*I*\sin(2*a))*\arctan2(\sin(a), x + \cos(a)) - \frac{1}{2}*(2*\cos(2*a) + 2*I*\sin(2*a))*\arctan2(\sin(a), x - \cos(a)) + \frac{1}{2}*(-I*\cos(2*a) + \sin(2*a))*\log(x^2 + 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) + \frac{1}{2}*(-I*\cos(2*a) + \sin(2*a))*\log(x^2 - 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ix e^{(2i a - 2 \log(x))} + ix}{e^{(2i a - 2 \log(x))} - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cot(a+I\*log(x)),x, algorithm="fricas")

[Out] integral((I\*x\*e^(2\*I\*a - 2\*log(x)) + I\*x)/(e^(2\*I\*a - 2\*log(x)) - 1), x)

---

**Sympy [A]** time = 0.505025, size = 27, normalized size = 0.77

$$-\frac{ix^2}{2} - ie^{2ia} \log(x^2 - e^{2ia})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cot(a+I\*ln(x)),x)

[Out]  $-I*x**2/2 - I*\exp(2*I*a)*\log(x**2 - \exp(2*I*a))$

---

**Giac [A]** time = 1.37946, size = 55, normalized size = 1.57

$$-\frac{1}{2}ix^2 + \frac{1}{2}\pi e^{(2ia)} - ie^{(2ia)} \log(x + e^{(ia)}) - ie^{(2ia)} \log(-x + e^{(ia)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cot(a+I\*log(x)),x, algorithm="giac")

[Out]  $-\frac{1}{2}I*x^2 + \frac{1}{2}*\pi*e^{(2*I*a)} - I*e^{(2*I*a)}*\log(x + e^{(I*a)}) - I*e^{(2*I*a)}*\log(-x + e^{(I*a)})$



### 3.189 $\int \cot(a + i \log(x)) dx$

**Optimal.** Leaf size=27

$$2ie^{ia} \tanh^{-1}(e^{-ia}x) - ix$$

[Out]  $(-I)*x + (2*I)*E^{(I*a)}*ArcTanh[x/E^{(I*a)}]$

**Rubi [F]** time = 0.0072593, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[\text{Cot}[a + I*\text{Log}[x]], x]$

[Out]  $\text{Defer}[\text{Int}][\text{Cot}[a + I*\text{Log}[x]], x]$

Rubi steps

$$\int \cot(a + i \log(x)) dx = \int \cot(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.0083278, size = 42, normalized size = 1.56

$$2i \cos(a) \tanh^{-1}(x \cos(a) - ix \sin(a)) - 2 \sin(a) \tanh^{-1}(x \cos(a) - ix \sin(a)) - ix$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Cot}[a + I*\text{Log}[x]], x]$

[Out]  $(-I)*x + (2*I)*ArcTanh[x*\text{Cos}[a] - I*x*\text{Sin}[a]]*\text{Cos}[a] - 2*ArcTanh[x*\text{Cos}[a] - I*x*\text{Sin}[a]]*\text{Sin}[a]$

**Maple [A]** time = 0.071, size = 44, normalized size = 1.6

$$ix + i(-2x + e^{ia} \ln(e^{ia} + x) - e^{ia} \ln(e^{ia} - x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(a+I*\ln(x)), x)$

[Out]  $I*x+I*(-2*x+\exp(I*a)*\ln(\exp(I*a)+x)-\exp(I*a)*\ln(\exp(I*a)-x))$

**Maxima [B]** time = 1.0568, size = 132, normalized size = 4.89

$$-\frac{1}{2}(2 \cos(a) + 2i \sin(a)) \arctan(\sin(a), x + \cos(a)) - \frac{1}{2}(2 \cos(a) + 2i \sin(a)) \arctan(\sin(a), x - \cos(a)) - \frac{1}{2}(-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x)),x, algorithm="maxima")

[Out]  $-1/2*(2*\cos(a) + 2*I*\sin(a))*\arctan2(\sin(a), x + \cos(a)) - 1/2*(2*\cos(a) + 2*I*\sin(a))*\arctan2(\sin(a), x - \cos(a)) - 1/2*(-I*\cos(a) + \sin(a))*\log(x^2 + 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) - 1/2*(I*\cos(a) - \sin(a))*\log(x^2 - 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) - I*x$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i e^{(2i a - 2 \log(x))} + i}{e^{(2i a - 2 \log(x))} - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x)),x, algorithm="fricas")

[Out] integral((I\*e^(2\*I\*a - 2\*log(x)) + I)/(e^(2\*I\*a - 2\*log(x)) - 1), x)

**Sympy [A]** time = 0.494968, size = 29, normalized size = 1.07

$$-ix - (i \log(x - e^{ia}) - i \log(x + e^{ia})) e^{ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*ln(x)),x)

[Out]  $-I*x - (I*\log(x - \exp(I*a)) - I*\log(x + \exp(I*a)))*\exp(I*a)$

**Giac [B]** time = 1.34179, size = 51, normalized size = 1.89

$$i e^{(ia)} \log(ix + i e^{(ia)}) - i e^{(ia)} \log(-ix + i e^{(ia)}) - ix$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x)),x, algorithm="giac")

[Out]  $I*e^{(I*a)}*\log(I*x + I*e^{(I*a)}) - I*e^{(I*a)}*\log(-I*x + I*e^{(I*a)}) - I*x$

$$3.190 \quad \int \frac{\cot(a+i \log(x))}{x} dx$$

**Optimal.** Leaf size=14

$$-i \log(\sin(a + i \log(x)))$$

[Out] (-I)\*Log[Sin[a + I\*Log[x]]]

**Rubi [A]** time = 0.012911, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {3475}

$$-i \log(\sin(a + i \log(x)))$$

Antiderivative was successfully verified.

[In] Int[Cot[a + I\*Log[x]]/x,x]

[Out] (-I)\*Log[Sin[a + I\*Log[x]]]

Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot(a + i \log(x))}{x} dx &= \text{Subst}\left(\int \cot(a + ix) dx, x, \log(x)\right) \\ &= -i \log(\sin(a + i \log(x))) \end{aligned}$$

**Mathematica [A]** time = 0.0254794, size = 25, normalized size = 1.79

$$-i(\log(\tan(a + i \log(x))) + \log(\cos(a + i \log(x))))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + I\*Log[x]]/x,x]

[Out] (-I)\*(Log[Cos[a + I\*Log[x]]] + Log[Tan[a + I\*Log[x]]])

**Maple [A]** time = 0.017, size = 17, normalized size = 1.2

$$\frac{i}{2} \ln((\cot(a + i \ln(x)))^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+I\*ln(x))/x,x)

[Out]  $\frac{1}{2}i \ln(\cot(a + i \ln(x))^{2+1})$

**Maxima [A]** time = 1.12153, size = 14, normalized size = 1.

$$-i \log(\sin(a + i \log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))/x,x, algorithm="maxima")`

[Out]  $-I \log(\sin(a + I \log(x)))$

**Fricas [A]** time = 0.482536, size = 61, normalized size = 4.36

$$-i \log(x) - i \log(e^{(2i a - 2 \log(x))} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))/x,x, algorithm="fricas")`

[Out]  $-I \log(x) - I \log(e^{(2I a - 2 \log(x))} - 1)$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*ln(x))/x,x)`

[Out] Exception raised: PolynomialError

**Giac [B]** time = 1.22991, size = 78, normalized size = 5.57

$$-\frac{1}{2}i \log\left(-\frac{1}{8}\left(\frac{(x^2+1)^2}{x^2} - \frac{(x^2-1)^2}{x^2}\right)\cos(2a) + \frac{(x^2+1)^2}{8x^2} + \frac{(x^2-1)^2}{8x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))/x,x, algorithm="giac")`

[Out]  $-\frac{1}{2}i \log\left(-\frac{1}{8} \left(\frac{(x^2+1)^2}{x^2} - \frac{(x^2-1)^2}{x^2}\right) \cos(2a) + \frac{1}{8} \frac{(x^2+1)^2}{x^2} + \frac{1}{8} \frac{(x^2-1)^2}{x^2}\right)$

$$3.191 \quad \int \frac{\cot(a+i \log(x))}{x^2} dx$$

**Optimal.** Leaf size=29

$$2ie^{-ia} \tanh^{-1}(e^{-ia}x) - \frac{i}{x}$$

[Out]  $(-I)/x + ((2*I)*\text{ArcTanh}[x/E^{(I*a)}])/E^{(I*a)}$

**Rubi [F]** time = 0.0246961, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot(a + i \log(x))}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[\text{Cot}[a + I*\text{Log}[x]]/x^2, x]$

[Out]  $\text{Defer}[\text{Int}][\text{Cot}[a + I*\text{Log}[x]]/x^2, x]$

Rubi steps

$$\int \frac{\cot(a + i \log(x))}{x^2} dx = \int \frac{\cot(a + i \log(x))}{x^2} dx$$

**Mathematica [A]** time = 0.0226852, size = 44, normalized size = 1.52

$$2i \cos(a) \tanh^{-1}(x \cos(a) - ix \sin(a)) + 2 \sin(a) \tanh^{-1}(x \cos(a) - ix \sin(a)) - \frac{i}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Cot}[a + I*\text{Log}[x]]/x^2, x]$

[Out]  $(-I)/x + (2*I)*\text{ArcTanh}[x*\text{Cos}[a] - I*x*\text{Sin}[a]]*\text{Cos}[a] + 2*\text{ArcTanh}[x*\text{Cos}[a] - I*x*\text{Sin}[a]]*\text{Sin}[a]$

**Maple [A]** time = 0.06, size = 47, normalized size = 1.6

$$\frac{-i}{x} + i \left( \frac{\ln(e^{ia} + x)}{e^{ia}} - \frac{\ln(e^{ia} - x)}{e^{ia}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cot(a+I*\ln(x))/x^2, x)$

[Out]  $-I/x + I*(1/\exp(I*a)*\ln(\exp(I*a)+x) - 1/\exp(I*a)*\ln(\exp(I*a)-x))$

---

**Maxima [B]** time = 1.04772, size = 139, normalized size = 4.79

$$\frac{x(i \cos(a) + \sin(a)) \log(x^2 + 2x \cos(a) + \cos(a)^2 + \sin(a)^2) + x(-i \cos(a) - \sin(a)) \log(x^2 - 2x \cos(a) + \cos(a)^2 + \sin(a)^2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))/x^2,x, algorithm="maxima")

[Out] 1/2\*(x\*(I\*cos(a) + sin(a))\*log(x^2 + 2\*x\*cos(a) + cos(a)^2 + sin(a)^2) + x\*(-I\*cos(a) - sin(a))\*log(x^2 - 2\*x\*cos(a) + cos(a)^2 + sin(a)^2) - ((2\*cos(a) - 2\*I\*sin(a))\*arctan2(sin(a), x + cos(a)) + (2\*cos(a) - 2\*I\*sin(a))\*arctan2(sin(a), x - cos(a)))\*x - 2\*I)/x

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i e^{(2i a - 2 \log(x))} + i}{x^2 e^{(2i a - 2 \log(x))} - x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))/x^2,x, algorithm="fricas")

[Out] integral((I\*e^(2\*I\*a - 2\*log(x)) + I)/(x^2\*e^(2\*I\*a - 2\*log(x)) - x^2), x)

---

**Sympy [A]** time = 0.474434, size = 29, normalized size = 1.

$$-(i \log(x - e^{ia}) - i \log(x + e^{ia})) e^{-ia} - \frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*ln(x))/x\*\*2,x)

[Out] -(I\*log(x - exp(I\*a)) - I\*log(x + exp(I\*a)))\*exp(-I\*a) - I/x

---

**Giac [B]** time = 1.26395, size = 54, normalized size = 1.86

$$i e^{(-ia)} \log(ix + i e^{(ia)}) - i e^{(-ia)} \log(-ix + i e^{(ia)}) - \frac{i}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))/x^2,x, algorithm="giac")

[Out] I\*e^(-I\*a)\*log(I\*x + I\*e^(I\*a)) - I\*e^(-I\*a)\*log(-I\*x + I\*e^(I\*a)) - I/x

$$3.192 \quad \int \frac{\cot(a+i \log(x))}{x^3} dx$$

**Optimal.** Leaf size=36

$$-ie^{-2ia} \log\left(1 - \frac{e^{2ia}}{x^2}\right) - \frac{i}{2x^2}$$

[Out]  $(-I/2)/x^2 - (I*\text{Log}[1 - E^{((2*I)*a)/x^2}])/E^{((2*I)*a)}$

**Rubi [F]** time = 0.0240748, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot(a + i \log(x))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + I\*Log[x]]/x^3,x]

[Out] Defer[Int][Cot[a + I\*Log[x]]/x^3, x]

Rubi steps

$$\int \frac{\cot(a + i \log(x))}{x^3} dx = \int \frac{\cot(a + i \log(x))}{x^3} dx$$

**Mathematica [B]** time = 0.0297716, size = 136, normalized size = 3.78

$$-\frac{1}{2}i \cos(2a) \log(-2x^2 \cos(2a) + x^4 + 1) - \frac{1}{2} \sin(2a) \log(-2x^2 \cos(2a) + x^4 + 1) + \cos(2a) \left( -\tan^{-1} \left( \frac{(x^2 - 1) \cos(a)}{x^2(-\sin(a)) - \sin(a)} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + I\*Log[x]]/x^3,x]

[Out]  $(-I/2)/x^2 - \text{ArcTan}[\frac{(-1 + x^2)*\text{Cos}[a]}{(-\text{Sin}[a] - x^2*\text{Sin}[a])}] * \text{Cos}[2*a] + (2*I)*\text{Cos}[2*a]*\text{Log}[x] - (I/2)*\text{Cos}[2*a]*\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]] + I*\text{ArcTan}[\frac{(-1 + x^2)*\text{Cos}[a]}{(-\text{Sin}[a] - x^2*\text{Sin}[a])}] * \text{Sin}[2*a] + 2*\text{Log}[x]*\text{Sin}[2*a] - (\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]] * \text{Sin}[2*a])/2$

**Maple [A]** time = 0.066, size = 59, normalized size = 1.6

$$\frac{-i}{x^2} + i \left( 2 \frac{\ln(x)}{(e^{ia})^2} - \frac{\ln(e^{ia} - x)}{(e^{ia})^2} - \frac{\ln(e^{ia} + x)}{(e^{ia})^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+I\*ln(x))/x^3,x)

[Out]  $-1/2*I/x^2+I*(2/\exp(I*a)^2*\ln(x)-1/\exp(I*a)^2*\ln(\exp(I*a)-x)-1/\exp(I*a)^2*\ln(\exp(I*a)+x))$

**Maxima [B]** time = 1.38878, size = 188, normalized size = 5.22

$$x^2(i \cos(2a) + \sin(2a)) \log(x^2 + 2x \cos(a) + \cos(a)^2 + \sin(a)^2) + x^2(i \cos(2a) + \sin(2a)) \log(x^2 - 2x \cos(a) + \cos(a)^2 + \sin(a)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))/x^3,x, algorithm="maxima")`

[Out]  $-1/2*(x^2*(I*\cos(2*a) + \sin(2*a))*\log(x^2 + 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) + x^2*(I*\cos(2*a) + \sin(2*a))*\log(x^2 - 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) - ((2*\cos(2*a) - 2*I*\sin(2*a))*\arctan2(\sin(a), x + \cos(a)) - (2*\cos(2*a) - 2*I*\sin(2*a))*\arctan2(\sin(a), x - \cos(a)) + 4*(I*\cos(2*a) + \sin(2*a))*\log(x))*x^2 + I)/x^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{i e^{(2i a - 2 \log(x))} + i}{x^3 e^{(2i a - 2 \log(x))} - x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))/x^3,x, algorithm="fricas")`

[Out] `integral((I*e^(2*I*a - 2*log(x)) + I)/(x^3*e^(2*I*a - 2*log(x)) - x^3), x)`

**Sympy [A]** time = 0.697061, size = 39, normalized size = 1.08

$$2ie^{-2ia} \log(x) - ie^{-2ia} \log(x^2 - e^{2ia}) - \frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*ln(x))/x**3,x)`

[Out] `2*I*exp(-2*I*a)*log(x) - I*exp(-2*I*a)*log(x**2 - exp(2*I*a)) - I/(2*x**2)`

**Giac [B]** time = 1.3175, size = 66, normalized size = 1.83

$$\frac{1}{2} \pi e^{(-2ia)} - i e^{(-2ia)} \log(x + e^{ia}) + 2i e^{(-2ia)} \log(x) - i e^{(-2ia)} \log(-x + e^{ia}) - \frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))/x^3,x, algorithm="giac")`

[Out] `1/2*pi*e^(-2*I*a) - I*e^(-2*I*a)*log(x + e^(I*a)) + 2*I*e^(-2*I*a)*log(x) - I*e^(-2*I*a)*log(-x + e^(I*a)) - 1/2*I/x^2`



$$3.193 \quad \int \frac{\cot(a+i \log(x))}{x^4} dx$$

**Optimal.** Leaf size=45

$$-\frac{2ie^{-2ia}}{x} + 2ie^{-3ia} \tanh^{-1}(e^{-ia}x) - \frac{i}{3x^3}$$

[Out]  $(-I/3)/x^3 - (2*I)/(E^((2*I)*a)*x) + ((2*I)*ArcTanh[x/E^(I*a)])/E^((3*I)*a)$

**Rubi [F]** time = 0.025074, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot(a + i \log(x))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + I\*Log[x]]/x^4,x]

[Out] Defer[Int][Cot[a + I\*Log[x]]/x^4, x]

Rubi steps

$$\int \frac{\cot(a + i \log(x))}{x^4} dx = \int \frac{\cot(a + i \log(x))}{x^4} dx$$

**Mathematica [A]** time = 0.0215919, size = 70, normalized size = 1.56

$$-\frac{2 \sin(2a)}{x} - \frac{2i \cos(2a)}{x} + 2i \cos(3a) \tanh^{-1}(x \cos(a) - ix \sin(a)) + 2 \sin(3a) \tanh^{-1}(x \cos(a) - ix \sin(a)) - \frac{i}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + I\*Log[x]]/x^4,x]

[Out]  $(-I/3)/x^3 - ((2*I)*Cos[2*a])/x + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Cos[3*a] - (2*Sin[2*a])/x + 2*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]$

**Maple [A]** time = 0.061, size = 59, normalized size = 1.3

$$\frac{i}{x^3} + i \left( -2 \frac{1}{(e^{ia})^2 x} + \frac{\ln(e^{ia} + x)}{(e^{ia})^3} - \frac{\ln(e^{ia} - x)}{(e^{ia})^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+I\*ln(x))/x^4,x)

[Out]  $-1/3*I/x^3 + I*(-2/\exp(I*a)^2/x + 1/\exp(I*a)^3*\ln(\exp(I*a)+x) - 1/\exp(I*a)^3*\ln(\exp(I*a)-x))$

---

**Maxima [B]** time = 1.14161, size = 192, normalized size = 4.27

$$3x^3(-i \cos(3a) - \sin(3a)) \log(x^2 + 2x \cos(a) + \cos(a)^2 + \sin(a)^2) + 3x^3(i \cos(3a) + \sin(3a)) \log(x^2 - 2x \cos(a) + \cos(a)^2 + \sin(a)^2)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))/x^4,x, algorithm="maxima")

[Out]  $-1/6*(3*x^3*(-I*\cos(3*a) - \sin(3*a))*\log(x^2 + 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) + 3*x^3*(I*\cos(3*a) + \sin(3*a))*\log(x^2 - 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) + ((6*\cos(3*a) - 6*I*\sin(3*a))*\arctan2(\sin(a), x + \cos(a)) + (6*\cos(3*a) - 6*I*\sin(3*a))*\arctan2(\sin(a), x - \cos(a)))*x^3 + 12*x^2*(I*\cos(2*a) + \sin(2*a)) + 2*I)/x^3$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ie^{(2ia-2\log(x))} + i}{x^4e^{(2ia-2\log(x))} - x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))/x^4,x, algorithm="fricas")

[Out] integral((I\*e^(2\*I\*a - 2\*log(x)) + I)/(x^4\*e^(2\*I\*a - 2\*log(x)) - x^4), x)

---

**Sympy [A]** time = 0.626059, size = 54, normalized size = 1.2

$$-(i \log(x - e^{ia}) - i \log(x + e^{ia}))e^{-3ia} - \frac{(6ix^2 + ie^{2ia})e^{-2ia}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*ln(x))/x\*\*4,x)

[Out]  $-(I*\log(x - \exp(I*a)) - I*\log(x + \exp(I*a)))*\exp(-3*I*a) - (6*I*x**2 + I*\exp(2*I*a))*\exp(-2*I*a)/(3*x**3)$

---

**Giac [A]** time = 1.37743, size = 66, normalized size = 1.47

$$ie^{(-3ia)} \log(ix + ie^{(ia)}) - ie^{(-3ia)} \log(-ix + ie^{(ia)}) - \frac{2ie^{(-2ia)}}{x} - \frac{i}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))/x^4,x, algorithm="giac")

[Out]  $I*e^{(-3*I*a)}*\log(I*x + I*e^{(I*a)}) - I*e^{(-3*I*a)}*\log(-I*x + I*e^{(I*a)}) - 2*I*e^{(-2*I*a)}/x - 1/3*I/x^3$

### 3.194 $\int x^3 \cot^2(a + i \log(x)) dx$

**Optimal.** Leaf size=67

$$-2e^{2ia}x^2 - \frac{2e^{6ia}}{-x^2 + e^{2ia}} - 4e^{4ia} \log(-x^2 + e^{2ia}) - \frac{x^4}{4}$$

[Out]  $-2E^{((2*I)*a)}*x^2 - x^4/4 - (2E^{((6*I)*a)})/(E^{((2*I)*a)} - x^2) - 4E^{((4*I)*a)}*Log[E^{((2*I)*a)} - x^2]$

**Rubi [F]** time = 0.0650718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \cot^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*Cot[a + I\*Log[x]]^2,x]

[Out] Defer[Int][x^3\*Cot[a + I\*Log[x]]^2, x]

Rubi steps

$$\int x^3 \cot^2(a + i \log(x)) dx = \int x^3 \cot^2(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.174936, size = 162, normalized size = 2.42

$$-2ix^2 \sin(2a) - 2x^2 \cos(2a) - 2 \cos(4a) \log(-2x^2 \cos(2a) + x^4 + 1) + \frac{2 \cos(5a) + 2i \sin(5a)}{(x^2 - 1) \cos(a) - i(x^2 + 1) \sin(a)} - 2i \sin(4a) \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Cot[a + I\*Log[x]]^2,x]

[Out]  $-x^4/4 - 2x^2 \cos[2a] + (4I) \text{ArcTan}[(\cot[a] - x^2 \cot[a])/(1 + x^2)] \cos[4a] - 2 \cos[4a] \log[1 + x^4 - 2x^2 \cos[2a]] - (2I)x^2 \sin[2a] - 4 \text{ArcTan}[(\cot[a] - x^2 \cot[a])/(1 + x^2)] \sin[4a] - (2I) \log[1 + x^4 - 2x^2 \cos[2a]] \sin[4a] + (2 \cos[5a] + (2I) \sin[5a])/((-1 + x^2) \cos[a] - I(1 + x^2) \sin[a])$

**Maple [A]** time = 0.09, size = 77, normalized size = 1.2

$$-\frac{9x^4}{4} - 2 \frac{x^4}{(e^{i(a+i \ln(x))})^2 - 1} - 4x^2 (e^{ia})^2 - 4 (e^{ia})^4 \ln(e^{ia} - x) - 4 (e^{ia})^4 \ln(e^{ia} + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cot(a+I\*ln(x))^2,x)

[Out]  $-9/4*x^4-2*x^4/(\exp(I*(a+I*\ln(x)))^2-1)-4*x^2*\exp(I*a)^2-4*\exp(I*a)^4*\ln(\exp(I*a)-x)-4*\exp(I*a)^4*\ln(\exp(I*a)+x)$

**Maxima [B]** time = 1.19729, size = 489, normalized size = 7.3

$$x^6 + x^4(7 \cos(2a) + 7i \sin(2a)) - (16(-i \cos(4a) + \sin(4a)) \arctan(\sin(a), x + \cos(a)) + 16(i \cos(4a) - \sin(4a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cot(a+I*log(x))^2,x, algorithm="maxima")`

[Out]  $-(x^6 + x^4(7*\cos(2*a) + 7*I*\sin(2*a)) - (16*(-I*\cos(4*a) + \sin(4*a))*\arctan2(\sin(a), x + \cos(a)) + 16*(I*\cos(4*a) - \sin(4*a))*\arctan2(\sin(a), x - \cos(a)) + 8*\cos(4*a) + 8*I*\sin(4*a))*x^2 - (16*(I*\cos(2*a) - \sin(2*a))*\cos(4*a) - (16*\cos(2*a) + 16*I*\sin(2*a))*\sin(4*a))*\arctan2(\sin(a), x + \cos(a)) - (16*(-I*\cos(2*a) + \sin(2*a))*\cos(4*a) + (16*\cos(2*a) + 16*I*\sin(2*a))*\sin(4*a))*\arctan2(\sin(a), x - \cos(a)) + (x^2*(8*\cos(4*a) + 8*I*\sin(4*a)) - (8*\cos(2*a) + 8*I*\sin(2*a))*\cos(4*a) - 8*(I*\cos(2*a) - \sin(2*a))*\sin(4*a))*\log(x^2 + 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) + (x^2*(8*\cos(4*a) + 8*I*\sin(4*a)) - (8*\cos(2*a) + 8*I*\sin(2*a))*\cos(4*a) - 8*(I*\cos(2*a) - \sin(2*a))*\sin(4*a))*\log(x^2 - 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) - 8*\cos(6*a) - 8*I*\sin(6*a))/(4*x^2 - 4*\cos(2*a) - 4*I*\sin(2*a))$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x^4 - (e^{(2ia-2 \log(x))} - 1) \operatorname{integral}\left(-\frac{x^3 e^{(2ia-2 \log(x))} - 9x^3}{e^{(2ia-2 \log(x))} - 1}, x\right)}{e^{(2ia-2 \log(x))} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cot(a+I*log(x))^2,x, algorithm="fricas")`

[Out]  $-(2*x^4 - (e^{(2*I*a - 2*\log(x))} - 1)*\operatorname{integral}(-(x^3*e^{(2*I*a - 2*\log(x))} - 9*x^3)/(e^{(2*I*a - 2*\log(x))} - 1), x))/(e^{(2*I*a - 2*\log(x))} - 1)$

**Sympy [A]** time = 0.750358, size = 54, normalized size = 0.81

$$-\frac{x^4}{4} - 2x^2 e^{2ia} - 4e^{4ia} \log(x^2 - e^{2ia}) + \frac{2e^{6ia}}{x^2 - e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cot(a+I*ln(x))**2,x)`

[Out]  $-x**4/4 - 2*x**2*\exp(2*I*a) - 4*\exp(4*I*a)*\log(x**2 - \exp(2*I*a)) + 2*\exp(6*I*a)/(x**2 - \exp(2*I*a))$

**Giac [B]** time = 1.43341, size = 188, normalized size = 2.81

$$\frac{x^6}{4(x^2 - e^{2ia})} - \frac{7x^4 e^{2ia}}{4(x^2 - e^{2ia})} - \frac{4x^2 e^{4ia} \log(-x^2 + e^{2ia})}{x^2 - e^{2ia}} + \frac{2x^2 e^{4ia}}{x^2 - e^{2ia}} + \frac{4e^{6ia} \log(-x^2 + e^{2ia})}{x^2 - e^{2ia}} + \frac{2e^{6ia}}{x^2 - e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cot(a+I\*log(x))^2,x, algorithm="giac")

[Out]  $-1/4*x^6/(x^2 - e^{(2*I*a)}) - 7/4*x^4*e^{(2*I*a)}/(x^2 - e^{(2*I*a)}) - 4*x^2*e^{(4*I*a)*\log(-x^2 + e^{(2*I*a)})}/(x^2 - e^{(2*I*a)}) + 2*x^2*e^{(4*I*a)}/(x^2 - e^{(2*I*a)}) + 4*e^{(6*I*a)*\log(-x^2 + e^{(2*I*a)})}/(x^2 - e^{(2*I*a)}) + 2*e^{(6*I*a)}/(x^2 - e^{(2*I*a)})$

### 3.195 $\int x^2 \cot^2(a + i \log(x)) dx$

**Optimal.** Leaf size=64

$$-\frac{2e^{2ia}x^3}{-x^2 + e^{2ia}} - 6e^{2ia}x + 6e^{3ia} \tanh^{-1}(e^{-ia}x) - \frac{x^3}{3}$$

[Out]  $-6E^{((2*I)*a)}*x - x^3/3 - (2E^{((2*I)*a)}*x^3)/(E^{((2*I)*a)} - x^2) + 6E^{((3*I)*a)}*ArcTanh[x/E^{(I*a)}]$

**Rubi [F]** time = 0.0509217, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ , Rules used = {}

$$\int x^2 \cot^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Cot[a + I\*Log[x]]^2,x]

[Out] Defer[Int][x^2\*Cot[a + I\*Log[x]]^2, x]

Rubi steps

$$\int x^2 \cot^2(a + i \log(x)) dx = \int x^2 \cot^2(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.126752, size = 100, normalized size = 1.56

$$\frac{2x(\cos(3a) + i \sin(3a))}{(x^2 - 1)\cos(a) - i(x^2 + 1)\sin(a)} - 4ix \sin(2a) - 4x \cos(2a) + 6 \cos(3a) \tanh^{-1}(x(\cos(a) - i \sin(a))) + 6i \sin(3a) \tanh^{-1}(x(\cos(a) - i \sin(a)))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cot[a + I\*Log[x]]^2,x]

[Out]  $-x^3/3 - 4*x*\cos[2*a] + 6*ArcTanh[x*(\cos[a] - I*\sin[a])]*\cos[3*a] - (4*I)*x*\sin[2*a] + (2*x*(\cos[3*a] + I*\sin[3*a]))/((-1 + x^2)*\cos[a] - I*(1 + x^2)*\sin[a]) + (6*I)*ArcTanh[x*(\cos[a] - I*\sin[a])]*\sin[3*a]$

**Maple [A]** time = 0.086, size = 75, normalized size = 1.2

$$-\frac{7x^3}{3} - 2 \frac{x^3}{(e^{i(a+i \ln(x))})^2 - 1} - 6(e^{ia})^2 x - 3(e^{ia})^3 \ln(e^{ia} - x) + 3(e^{ia})^3 \ln(e^{ia} + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cot(a+I\*ln(x))^2,x)

[Out]  $-7/3*x^3-2*x^3/(exp(I*(a+I*ln(x)))^2-1)-6*exp(I*a)^2*x-3*exp(I*a)^3*ln(exp(I*a)-x)+3*exp(I*a)^3*ln(exp(I*a)+x)$

**Maxima [B]** time = 1.23497, size = 475, normalized size = 7.42

$2x^5 + x^3(22 \cos(2a) + 22i \sin(2a)) + 18((-i \cos(3a) + \sin(3a)) \arctan(\sin(a), x + \cos(a)) + (-i \cos(3a) + \sin(3a)) \arctan(\sin(a), x - \cos(a)))x^2 - x(36 \cos(4a) + 36I \sin(4a)) + (18(I \cos(2a) - \sin(2a)) \cos(3a) - (18 \cos(2a) + 18I \sin(2a)) \sin(3a)) \arctan(\sin(a), x + \cos(a)) + (18(I \cos(2a) - \sin(2a)) \cos(3a) - (18 \cos(2a) + 18I \sin(2a)) \sin(3a)) \arctan(\sin(a), x - \cos(a)) - (x^2(9 \cos(3a) + 9I \sin(3a)) - (9 \cos(2a) + 9I \sin(2a)) \cos(3a) - 9(I \cos(2a) - \sin(2a)) \sin(3a)) \log(x^2 + 2x \cos(a) + \cos(a)^2 + \sin(a)^2) + (x^2(9 \cos(3a) + 9I \sin(3a)) - (9 \cos(2a) + 9I \sin(2a)) \cos(3a) + 9(-I \cos(2a) + \sin(2a)) \sin(3a)) \log(x^2 - 2x \cos(a) + \cos(a)^2 + \sin(a)^2) / (6x^2 - 6 \cos(2a) - 6I \sin(2a))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cot(a+I\*log(x))^2,x, algorithm="maxima")

[Out]  $-(2*x^5 + x^3(22*\cos(2*a) + 22*I*\sin(2*a)) + 18*((-I*\cos(3*a) + \sin(3*a))*\arctan2(\sin(a), x + \cos(a)) + (-I*\cos(3*a) + \sin(3*a))*\arctan2(\sin(a), x - \cos(a))))*x^2 - x*(36*\cos(4*a) + 36*I*\sin(4*a)) + (18*(I*\cos(2*a) - \sin(2*a))*\cos(3*a) - (18*\cos(2*a) + 18*I*\sin(2*a))*\sin(3*a))*\arctan2(\sin(a), x + \cos(a)) + (18*(I*\cos(2*a) - \sin(2*a))*\cos(3*a) - (18*\cos(2*a) + 18*I*\sin(2*a))*\sin(3*a))*\arctan2(\sin(a), x - \cos(a)) - (x^2*(9*\cos(3*a) + 9*I*\sin(3*a)) - (9*\cos(2*a) + 9*I*\sin(2*a))*\cos(3*a) - 9*(I*\cos(2*a) - \sin(2*a))*\sin(3*a))*\log(x^2 + 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) + (x^2*(9*\cos(3*a) + 9*I*\sin(3*a)) - (9*\cos(2*a) + 9*I*\sin(2*a))*\cos(3*a) + 9*(-I*\cos(2*a) + \sin(2*a))*\sin(3*a))*\log(x^2 - 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2)/(6*x^2 - 6*\cos(2*a) - 6*I*\sin(2*a))$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x^3 - (e^{(2ia-2 \log(x))} - 1) \operatorname{integral}\left(-\frac{x^2 e^{(2ia-2 \log(x))} - 7x^2}{e^{(2ia-2 \log(x))} - 1}, x\right)}{e^{(2ia-2 \log(x))} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cot(a+I\*log(x))^2,x, algorithm="fricas")

[Out]  $-(2*x^3 - (e^{(2*I*a - 2*\log(x))} - 1)*\operatorname{integral}(-(x^2*e^{(2*I*a - 2*\log(x))} - 7*x^2)/(e^{(2*I*a - 2*\log(x))} - 1), x))/(e^{(2*I*a - 2*\log(x))} - 1)$

**Sympy [A]** time = 0.949005, size = 60, normalized size = 0.94

$$-\frac{x^3}{3} - 4xe^{2ia} + \frac{2xe^{4ia}}{x^2 - e^{2ia}} - 3(\log(x - e^{ia}) - \log(x + e^{ia}))e^{3ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cot(a+I\*ln(x))\*\*2,x)

[Out]  $-x**3/3 - 4*x*exp(2*I*a) + 2*x*exp(4*I*a)/(x**2 - exp(2*I*a)) - 3*(\log(x - exp(I*a)) - \log(x + exp(I*a)))*exp(3*I*a)$

**Giac [A]** time = 1.31541, size = 112, normalized size = 1.75

$$-\frac{x^5}{3(x^2 - e^{2ia})} - \frac{11x^3e^{2ia}}{3(x^2 - e^{2ia})} - \frac{6 \arctan\left(\frac{x}{\sqrt{-e^{2ia}}}\right)e^{4ia}}{\sqrt{-e^{2ia}}} + \frac{10xe^{4ia}}{x^2 - e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cot(a+I\*log(x))^2,x, algorithm="giac")

[Out] -1/3\*x^5/(x^2 - e^(2\*I\*a)) - 11/3\*x^3\*e^(2\*I\*a)/(x^2 - e^(2\*I\*a)) - 6\*arctan(x/sqrt(-e^(2\*I\*a)))\*e^(4\*I\*a)/sqrt(-e^(2\*I\*a)) + 10\*x\*e^(4\*I\*a)/(x^2 - e^(2\*I\*a))



### 3.196 $\int x \cot^2(a + i \log(x)) dx$

**Optimal.** Leaf size=55

$$-\frac{2e^{4ia}}{-x^2 + e^{2ia}} - 2e^{2ia} \log(-x^2 + e^{2ia}) - \frac{x^2}{2}$$

[Out]  $-x^2/2 - (2E^{(4*I)*a})/(E^{(2*I)*a} - x^2) - 2E^{(2*I)*a} \text{Log}[E^{(2*I)*a} - x^2]$

**Rubi [F]** time = 0.0356521, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \cot^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[x\*Cot[a + I\*Log[x]]^2,x]

[Out] Defer[Int][x\*Cot[a + I\*Log[x]]^2, x]

Rubi steps

$$\int x \cot^2(a + i \log(x)) dx = \int x \cot^2(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.12394, size = 142, normalized size = 2.58

$$-\cos(2a) \log(-2x^2 \cos(2a) + x^4 + 1) + \frac{2 \cos(3a) + 2i \sin(3a)}{(x^2 - 1) \cos(a) - i(x^2 + 1) \sin(a)} - i \sin(2a) \log(-2x^2 \cos(2a) + x^4 + 1) + 2$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cot[a + I\*Log[x]]^2,x]

[Out]  $-x^2/2 + (2*I)*\text{ArcTan}[(\text{Cot}[a] - x^2*\text{Cot}[a])/(1 + x^2)]*\text{Cos}[2*a] - \text{Cos}[2*a]*\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]] - 4*\text{ArcTan}[(\text{Cot}[a] - x^2*\text{Cot}[a])/(1 + x^2)]*\text{Cos}[a]*\text{Sin}[a] - I*\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]]*\text{Sin}[2*a] + (2*\text{Cos}[3*a] + (2*I)*\text{Sin}[3*a])/((-1 + x^2)*\text{Cos}[a] - I*(1 + x^2)*\text{Sin}[a])$

**Maple [A]** time = 0.083, size = 65, normalized size = 1.2

$$-\frac{5x^2}{2} - 2 \frac{x^2}{(e^{i(a+i \ln(x))})^2 - 1} - 2 (e^{ia})^2 \ln(e^{ia} - x) - 2 (e^{ia})^2 \ln(e^{ia} + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cot(a+I\*ln(x))^2,x)

[Out]  $-5/2*x^2-2*x^2/(\exp(I*(a+I*\ln(x)))^2-1)-2*\exp(I*a)^2*\ln(\exp(I*a)-x)-2*\exp(I*a)^2*\ln(\exp(I*a)+x)$

**Maxima [B]** time = 1.08836, size = 400, normalized size = 7.27

$$x^4 - (4(-i \cos(2a) + \sin(2a)) \arctan(\sin(a), x + \cos(a)) + 4(i \cos(2a) - \sin(2a)) \arctan(\sin(a), x - \cos(a)) + c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(a+I*log(x))^2,x, algorithm="maxima")`

[Out]  $-(x^4 - (4*(-I*\cos(2*a) + \sin(2*a))*\arctan2(\sin(a), x + \cos(a)) + 4*(I*\cos(2*a) - \sin(2*a))*\arctan2(\sin(a), x - \cos(a)) + \cos(2*a) + I*\sin(2*a))*x^2 + (-4*I*\cos(2*a)^2 + 8*\cos(2*a)*\sin(2*a) + 4*I*\sin(2*a)^2)*\arctan2(\sin(a), x + \cos(a)) + (4*I*\cos(2*a)^2 - 8*\cos(2*a)*\sin(2*a) - 4*I*\sin(2*a)^2)*\arctan2(\sin(a), x - \cos(a)) + (x^2*(2*\cos(2*a) + 2*I*\sin(2*a)) - 2*\cos(2*a)^2 - 4*I*\cos(2*a)*\sin(2*a) + 2*\sin(2*a)^2)*\log(x^2 + 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) + (x^2*(2*\cos(2*a) + 2*I*\sin(2*a)) - 2*\cos(2*a)^2 - 4*I*\cos(2*a)*\sin(2*a) + 2*\sin(2*a)^2)*\log(x^2 - 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) - 4*\cos(4*a) - 4*I*\sin(4*a))/(2*x^2 - 2*\cos(2*a) - 2*I*\sin(2*a))$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x^2 - (e^{(2ia-2 \log(x))} - 1) \operatorname{integral}\left(-\frac{xe^{(2ia-2 \log(x))-5x}}{e^{(2ia-2 \log(x))-1}}, x\right)}{e^{(2ia-2 \log(x))} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(a+I*log(x))^2,x, algorithm="fricas")`

[Out]  $-(2*x^2 - (e^{(2*I*a - 2*\log(x))} - 1)*\operatorname{integral}(-(x*e^{(2*I*a - 2*\log(x))} - 5*x)/(e^{(2*I*a - 2*\log(x))} - 1), x))/(e^{(2*I*a - 2*\log(x))} - 1)$

**Sympy [A]** time = 0.602831, size = 42, normalized size = 0.76

$$-\frac{x^2}{2} - 2e^{2ia} \log(x^2 - e^{2ia}) + \frac{2e^{4ia}}{x^2 - e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(a+I*ln(x))**2,x)`

[Out]  $-x**2/2 - 2*\exp(2*I*a)*\log(x**2 - \exp(2*I*a)) + 2*\exp(4*I*a)/(x**2 - \exp(2*I*a))$

**Giac [B]** time = 1.31924, size = 159, normalized size = 2.89

$$\frac{x^4}{2(x^2 - e^{(2ia)})} - \frac{2x^2e^{(2ia)} \log(-x^2 + e^{(2ia)})}{x^2 - e^{(2ia)}} + \frac{x^2e^{(2ia)}}{2(x^2 - e^{(2ia)})} + \frac{2e^{(4ia)} \log(-x^2 + e^{(2ia)})}{x^2 - e^{(2ia)}} + \frac{2e^{(4ia)}}{x^2 - e^{(2ia)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*x^4/(x^2 - e^(2*I*a)) - 2*x^2*e^(2*I*a)*log(-x^2 + e^(2*I*a))/(x^2 - e^(2*I*a)) + 1/2*x^2*e^(2*I*a)/(x^2 - e^(2*I*a)) + 2*e^(4*I*a)*log(-x^2 + e^(2*I*a))/(x^2 - e^(2*I*a)) + 2*e^(4*I*a)/(x^2 - e^(2*I*a))
```

### 3.197 $\int \cot^2(a + i \log(x)) dx$

**Optimal.** Leaf size=48

$$-\frac{2e^{2ia}x}{-x^2 + e^{2ia}} + 2e^{ia} \tanh^{-1}(e^{-ia}x) - x$$

[Out]  $-x - (2E^{((2I)*a)*x})/(E^{((2I)*a)} - x^2) + 2E^{(I*a)}*ArcTanh[x/E^{(I*a)}]$

**Rubi [F]** time = 0.0105502, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + I\*Log[x]]^2,x]

[Out] Defer[Int][Cot[a + I\*Log[x]]^2, x]

Rubi steps

$$\int \cot^2(a + i \log(x)) dx = \int \cot^2(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.0845853, size = 70, normalized size = 1.46

$$\frac{-x(x^2 - 3)\cos(a) + ix(x^2 + 3)\sin(a)}{(x^2 - 1)\cos(a) - i(x^2 + 1)\sin(a)} + 2(\cos(a) + i\sin(a))\tanh^{-1}(x(\cos(a) - i\sin(a)))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + I\*Log[x]]^2,x]

[Out]  $2*ArcTanh[x*(Cos[a] - I*Sin[a])]*(Cos[a] + I*Sin[a]) + (-x*(-3 + x^2)*Cos[a]) + I*x*(3 + x^2)*Sin[a])/((-1 + x^2)*Cos[a] - I*(1 + x^2)*Sin[a])$

**Maple [A]** time = 0.073, size = 56, normalized size = 1.2

$$-3x - 2 \frac{x}{(e^{i(a+i\ln(x))})^2 - 1} - e^{ia} \ln(e^{ia} - x) + e^{ia} \ln(e^{ia} + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+I\*ln(x))^2,x)

[Out]  $-3*x-2*x/(exp(I*(a+I*ln(x)))^2-1)-exp(I*a)*ln(exp(I*a)-x)+exp(I*a)*ln(exp(I*a)+x)$

---

**Maxima [B]** time = 1.2429, size = 375, normalized size = 7.81

$$2((-i \cos(a) + \sin(a)) \arctan(\sin(a), x + \cos(a)) + (-i \cos(a) + \sin(a)) \arctan(\sin(a), x - \cos(a)))x^2 + 2x^3 -$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))^2,x, algorithm="maxima")

[Out]  $-(2*((-I*\cos(a) + \sin(a))*\arctan2(\sin(a), x + \cos(a)) + (-I*\cos(a) + \sin(a))\arctan2(\sin(a), x - \cos(a)))*x^2 + 2*x^3 - x*(6*\cos(2*a) + 6*I*\sin(2*a)) + (2*(I*\cos(a) - \sin(a))*\cos(2*a) - (2*\cos(a) + 2*I*\sin(a))*\sin(2*a))*\arctan2(\sin(a), x + \cos(a)) + (2*(I*\cos(a) - \sin(a))*\cos(2*a) - (2*\cos(a) + 2*I*\sin(a))*\sin(2*a))*\arctan2(\sin(a), x - \cos(a)) - (x^2*(\cos(a) + I*\sin(a)) - (\cos(a) + I*\sin(a))*\cos(2*a) + (-I*\cos(a) + \sin(a))*\sin(2*a))*\log(x^2 + 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2) + (x^2*(\cos(a) + I*\sin(a)) - (\cos(a) + I*\sin(a))*\cos(2*a) - (I*\cos(a) - \sin(a))*\sin(2*a))*\log(x^2 - 2*x*\cos(a) + \cos(a)^2 + \sin(a)^2))/(2*x^2 - 2*\cos(2*a) - 2*I*\sin(2*a))$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(e^{(2i a - 2 \log(x))} - 1) \operatorname{integral}\left(-\frac{e^{(2i a - 2 \log(x)) - 3}}{e^{(2i a - 2 \log(x)) - 1}}, x\right) - 2x}{e^{(2i a - 2 \log(x))} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))^2,x, algorithm="fricas")

[Out]  $((e^{(2*I*a - 2*\log(x))} - 1)*\operatorname{integral}(-(e^{(2*I*a - 2*\log(x))} - 3)/(e^{(2*I*a - 2*\log(x))} - 1), x) - 2*x)/(e^{(2*I*a - 2*\log(x))} - 1)$

---

**Sympy [A]** time = 0.480368, size = 42, normalized size = 0.88

$$-x + \frac{2xe^{2ia}}{x^2 - e^{2ia}} - (\log(x - e^{ia}) - \log(x + e^{ia}))e^{ia}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*ln(x))\*\*2,x)

[Out]  $-x + 2*x*\exp(2*I*a)/(x**2 - \exp(2*I*a)) - (\log(x - \exp(I*a)) - \log(x + \exp(I*a)))*\exp(I*a)$

---

**Giac [B]** time = 1.33802, size = 107, normalized size = 2.23

$$-\frac{x^3}{x^2 - e^{(2i a)}} - 2 \left( \frac{\arctan\left(\frac{x}{\sqrt{-e^{(2i a)}}}\right)}{\sqrt{-e^{(2i a)}}} - \frac{x}{x^2 - e^{(2i a)}} \right) e^{(2i a)} + \frac{5xe^{(2i a)}}{x^2 - e^{(2i a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2,x, algorithm="giac")
```

```
[Out] -x^3/(x^2 - e^(2*I*a)) - 2*(arctan(x/sqrt(-e^(2*I*a)))/sqrt(-e^(2*I*a)) - x  
/(x^2 - e^(2*I*a)))*e^(2*I*a) + 5*x*e^(2*I*a)/(x^2 - e^(2*I*a))
```

$$3.198 \quad \int \frac{\cot^2(a+i \log(x))}{x} dx$$

**Optimal.** Leaf size=18

$$-\log(x) + i \cot(a + i \log(x))$$

[Out] I\*Cot[a + I\*Log[x]] - Log[x]

**Rubi [A]** time = 0.0235847, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {3473, 8}

$$-\log(x) + i \cot(a + i \log(x))$$

Antiderivative was successfully verified.

[In] Int[Cot[a + I\*Log[x]]^2/x,x]

[Out] I\*Cot[a + I\*Log[x]] - Log[x]

#### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cot^2(a + i \log(x))}{x} dx &= \text{Subst} \left( \int \cot^2(a + ix) dx, x, \log(x) \right) \\ &= i \cot(a + i \log(x)) - \text{Subst} \left( \int 1 dx, x, \log(x) \right) \\ &= i \cot(a + i \log(x)) - \log(x) \end{aligned}$$

**Mathematica [C]** time = 0.0498985, size = 34, normalized size = 1.89

$$i \cot(a + i \log(x)) \text{Hypergeometric2F1} \left( -\frac{1}{2}, 1, \frac{1}{2}, -\tan^2(a + i \log(x)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + I\*Log[x]]^2/x,x]

[Out] I\*Cot[a + I\*Log[x]]\*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + I\*Log[x]]^2]

**Maple [A]** time = 0.018, size = 27, normalized size = 1.5

$$i \cot(a + i \ln(x)) - \frac{i}{2} \pi + i(a + i \ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+I\*ln(x))^2/x,x)

[Out] I\*cot(a+I\*ln(x))-1/2\*I\*Pi+I\*(a+I\*ln(x))

**Maxima [A]** time = 1.72516, size = 26, normalized size = 1.44

$$i a + \frac{i}{\tan(a + i \log(x))} - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))^2/x,x, algorithm="maxima")

[Out] I\*a + I/tan(a + I\*log(x)) - log(x)

**Fricas [B]** time = 0.476125, size = 97, normalized size = 5.39

$$-\frac{e^{(2ia-2 \log(x))} \log(x) - \log(x) + 2}{e^{(2ia-2 \log(x))} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*log(x))^2/x,x, algorithm="fricas")

[Out] -(e^(2\*I\*a - 2\*log(x))\*log(x) - log(x) + 2)/(e^(2\*I\*a - 2\*log(x)) - 1)

**Sympy [A]** time = 0.538921, size = 20, normalized size = 1.11

$$-\log(x) + \frac{2e^{2ia}}{x^2 - e^{2ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+I\*ln(x))\*\*2/x,x)

[Out] -log(x) + 2\*exp(2\*I\*a)/(x\*\*2 - exp(2\*I\*a))

**Giac [B]** time = 1.31752, size = 43, normalized size = 2.39

$$i a + \frac{i}{2 \tan\left(\frac{1}{2} a + \frac{1}{2} i \log(x)\right)} - \log(x) - \frac{1}{2} i \tan\left(\frac{1}{2} a + \frac{1}{2} i \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(cot(a+I*log(x))^2/x,x, algorithm="giac")
```

```
[Out] I*a + 1/2*I/tan(1/2*a + 1/2*I*log(x)) - log(x) - 1/2*I*tan(1/2*a + 1/2*I*log(x))
```

$$3.199 \quad \int \frac{\cot^2(a+i \log(x))}{x^2} dx$$

**Optimal.** Leaf size=64

$$-\frac{3x}{-x^2 + e^{2ia}} + \frac{e^{2ia}}{x(-x^2 + e^{2ia})} - 2e^{-ia} \tanh^{-1}(e^{-ia}x)$$

[Out]  $E^{\left(\left(2 \cdot I\right) \cdot a\right)} / \left(x \cdot \left(E^{\left(\left(2 \cdot I\right) \cdot a\right)} - x^2\right)\right) - \left(3 \cdot x\right) / \left(E^{\left(\left(2 \cdot I\right) \cdot a\right)} - x^2\right) - \left(2 \cdot \text{ArcTan} \left[x / E^{\left(I \cdot a\right)}\right]\right) / E^{\left(I \cdot a\right)}$

**Rubi [F]** time = 0.0478845, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot^2(a + i \log(x))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + I\*Log[x]]^2/x^2, x]

[Out] Defer[Int][Cot[a + I\*Log[x]]^2/x^2, x]

Rubi steps

$$\int \frac{\cot^2(a + i \log(x))}{x^2} dx = \int \frac{\cot^2(a + i \log(x))}{x^2} dx$$

**Mathematica [A]** time = 0.121487, size = 72, normalized size = 1.12

$$\frac{2x(\cos(a) - i \sin(a))}{(x^2 - 1)\cos(a) - i(x^2 + 1)\sin(a)} - 2\cos(a) \tanh^{-1}(x(\cos(a) - i \sin(a))) + 2i \sin(a) \tanh^{-1}(x(\cos(a) - i \sin(a))) + \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + I\*Log[x]]^2/x^2, x]

[Out]  $x^{(-1)} - 2 \cdot \text{ArcTanh}[x \cdot (\text{Cos}[a] - I \cdot \text{Sin}[a])] \cdot \text{Cos}[a] + (2 \cdot I) \cdot \text{ArcTanh}[x \cdot (\text{Cos}[a] - I \cdot \text{Sin}[a])] \cdot \text{Sin}[a] + (2 \cdot x \cdot (\text{Cos}[a] - I \cdot \text{Sin}[a])) / ((-1 + x^2) \cdot \text{Cos}[a] - I \cdot (1 + x^2) \cdot \text{Sin}[a])$

**Maple [A]** time = 0.069, size = 62, normalized size = 1.

$$x^{-1} - 2 \frac{1}{x \left( \left( e^{i(a+i \ln(x))} \right)^2 - 1 \right)} - \frac{\ln(e^{ia} + x)}{e^{ia}} + \frac{\ln(e^{ia} - x)}{e^{ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+I\*ln(x))^2/x^2, x)



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+I*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] 2*(arctan(x/sqrt(-e^(2*I*a))))*e^(-2*I*a)/sqrt(-e^(2*I*a)) + x*e^(-2*I*a)/(x^2 - e^(2*I*a))*e^(2*I*a) + 5*x^2/(x^3 - x*e^(2*I*a)) - e^(2*I*a)/(x^3 - x*e^(2*I*a))
```

$$3.200 \quad \int \frac{\cot^2(a+i \log(x))}{x^3} dx$$

**Optimal.** Leaf size=57

$$\frac{2e^{-2ia}}{1 - \frac{e^{2ia}}{x^2}} + 2e^{-2ia} \log\left(1 - \frac{e^{2ia}}{x^2}\right) + \frac{1}{2x^2}$$

[Out]  $2/(E^{((2*I)*a)}*(1 - E^{((2*I)*a)/x^2})) + 1/(2*x^2) + (2*Log[1 - E^{((2*I)*a)/x^2}])/E^{((2*I)*a)}$

**Rubi [F]** time = 0.050647, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot^2(a + i \log(x))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + I\*Log[x]]^2/x^3, x]

[Out] Defer[Int][Cot[a + I\*Log[x]]^2/x^3, x]

Rubi steps

$$\int \frac{\cot^2(a + i \log(x))}{x^3} dx = \int \frac{\cot^2(a + i \log(x))}{x^3} dx$$

**Mathematica [B]** time = 0.221049, size = 153, normalized size = 2.68

$$\cos(2a) \left( \log(-2x^2 \cos(2a) + x^4 + 1) - 4 \log(x) \right) + \frac{2 \cos(a)}{(x^2 - 1) \cos(a) - i(x^2 + 1) \sin(a)} + \frac{2 \sin(a)}{(x^2 + 1) \sin(a) + i(x^2 - 1) \cos(a)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + I\*Log[x]]^2/x^3, x]

[Out]  $1/(2*x^2) + \text{Cos}[2*a]*(-4*\text{Log}[x] + \text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]]) + (2*\text{Cos}[a])/((-1 + x^2)*\text{Cos}[a] - I*(1 + x^2)*\text{Sin}[a]) + (2*\text{Sin}[a])/(I*(-1 + x^2)*\text{Cos}[a] + (1 + x^2)*\text{Sin}[a]) + \text{ArcTan}[(\text{Cot}[a] - x^2*\text{Cot}[a])/(1 + x^2)]*((-2*I)*\text{Cos}[2*a] - 4*\text{Cos}[a]*\text{Sin}[a]) + (4*I)*\text{Log}[x]*\text{Sin}[2*a] - I*\text{Log}[1 + x^4 - 2*x^2*\text{Cos}[2*a]]*\text{Sin}[2*a]$

**Maple [A]** time = 0.074, size = 76, normalized size = 1.3

$$\frac{1}{2x^2} - 2 \frac{1}{x^2 \left( (e^{i(a+i \ln(x))})^2 - 1 \right)} - 4 \frac{\ln(x)}{(e^{ia})^2} + 2 \frac{\ln(e^{ia} - x)}{(e^{ia})^2} + 2 \frac{\ln(e^{ia} + x)}{(e^{ia})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(a+I*ln(x))^2/x^3,x)`

[Out]  $\frac{1}{2}x^{-2} - \frac{2}{x^2} \frac{1}{(\exp(I(a+I\ln(x))))^2 - 1} - \frac{4}{\exp(Ia)^2 \ln(x) + 2} \frac{1}{\exp(Ia) - x} + \frac{2}{\exp(Ia)^2 \ln(\exp(Ia) + x)}$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))^2/x^3,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(x^2 e^{2ia - 2 \log(x)} - x^2) \operatorname{integral}\left(-\frac{e^{2ia - 2 \log(x)} + 3}{x^3 e^{2ia - 2 \log(x)} - x^3}, x\right) - 2}{x^2 e^{2ia - 2 \log(x)} - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))^2/x^3,x, algorithm="fricas")`

[Out]  $((x^2 e^{2ia - 2 \log(x)} - x^2) \operatorname{integral}(-(e^{2ia - 2 \log(x)} + 3)/(x^3 e^{2ia - 2 \log(x)} - x^3), x) - 2)/(x^2 e^{2ia - 2 \log(x)} - x^2)$

**Sympy [A]** time = 1.17795, size = 60, normalized size = 1.05

$$\frac{5x^2 - e^{2ia}}{2x^4 - 2x^2 e^{2ia}} - 4e^{-2ia} \log(x) + 2e^{-2ia} \log(x^2 - e^{2ia})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*ln(x))**2/x**3,x)`

[Out]  $(5x^{**2} - \exp(2Ia))/(2x^{**4} - 2x^{**2}\exp(2Ia)) - 4\exp(-2Ia)\log(x) + 2\exp(-2Ia)\log(x^{**2} - \exp(2Ia))$

**Giac [B]** time = 1.34441, size = 257, normalized size = 4.51

$$\frac{2x^4 \log(x^2 - e^{2ia})}{x^4 e^{2ia} - x^2 e^{4ia}} - \frac{4x^4 \log(x)}{x^4 e^{2ia} - x^2 e^{4ia}} - \frac{2x^2 e^{2ia} \log(x^2 - e^{2ia})}{x^4 e^{2ia} - x^2 e^{4ia}} + \frac{4x^2 e^{2ia} \log(x)}{x^4 e^{2ia} - x^2 e^{4ia}} + \frac{5x^2 e^{2ia}}{2(x^4 e^{2ia} - x^2 e^{4ia})} - \frac{1}{2(x^4 e^{2ia} - x^2 e^{4ia})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+I*log(x))^2/x^3,x, algorithm="giac")`

```
[Out] 2*x^4*log(x^2 - e^(2*I*a))/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 4*x^4*log(x)/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 2*x^2*e^(2*I*a)*log(x^2 - e^(2*I*a))/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) + 4*x^2*e^(2*I*a)*log(x)/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) + 5/2*x^2*e^(2*I*a)/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 1/2*e^(4*I*a)/(x^4*e^(2*I*a) - x^2*e^(4*I*a))
```

### 3.201 $\int (ex)^m \cot(a + i \log(x)) dx$

**Optimal.** Leaf size=70

$$\frac{i(ex)^{m+1}}{e(m+1)} - \frac{2i(ex)^{m+1} \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m-1), \frac{1-m}{2}, \frac{e^{2ia}}{x^2}\right)}{e(m+1)}$$

[Out]  $(I*(e*x)^{(1+m)})/(e*(1+m)) - ((2*I)*(e*x)^{(1+m)}*\text{Hypergeometric2F1}[1, (-1-m)/2, (1-m)/2, E^{((2*I)*a)/x^2}])/(e*(1+m))$

**Rubi [F]** time = 0.040328, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \cot(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[(e*x)^m*\text{Cot}[a + I*\text{Log}[x]], x]$

[Out]  $\text{Defer}[\text{Int}[(e*x)^m*\text{Cot}[a + I*\text{Log}[x]], x]$

Rubi steps

$$\int (ex)^m \cot(a + i \log(x)) dx = \int (ex)^m \cot(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.241067, size = 103, normalized size = 1.47

$$ix(ex)^m \left( \frac{x^2(\cos(a) - i \sin(a))^2 \text{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, x^2(\cos(2a) - i \sin(2a))\right)}{m+3} + \frac{\text{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, x^2(\cos(2a) - i \sin(2a))\right)}{m+3} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(e*x)^m*\text{Cot}[a + I*\text{Log}[x]], x]$

[Out]  $I*x*(e*x)^m*(\text{Hypergeometric2F1}[1, (1+m)/2, (3+m)/2, x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a])])/(1+m) + (x^2*\text{Hypergeometric2F1}[1, (3+m)/2, (5+m)/2, x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a])])*(\text{Cos}[a] - I*\text{Sin}[a])^2/(3+m)$

**Maple [F]** time = 0.158, size = 0, normalized size = 0.

$$\int (ex)^m \cot(a + i \ln(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e*x)^m*\cot(a+I*\ln(x)), x)$



[Out] `int((e*x)^m*cot(a+I*ln(x)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(a + i \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="maxima")`

[Out] `integrate((e*x)^m*cot(a + I*log(x)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex)^m (i e^{2ia-2 \log(x)} + i)}{e^{2ia-2 \log(x)} - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="fricas")`

[Out] `integral((e*x)^m*(I*e^(2*I*a - 2*log(x)) + I)/(e^(2*I*a - 2*log(x)) - 1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(a + i \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*cot(a+I*ln(x)),x)`

[Out] `Integral((e*x)**m*cot(a + I*log(x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(a + i \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="giac")`

[Out] `integrate((e*x)^m*cot(a + I*log(x)), x)`

### 3.202 $\int (ex)^m \cot^2(a + i \log(x)) dx$

**Optimal.** Leaf size=77

$$-2x(ex)^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m-1), \frac{1-m}{2}, \frac{e^{2ia}}{x^2}\right) + \frac{2x(ex)^m}{1 - \frac{e^{2ia}}{x^2}} - \frac{x(ex)^m}{m+1}$$

[Out]  $-\left(\frac{x(e^x)^m}{1+m}\right) + \frac{2x(e^x)^m}{1 - E^{\left(\frac{2I}{x^2}\right)a}} - \frac{x(e^x)^m}{m+1}$   
 Hypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, E^{\left(\frac{2I}{x^2}\right)a}]

**Rubi [F]** time = 0.0695071, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \cot^2(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Cot[a + I\*Log[x]]^2,x]

[Out] Defer[Int] [(e\*x)^m\*Cot[a + I\*Log[x]]^2, x]

Rubi steps

$$\int (ex)^m \cot^2(a + i \log(x)) dx = \int (ex)^m \cot^2(a + i \log(x)) dx$$

**Mathematica [B]** time = 0.429795, size = 169, normalized size = 2.19

$$x(ex)^m \left( -\frac{x^4(\cos(a)-i\sin(a))^2 \text{Hypergeometric2F1}\left(2, \frac{m+5}{2}, \frac{m+7}{2}, x^2(\cos(2a)-i\sin(2a))\right)}{m+5} - \frac{2x^2 \text{Hypergeometric2F1}\left(2, \frac{m+3}{2}, \frac{m+5}{2}, x^2(\cos(2a)-i\sin(2a))\right)}{m+3} \right) - \frac{1}{(\cos(a) + i \sin(a))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Cot[a + I\*Log[x]]^2,x]

[Out]  $(x(e^x)^m \left( (-2x^2 \text{Hypergeometric2F1}\left[2, \frac{3+m}{2}, \frac{5+m}{2}, x^2(\cos[2a] - I\sin[2a])\right]\right) / (3+m) - (x^4 \text{Hypergeometric2F1}\left[2, \frac{5+m}{2}, \frac{7+m}{2}, x^2(\cos[2a] - I\sin[2a])\right]\right) * (\cos[a] - I\sin[a])^2 / (5+m) - (\text{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, x^2(\cos[2a] - I\sin[2a])\right]\right) * (\cos[2a] + (2I)\cos[a]\sin[a]) / (1+m) \right) / (\cos[a] + I\sin[a])^2$

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int (ex)^m (\cot(a + i \ln(x)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*cot(a+I*ln(x))^2,x)`

[Out] `int((e*x)^m*cot(a+I*ln(x))^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(a + i \log(x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*cot(a + I*log(x))^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2 (ex)^m x - (e^{(2i a - 2 \log(x))} - 1) \operatorname{integral}\left(\frac{(ex)^m (2 m - e^{(2i a - 2 \log(x)) + 3})}{e^{(2i a - 2 \log(x))} - 1}, x\right)}{e^{(2i a - 2 \log(x))} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="fricas")`

[Out] `-(2*(e*x)^m*x - (e^(2*I*a - 2*log(x)) - 1)*integral((e*x)^m*(2*m - e^(2*I*a - 2*log(x)) + 3)/(e^(2*I*a - 2*log(x)) - 1), x))/(e^(2*I*a - 2*log(x)) - 1)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot^2(a + i \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*cot(a+I*ln(x))**2,x)`

[Out] `Integral((e*x)**m*cot(a + I*log(x))**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(a + i \log(x))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="giac")`

[Out] `integrate((e*x)^m*cot(a + I*log(x))^2, x)`

### 3.203 $\int (ex)^m \cot^3(a + i \log(x)) dx$

**Optimal.** Leaf size=169

$$\frac{i(m^2 + 2m + 3)x(ex)^m \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m - 1), \frac{1-m}{2}, \frac{e^{2ia}}{x^2}\right)}{m + 1} - \frac{ix\left(1 + \frac{e^{2ia}}{x^2}\right)^2 (ex)^m}{2\left(1 - \frac{e^{2ia}}{x^2}\right)^2} - \frac{ix\left(-\frac{e^{2ia}(1-m)}{x^2} + m + 3\right)(ex)^m}{2\left(1 - \frac{e^{2ia}}{x^2}\right)}$$

[Out]  $((I/2)*(1 - m)*m*x*(e*x)^m)/(1 + m) - ((I/2)*(1 + E^((2*I)*a)/x^2)^2*x*(e*x)^m)/(1 - E^((2*I)*a)/x^2)^2 - ((I/2)*(3 + m - (E^((2*I)*a)*(1 - m))/x^2)*x*(e*x)^m)/(1 - E^((2*I)*a)/x^2) + (I*(3 + 2*m + m^2)*x*(e*x)^m*\text{Hypergeometric2F1}[1, (-1 - m)/2, (1 - m)/2, E^((2*I)*a)/x^2])/(1 + m)$

**Rubi [F]** time = 0.0772087, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \cot^3(a + i \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Cot[a + I\*Log[x]]^3,x]

[Out] Defer[Int] [(e\*x)^m\*Cot[a + I\*Log[x]]^3, x]

Rubi steps

$$\int (ex)^m \cot^3(a + i \log(x)) dx = \int (ex)^m \cot^3(a + i \log(x)) dx$$

**Mathematica [A]** time = 0.950914, size = 250, normalized size = 1.48

$$x(ex)^m \left( -\frac{ix^4(\cos(2a)-i\sin(2a))(m+5)x^2(\cos(a)-i\sin(a))\text{Hypergeometric2F1}\left(3, \frac{m+7}{2}, \frac{m+9}{2}, x^2(\cos(2a)-i\sin(2a))\right)+3(m+7)(\cos(a)+i\sin(a))\text{Hypergeometric2F1}\left(3, \frac{m+7}{2}, \frac{m+9}{2}, x^2(\cos(2a)-i\sin(2a))\right)}{(m+5)(m+7)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Cot[a + I\*Log[x]]^3,x]

[Out]  $(x*(e*x)^m*((\text{Hypergeometric2F1}[3, (1 + m)/2, (3 + m)/2, x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a])])*(I*\text{Cos}[a] - \text{Sin}[a])^3)/(1 + m) + (3*x^2*\text{Hypergeometric2F1}[3, (3 + m)/2, (5 + m)/2, x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a])])*((-I)*\text{Cos}[a] + \text{Sin}[a]))/(3 + m) - (I*x^4*((5 + m)*x^2*\text{Hypergeometric2F1}[3, (7 + m)/2, (9 + m)/2, x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a])])*(\text{Cos}[a] - I*\text{Sin}[a]) + 3*(7 + m)*\text{Hypergeometric2F1}[3, (5 + m)/2, (7 + m)/2, x^2*(\text{Cos}[2*a] - I*\text{Sin}[2*a])])*(\text{Cos}[a] + I*\text{Sin}[a]))*(\text{Cos}[2*a] - I*\text{Sin}[2*a]))/((5 + m)*(7 + m)))/(\text{Cos}[a] + I*\text{Sin}[a])^3$

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int (ex)^m (\cot(a + i \ln(x)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*cot(a+I*ln(x))^3,x)`

[Out] `int((e*x)^m*cot(a+I*ln(x))^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(a + i \log(x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*cot(a + I*log(x))^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{((im - i)xe^{(2ia-2 \log(x))} + (-im - i)x)(ex)^m + (e^{(4ia-4 \log(x))} - 2e^{(2ia-2 \log(x))} + 1)\text{integral}\left(\frac{(-im^2 - 2im - ie^{(2ia-2 \log(x))} - 2i)(ex)}{e^{(2ia-2 \log(x))} - 1}\right)}{e^{(4ia-4 \log(x))} - 2e^{(2ia-2 \log(x))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="fricas")`

[Out] `((I*m - I)*x*e^(2*I*a - 2*log(x)) + (-I*m - I)*x)*(e*x)^m + (e^(4*I*a - 4*log(x)) - 2*e^(2*I*a - 2*log(x)) + 1)*integral((-I*m^2 - 2*I*m - I*e^(2*I*a - 2*log(x)) - 2*I)*(e*x)^m/(e^(2*I*a - 2*log(x)) - 1), x)/(e^(4*I*a - 4*log(x)) - 2*e^(2*I*a - 2*log(x)) + 1)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*cot(a+I*ln(x))**3,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(a + i \log(x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="giac")`

[Out] `integrate((e*x)^m*cot(a + I*log(x))^3, x)`

### 3.204 $\int \cot^p(a + b \log(x)) dx$

**Optimal.** Leaf size=142

$$x(1 - e^{2ia}x^{2ib})^p (1 + e^{2ia}x^{2ib})^{-p} \left( -\frac{i(1 + e^{2ia}x^{2ib})}{1 - e^{2ia}x^{2ib}} \right)^p F_1 \left( -\frac{i}{2b}; p, -p; 1 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right)$$

[Out] (x\*(1 - E^((2\*I)\*a)\*x^((2\*I)\*b)))^p\*(((-I)\*(1 + E^((2\*I)\*a)\*x^((2\*I)\*b)))/(1 - E^((2\*I)\*a)\*x^((2\*I)\*b)))^p\*AppellF1[(-I/2)/b, p, -p, 1 - (I/2)/b, E^((2\*I)\*a)\*x^((2\*I)\*b), -(E^((2\*I)\*a)\*x^((2\*I)\*b))]/(1 + E^((2\*I)\*a)\*x^((2\*I)\*b))^p

**Rubi [F]** time = 0.0215861, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot^p(a + b \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + b\*Log[x]]^p,x]

[Out] Defer[Int][Cot[a + b\*Log[x]]^p, x]

Rubi steps

$$\int \cot^p(a + b \log(x)) dx = \int \cot^p(a + b \log(x)) dx$$

**Mathematica [B]** time = 0.593524, size = 330, normalized size = 2.32

$$\frac{(2b - i)x \left( \frac{i(1 + e^{2ia}x^{2ib})}{-1 + e^{2ia}x^{2ib}} \right)^p F_1 \left( -\frac{i}{2b}; p, -p; 1 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right)}{2e^{2ia}bp x^{2ib} F_1 \left( 1 - \frac{i}{2b}; p, 1 - p; 2 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right) + 2e^{2ia}bp x^{2ib} F_1 \left( 1 - \frac{i}{2b}; p + 1, -p; 2 - \frac{i}{2b}; e^{2ia}x^{2ib}, -e^{2ia}x^{2ib} \right) + (2b - i)x}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[a + b\*Log[x]]^p,x]

[Out] ((-I + 2\*b)\*x\*((I\*(1 + E^((2\*I)\*a)\*x^((2\*I)\*b)))/(-1 + E^((2\*I)\*a)\*x^((2\*I)\*b)))^p\*AppellF1[(-I/2)/b, p, -p, 1 - (I/2)/b, E^((2\*I)\*a)\*x^((2\*I)\*b), -(E^((2\*I)\*a)\*x^((2\*I)\*b))]/(2\*b\*E^((2\*I)\*a)\*p\*x^((2\*I)\*b)\*AppellF1[1 - (I/2)/b, p, 1 - p, 2 - (I/2)/b, E^((2\*I)\*a)\*x^((2\*I)\*b), -(E^((2\*I)\*a)\*x^((2\*I)\*b))] + 2\*b\*E^((2\*I)\*a)\*p\*x^((2\*I)\*b)\*AppellF1[1 - (I/2)/b, 1 + p, -p, 2 - (I/2)/b, E^((2\*I)\*a)\*x^((2\*I)\*b), -(E^((2\*I)\*a)\*x^((2\*I)\*b))] + (-I + 2\*b)\*AppellF1[(-I/2)/b, p, -p, 1 - (I/2)/b, E^((2\*I)\*a)\*x^((2\*I)\*b), -(E^((2\*I)\*a)\*x^((2\*I)\*b))]

**Maple [F]** time = 0.372, size = 0, normalized size = 0.

$$\int (\cot(a + b \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(a+b*ln(x))^p,x)`

[Out] `int(cot(a+b*ln(x))^p,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*log(x))^p,x, algorithm="maxima")`

[Out] `integrate(cot(b*log(x) + a)^p, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\cot(b \log(x) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*log(x))^p,x, algorithm="fricas")`

[Out] `integral(cot(b*log(x) + a)^p, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot^p(a + b \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*ln(x))**p,x)`

[Out] `Integral(cot(a + b*log(x))**p, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*log(x))^p,x, algorithm="giac")`

[Out] `integrate(cot(b*log(x) + a)^p, x)`

### 3.205 $\int (ex)^m \cot^p(a + b \log(x)) dx$

**Optimal.** Leaf size=162

$$\frac{(ex)^{m+1} (1 - e^{2ia} x^{2ib})^p (1 + e^{2ia} x^{2ib})^{-p} \left( \frac{i(1+e^{2ia} x^{2ib})}{1-e^{2ia} x^{2ib}} \right)^p F_1 \left( -\frac{i(m+1)}{2b}; p, -p; 1 - \frac{i(m+1)}{2b}; e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right)}{e(m+1)}$$

[Out]  $((e*x)^{(1+m)}*(1 - E^{((2*I)*a)}*x^{((2*I)*b)})^p*(((-I)*(1 + E^{((2*I)*a)}*x^{((2*I)*b)})))/(1 - E^{((2*I)*a)}*x^{((2*I)*b)})^p*AppellF1[(((I/2)*(1 + m))/b, p, -p, 1 - ((I/2)*(1 + m))/b, E^{((2*I)*a)}*x^{((2*I)*b)}, -(E^{((2*I)*a)}*x^{((2*I)*b)}))]/(e*(1 + m)*(1 + E^{((2*I)*a)}*x^{((2*I)*b)})^p)$

**Rubi [F]** time = 0.117339, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \cot^p(a + b \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Cot[a + b\*Log[x]]^p,x]

[Out] Defer[Int][(e\*x)^m\*Cot[a + b\*Log[x]]^p, x]

Rubi steps

$$\int (ex)^m \cot^p(a + b \log(x)) dx = \int (ex)^m \cot^p(a + b \log(x)) dx$$

**Mathematica [A]** time = 0.637541, size = 157, normalized size = 0.97

$$\frac{x(ex)^m (1 - e^{2ia} x^{2ib})^p (1 + e^{2ia} x^{2ib})^{-p} \left( \frac{i(1+e^{2ia} x^{2ib})}{-1+e^{2ia} x^{2ib}} \right)^p F_1 \left( -\frac{i(m+1)}{2b}; p, -p; 1 - \frac{i(m+1)}{2b}; e^{2ia} x^{2ib}, -e^{2ia} x^{2ib} \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Cot[a + b\*Log[x]]^p,x]

[Out]  $(x*(e*x)^m*(1 - E^{((2*I)*a)}*x^{((2*I)*b)})^p*((I*(1 + E^{((2*I)*a)}*x^{((2*I)*b)})))/(-1 + E^{((2*I)*a)}*x^{((2*I)*b)})^p*AppellF1[(((I/2)*(1 + m))/b, p, -p, 1 - ((I/2)*(1 + m))/b, E^{((2*I)*a)}*x^{((2*I)*b)}, -(E^{((2*I)*a)}*x^{((2*I)*b)}))]/((1 + m)*(1 + E^{((2*I)*a)}*x^{((2*I)*b)})^p)$

**Maple [F]** time = 0.409, size = 0, normalized size = 0.

$$\int (ex)^m (\cot(a + b \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((e*x)^m*cot(a+b*ln(x))^p,x)`

[Out] `int((e*x)^m*cot(a+b*ln(x))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*cot(b*log(x) + a)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \cot(b \log(x) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="fricas")`

[Out] `integral((e*x)^m*cot(b*log(x) + a)^p, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot^p(a + b \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*cot(a+b*ln(x))**p,x)`

[Out] `Integral((e*x)**m*cot(a + b*log(x))**p, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(b \log(x) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="giac")`

[Out] `integrate((e*x)^m*cot(b*log(x) + a)^p, x)`

### 3.206 $\int \cot^p(a + \log(x)) dx$

**Optimal.** Leaf size=120

$$x(1 - e^{2iax^{2i}})^p (1 + e^{2iax^{2i}})^{-p} \left( -\frac{i(1 + e^{2iax^{2i}})}{1 - e^{2iax^{2i}}} \right)^p F_1 \left( -\frac{i}{2}; p, -p; 1 - \frac{i}{2}; e^{2iax^{2i}}, -e^{2iax^{2i}} \right)$$

[Out]  $((1 - E^((2*I)*a)*x^(2*I)))^p * (((-I)*(1 + E^((2*I)*a)*x^(2*I)))) / (1 - E^((2*I)*a)*x^(2*I)))^p * x * \text{AppellF1}[-I/2, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))]/(1 + E^((2*I)*a)*x^(2*I))^p$

**Rubi [F]** time = 0.0200752, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot^p(a + \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + Log[x]]^p, x]

[Out] Defer[Int][Cot[a + Log[x]]^p, x]

Rubi steps

$$\int \cot^p(a + \log(x)) dx = \int \cot^p(a + \log(x)) dx$$

**Mathematica [A]** time = 0.479071, size = 238, normalized size = 1.98

$$\frac{(2 - i)x \left( \frac{i(1 + e^{2iax^{2i}})}{-1 + e^{2iax^{2i}}} \right)^p F_1 \left( -\frac{i}{2}; p, -p; 1 - \frac{i}{2}; e^{2iax^{2i}}, -e^{2iax^{2i}} \right)}{(2 - i)F_1 \left( -\frac{i}{2}; p, -p; 1 - \frac{i}{2}; e^{2iax^{2i}}, -e^{2iax^{2i}} \right) + 2e^{2ia}px^{2i} \left( F_1 \left( 1 - \frac{i}{2}; p, 1 - p; 2 - \frac{i}{2}; e^{2iax^{2i}}, -e^{2iax^{2i}} \right) + F_1 \left( 1 - \frac{i}{2}; p + 1, -p; 2 - \frac{i}{2}; e^{2iax^{2i}}, -e^{2iax^{2i}} \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[a + Log[x]]^p, x]

[Out]  $((2 - I)*((I*(1 + E^((2*I)*a)*x^(2*I))))/(-1 + E^((2*I)*a)*x^(2*I)))^p * x * \text{AppellF1}[-I/2, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))]/((2 - I)*\text{AppellF1}[-I/2, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))] + 2*E^((2*I)*a)*p*x^(2*I)*(\text{AppellF1}[1 - I/2, p, 1 - p, 2 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))] + \text{AppellF1}[1 - I/2, 1 + p, -p, 2 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))])$

**Maple [F]** time = 0.322, size = 0, normalized size = 0.

$$\int (\cot(a + \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(a+ln(x))^p,x)`

[Out] `int(cot(a+ln(x))^p,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot(a + \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+log(x))^p,x, algorithm="maxima")`

[Out] `integrate(cot(a + log(x))^p, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\cot(a + \log(x))^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+log(x))^p,x, algorithm="fricas")`

[Out] `integral(cot(a + log(x))^p, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot^p(a + \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+ln(x))**p,x)`

[Out] `Integral(cot(a + log(x))**p, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot(a + \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+log(x))^p,x, algorithm="giac")`

[Out] `integrate(cot(a + log(x))^p, x)`

### 3.207 $\int \cot^p(a + 2 \log(x)) dx$

**Optimal.** Leaf size=120

$$x \left(1 - e^{2iax^{4i}}\right)^p \left(1 + e^{2iax^{4i}}\right)^{-p} \left(-\frac{i(1 + e^{2iax^{4i}})}{1 - e^{2iax^{4i}}}\right)^p F_1\left(-\frac{i}{4}; p, -p; 1 - \frac{i}{4}; e^{2iax^{4i}}, -e^{2iax^{4i}}\right)$$

[Out]  $((1 - E^((2*I)*a)*x^{(4*I)})^p * ((-I)*(1 + E^((2*I)*a)*x^{(4*I)})) / (1 - E^((2*I)*a)*x^{(4*I)}))^p * x * \text{AppellF1}[-I/4, p, -p, 1 - I/4, E^((2*I)*a)*x^{(4*I)}, -(E^((2*I)*a)*x^{(4*I)})] / (1 + E^((2*I)*a)*x^{(4*I)})^p$

**Rubi [F]** time = 0.0202924, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot^p(a + 2 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + 2\*Log[x]]^p, x]

[Out] Defer[Int][Cot[a + 2\*Log[x]]^p, x]

Rubi steps

$$\int \cot^p(a + 2 \log(x)) dx = \int \cot^p(a + 2 \log(x)) dx$$

**Mathematica [A]** time = 0.465129, size = 238, normalized size = 1.98

$$\frac{(4 - i)x \left(\frac{i(1 + e^{2iax^{4i}})}{-1 + e^{2iax^{4i}}}\right)^p F_1\left(-\frac{i}{4}; p, -p; 1 - \frac{i}{4}; e^{2iax^{4i}}, -e^{2iax^{4i}}\right)}{(4 - i)F_1\left(-\frac{i}{4}; p, -p; 1 - \frac{i}{4}; e^{2iax^{4i}}, -e^{2iax^{4i}}\right) + 4e^{2ia}px^{4i} \left(F_1\left(1 - \frac{i}{4}; p, 1 - p; 2 - \frac{i}{4}; e^{2iax^{4i}}, -e^{2iax^{4i}}\right) + F_1\left(1 - \frac{i}{4}; p + 1, -p; 2 - \frac{i}{4}; e^{2iax^{4i}}, -e^{2iax^{4i}}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[a + 2\*Log[x]]^p, x]

[Out]  $((4 - I)*((I*(1 + E^((2*I)*a)*x^{(4*I)})) / (-1 + E^((2*I)*a)*x^{(4*I)}))^p * x * \text{AppellF1}[-I/4, p, -p, 1 - I/4, E^((2*I)*a)*x^{(4*I)}, -(E^((2*I)*a)*x^{(4*I)})] / ((4 - I)*\text{AppellF1}[-I/4, p, -p, 1 - I/4, E^((2*I)*a)*x^{(4*I)}, -(E^((2*I)*a)*x^{(4*I)})] + 4*E^((2*I)*a)*p*x^{(4*I)}*(\text{AppellF1}[1 - I/4, p, 1 - p, 2 - I/4, E^((2*I)*a)*x^{(4*I)}, -(E^((2*I)*a)*x^{(4*I)})] + \text{AppellF1}[1 - I/4, 1 + p, -p, 2 - I/4, E^((2*I)*a)*x^{(4*I)}, -(E^((2*I)*a)*x^{(4*I)})]))$

**Maple [F]** time = 0.357, size = 0, normalized size = 0.

$$\int (\cot(a + 2 \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(a+2*ln(x))^p,x)`

[Out] `int(cot(a+2*ln(x))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot (a + 2 \log (x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+2*log(x))^p,x, algorithm="maxima")`

[Out] `integrate(cot(a + 2*log(x))^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cot (a + 2 \log (x))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+2*log(x))^p,x, algorithm="fricas")`

[Out] `integral(cot(a + 2*log(x))^p, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot^p (a + 2 \log (x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+2*ln(x))**p,x)`

[Out] `Integral(cot(a + 2*log(x))**p, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot (a + 2 \log (x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+2*log(x))^p,x, algorithm="giac")`

[Out] `integrate(cot(a + 2*log(x))^p, x)`

### 3.208 $\int \cot^p(a + 3 \log(x)) dx$

**Optimal.** Leaf size=120

$$x \left(1 - e^{2iax^{6i}}\right)^p \left(1 + e^{2iax^{6i}}\right)^{-p} \left(-\frac{i(1 + e^{2iax^{6i}})}{1 - e^{2iax^{6i}}}\right)^p F_1\left(-\frac{i}{6}; p, -p; 1 - \frac{i}{6}; e^{2iax^{6i}}, -e^{2iax^{6i}}\right)$$

[Out]  $((1 - E^((2*I)*a)*x^{(6*I)})^p * ((-I)*(1 + E^((2*I)*a)*x^{(6*I)})) / (1 - E^((2*I)*a)*x^{(6*I)}))^p * x * \text{AppellF1}[-I/6, p, -p, 1 - I/6, E^((2*I)*a)*x^{(6*I)}, -(E^((2*I)*a)*x^{(6*I)})] / (1 + E^((2*I)*a)*x^{(6*I)})^p$

**Rubi [F]** time = 0.0210843, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot^p(a + 3 \log(x)) dx$$

Verification is Not applicable to the result.

[In] Int[Cot[a + 3\*Log[x]]^p, x]

[Out] Defer[Int][Cot[a + 3\*Log[x]]^p, x]

Rubi steps

$$\int \cot^p(a + 3 \log(x)) dx = \int \cot^p(a + 3 \log(x)) dx$$

**Mathematica [A]** time = 0.464693, size = 238, normalized size = 1.98

$$\frac{(6-i)x \left(\frac{i(1+e^{2iax^{6i}})}{-1+e^{2iax^{6i}}}\right)^p F_1\left(-\frac{i}{6}; p, -p; 1 - \frac{i}{6}; e^{2iax^{6i}}, -e^{2iax^{6i}}\right)}{(6-i)F_1\left(-\frac{i}{6}; p, -p; 1 - \frac{i}{6}; e^{2iax^{6i}}, -e^{2iax^{6i}}\right) + 6e^{2ia}px^{6i} \left(F_1\left(1 - \frac{i}{6}; p, 1 - p; 2 - \frac{i}{6}; e^{2iax^{6i}}, -e^{2iax^{6i}}\right) + F_1\left(1 - \frac{i}{6}; p + 1, -p; 2 - \frac{i}{6}; e^{2iax^{6i}}, -e^{2iax^{6i}}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[a + 3\*Log[x]]^p, x]

[Out]  $((6 - I)*((I*(1 + E^((2*I)*a)*x^{(6*I)})) / (-1 + E^((2*I)*a)*x^{(6*I)}))^p * x * \text{AppellF1}[-I/6, p, -p, 1 - I/6, E^((2*I)*a)*x^{(6*I)}, -(E^((2*I)*a)*x^{(6*I)})] / ((6 - I)*\text{AppellF1}[-I/6, p, -p, 1 - I/6, E^((2*I)*a)*x^{(6*I)}, -(E^((2*I)*a)*x^{(6*I)})] + 6*E^((2*I)*a)*p*x^{(6*I)}*(\text{AppellF1}[1 - I/6, p, 1 - p, 2 - I/6, E^((2*I)*a)*x^{(6*I)}, -(E^((2*I)*a)*x^{(6*I)})] + \text{AppellF1}[1 - I/6, 1 + p, -p, 2 - I/6, E^((2*I)*a)*x^{(6*I)}, -(E^((2*I)*a)*x^{(6*I)})]))$

**Maple [F]** time = 0.336, size = 0, normalized size = 0.

$$\int (\cot(a + 3 \ln(x)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(a+3*ln(x))^p,x)`

[Out] `int(cot(a+3*ln(x))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot(a + 3 \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+3*log(x))^p,x, algorithm="maxima")`

[Out] `integrate(cot(a + 3*log(x))^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\cot(a + 3 \log(x))^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+3*log(x))^p,x, algorithm="fricas")`

[Out] `integral(cot(a + 3*log(x))^p, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot^p(a + 3 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+3*ln(x))**p,x)`

[Out] `Integral(cot(a + 3*log(x))**p, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot(a + 3 \log(x))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+3*log(x))^p,x, algorithm="giac")`

[Out] `integrate(cot(a + 3*log(x))^p, x)`

### 3.209 $\int x^3 \cot(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=70

$$\frac{ix^4}{4} - \frac{1}{2}ix^4 \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, e^{2iad}(cx^n)^{2ibd}\right)$$

[Out] (I/4)\*x^4 - (I/2)\*x^4\*Hypergeometric2F1[1, (-2\*I)/(b\*d\*n), 1 - (2\*I)/(b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]

**Rubi [F]** time = 0.0352325, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \cot(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] Defer[Int][x^3\*Cot[d\*(a + b\*Log[c\*x^n])], x]

Rubi steps

$$\int x^3 \cot(d(a + b \log(cx^n))) dx = \int x^3 \cot(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 5.39956, size = 220, normalized size = 3.14

$$x^4 \left( 2e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{2i}{bdn}, 2 - \frac{2i}{bdn}, e^{2id(a+b \log(cx^n))}\right) + (bdn - 2i) \left( i \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, e^{2id(a+b \log(cx^n))}\right) + (-2i + bdn) * (\text{Cot}[d*(a + b*Log[c*x^n])] - \text{Cot}[d*(a - b*n*Log[x] + b*Log[c*x^n])] + I*\text{Hypergeometric2F1}[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n])]) + \text{Csc}[d*(a + b*Log[c*x^n])] * \text{Csc}[d*(a - b*n*Log[x] + b*Log[c*x^n])] * \text{Sin}[b*d*n*Log[x]]) \right) \right) / (-8*I + 4*b*d*n)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] -((x^4\*(2\*E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*Hypergeometric2F1[1, 1 - (2\*I)/(b\*d\*n), 2 - (2\*I)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n])]) + (-2\*I + b\*d\*n)\*(Cot[d\*(a + b\*Log[c\*x^n])] - Cot[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] + I\*Hypergeometric2F1[1, (-2\*I)/(b\*d\*n), 1 - (2\*I)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n])]) + Csc[d\*(a + b\*Log[c\*x^n])] \* Csc[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] \* Sin[b\*d\*n\*Log[x]])))/(-8\*I + 4\*b\*d\*n)

**Maple [F]** time = 1.831, size = 0, normalized size = 0.

$$\int x^3 \cot(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cot(d\*(a+b\*ln(c\*x^n))), x)



[Out] `int(x^3*cot(d*(a+b*ln(c*x^n))),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \cot((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(x^3*cot((b*log(c*x^n) + a)*d), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \cot(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(x^3*cot(b*d*log(c*x^n) + a*d), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cot(d*(a+b*ln(c*x**n))),x)`

[Out] Timed out

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Exception raised: TypeError

### 3.210 $\int x^2 \cot(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=74

$$\frac{ix^3}{3} - \frac{2}{3}ix^3 \text{Hypergeometric2F1}\left(1, -\frac{3i}{2bdn}, 1 - \frac{3i}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right)$$

[Out] (I/3)\*x^3 - ((2\*I)/3)\*x^3\*Hypergeometric2F1[1, ((-3\*I)/2)/(b\*d\*n), 1 - ((3\*I)/2)/(b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]

**Rubi [F]** time = 0.0290022, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \cot(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] Defer[Int][x^2\*Cot[d\*(a + b\*Log[c\*x^n])], x]

Rubi steps

$$\int x^2 \cot(d(a + b \log(cx^n))) dx = \int x^2 \cot(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 5.74146, size = 229, normalized size = 3.09

$$x^3 \left( 3e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{3i}{2bdn}, 2 - \frac{3i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) + (2bdn - 3i) \left( i \text{Hypergeometric2F1}\left(1, 1 - \frac{3i}{2bdn}, 2 - \frac{3i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) + (-3i + 2b*d*n) * (\text{Cot}[d*(a + b*Log[c*x^n])] - \text{Cot}[d*(a - b*n*Log[x] + b*Log[c*x^n])]) + I * \text{Hypergeometric2F1}\left[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n])]\right] + \text{Csc}[d*(a + b*Log[c*x^n])] * \text{Csc}[d*(a - b*n*Log[x] + b*Log[c*x^n])] * \text{Sin}[b*d*n*Log[x]] \right) \right) / (-9*I + 6*b*d*n)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] -((x^3\*(3\*E^((2\*I)\*d\*(a + b\*Log[c\*x^n])))\*Hypergeometric2F1[1, 1 - ((3\*I)/2)/(b\*d\*n), 2 - ((3\*I)/2)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n])]) + (-3\*I + 2\*b\*d\*n)\*(Cot[d\*(a + b\*Log[c\*x^n])] - Cot[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]) + I\*Hypergeometric2F1[1, ((-3\*I)/2)/(b\*d\*n), 1 - ((3\*I)/2)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n])]) + Csc[d\*(a + b\*Log[c\*x^n])]\*Csc[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]\*Sin[b\*d\*n\*Log[x]]))/(-9\*I + 6\*b\*d\*n)

**Maple [F]** time = 1.595, size = 0, normalized size = 0.

$$\int x^2 \cot(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cot(d\*(a+b\*ln(c\*x^n))), x)

[Out] `int(x^2*cot(d*(a+b*ln(c*x^n))),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \cot((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(x^2*cot((b*log(c*x^n) + a)*d), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \cot(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(x^2*cot(b*d*log(c*x^n) + a*d), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \cot(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cot(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral(x**2*cot(a*d + b*d*log(c*x**n)), x)`

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Exception raised: TypeError

### 3.211 $\int x \cot(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=68

$$\frac{ix^2}{2} - ix^2 \text{Hypergeometric2F1}\left(1, -\frac{i}{bdn}, 1 - \frac{i}{bdn}, e^{2iad}(cx^n)^{2ibd}\right)$$

[Out] (I/2)\*x^2 - I\*x^2\*Hypergeometric2F1[1, (-I)/(b\*d\*n), 1 - I/(b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]

**Rubi [F]** time = 0.0225825, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ ., Rules used = {}

$$\int x \cot(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x\*Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] Defer[Int][x\*Cot[d\*(a + b\*Log[c\*x^n])], x]

Rubi steps

$$\int x \cot(d(a + b \log(cx^n))) dx = \int x \cot(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 5.71875, size = 219, normalized size = 3.22

$$x^2 \left( e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{i}{bdn}, 2 - \frac{i}{bdn}, e^{2id(a+b \log(cx^n))}\right) + (bdn - i) \left( i \text{Hypergeometric2F1}\left(1, -\frac{i}{bdn}, 2 - \frac{i}{bdn}, e^{2id(a+b \log(cx^n))}\right) + (-I + b*d*n) * (\text{Cot}[d*(a + b*Log[c*x^n])] - \text{Cot}[d*(a - b*n*Log[x] + b*Log[c*x^n])] + I*\text{Hypergeometric2F1}[1, (-I)/(b*d*n), 1 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n])]) + \text{Csc}[d*(a + b*Log[c*x^n])] * \text{Csc}[d*(a - b*n*Log[x] + b*Log[c*x^n])] * \text{Sin}[b*d*n*Log[x]]) / (-2*I + 2*b*d*n) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] -((x^2\*(E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*Hypergeometric2F1[1, 1 - I/(b\*d\*n), 2 - I/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n])]) + (-I + b\*d\*n)\*(Cot[d\*(a + b\*Log[c\*x^n])] - Cot[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] + I\*Hypergeometric2F1[1, (-I)/(b\*d\*n), 1 - I/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n])]) + Csc[d\*(a + b\*Log[c\*x^n])] \* Csc[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] \* Sin[b\*d\*n\*Log[x]])))/(-2\*I + 2\*b\*d\*n))

**Maple [F]** time = 1.45, size = 0, normalized size = 0.

$$\int x \cot(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cot(d\*(a+b\*ln(c\*x^n))), x)

[Out] `int(x*cot(d*(a+b*ln(c*x^n))),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \cot((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(x*cot((b*log(c*x^n) + a)*d), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \cot(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(x*cot(b*d*log(c*x^n) + a*d), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \cot(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral(x*cot(a*d + b*d*log(c*x**n)), x)`

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Exception raised: TypeError

### 3.212 $\int \cot(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=66

$$ix - 2ix \operatorname{Hypergeometric2F1}\left(1, -\frac{i}{2bdn}, 1 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right)$$

[Out] I\*x - (2\*I)\*x\*Hypergeometric2F1[1, (-I/2)/(b\*d\*n), 1 - (I/2)/(b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]

**Rubi [F]** time = 0.01151, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] Defer[Int][Cot[d\*(a + b\*Log[c\*x^n])], x]

Rubi steps

$$\int \cot(d(a + b \log(cx^n))) dx = \int \cot(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 10.6454, size = 141, normalized size = 2.14

$$x \left( \frac{e^{2id(a+b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(1, 1 - \frac{i}{2bdn}, 2 - \frac{i}{2bdn}, e^{2id(a+b \log(cx^n))}\right)}{2bdn - i} - i \operatorname{Hypergeometric2F1}\left(1, -\frac{i}{2bdn}, 1 - \frac{i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] x\*(-((E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*Hypergeometric2F1[1, 1 - (I/2)/(b\*d\*n), 2 - (I/2)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))])/(-I + 2\*b\*d\*n)) - I\*Hypergeometric2F1[1, (-I/2)/(b\*d\*n), 1 - (I/2)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))])

**Maple [F]** time = 1.309, size = 0, normalized size = 0.

$$\int \cot(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*(a+b\*ln(c\*x^n))), x)

[Out] `int(cot(d*(a+b*ln(c*x^n))),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")`

[Out] `integrate(cot((b*log(c*x^n) + a)*d), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\cot(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")`

[Out] `integral(cot(b*d*log(c*x^n) + a*d), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*ln(c*x**n))),x)`

[Out] `Integral(cot(d*(a + b*log(c*x**n))), x)`

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.213 \quad \int \frac{\cot(d(a+b \log(cx^n)))}{x} dx$$

**Optimal.** Leaf size=25

$$\frac{\log(\sin(ad + bd \log(cx^n)))}{bdn}$$

[Out] Log[Sin[a\*d + b\*d\*Log[c\*x^n]]]/(b\*d\*n)

**Rubi [A]** time = 0.0184988, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {3475}

$$\frac{\log(\sin(ad + bd \log(cx^n)))}{bdn}$$

Antiderivative was successfully verified.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])]/x, x]

[Out] Log[Sin[a\*d + b\*d\*Log[c\*x^n]]]/(b\*d\*n)

Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot(d(a + b \log(cx^n)))}{x} dx &= \frac{\text{Subst}\left(\int \cot(d(a + bx)) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\log(\sin(ad + bd \log(cx^n)))}{bdn} \end{aligned}$$

**Mathematica [A]** time = 0.0591389, size = 40, normalized size = 1.6

$$\frac{\log(\tan(ad + bd \log(cx^n))) + \log(\cos(d(a + b \log(cx^n))))}{bdn}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])]/x, x]

[Out] (Log[Cos[d\*(a + b\*Log[c\*x^n])]] + Log[Tan[a\*d + b\*d\*Log[c\*x^n]]])/(b\*d\*n)

**Maple [A]** time = 0.019, size = 30, normalized size = 1.2

$$\frac{\ln\left(\cot(d(a + b \ln(cx^n)))^2 + 1\right)}{2 b d n}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(cot(d*(a+b*ln(c*x^n)))/x,x)`

[Out]  $-1/2/n/d/b*\ln(\cot(d*(a+b*\ln(c*x^n)))^2+1)$

**Maxima [A]** time = 0.971066, size = 32, normalized size = 1.28

$$\frac{\log(\sin((b \log(cx^n) + a)d))}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")`

[Out]  $\log(\sin((b*\log(c*x^n) + a)*d))/(b*d*n)$

**Fricas [A]** time = 0.502593, size = 97, normalized size = 3.88

$$\frac{\log\left(-\frac{1}{2} \cos(2 b d n \log(x) + 2 b d \log(c) + 2 a d) + \frac{1}{2}\right)}{2 b d n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")`

[Out]  $1/2*\log(-1/2*\cos(2*b*d*n*\log(x) + 2*b*d*\log(c) + 2*a*d) + 1/2)/(b*d*n)$

**Sympy [A]** time = 5.00643, size = 46, normalized size = 1.84

$$\begin{cases} \log(x) \cot(ad) & \text{for } b = 0 \\ \infty \log(x) & \text{for } d = 0 \\ \log(x) \cot(ad + bd \log(c)) & \text{for } n = 0 \\ \frac{\log(\sin(ad + bd \log(cx^n)))}{bdn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*ln(c*x**n)))/x,x)`

[Out] `Piecewise((log(x)*cot(a*d), Eq(b, 0)), (zoo*log(x), Eq(d, 0)), (log(x)*cot(a*d + b*d*log(c)), Eq(n, 0)), (log(sin(a*d + b*d*log(c*x**n)))/(b*d*n), True))`

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.214 \quad \int \frac{\cot(d(a+b \log(cx^n)))}{x^2} dx$$

**Optimal.** Leaf size=70

$$\frac{{}_2F_1\left(1, \frac{i}{2bdn}, 1 + \frac{i}{2bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{x} - \frac{i}{x}$$

[Out] (-I)/x + ((2\*I)\*Hypergeometric2F1[1, (I/2)/(b\*d\*n), 1 + (I/2)/(b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)])/x

**Rubi [F]** time = 0.0295843, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot(d(a+b \log(cx^n)))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])]/x^2, x]

[Out] Defer[Int][Cot[d\*(a + b\*Log[c\*x^n])]/x^2, x]

Rubi steps

$$\int \frac{\cot(d(a+b \log(cx^n)))}{x^2} dx = \int \frac{\cot(d(a+b \log(cx^n)))}{x^2} dx$$

**Mathematica [B]** time = 4.67465, size = 217, normalized size = 3.1

$$\frac{e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 + \frac{i}{2bdn}, 2 + \frac{i}{2bdn}, e^{2id(a+b \log(cx^n))}\right)}{2bdn+i} + i \text{Hypergeometric2F1}\left(1, \frac{i}{2bdn}, 1 + \frac{i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) +$$

Antiderivative was successfully verified.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])]/x^2, x]

[Out] (Cot[d\*(a + b\*Log[c\*x^n])] - Cot[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])] - (E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*Hypergeometric2F1[1, 1 + (I/2)/(b\*d\*n), 2 + (I/2)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))])/(I + 2\*b\*d\*n) + I\*Hypergeometric2F1[1, (I/2)/(b\*d\*n), 1 + (I/2)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) + Csc[d\*(a + b\*Log[c\*x^n])]\*Csc[d\*(a - b\*n\*Log[x] + b\*Log[c\*x^n])]\*Sin[b\*d\*n\*Log[x]])/x

**Maple [F]** time = 1.589, size = 0, normalized size = 0.

$$\int \frac{\cot(d(a+b \ln(cx^n)))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*(a+b*ln(c*x^n)))/x^2,x)`

[Out] `int(cot(d*(a+b*ln(c*x^n)))/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot((b \log(cx^n) + a)d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")`

[Out] `integrate(cot((b*log(c*x^n) + a)*d)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot(bd \log(cx^n) + ad)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")`

[Out] `integral(cot(b*d*log(c*x^n) + a*d)/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(ad + bd \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*ln(c*x**n)))/x**2,x)`

[Out] `Integral(cot(a*d + b*d*log(c*x**n))/x**2, x)`

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")`

[Out] Exception raised: RuntimeError

$$3.215 \quad \int \frac{\cot(d(a+b \log(cx^n)))}{x^3} dx$$

**Optimal.** Leaf size=68

$$\frac{i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{x^2} - \frac{i}{2x^2}$$

[Out]  $(-I/2)/x^2 + (I*\operatorname{Hypergeometric2F1}[1, I/(b*d*n), 1 + I/(b*d*n), E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}])/x^2$

**Rubi [F]** time = 0.029018, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot(d(a+b \log(cx^n)))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])]/x^3,x]

[Out] Defer[Int][Cot[d\*(a + b\*Log[c\*x^n])]/x^3, x]

Rubi steps

$$\int \frac{\cot(d(a+b \log(cx^n)))}{x^3} dx = \int \frac{\cot(d(a+b \log(cx^n)))}{x^3} dx$$

**Mathematica [B]** time = 4.3129, size = 211, normalized size = 3.1

$$\frac{e^{2id(a+b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(1, 1 + \frac{i}{bdn}, 2 + \frac{i}{bdn}, e^{2id(a+b \log(cx^n))}\right)}{bdn+i} + i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, e^{2id(a+b \log(cx^n))}\right) + \cot$$

Antiderivative was successfully verified.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])]/x^3,x]

[Out]  $(\operatorname{Cot}[d*(a + b*\operatorname{Log}[c*x^n])] - \operatorname{Cot}[d*(a - b*n*\operatorname{Log}[x] + b*\operatorname{Log}[c*x^n])]) - (E^{((2*I)*d*(a + b*\operatorname{Log}[c*x^n]))}*\operatorname{Hypergeometric2F1}[1, 1 + I/(b*d*n), 2 + I/(b*d*n), E^{((2*I)*d*(a + b*\operatorname{Log}[c*x^n]))}])/(I + b*d*n) + I*\operatorname{Hypergeometric2F1}[1, I/(b*d*n), 1 + I/(b*d*n), E^{((2*I)*d*(a + b*\operatorname{Log}[c*x^n]))}] + \operatorname{Csc}[d*(a + b*\operatorname{Log}[c*x^n])] * \operatorname{Csc}[d*(a - b*n*\operatorname{Log}[x] + b*\operatorname{Log}[c*x^n])] * \operatorname{Sin}[b*d*n*\operatorname{Log}[x]])/(2*x^2)$

**Maple [F]** time = 1.779, size = 0, normalized size = 0.

$$\int \frac{\cot(d(a+b \ln(cx^n)))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*(a+b*ln(c*x^n)))/x^3,x)`

[Out] `int(cot(d*(a+b*ln(c*x^n)))/x^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")`

[Out] `integrate(cot((b*log(c*x^n) + a)*d)/x^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot(bd \log(cx^n) + ad)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")`

[Out] `integral(cot(b*d*log(c*x^n) + a*d)/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(ad + bd \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*ln(c*x**n)))/x**3,x)`

[Out] `Integral(cot(a*d + b*d*log(c*x**n))/x**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot((b \log(cx^n) + a)d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")`

[Out] `integrate(cot((b*log(c*x^n) + a)*d)/x^3, x)`

### 3.216 $\int x^3 \cot^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=158

$$-\frac{2ix^4 \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{bdn} + \frac{ix^4 (1 + e^{2iad} (cx^n)^{2ibd})}{bdn (1 - e^{2iad} (cx^n)^{2ibd})} + \frac{x^4(-bdn + 4i)}{4bdn}$$

[Out]  $((4*I - b*d*n)*x^4)/(4*b*d*n) + (I*x^4*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^4*\text{Hypergeometric2F1}[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(b*d*n)$

**Rubi [F]** time = 0.0853566, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \cot^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*Cot[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out] Defer[Int][x^3\*Cot[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int x^3 \cot^2(d(a + b \log(cx^n))) dx = \int x^3 \cot^2(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 4.99133, size = 175, normalized size = 1.11

$$\frac{x^4 \left( 8e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{2i}{bdn}, 2 - \frac{2i}{bdn}, e^{2id(a+b \log(cx^n))}\right) + (bdn - 2i) \left( 4i \text{Hypergeometric2F1}\left(1, \right. \right. \right.}{4bdn(bdn - 2i)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Cot[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out]  $-(x^4*(8*E^((2*I)*d*(a + b*Log[c*x^n]))*\text{Hypergeometric2F1}[1, 1 - (2*I)/(b*d*n), 2 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))]) + (-2*I + b*d*n)*(b*d*n + 4*Cot[d*(a + b*Log[c*x^n])] + (4*I)*\text{Hypergeometric2F1}[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])))/(4*b*d*n*(-2*I + b*d*n))$

**Maple [F]** time = 1.815, size = 0, normalized size = 0.

$$\int x^3 (\cot(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cot(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(x^3*cot(d*(a+b*ln(c*x^n)))^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \cot (bd \log (cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(x^3*cot(b*d*log(c*x^n) + a*d)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cot(d*(a+b*ln(c*x**n)))**2,x)`

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] Timed out

### 3.217 $\int x^2 \cot^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=162

$$\frac{2ix^3 \text{Hypergeometric2F1}\left(1, -\frac{3i}{2bdn}, 1 - \frac{3i}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right)}{bdn} + \frac{ix^3(1 + e^{2iad}(cx^n)^{2ibd})}{bdn(1 - e^{2iad}(cx^n)^{2ibd})} + \frac{x^3(-bdn + 3i)}{3bdn}$$

[Out]  $((3I - b*d*n)*x^3)/(3*b*d*n) + (I*x^3*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^3*\text{Hypergeometric2F1}[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(b*d*n)$

**Rubi [F]** time = 0.0611011, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \cot^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Cot[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out] Defer[Int][x^2\*Cot[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int x^2 \cot^2(d(a + b \log(cx^n))) dx = \int x^2 \cot^2(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 5.32856, size = 185, normalized size = 1.14

$$\frac{x^3 \left( 9e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{3i}{2bdn}, 2 - \frac{3i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) + (2bdn - 3i) \left( 3i \text{Hypergeometric2F1}\left(1, 1 - \frac{3i}{2bdn}, 2 - \frac{3i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) + (-3I + 2*b*d*n)*(b*d*n + 3*\text{Cot}[d*(a + b*\text{Log}[c*x^n]]) + (3*I)*\text{Hypergeometric2F1}[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*\text{Log}[c*x^n]))])\right) \right)}{3bdn(2bdn - 3i)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Cot[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out]  $-(x^3*(9*E^((2*I)*d*(a + b*\text{Log}[c*x^n]))*\text{Hypergeometric2F1}[1, 1 - ((3*I)/2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*\text{Log}[c*x^n]))]) + (-3*I + 2*b*d*n)*(b*d*n + 3*\text{Cot}[d*(a + b*\text{Log}[c*x^n]]) + (3*I)*\text{Hypergeometric2F1}[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*\text{Log}[c*x^n]))])\right)/(3*b*d*n*(-3*I + 2*b*d*n))$

**Maple [F]** time = 1.632, size = 0, normalized size = 0.

$$\int x^2 (\cot(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^2*cot(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(x^2*cot(d*(a+b*ln(c*x^n)))^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \cot (bd \log (cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(x^2*cot(b*d*log(c*x^n) + a*d)^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cot(d*(a+b*ln(c*x**n)))**2,x)`

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] Timed out

### 3.218 $\int x \cot^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=158

$$\frac{2ix^2 \text{Hypergeometric2F1}\left(1, -\frac{i}{bdn}, 1 - \frac{i}{bdn}, e^{2iad}(cx^n)^{2ibd}\right)}{bdn} + \frac{ix^2(1 + e^{2iad}(cx^n)^{2ibd})}{bdn(1 - e^{2iad}(cx^n)^{2ibd})} + \frac{x^2(-bdn + 2i)}{2bdn}$$

[Out]  $((2I - b*d*n)*x^2)/(2*b*d*n) + (I*x^2*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*x^2*\text{Hypergeometric2F1}[1, (-I)/(b*d*n), 1 - I/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(b*d*n)$

**Rubi [F]** time = 0.0439753, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \cot^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[x\*Cot[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out] Defer[Int][x\*Cot[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int x \cot^2(d(a + b \log(cx^n))) dx = \int x \cot^2(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 5.12953, size = 175, normalized size = 1.11

$$\frac{x^2 \left( 2e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{i}{bdn}, 2 - \frac{i}{bdn}, e^{2id(a+b \log(cx^n))}\right) + (bdn - i) \left( 2i \text{Hypergeometric2F1}\left(1, -\frac{i}{bdn}, 1 - \frac{i}{bdn}, e^{2id(a+b \log(cx^n))}\right) \right) \right)}{2bdn(bdn - i)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cot[d\*(a + b\*Log[c\*x^n])]^2,x]

[Out]  $-(x^2*(2*E^((2*I)*d*(a + b*Log[c*x^n]))*\text{Hypergeometric2F1}[1, 1 - I/(b*d*n), 2 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))]) + (-I + b*d*n)*(b*d*n + 2*\text{Cot}[d*(a + b*Log[c*x^n])] + (2*I)*\text{Hypergeometric2F1}[1, (-I)/(b*d*n), 1 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])))/(2*b*d*n*(-I + b*d*n))$

**Maple [F]** time = 1.48, size = 0, normalized size = 0.

$$\int x (\cot(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cot(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(x*cot(d*(a+b*ln(c*x^n)))^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \cot(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(x*cot(b*d*log(c*x^n) + a*d)^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \cot^2(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(d*(a+b*ln(c*x**n)))**2,x)`

[Out] `Integral(x*cot(a*d + b*d*log(c*x**n))**2, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] Timed out

### 3.219 $\int \cot^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=153

$$\frac{2ix \operatorname{Hypergeometric2F1}\left(1, -\frac{i}{2bdn}, 1 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right)}{bdn} + \frac{ix(1 + e^{2iad}(cx^n)^{2ibd})}{bdn(1 - e^{2iad}(cx^n)^{2ibd})} + \frac{x(-bdn + i)}{bdn}$$

[Out] ((I - b\*d\*n)\*x)/(b\*d\*n) + (I\*x\*(1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))/(b\*d\*n\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))) - ((2\*I)\*x\*Hypergeometric2F1[1, (-I/2)/(b\*d\*n), 1 - (I/2)/(b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)])/(b\*d\*n)

**Rubi [F]** time = 0.0139643, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot^2(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])]^2, x]

[Out] Defer[Int][Cot[d\*(a + b\*Log[c\*x^n])]^2, x]

Rubi steps

$$\int \cot^2(d(a + b \log(cx^n))) dx = \int \cot^2(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 9.63724, size = 178, normalized size = 1.16

$$\frac{x \left( e^{2id(a+b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(1, 1 - \frac{i}{2bdn}, 2 - \frac{i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) + (2bdn - i) \left( i \operatorname{Hypergeometric2F1}\left(1, -\frac{i}{2bdn}, 1 - \frac{i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) + (-I + 2*b*d*n)*(b*d*n + \operatorname{Cot}[d*(a + b*\operatorname{Log}[c*x^n])]) + I*\operatorname{Hypergeometric2F1}\left[1, (-I/2)/(b*d*n), 1 - (I/2)/(b*d*n), E^((2*I)*d*(a + b*\operatorname{Log}[c*x^n]))\right]\right) \right)}{bdn(2bdn - i)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])]^2, x]

[Out] -((x\*(E^((2\*I)\*d\*(a + b\*Log[c\*x^n])))\*Hypergeometric2F1[1, 1 - (I/2)/(b\*d\*n), 2 - (I/2)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]) + (-I + 2\*b\*d\*n)\*(b\*d\*n + Cot[d\*(a + b\*Log[c\*x^n])]) + I\*Hypergeometric2F1[1, (-I/2)/(b\*d\*n), 1 - (I/2)/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]))/(b\*d\*n\*(-I + 2\*b\*d\*n))

**Maple [F]** time = 1.293, size = 0, normalized size = 0.

$$\int (\cot(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*(a+b*ln(c*x^n)))^2,x)`

[Out] `int(cot(d*(a+b*ln(c*x^n)))^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\cot(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")`

[Out] `integral(cot(b*d*log(c*x^n) + a*d)^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot^2(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*ln(c*x**n)))**2,x)`

[Out] `Integral(cot(d*(a + b*log(c*x**n)))**2, x)`

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.220 \quad \int \frac{\cot^2(d(a+b \log(cx^n)))}{x} dx$$

**Optimal.** Leaf size=30

$$-\frac{\cot(ad + bd \log(cx^n))}{bdn} - \log(x)$$

[Out] -(Cot[a\*d + b\*d\*Log[c\*x^n]]/(b\*d\*n)) - Log[x]

**Rubi [A]** time = 0.0300002, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3473, 8}

$$-\frac{\cot(ad + bd \log(cx^n))}{bdn} - \log(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])]^2/x,x]

[Out] -(Cot[a\*d + b\*d\*Log[c\*x^n]]/(b\*d\*n)) - Log[x]

#### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cot^2(d(a+b \log(cx^n)))}{x} dx &= \frac{\text{Subst}\left(\int \cot^2(d(a+bx)) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cot(ad + bd \log(cx^n))}{bdn} - \frac{\text{Subst}\left(\int 1 dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cot(ad + bd \log(cx^n))}{bdn} - \log(x) \end{aligned}$$

**Mathematica [C]** time = 0.109699, size = 51, normalized size = 1.7

$$\frac{\cot(ad + bd \log(cx^n)) \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\tan^2(ad + bd \log(cx^n))\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])]^2/x,x]

[Out] -((Cot[a\*d + b\*d\*Log[c\*x^n]]\*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a\*d + b\*d\*Log[c\*x^n]]^2])/(b\*d\*n))

---

**Maple [B]** time = 0.022, size = 63, normalized size = 2.1

$$-\frac{\cot(d(a+b\ln(cx^n)))}{bdn} + \frac{\pi}{2bdn} - \frac{\operatorname{arccot}(\cot(d(a+b\ln(cx^n))))}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*(a+b\*ln(c\*x^n)))^2/x,x)

[Out] -1/b/d/n\*cot(d\*(a+b\*ln(c\*x^n)))+1/2/b/d/n\*Pi-1/b/d/n\*arccot(cot(d\*(a+b\*ln(c\*x^n))))

---

**Maxima [B]** time = 1.22975, size = 435, normalized size = 14.5

$$\frac{(bd \cos(2bd \log(c))^2 + bd \sin(2bd \log(c))^2)n \cos(2bd \log(x^n) + 2ad)^2 \log(x) + (bd \cos(2bd \log(c))^2 + bd \sin(2bd \log(c))^2)n \cos(2bd \log(x^n) + 2ad) - 2bdn \sin(2bd \log(c)) \sin(2bd \log(x^n) + 2ad)}{2bdn \cos(2bd \log(c)) \cos(2bd \log(x^n) + 2ad) - 2bdn \sin(2bd \log(c)) \sin(2bd \log(x^n) + 2ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*(a+b\*log(c\*x^n)))^2/x,x, algorithm="maxima")

[Out] ((b\*d\*cos(2\*b\*d\*log(c))^2 + b\*d\*sin(2\*b\*d\*log(c))^2)\*n\*cos(2\*b\*d\*log(x^n) + 2\*a\*d)^2\*log(x) + (b\*d\*cos(2\*b\*d\*log(c))^2 + b\*d\*sin(2\*b\*d\*log(c))^2)\*n\*log(x)\*sin(2\*b\*d\*log(x^n) + 2\*a\*d)^2 + b\*d\*n\*log(x) - 2\*(b\*d\*n\*cos(2\*b\*d\*log(c))\*log(x) - sin(2\*b\*d\*log(c)))\*cos(2\*b\*d\*log(x^n) + 2\*a\*d) + 2\*(b\*d\*n\*log(x)\*sin(2\*b\*d\*log(c)) + cos(2\*b\*d\*log(c)))\*sin(2\*b\*d\*log(x^n) + 2\*a\*d))/(2\*b\*d\*n\*cos(2\*b\*d\*log(c))\*cos(2\*b\*d\*log(x^n) + 2\*a\*d) - 2\*b\*d\*n\*sin(2\*b\*d\*log(c))\*sin(2\*b\*d\*log(x^n) + 2\*a\*d) - (b\*d\*cos(2\*b\*d\*log(c))^2 + b\*d\*sin(2\*b\*d\*log(c))^2)\*n\*cos(2\*b\*d\*log(x^n) + 2\*a\*d)^2 - (b\*d\*cos(2\*b\*d\*log(c))^2 + b\*d\*sin(2\*b\*d\*log(c))^2)\*n\*sin(2\*b\*d\*log(x^n) + 2\*a\*d)^2 - b\*d\*n)

---

**Fricas [B]** time = 0.490541, size = 216, normalized size = 7.2

$$\frac{bdn \log(x) \sin(2bdn \log(x) + 2bd \log(c) + 2ad) + \cos(2bdn \log(x) + 2bd \log(c) + 2ad) + 1}{bdn \sin(2bdn \log(x) + 2bd \log(c) + 2ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*(a+b\*log(c\*x^n)))^2/x,x, algorithm="fricas")

[Out] -(b\*d\*n\*log(x)\*sin(2\*b\*d\*n\*log(x) + 2\*b\*d\*log(c) + 2\*a\*d) + cos(2\*b\*d\*n\*log(x) + 2\*b\*d\*log(c) + 2\*a\*d) + 1)/(b\*d\*n\*sin(2\*b\*d\*n\*log(x) + 2\*b\*d\*log(c) + 2\*a\*d))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(ad + bd \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] Integral(cot(a*d + b*d*log(c*x**n))**2/x, x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

```
[Out] Timed out
```



$$3.221 \quad \int \frac{\cot^2(d(a+b \log(cx^n)))}{x^2} dx$$

**Optimal.** Leaf size=156

$$\frac{{}_2F_1\left(1, \frac{i}{2bdn}, 1 + \frac{i}{2bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{bdnx} + \frac{i(1 + e^{2iad} (cx^n)^{2ibd})}{bdnx(1 - e^{2iad} (cx^n)^{2ibd})} + \frac{1 + \frac{i}{bdn}}{x}$$

[Out]  $(1 + I/(b*d*n))/x + (I*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*n*x*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*Hypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(b*d*n*x)$

**Rubi [F]** time = 0.0569329, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot^2(d(a+b \log(cx^n)))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])]^2/x^2, x]

[Out] Defer[Int][Cot[d\*(a + b\*Log[c\*x^n])]^2/x^2, x]

Rubi steps

$$\int \frac{\cot^2(d(a+b \log(cx^n)))}{x^2} dx = \int \frac{\cot^2(d(a+b \log(cx^n)))}{x^2} dx$$

**Mathematica [A]** time = 4.36142, size = 181, normalized size = 1.16

$$\frac{e^{2id(a+b \log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 + \frac{i}{2bdn}, 2 + \frac{i}{2bdn}, e^{2id(a+b \log(cx^n))}\right) + (2bdn + i) \left(-i \text{Hypergeometric2F1}\left(1, \frac{I}{2}, 1 + \frac{I}{2}, E^{\frac{I}{2} d(a+b \log(cx^n))}\right)\right)}{bdnx(2bdn + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])]^2/x^2, x]

[Out]  $(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n), 2 + (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (I + 2*b*d*n)*(b*d*n - Cot[d*(a + b*Log[c*x^n])] - I*Hypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))]))/(b*d*n*(I + 2*b*d*n)*x)$

**Maple [F]** time = 1.628, size = 0, normalized size = 0.

$$\int \frac{(\cot(d(a+b \ln(cx^n))))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*(a+b*ln(c*x^n)))^2/x^2,x)`

[Out] `int(cot(d*(a+b*ln(c*x^n)))^2/x^2,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot(bd \log(cx^n) + ad)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")`

[Out] `integral(cot(b*d*log(c*x^n) + a*d)^2/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(ad + bd \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*ln(c*x**n)))**2/x**2,x)`

[Out] `Integral(cot(a*d + b*d*log(c*x**n))**2/x**2, x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")`

[Out] Timed out

$$3.222 \quad \int \frac{\cot^2(d(a+b \log(cx^n)))}{x^3} dx$$

**Optimal.** Leaf size=155

$$\frac{2i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{bdnx^2} + \frac{i(1 + e^{2iad} (cx^n)^{2ibd})}{bdnx^2(1 - e^{2iad} (cx^n)^{2ibd})} + \frac{1 + \frac{2i}{bdn}}{2x^2}$$

[Out] (1 + (2\*I)/(b\*d\*n))/(2\*x^2) + (I\*(1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))/(b\*d\*n\*x^2\*(1 - E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))) - ((2\*I)\*Hypergeometric2F1[1, I/(b\*d\*n), 1 + I/(b\*d\*n), E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)])/(b\*d\*n\*x^2)

**Rubi [F]** time = 0.0533164, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cot^2(d(a+b \log(cx^n)))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])]^2/x^3, x]

[Out] Defer[Int][Cot[d\*(a + b\*Log[c\*x^n])]^2/x^3, x]

Rubi steps

$$\int \frac{\cot^2(d(a+b \log(cx^n)))}{x^3} dx = \int \frac{\cot^2(d(a+b \log(cx^n)))}{x^3} dx$$

**Mathematica [A]** time = 4.02793, size = 175, normalized size = 1.13

$$\frac{2e^{2id(a+b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(1, 1 + \frac{i}{bdn}, 2 + \frac{i}{bdn}, e^{2id(a+b \log(cx^n))}\right) + (bdn + i) \left(-2i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bdn}, 1 + \frac{i}{bdn}, e^{2id(a+b \log(cx^n))}\right)\right)}{2bdnx^2(bdn + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])]^2/x^3, x]

[Out] (2\*E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))\*Hypergeometric2F1[1, 1 + I/(b\*d\*n), 2 + I/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))] + (I + b\*d\*n)\*(b\*d\*n - 2\*Cot[d\*(a + b\*Log[c\*x^n])]) - (2\*I)\*Hypergeometric2F1[1, I/(b\*d\*n), 1 + I/(b\*d\*n), E^((2\*I)\*d\*(a + b\*Log[c\*x^n]))]))/(2\*b\*d\*n\*(I + b\*d\*n)\*x^2)

**Maple [F]** time = 1.863, size = 0, normalized size = 0.

$$\int \frac{(\cot(d(a+b \ln(cx^n))))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(d*(a+b*ln(c*x^n)))^2/x^3,x)`

[Out] `int(cot(d*(a+b*ln(c*x^n)))^2/x^3,x)`

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")`

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot(bd \log(cx^n) + ad)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")`

[Out] `integral(cot(b*d*log(c*x^n) + a*d)^2/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(ad + bd \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*ln(c*x**n)))**2/x**3,x)`

[Out] `Integral(cot(a*d + b*d*log(c*x**n))**2/x**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot((b \log(cx^n) + a)d)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")`

[Out] `integrate(cot((b*log(c*x^n) + a)*d)^2/x^3, x)`

$$3.223 \quad \int \frac{\cot^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=44

$$-\frac{\log(\sin(a+b \log(cx^n)))}{bn} - \frac{\cot^2(a+b \log(cx^n))}{2bn}$$

[Out]  $-\text{Cot}[a + b \cdot \text{Log}[c \cdot x^n]]^2 / (2 \cdot b \cdot n) - \text{Log}[\text{Sin}[a + b \cdot \text{Log}[c \cdot x^n]]] / (b \cdot n)$

**Rubi [A]** time = 0.0365938, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3473, 3475}

$$-\frac{\log(\sin(a+b \log(cx^n)))}{bn} - \frac{\cot^2(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] Int[Cot[a + b\*Log[c\*x^n]]^3/x, x]

[Out]  $-\text{Cot}[a + b \cdot \text{Log}[c \cdot x^n]]^2 / (2 \cdot b \cdot n) - \text{Log}[\text{Sin}[a + b \cdot \text{Log}[c \cdot x^n]]] / (b \cdot n)$

**Rule 3473**

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

**Rule 3475**

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\cot^3(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cot^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cot^2(a+b \log(cx^n))}{2bn} - \frac{\text{Subst}\left(\int \cot(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cot^2(a+b \log(cx^n))}{2bn} - \frac{\log(\sin(a+b \log(cx^n)))}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.223971, size = 52, normalized size = 1.18

$$\frac{2 \log(\tan(a+b \log(cx^n))) + 2 \log(\cos(a+b \log(cx^n))) + \cot^2(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b\*Log[c\*x^n]]^3/x, x]

[Out]  $-(\text{Cot}[a + b \cdot \text{Log}[c \cdot x^n]]^2 + 2 \cdot \text{Log}[\text{Cos}[a + b \cdot \text{Log}[c \cdot x^n]]] + 2 \cdot \text{Log}[\text{Tan}[a + b \cdot \text{Log}[c \cdot x^n]]]) / (2 \cdot b \cdot n)$



a) - (cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*sin(2\*b\*log(x^n) + 2\*a))\*sin(4\*b\*log(x^n) + 4\*a) + 4\*sin(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a))/((b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*cos(4\*b\*log(x^n) + 4\*a)^2 - 4\*b\*n\*cos(2\*b\*log(c))\*cos(2\*b\*log(x^n) + 2\*a) + 4\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*cos(2\*b\*log(x^n) + 2\*a)^2 + (b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*sin(4\*b\*log(x^n) + 4\*a)^2 + 4\*b\*n\*sin(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a) + 4\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*sin(2\*b\*log(x^n) + 2\*a)^2 + b\*n + 2\*(b\*n\*cos(4\*b\*log(c)) - 2\*(b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c))))\*n\*cos(2\*b\*log(x^n) + 2\*a) - 2\*(b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n\*sin(2\*b\*log(x^n) + 2\*a))\*cos(4\*b\*log(x^n) + 4\*a) + 2\*(2\*(b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n\*cos(2\*b\*log(x^n) + 2\*a) - b\*n\*sin(4\*b\*log(c)) - 2\*(b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n\*sin(2\*b\*log(x^n) + 2\*a))\*sin(4\*b\*log(x^n) + 4\*a))

**Fricas [A]** time = 0.49762, size = 212, normalized size = 4.82

$$\frac{(\cos(2bn \log(x) + 2b \log(c) + 2a) - 1) \log\left(-\frac{1}{2} \cos(2bn \log(x) + 2b \log(c) + 2a) + \frac{1}{2}\right) - 2}{2(bn \cos(2bn \log(x) + 2b \log(c) + 2a) - bn)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*log(c\*x^n))^3/x,x, algorithm="fricas")

[Out] -1/2\*((cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) - 1)\*log(-1/2\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) + 1/2) - 2)/(b\*n\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) - b\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*ln(c\*x\*\*n))\*\*3/x,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*log(c\*x^n))^3/x,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.224 \quad \int \frac{\cot^4(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=44

$$-\frac{\cot^3(a+b \log(cx^n))}{3bn} + \frac{\cot(a+b \log(cx^n))}{bn} + \log(x)$$

[Out] Cot[a + b\*Log[c\*x^n]]/(b\*n) - Cot[a + b\*Log[c\*x^n]]^3/(3\*b\*n) + Log[x]

**Rubi [A]** time = 0.038864, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3473, 8}

$$-\frac{\cot^3(a+b \log(cx^n))}{3bn} + \frac{\cot(a+b \log(cx^n))}{bn} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[a + b\*Log[c\*x^n]]^4/x,x]

[Out] Cot[a + b\*Log[c\*x^n]]/(b\*n) - Cot[a + b\*Log[c\*x^n]]^3/(3\*b\*n) + Log[x]

#### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_.)]^(n\_.), x\_Symbol] :> Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cot^4(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cot^4(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cot^3(a+b \log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \cot^2(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\cot(a+b \log(cx^n))}{bn} - \frac{\cot^3(a+b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int 1 dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\cot(a+b \log(cx^n))}{bn} - \frac{\cot^3(a+b \log(cx^n))}{3bn} + \log(x) \end{aligned}$$

**Mathematica [C]** time = 0.115519, size = 46, normalized size = 1.05

$$-\frac{\cot^3(a+b \log(cx^n)) \text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, -\tan^2(a+b \log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b\*Log[c\*x^n]]^4/x,x]



[Out]  $-(\text{Cot}[a + b \cdot \text{Log}[c \cdot x^n]]^3 \text{Hypergeometric2F1}[-3/2, 1, -1/2, -\text{Tan}[a + b \cdot \text{Log}[c \cdot x^n]]^2]) / (3 \cdot b \cdot n)$

**Maple [A]** time = 0.021, size = 69, normalized size = 1.6

$$-\frac{(\cot(a + b \ln(cx^n)))^3}{3bn} + \frac{\cot(a + b \ln(cx^n))}{bn} - \frac{\pi}{2bn} + \frac{\text{arccot}(\cot(a + b \ln(cx^n)))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(a+b*ln(c*x^n))^4/x,x)`

[Out]  $-1/3 \cdot \cot(a + b \cdot \ln(c \cdot x^n))^3 / b \cdot n + \cot(a + b \cdot \ln(c \cdot x^n)) / b \cdot n - 1/2 \cdot n / b \cdot \pi + 1/n \cdot b \cdot \text{arccot}(\cot(a + b \cdot \ln(c \cdot x^n)))$

**Maxima [B]** time = 1.41196, size = 2932, normalized size = 66.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*log(c*x^n))^4/x,x, algorithm="maxima")`

[Out]  $1/3 \cdot (3 \cdot (b \cdot \cos(6 \cdot b \cdot \log(c)))^2 + b \cdot \sin(6 \cdot b \cdot \log(c))^2) \cdot n \cdot \cos(6 \cdot b \cdot \log(x^n) + 6 \cdot a)^2 \cdot \log(x) + 27 \cdot (b \cdot \cos(4 \cdot b \cdot \log(c)))^2 + b \cdot \sin(4 \cdot b \cdot \log(c))^2) \cdot n \cdot \cos(4 \cdot b \cdot \log(x^n) + 4 \cdot a)^2 \cdot \log(x) + 27 \cdot (b \cdot \cos(2 \cdot b \cdot \log(c)))^2 + b \cdot \sin(2 \cdot b \cdot \log(c))^2) \cdot n \cdot \cos(2 \cdot b \cdot \log(x^n) + 2 \cdot a)^2 \cdot \log(x) + 3 \cdot (b \cdot \cos(6 \cdot b \cdot \log(c)))^2 + b \cdot \sin(6 \cdot b \cdot \log(c))^2) \cdot n \cdot \log(x) \cdot \sin(6 \cdot b \cdot \log(x^n) + 6 \cdot a)^2 + 27 \cdot (b \cdot \cos(4 \cdot b \cdot \log(c)))^2 + b \cdot \sin(4 \cdot b \cdot \log(c))^2) \cdot n \cdot \log(x) \cdot \sin(4 \cdot b \cdot \log(x^n) + 4 \cdot a)^2 + 27 \cdot (b \cdot \cos(2 \cdot b \cdot \log(c)))^2 + b \cdot \sin(2 \cdot b \cdot \log(c))^2) \cdot n \cdot \log(x) \cdot \sin(2 \cdot b \cdot \log(x^n) + 2 \cdot a)^2 + 3 \cdot b \cdot n \cdot \log(x) - 2 \cdot (3 \cdot b \cdot n \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \log(x) + 3 \cdot (3 \cdot (b \cdot \cos(6 \cdot b \cdot \log(c))) \cdot \cos(4 \cdot b \cdot \log(c)) + b \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) - 2 \cdot \cos(4 \cdot b \cdot \log(c)) \cdot \sin(6 \cdot b \cdot \log(c))) + 2 \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c))) \cdot \cos(4 \cdot b \cdot \log(x^n) + 4 \cdot a) - 3 \cdot (3 \cdot (b \cdot \cos(6 \cdot b \cdot \log(c))) \cdot \cos(2 \cdot b \cdot \log(c)) + b \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) - 2 \cdot \cos(2 \cdot b \cdot \log(c)) \cdot \sin(6 \cdot b \cdot \log(c)) + 2 \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot \cos(2 \cdot b \cdot \log(x^n) + 2 \cdot a) + 3 \cdot (3 \cdot (b \cdot \cos(4 \cdot b \cdot \log(c))) \cdot \sin(6 \cdot b \cdot \log(c)) - b \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) + 2 \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \cos(4 \cdot b \cdot \log(c)) + 2 \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c))) \cdot \sin(4 \cdot b \cdot \log(x^n) + 4 \cdot a) - 3 \cdot (3 \cdot (b \cdot \cos(2 \cdot b \cdot \log(c))) \cdot \sin(6 \cdot b \cdot \log(c)) - b \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) + 2 \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \cos(2 \cdot b \cdot \log(c)) + 2 \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot \sin(2 \cdot b \cdot \log(x^n) + 2 \cdot a) - 4 \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \cos(6 \cdot b \cdot \log(x^n) + 6 \cdot a) + 6 \cdot (3 \cdot b \cdot n \cdot \cos(4 \cdot b \cdot \log(c)) \cdot \log(x) - 9 \cdot (b \cdot \cos(4 \cdot b \cdot \log(c))) \cdot \cos(2 \cdot b \cdot \log(c)) + b \cdot \sin(4 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot n \cdot \cos(2 \cdot b \cdot \log(x^n) + 2 \cdot a) \cdot \log(x) - 9 \cdot (b \cdot \cos(2 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c)) - b \cdot \cos(4 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) \cdot \sin(2 \cdot b \cdot \log(x^n) + 2 \cdot a) - 2 \cdot \sin(4 \cdot b \cdot \log(c)) \cdot \cos(4 \cdot b \cdot \log(x^n) + 4 \cdot a) - 6 \cdot (3 \cdot b \cdot n \cdot \cos(2 \cdot b \cdot \log(c)) \cdot \log(x) - 2 \cdot \sin(2 \cdot b \cdot \log(c))) \cdot \cos(2 \cdot b \cdot \log(x^n) + 2 \cdot a) + 2 \cdot (3 \cdot b \cdot n \cdot \log(x) \cdot \sin(6 \cdot b \cdot \log(c)) + 3 \cdot (3 \cdot (b \cdot \cos(4 \cdot b \cdot \log(c))) \cdot \sin(6 \cdot b \cdot \log(c)) - b \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) + 2 \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \cos(4 \cdot b \cdot \log(c)) + 2 \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c))) \cdot \cos(4 \cdot b \cdot \log(x^n) + 4 \cdot a) - 3 \cdot (3 \cdot (b \cdot \cos(2 \cdot b \cdot \log(c))) \cdot \sin(6 \cdot b \cdot \log(c)) - b \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) + 2 \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \cos(2 \cdot b \cdot \log(c)) + 2 \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot \cos(2 \cdot b \cdot \log(x^n) + 2 \cdot a) - 3 \cdot (3 \cdot (b \cdot \cos(6 \cdot b \cdot \log(c))) \cdot \cos(4 \cdot b \cdot \log(c)) + b \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) - 2 \cdot \cos(4 \cdot b \cdot \log(c)) \cdot \sin(6 \cdot b \cdot \log(c)) + 2 \cdot \cos(6 \cdot b \cdot \log(c)) \cdot \sin(4 \cdot b \cdot \log(c))) \cdot \sin(4 \cdot b \cdot \log(x^n) + 4 \cdot a) + 3 \cdot (3 \cdot (b \cdot \cos(6 \cdot b \cdot \log(c))) \cdot \cos(2 \cdot b \cdot \log(c)) + b \cdot \sin(6 \cdot b \cdot \log(c)) \cdot \sin(2 \cdot b \cdot \log(c))) \cdot n \cdot \log(x) - 2 \cdot \cos($

$$2*b*\log(c))*\sin(6*b*\log(c)) + 2*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + 4*\cos(6*b*\log(c))*\sin(6*b*\log(x^n) + 6*a) + 6*(9*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n*\cos(2*b*\log(x^n) + 2*a)*\log(x) - 3*b*n*\log(x)*\sin(4*b*\log(c)) - 9*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n*\log(x)*\sin(2*b*\log(x^n) + 2*a) - 2*\cos(4*b*\log(c))*\sin(4*b*\log(x^n) + 4*a) + 6*(3*b*n*\log(x)*\sin(2*b*\log(c)) + 2*\cos(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a))/((b*\cos(6*b*\log(c))^2 + b*\sin(6*b*\log(c))^2)*n*\cos(6*b*\log(x^n) + 6*a)^2 + 9*(b*\cos(4*b*\log(c))^2 + b*\sin(4*b*\log(c))^2)*n*\cos(4*b*\log(x^n) + 4*a)^2 - 6*b*n*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 9*(b*\cos(2*b*\log(c))^2 + b*\sin(2*b*\log(c))^2)*n*\cos(2*b*\log(x^n) + 2*a)^2 + (b*\cos(6*b*\log(c))^2 + b*\sin(6*b*\log(c))^2)*n*\sin(6*b*\log(x^n) + 6*a)^2 + 9*(b*\cos(4*b*\log(c))^2 + b*\sin(4*b*\log(c))^2)*n*\sin(4*b*\log(x^n) + 4*a)^2 + 6*b*n*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 9*(b*\cos(2*b*\log(c))^2 + b*\sin(2*b*\log(c))^2)*n*\sin(2*b*\log(x^n) + 2*a)^2 + b*n - 2*(b*n*\cos(6*b*\log(c)) + 3*(b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n*\cos(4*b*\log(x^n) + 4*a) - 3*(b*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n*\cos(2*b*\log(x^n) + 2*a) + 3*(b*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n*\sin(4*b*\log(x^n) + 4*a) - 3*(b*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n*\sin(2*b*\log(x^n) + 2*a))*\cos(6*b*\log(x^n) + 6*a) + 6*(b*n*\cos(4*b*\log(c)) - 3*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n*\cos(2*b*\log(x^n) + 2*a) - 3*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n*\sin(2*b*\log(x^n) + 2*a))*\cos(4*b*\log(x^n) + 4*a) + 2*(3*(b*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n*\cos(4*b*\log(x^n) + 4*a) - 3*(b*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n*\cos(2*b*\log(x^n) + 2*a) + b*n*\sin(6*b*\log(c)) - 3*(b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n*\sin(4*b*\log(x^n) + 4*a) + 3*(b*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n*\sin(2*b*\log(x^n) + 2*a))*\sin(6*b*\log(x^n) + 6*a) + 6*(3*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n*\cos(2*b*\log(x^n) + 2*a) - b*n*\sin(4*b*\log(c)) - 3*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n*\sin(2*b*\log(x^n) + 2*a))*\sin(4*b*\log(x^n) + 4*a))$$

**Fricas [B]** time = 0.490334, size = 385, normalized size = 8.75

$$\frac{4 \cos(2bn \log(x) + 2b \log(c) + 2a)^2 + 3(bn \cos(2bn \log(x) + 2b \log(c) + 2a) \log(x) - bn \log(x)) \sin(2bn \log(x) + 2b \log(c) + 2a)}{3(bn \cos(2bn \log(x) + 2b \log(c) + 2a) - bn) \sin(2bn \log(x) + 2b \log(c) + 2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*log(c\*x^n))^4/x,x, algorithm="fricas")

[Out]  $\frac{1}{3} * (4 * \cos(2 * b * n * \log(x) + 2 * b * \log(c) + 2 * a)^2 + 3 * (b * n * \cos(2 * b * n * \log(x) + 2 * b * \log(c) + 2 * a) * \log(x) - b * n * \log(x)) * \sin(2 * b * n * \log(x) + 2 * b * \log(c) + 2 * a) + 2 * \cos(2 * b * n * \log(x) + 2 * b * \log(c) + 2 * a) - 2) / ((b * n * \cos(2 * b * n * \log(x) + 2 * b * \log(c) + 2 * a) - b * n) * \sin(2 * b * n * \log(x) + 2 * b * \log(c) + 2 * a))$

**Sympy [A]** time = 15.288, size = 66, normalized size = 1.5

$$\begin{cases} \log(x) \cot^4(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x) \cot^4(a + b \log(c)) & \text{for } n = 0 \\ \log(x) - \frac{\cot^3(a + bn \log(x) + b \log(c))}{3bn} + \frac{\cot(a + bn \log(x) + b \log(c))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Piecewise((log(x)*cot(a)**4, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cot
(a + b*log(c))**4, Eq(n, 0)), (log(x) - cot(a + b*n*log(x) + b*log(c))**3/(
3*b*n) + cot(a + b*n*log(x) + b*log(c))/(b*n), True))
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.225 \quad \int \frac{\cot^5(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=66

$$\frac{\log(\sin(a+b \log(cx^n)))}{bn} - \frac{\cot^4(a+b \log(cx^n))}{4bn} + \frac{\cot^2(a+b \log(cx^n))}{2bn}$$

[Out] Cot[a + b\*Log[c\*x^n]]^2/(2\*b\*n) - Cot[a + b\*Log[c\*x^n]]^4/(4\*b\*n) + Log[Sin[a + b\*Log[c\*x^n]]]/(b\*n)

**Rubi [A]** time = 0.0455594, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3473, 3475}

$$\frac{\log(\sin(a+b \log(cx^n)))}{bn} - \frac{\cot^4(a+b \log(cx^n))}{4bn} + \frac{\cot^2(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] Int[Cot[a + b\*Log[c\*x^n]]^5/x, x]

[Out] Cot[a + b\*Log[c\*x^n]]^2/(2\*b\*n) - Cot[a + b\*Log[c\*x^n]]^4/(4\*b\*n) + Log[Sin[a + b\*Log[c\*x^n]]]/(b\*n)

#### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cot^5(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cot^5(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cot^4(a+b \log(cx^n))}{4bn} - \frac{\text{Subst}\left(\int \cot^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\cot^2(a+b \log(cx^n))}{2bn} - \frac{\cot^4(a+b \log(cx^n))}{4bn} + \frac{\text{Subst}\left(\int \cot(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\cot^2(a+b \log(cx^n))}{2bn} - \frac{\cot^4(a+b \log(cx^n))}{4bn} + \frac{\log(\sin(a+b \log(cx^n)))}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.230863, size = 69, normalized size = 1.05

$$\frac{4 \log(\tan(a+b \log(cx^n))) + 4 \log(\cos(a+b \log(cx^n))) - \cot^4(a+b \log(cx^n)) + 2 \cot^2(a+b \log(cx^n))}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b\*Log[c\*x^n]]^5/x,x]

[Out] (2\*Cot[a + b\*Log[c\*x^n]]^2 - Cot[a + b\*Log[c\*x^n]]^4 + 4\*Log[Cos[a + b\*Log[c\*x^n]]] + 4\*Log[Tan[a + b\*Log[c\*x^n]]])/(4\*b\*n)

**Maple [A]** time = 0.025, size = 68, normalized size = 1.

$$-\frac{(\cot(a + b \ln(cx^n)))^4}{4bn} + \frac{(\cot(a + b \ln(cx^n)))^2}{2bn} - \frac{\ln((\cot(a + b \ln(cx^n)))^2 + 1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+b\*ln(c\*x^n))^5/x,x)

[Out] -1/4\*cot(a+b\*ln(c\*x^n))^4/b/n+1/2\*cot(a+b\*ln(c\*x^n))^2/b/n-1/2/n/b\*ln(cot(a+b\*ln(c\*x^n))^2+1)

**Maxima [B]** time = 1.88761, size = 8097, normalized size = 122.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*log(c\*x^n))^5/x,x, algorithm="maxima")

[Out] 1/2\*(32\*(cos(6\*b\*log(c))^2 + sin(6\*b\*log(c))^2)\*cos(6\*b\*log(x^n) + 6\*a)^2 + 48\*(cos(4\*b\*log(c))^2 + sin(4\*b\*log(c))^2)\*cos(4\*b\*log(x^n) + 4\*a)^2 + 32\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*cos(2\*b\*log(x^n) + 2\*a)^2 + 32\*(cos(6\*b\*log(c))^2 + sin(6\*b\*log(c))^2)\*sin(6\*b\*log(x^n) + 6\*a)^2 + 48\*(cos(4\*b\*log(c))^2 + sin(4\*b\*log(c))^2)\*sin(4\*b\*log(x^n) + 4\*a)^2 + 32\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*sin(2\*b\*log(x^n) + 2\*a)^2 - 8\*((cos(8\*b\*log(c))\*cos(6\*b\*log(c)) + sin(8\*b\*log(c))\*sin(6\*b\*log(c)))\*cos(6\*b\*log(x^n) + 6\*a) - (cos(8\*b\*log(c))\*cos(4\*b\*log(c)) + sin(8\*b\*log(c))\*sin(4\*b\*log(c)))\*cos(4\*b\*log(x^n) + 4\*a) + (cos(8\*b\*log(c))\*cos(2\*b\*log(c)) + sin(8\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(2\*b\*log(x^n) + 2\*a) + (cos(6\*b\*log(c))\*sin(8\*b\*log(c)) - cos(8\*b\*log(c))\*sin(6\*b\*log(c)))\*sin(6\*b\*log(x^n) + 6\*a) - (cos(4\*b\*log(c))\*sin(8\*b\*log(c)) - cos(8\*b\*log(c))\*sin(4\*b\*log(c)))\*sin(4\*b\*log(x^n) + 4\*a) + (cos(2\*b\*log(c))\*sin(8\*b\*log(c)) - cos(8\*b\*log(c))\*sin(2\*b\*log(c)))\*sin(2\*b\*log(x^n) + 2\*a)\*cos(8\*b\*log(x^n) + 8\*a) - 8\*(10\*(cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + sin(6\*b\*log(c))\*sin(4\*b\*log(c)))\*cos(4\*b\*log(x^n) + 4\*a) - 8\*(cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + sin(6\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(2\*b\*log(x^n) + 2\*a) + 10\*(cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - cos(6\*b\*log(c))\*sin(4\*b\*log(c)))\*sin(4\*b\*log(x^n) + 4\*a) - 8\*(cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - cos(6\*b\*log(c))\*sin(2\*b\*log(c)))\*sin(2\*b\*log(x^n) + 2\*a) + cos(6\*b\*log(c))\*cos(6\*b\*log(x^n) + 6\*a) - 8\*(10\*(cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*cos(2\*b\*log(x^n) + 2\*a) + 10\*(cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*sin(2\*b\*log(x^n) + 2\*a) - cos(4\*b\*log(c))\*cos(4\*b\*log(x^n) + 4\*a) - 8\*cos(2\*b\*log(c))\*cos(2\*b\*log(x^n) + 2\*a) + ((cos(8\*b\*log(c))^2 + sin(8\*b\*log(c))^2)\*cos(8\*b\*log(x^n) + 8\*a)^2 + 16\*(cos(6\*b\*log(c))^2 + sin(6\*b\*log(c))^2)\*cos(6\*b\*log(x^n) + 6\*a)^2 + 36\*(cos(4\*b\*log(c))^2 + sin(4\*b\*log(c))^2)\*cos(4\*b\*log(x^n) + 4\*a)^2 + 16\*(cos(2\*b\*log(c))^2 + sin(2\*b\*log(c))^2)\*cos(2\*b\*log(x^n) + 2\*a)^2 + (cos(8\*b\*log(c))^2 + sin(8\*b\*log(c))^2)\*sin(8\*b\*log(x^n) + 8\*a)^2 + 16\*(cos(6\*b\*log(c))^2 + sin(6\*b\*log(c))^2)\*sin(6\*b\*log(x^n) + 6\*a)^2 + 36\*(cos(4\*b\*log

$$\begin{aligned}
& (c)^2 + \sin(4*b*\log(c))^2*\sin(4*b*\log(x^n) + 4*a)^2 + 16*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\sin(2*b*\log(x^n) + 2*a)^2 - 2*(4*(\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) - 6*(\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 4*(\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 4*(\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) - 6*(\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 4*(\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \cos(8*b*\log(c))*\cos(8*b*\log(x^n) + 8*a) - 8*(6*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) - 4*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 6*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - 4*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \cos(6*b*\log(c))*\cos(6*b*\log(x^n) + 6*a) - 12*(4*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 4*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \cos(4*b*\log(c))*\cos(4*b*\log(x^n) + 4*a) - 8*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 2*(4*(\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) - 6*(\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 4*(\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 4*(\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) + 6*(\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - 4*(\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \sin(8*b*\log(c))*\sin(8*b*\log(x^n) + 8*a) + 8*(6*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) - 4*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 6*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 4*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(6*b*\log(c))*\sin(6*b*\log(x^n) + 6*a) + 12*(4*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 4*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \sin(4*b*\log(c))*\sin(4*b*\log(x^n) + 4*a) + 8*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 1)*\log((\cos(a)^2 + \sin(a)^2)*\cos(b*\log(c))^2 + (\cos(a)^2 + \sin(a)^2)*\sin(b*\log(c))^2 + 2*(\cos(b*\log(c))*\cos(a) - \sin(b*\log(c))*\sin(a))*\cos(b*\log(x^n)) + \cos(b*\log(x^n))^2 - 2*(\cos(a)*\sin(b*\log(c)) + \cos(b*\log(c))*\sin(a))*\sin(b*\log(x^n)) + \sin(b*\log(x^n))^2) + ((\cos(8*b*\log(c))^2 + \sin(8*b*\log(c))^2)*\cos(8*b*\log(x^n) + 8*a)^2 + 16*(\cos(6*b*\log(c))^2 + \sin(6*b*\log(c))^2)*\cos(6*b*\log(x^n) + 6*a)^2 + 36*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\cos(4*b*\log(x^n) + 4*a)^2 + 16*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\cos(2*b*\log(x^n) + 2*a)^2 + (\cos(8*b*\log(c))^2 + \sin(8*b*\log(c))^2)*\sin(8*b*\log(x^n) + 8*a)^2 + 16*(\cos(6*b*\log(c))^2 + \sin(6*b*\log(c))^2)*\sin(6*b*\log(x^n) + 6*a)^2 + 36*(\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\sin(4*b*\log(x^n) + 4*a)^2 + 16*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\sin(2*b*\log(x^n) + 2*a)^2 - 2*(4*(\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) - 6*(\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 4*(\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 4*(\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) - 6*(\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 4*(\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \cos(8*b*\log(c))*\cos(8*b*\log(x^n) + 8*a) - 8*(6*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) - 4*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n)
\end{aligned}$$

$n) + 2*a) + 6*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - 4*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \cos(6*b*\log(c))*\cos(6*b*\log(x^n) + 6*a) - 12*(4*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 4*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \cos(4*b*\log(c))*\cos(4*b*\log(x^n) + 4*a) - 8*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 2*(4*(\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) - 6*(\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 4*(\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 4*(\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) + 6*(\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - 4*(\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \sin(8*b*\log(c))*\sin(8*b*\log(x^n) + 8*a) + 8*(6*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) - 4*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 6*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 4*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(6*b*\log(c))*\sin(6*b*\log(x^n) + 6*a) + 12*(4*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 4*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \sin(4*b*\log(c))*\sin(4*b*\log(x^n) + 4*a) + 8*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 1)*\log((\cos(a)^2 + \sin(a)^2)*\cos(b*\log(c))^2 + (\cos(a)^2 + \sin(a)^2)*\sin(b*\log(c))^2 - 2*(\cos(b*\log(c))*\cos(a) - \sin(b*\log(c))*\sin(a))*\cos(b*\log(x^n)) + \cos(b*\log(x^n))^2 + 2*(\cos(a)*\sin(b*\log(c)) + \cos(b*\log(c))*\sin(a))*\sin(b*\log(x^n)) + \sin(b*\log(x^n))^2) + 8*((\cos(6*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(6*b*\log(c)))*\cos(6*b*\log(x^n) + 6*a) - (\cos(4*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + (\cos(2*b*\log(c))*\sin(8*b*\log(c)) - \cos(8*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - (\cos(8*b*\log(c))*\cos(6*b*\log(c)) + \sin(8*b*\log(c))*\sin(6*b*\log(c)))*\sin(6*b*\log(x^n) + 6*a) + (\cos(8*b*\log(c))*\cos(4*b*\log(c)) + \sin(8*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - (\cos(8*b*\log(c))*\cos(2*b*\log(c)) + \sin(8*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a))*\sin(8*b*\log(x^n) + 8*a) + 8*(10*(\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) - 8*(\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 10*(\cos(6*b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 8*(\cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(6*b*\log(c))*\sin(6*b*\log(x^n) + 6*a) + 8*(10*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 10*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \sin(4*b*\log(c))*\sin(4*b*\log(x^n) + 4*a) + 8*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a))/((b*\cos(8*b*\log(c))^2 + b*\sin(8*b*\log(c))^2)*n*\cos(8*b*\log(x^n) + 8*a)^2 + 16*(b*\cos(6*b*\log(c))^2 + b*\sin(6*b*\log(c))^2)*n*\cos(6*b*\log(x^n) + 6*a)^2 + 36*(b*\cos(4*b*\log(c))^2 + b*\sin(4*b*\log(c))^2)*n*\cos(4*b*\log(x^n) + 4*a)^2 - 8*b*n*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 16*(b*\cos(2*b*\log(c))^2 + b*\sin(2*b*\log(c))^2)*n*\cos(2*b*\log(x^n) + 2*a)^2 + (b*\cos(8*b*\log(c))^2 + b*\sin(8*b*\log(c))^2)*n*\sin(8*b*\log(x^n) + 8*a)^2 + 16*(b*\cos(6*b*\log(c))^2 + b*\sin(6*b*\log(c))^2)*n*\sin(6*b*\log(x^n) + 6*a)^2 + 36*(b*\cos(4*b*\log(c))^2 + b*\sin(4*b*\log(c))^2)*n*\sin(4*b*\log(x^n) + 4*a)^2 + 8*b*n*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 16*(b*\cos(2*b*\log(c))^2 + b*\sin(2*b*\log(c))^2)*n*\sin(2*b*\log(x^n) + 2*a)^2 + b*n + 2*(b*n*\cos(8*b*\log(c)) - 4*(b*\cos(8*b*\log(c))*\cos(6*b*\log(c)) + b*\sin(8*b*\log(c))*\sin(6*b*\log(c)))*n*\cos(6*b*\log(x^n) + 6*a) + 6*(b*\cos(8*b*\log(c))*\cos(4*b*\log(c)) + b*\sin(8*b*\log(c))*\sin(4*b*\log(c)))*n*\cos(4*b*\log(x^n) + 4*a) - 4*(b*\cos(8*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(8*b*\log(c))*\sin(2*b*\log(c)))*n*\cos(2*b*\log(x^n) + 2*a) + b*n)$

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g(c))*sin(2*b*log(c))*n*cos(2*b*log(x^n) + 2*a) - 4*(b*cos(6*b*log(c))*sin
(8*b*log(c)) - b*cos(8*b*log(c))*sin(6*b*log(c)))*n*sin(6*b*log(x^n) + 6*a)
+ 6*(b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c))
)*n*sin(4*b*log(x^n) + 4*a) - 4*(b*cos(2*b*log(c))*sin(8*b*log(c)) - b*cos(
8*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*cos(8*b*log(x^n) +
8*a) - 8*(b*n*cos(6*b*log(c)) + 6*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*si
n(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) - 4*(b*cos(6*b*log
(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n
) + 2*a) + 6*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b
*log(c)))*n*sin(4*b*log(x^n) + 4*a) - 4*(b*cos(2*b*log(c))*sin(6*b*log(c))
- b*cos(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*cos(6*b*log
(x^n) + 6*a) + 12*(b*n*cos(4*b*log(c)) - 4*(b*cos(4*b*log(c))*cos(2*b*log(c
)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) - 4*(b*co
s(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*
b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) + 2*(4*(b*cos(6*b*log(c))*sin(8*
b*log(c)) - b*cos(8*b*log(c))*sin(6*b*log(c)))*n*cos(6*b*log(x^n) + 6*a) -
6*(b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)))*n
*cos(4*b*log(x^n) + 4*a) + 4*(b*cos(2*b*log(c))*sin(8*b*log(c)) - b*cos(8*b
*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) - b*n*sin(8*b*log(c)) -
4*(b*cos(8*b*log(c))*cos(6*b*log(c)) + b*sin(8*b*log(c))*sin(6*b*log(c)))*
n*sin(6*b*log(x^n) + 6*a) + 6*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*
b*log(c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) - 4*(b*cos(8*b*log(c))
*cos(2*b*log(c)) + b*sin(8*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) +
2*a))*sin(8*b*log(x^n) + 8*a) + 8*(6*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b
*cos(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) - 4*(b*cos(2*b*
log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(
x^n) + 2*a) + b*n*sin(6*b*log(c)) - 6*(b*cos(6*b*log(c))*cos(4*b*log(c)) +
b*sin(6*b*log(c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) + 4*(b*cos(6*b
*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log
(x^n) + 2*a))*sin(6*b*log(x^n) + 6*a) + 12*(4*(b*cos(2*b*log(c))*sin(4*b*lo
g(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) - b*n*
sin(4*b*log(c)) - 4*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*
sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4*a))

```

---

**Fricas [B]** time = 0.500595, size = 387, normalized size = 5.86

$$\frac{(\cos(2bn \log(x) + 2b \log(c) + 2a)^2 - 2 \cos(2bn \log(x) + 2b \log(c) + 2a) + 1) \log\left(-\frac{1}{2} \cos(2bn \log(x) + 2b \log(c) + 2a) + 1\right)}{2(bn \cos(2bn \log(x) + 2b \log(c) + 2a)^2 - 2bn \cos(2bn \log(x) + 2b \log(c) + 2a) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*log(c\*x^n))^5/x,x, algorithm="fricas")

[Out] 1/2\*((cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a)^2 - 2\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) + 1)\*log(-1/2\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) + 1/2) - 4\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) + 2)/(b\*n\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a)^2 - 2\*b\*n\*cos(2\*b\*n\*log(x) + 2\*b\*log(c) + 2\*a) + b\*n)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(cot(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.226 $\int (ex)^m \cot(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=100

$$\frac{i(ex)^{m+1}}{e(m+1)} - \frac{2i(ex)^{m+1} \text{Hypergeometric2F1}\left(1, -\frac{i(m+1)}{2bdn}, 1 - \frac{i(m+1)}{2bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{e(m+1)}$$

[Out]  $(I*(e*x)^{(1+m)})/(e*(1+m)) - ((2*I)*(e*x)^{(1+m)}*\text{Hypergeometric2F1}[1, (-I/2)*(1+m)/(b*d*n), 1 - ((I/2)*(1+m))/(b*d*n), E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}])/ (e*(1+m))$

**Rubi [F]** time = 0.0477099, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \cot(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out] Defer[Int][(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])], x]

Rubi steps

$$\int (ex)^m \cot(d(a + b \log(cx^n))) dx = \int (ex)^m \cot(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 13.9691, size = 182, normalized size = 1.82

$$\frac{ix(ex)^m \left( \frac{(m+1)e^{2iad}(cx^n)^{2ibd} \text{Hypergeometric2F1}\left(1, -\frac{i(2ibd+n+1)}{2bdn}, -\frac{i(4ibd+n+1)}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right)}{2ibd+n+1} + \text{Hypergeometric2F1}\left(1, -\frac{i(m+1)}{2bdn}, 1 - \frac{i(m+1)}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right) \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])], x]

[Out]  $((-I)*x*(e*x)^m*(\text{Hypergeometric2F1}[1, ((-I/2)*(1+m))/(b*d*n), 1 - ((I/2)*(1+m))/(b*d*n), E^{((2*I)*d*(a + b*Log[c*x^n])}] + (E^{((2*I)*a*d)}*(1+m)*(c*x^n)^{((2*I)*b*d)}*\text{Hypergeometric2F1}[1, ((-I/2)*(1+m + (2*I)*b*d*n))/(b*d*n), ((-I/2)*(1+m + (4*I)*b*d*n))/(b*d*n), E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}])/ (1+m + (2*I)*b*d*n)))/ (1+m)$

**Maple [F]** time = 1.946, size = 0, normalized size = 0.

$$\int (ex)^m \cot(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*cot(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*cot(d*(a+b*ln(c*x^n))),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*cot((b*log(c*x^n) + a)*d), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \cot(bd \log(cx^n) + ad), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*cot(b*d*log(c*x^n) + a*d), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*cot(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*cot(a*d + b*d*log(c*x**n)), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.227 $\int (ex)^m \cot^2 (d(a + b \log (cx^n))) dx$

**Optimal.** Leaf size=195

$$-\frac{2i(ex)^{m+1}\text{Hypergeometric2F1}\left(1, -\frac{i(m+1)}{2bdn}, 1 - \frac{i(m+1)}{2bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{bden} + \frac{i(ex)^{m+1} (1 + e^{2iad} (cx^n)^{2ibd})}{bden (1 - e^{2iad} (cx^n)^{2ibd})} + \frac{(ex)^{m+1}(-bdn + 1)}{bde(m + 1)}$$

```
[Out] ((I*(1 + m) - b*d*n)*(e*x)^(1 + m))/(b*d*e*(1 + m)*n) + (I*(e*x)^(1 + m)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*e*n*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[1, ((-I/2)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(b*d*e*n)
```

**Rubi [F]** time = 0.0784244, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \cot^2 (d(a + b \log (cx^n))) dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] Defer[Int] [(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^2, x]
```

Rubi steps

$$\int (ex)^m \cot^2 (d(a + b \log (cx^n))) dx = \int (ex)^m \cot^2 (d(a + b \log (cx^n))) dx$$

**Mathematica [B]** time = 16.5994, size = 547, normalized size = 2.81

$$(m + 1)x^{-m}(ex)^m \csc (d(a + b(\log (cx^n) - n \log (x)))) \left( \frac{x^{m+1} \sin(bdn \log(x)) \csc(d(a+b \log(cx^n)))}{m+1} - \frac{i \sin(d(a+b(\log(cx^n)-n \log(x)))) \exp(-\dots)}{\dots} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] -((x*(e*x)^m)/(1 + m)) + (x*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n])])*Csc[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]])/(b*d*n) - ((1 + m)*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*((x^(1 + m)*Csc[d*(a + b*Log[c*x^n])]*Sin[b*d*n*Log[x]])/(1 + m) - (I*(I*E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m + (2*I)*b*d*n)*Cot[d*(a + b*Log[c*x^n])] - E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-I/2)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] - E^((a*(1 + 2*m + (2*I)*b*d*n))/(b*n) + (1 + m + (2*I)*b*d*n)*Log[x] + ((1 + 2*m + (2*I)*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, ((-I/2)*(1 + m + (2*I)*b
```

$\frac{d^n}{dx^n} \left( \frac{(-1/2)(1+m+(4I)bd^n)}{bd^n} \right) E^{((2I)d(a+b \log[cx^n]))} \sin[d(a+b(-n \log[x] + \log[cx^n]))] / (E^{((1+2m)(a+b(-n \log[x] + \log[cx^n]))/(bn))(1+m+(2I)bd^n)}) / (bd^n x^m)$

**Maple [F]** time = 1.967, size = 0, normalized size = 0.

$$\int (ex)^m (\cot(d(a+b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*cot(d\*(a+b\*ln(c\*x^n)))^2,x)

[Out] int((e\*x)^m\*cot(d\*(a+b\*ln(c\*x^n)))^2,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*cot(d\*(a+b\*log(c\*x^n)))^2,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \cot(bd \log(cx^n) + ad)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*cot(d\*(a+b\*log(c\*x^n)))^2,x, algorithm="fricas")

[Out] integral((e\*x)^m\*cot(b\*d\*log(c\*x^n) + a\*d)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*cot(d\*(a+b\*ln(c\*x\*\*n)))\*\*2,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.228 $\int (ex)^m \cot^3(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=350

$$\frac{i(ex)^{m+1} \left(-2b^2d^2n^2 + m^2 + 2m + 1\right) \text{Hypergeometric2F1}\left(1, -\frac{i(m+1)}{2bdn}, 1 - \frac{i(m+1)}{2bdn}, e^{2iad} (cx^n)^{2ibd}\right)}{b^2d^2e(m+1)n^2} + \frac{ie^{-2iad}(ex)^{m+1} \left(\frac{e^{4i}}{2b^2}\right)}{2b^2}$$

[Out]  $((I*(1 + m) - b*d*n)*(1 + m + (2*I)*b*d*n)*(e*x)^{(1 + m)})/(2*b^2*d^2*e*(1 + m)*n^2) + ((e*x)^{(1 + m)*(1 + E^((2*I)*a*d)*(c*x^n)^{((2*I)*b*d)})^2})/(2*b*d*e*n*(1 - E^((2*I)*a*d)*(c*x^n)^{((2*I)*b*d)})^2) + ((I/2)*(e*x)^{(1 + m)*((E^((2*I)*a*d)*(1 + m - (2*I)*b*d*n))/n + (E^((4*I)*a*d)*(1 + m + (2*I)*b*d*n)*(c*x^n)^{((2*I)*b*d)})/n)))/(b^2*d^2*e*E^((2*I)*a*d)*n*(1 - E^((2*I)*a*d)*(c*x^n)^{((2*I)*b*d)})) - (I*(1 + 2*m + m^2 - 2*b^2*d^2*n^2)*(e*x)^{(1 + m)*Hypergeometric2F1[1, ((-I/2)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*a*d)*(c*x^n)^{((2*I)*b*d)}])/(b^2*d^2*e*(1 + m)*n^2)$

**Rubi [F]** time = 0.0726727, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \cot^3(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])]^3,x]

[Out] Defer[Int][(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])]^3, x]

Rubi steps

$$\int (ex)^m \cot^3(d(a + b \log(cx^n))) dx = \int (ex)^m \cot^3(d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 17.0389, size = 639, normalized size = 1.83

$$x^{-m}(ex)^m \left(2b^2d^2n^2 - m^2 - 2m - 1\right) \csc(d(a + b(\log(cx^n) - n \log(x)))) \left( \frac{x^{m+1} \sin(bdn \log(x)) \csc(d(a+b \log(cx^n)))}{m+1} - \frac{i \sin(d(a+b \log(cx^n)))}{2b^2d^2n^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])]^3,x]

[Out]  $-((x*(e*x)^m*Cot[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m)) - (x*(e*x)^m*Csc[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) + ((1 + m)*x*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csc[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]])/(2*b^2*d^2*n^2) + ((-1 - 2*m - m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(x^(1 + m)*Csc[d*(a + b*Log[c*x^n]))*Sin[b*d*n*Log[x]])/(1 + m) - (I*(I*E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m + (2*I)*b*d*n)*Cot[d*(a + b*Log[c*x^n])] - E^(($







$$\begin{aligned}
& * \log(x^n) + 4*a*d)^2 + 4*(2*(b^6*d^6*\cos(2*b*d*\log(c))^2 + b^6*d^6*\sin(2*b*d*\log(c))^2)*e^m*n^6 - ((b^4*d^4*\cos(2*b*d*\log(c))^2 + b^4*d^4*\sin(2*b*d*\log(c))^2)*e^m*m^2 + 2*(b^4*d^4*\cos(2*b*d*\log(c))^2 + b^4*d^4*\sin(2*b*d*\log(c))^2)*e^m*m + (b^4*d^4*\cos(2*b*d*\log(c))^2 + b^4*d^4*\sin(2*b*d*\log(c))^2)*e^m)*n^4)*\sin(2*b*d*\log(x^n) + 2*a*d)^2 + 2*(2*b^6*d^6*e^m*n^6*\cos(4*b*d*\log(c)) - (b^4*d^4*e^m*m^2*\cos(4*b*d*\log(c)) + 2*b^4*d^4*e^m*m*\cos(4*b*d*\log(c)) + b^4*d^4*e^m*\cos(4*b*d*\log(c)))*n^4 - 2*(2*(b^6*d^6*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^6*d^6*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*n^6 - ((b^4*d^4*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^4*d^4*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m^2 + 2*(b^4*d^4*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^4*d^4*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m + (b^4*d^4*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^4*d^4*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m)*n^4)*\cos(2*b*d*\log(x^n) + 2*a*d) - 2*(2*(b^6*d^6*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^6*d^6*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*n^6 - ((b^4*d^4*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^4*d^4*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m^2 + 2*(b^4*d^4*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^4*d^4*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m + (b^4*d^4*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^4*d^4*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m)*n^4)*\sin(2*b*d*\log(x^n) + 2*a*d)*\cos(4*b*d*\log(x^n) + 4*a*d) - 4*(2*b^6*d^6*e^m*n^6*\cos(2*b*d*\log(c)) - (b^4*d^4*e^m*m^2*\cos(2*b*d*\log(c)) + 2*b^4*d^4*e^m*m*\cos(2*b*d*\log(c)) + b^4*d^4*e^m*\cos(2*b*d*\log(c)))*n^4)*\cos(2*b*d*\log(x^n) + 2*a*d) - 2*(2*b^6*d^6*e^m*n^6*\sin(4*b*d*\log(c)) - (b^4*d^4*e^m*m^2*\sin(4*b*d*\log(c)) + 2*b^4*d^4*e^m*m*\sin(4*b*d*\log(c)) + b^4*d^4*e^m*\sin(4*b*d*\log(c)))*n^4 - 2*(2*(b^6*d^6*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^6*d^6*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*n^6 - ((b^4*d^4*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^4*d^4*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m^2 + 2*(b^4*d^4*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^4*d^4*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m + (b^4*d^4*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^4*d^4*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m)*n^4)*\cos(2*b*d*\log(x^n) + 2*a*d) + 2*(2*(b^6*d^6*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^6*d^6*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*n^6 - ((b^4*d^4*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^4*d^4*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m^2 + 2*(b^4*d^4*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^4*d^4*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m)*n^4)*\sin(2*b*d*\log(x^n) + 2*a*d) + 4*(2*b^6*d^6*e^m*n^6*\sin(2*b*d*\log(c)) - (b^4*d^4*e^m*m^2*\sin(2*b*d*\log(c)) + 2*b^4*d^4*e^m*m*\sin(2*b*d*\log(c)) + b^4*d^4*e^m*\sin(2*b*d*\log(c)))*n^4)*\sin(2*b*d*\log(x^n) + 2*a*d)*\int(-1/4*(x^m*\cos(b*d*\log(x^n) + a*d)*\sin(b*d*\log(c)) + x^m*\cos(b*d*\log(c))*\sin(b*d*\log(x^n) + a*d))/(2*b^4*d^4*n^4*\cos(b*d*\log(c))*\cos(b*d*\log(x^n) + a*d) - 2*b^4*d^4*n^4*\sin(b*d*\log(c))*\sin(b*d*\log(x^n) + a*d) - b^4*d^4*n^4 - (b^4*d^4*\cos(b*d*\log(c))^2 + b^4*d^4*\sin(b*d*\log(c))^2)*n^4*\cos(b*d*\log(x^n) + a*d)^2 - (b^4*d^4*\cos(b*d*\log(c))^2 + b^4*d^4*\sin(b*d*\log(c))^2)*n^4*\sin(b*d*\log(x^n) + a*d)^2), x) + (((\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + \sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m + 2*(b*d*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b*d*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*n + (\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + \sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m)*x*x^m*\cos(2*b*d*\log(x^n) + 2*a*d) + ((\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - \cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*m - 2*(b*d*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b*d*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m*n + (\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - \cos(4*b*d*\log(c))*\sin(2*b*d*\log(c)))*e^m)*x*x^m*\sin(2*b*d*\log(x^n) + 2*a*d) - (e^m*m*\cos(4*b*d*\log(c)) + e^m*\cos(4*b*d*\log(c)))*x*x^m*\sin(4*b*d*\log(x^n) + 4*a*d))/(4*b^2*d^2*n^2*\cos(2*b*d*\log(c))*\cos(2*b*d*\log(x^n) + 2*a*d) - 4*b^2*d^2*n^2*\sin(2*b*d*\log(c))*\sin(2*b*d*\log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*\cos(4*b*d*\log(c))^2 + b^2*d^2*\sin(4*b*d*\log(c))^2)*n^2*\cos(4*b*d*\log(x^n) + 4*a*d)^2 - 4*(b^2*d^2*\cos(2*b*d*\log(c))^2 + b^2*d^2*\sin(2*b*d*\log(c))^2)*n^2*\cos(2*b*d*\log(x^n) + 2*a*d)^2 - (b^2*d^2*\cos(4*b*d*\log(c))^2 + b^2*d^2*\sin(4*b*d*\log(c))^2)*n^2*\sin(4*b*d*\log(x^n) + 4*a*d)^2 - 4*(b^2*d^2*\cos(2*b*d*\log(c))^2 + b^2*d^2*\sin(2*b*d*\log(c))^2)*n^2*
\end{aligned}$$

$$\begin{aligned} & \sin(2*b*d*\log(x^n) + 2*a*d)^2 - 2*(b^2*d^2*n^2*\cos(4*b*d*\log(c)) - 2*(b^2*d^2*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^2*d^2*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c))) * n^2 * \cos(2*b*d*\log(x^n) + 2*a*d) - 2*(b^2*d^2*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^2*d^2*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c))) * n^2 * \sin(2*b*d*\log(x^n) + 2*a*d) * \cos(4*b*d*\log(x^n) + 4*a*d) + 2*(b^2*d^2*n^2*\sin(4*b*d*\log(c)) - 2*(b^2*d^2*\cos(2*b*d*\log(c))*\sin(4*b*d*\log(c)) - b^2*d^2*\cos(4*b*d*\log(c))*\sin(2*b*d*\log(c))) * n^2 * \cos(2*b*d*\log(x^n) + 2*a*d) + 2*(b^2*d^2*\cos(4*b*d*\log(c))*\cos(2*b*d*\log(c)) + b^2*d^2*\sin(4*b*d*\log(c))*\sin(2*b*d*\log(c))) * n^2 * \sin(2*b*d*\log(x^n) + 2*a*d) * \sin(4*b*d*\log(x^n) + 4*a*d) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \cot(bd \log(cx^n) + ad)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*cot(d\*(a+b\*log(c\*x^n)))^3,x, algorithm="fricas")

[Out] integral((e\*x)^m\*cot(b\*d\*log(c\*x^n) + a\*d)^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*cot(d\*(a+b\*ln(c\*x\*\*n)))\*\*3,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*cot(d\*(a+b\*log(c\*x^n)))^3,x, algorithm="giac")

[Out] Exception raised: TypeError

### 3.229 $\int \cot^p(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=190

$$x(1 - e^{2iad}(cx^n)^{2ibd})^p (1 + e^{2iad}(cx^n)^{2ibd})^{-p} \left( \frac{i(1 + e^{2iad}(cx^n)^{2ibd})}{1 - e^{2iad}(cx^n)^{2ibd}} \right)^p F_1 \left( -\frac{i}{2bdn}; p, -p; 1 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd} \right)$$

[Out]  $(x*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*(((-I)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1[(-I/2)/(b*d*n), p, -p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]/(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]^p$

**Rubi [F]** time = 0.0150207, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \cot^p(d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[Cot[d\*(a + b\*Log[c\*x^n])]^p, x]

[Out] Defer[Int][Cot[d\*(a + b\*Log[c\*x^n])]^p, x]

Rubi steps

$$\int \cot^p(d(a + b \log(cx^n))) dx = \int \cot^p(d(a + b \log(cx^n))) dx$$

**Mathematica [B]** time = 1.30346, size = 458, normalized size = 2.41

$$\frac{x(2bdn - i) \left( \frac{i(1 + e^{2iad}(cx^n)^{2ibd})}{-1 + e^{2iad}(cx^n)^{2ibd}} \right)^p F_1 \left( -\frac{i}{2bdn}; p, -p; 1 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd} \right)}{2bdnpe^{2iad}(cx^n)^{2ibd} F_1 \left( 1 - \frac{i}{2bdn}; p, 1 - p; 2 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd} \right) + 2bdnpe^{2iad}(cx^n)^{2ibd} F_1 \left( 1 - \frac{i}{2bdn}; p + 1, 1 - p; 2 - \frac{i}{2bdn}; e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd} \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cot[d\*(a + b\*Log[c\*x^n])]^p, x]

[Out]  $((-I + 2*b*d*n)*x*((I*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1[(-I/2)/(b*d*n), p, -p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]/(2*b*d*n*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*b*d)*AppellF1[1 - (I/2)/(b*d*n), p, 1 - p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] + 2*b*d*n*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*b*d)*AppellF1[1 - (I/2)/(b*d*n), 1 + p, -p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] + (-I + 2*b*d*n)*AppellF1[(-I/2)/(b*d*n), p, -p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]$

---

**Maple [F]** time = 0.233, size = 0, normalized size = 0.

$$\int (\cot(d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d\*(a+b\*ln(c\*x^n)))^p,x)

[Out] int(cot(d\*(a+b\*ln(c\*x^n)))^p,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="maxima")

[Out] integrate(cot((b\*log(c\*x^n) + a)\*d)^p, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\cot(bd \log(cx^n) + ad)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="fricas")

[Out] integral(cot(b\*d\*log(c\*x^n) + a\*d)^p, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \cot^p(d(a + b \log(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*(a+b\*ln(c\*x\*\*n)))\*\*p,x)

[Out] Integral(cot(d\*(a + b\*log(c\*x\*\*n)))\*\*p, x)

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="giac")

[Out] Exception raised: TypeError

### 3.230 $\int (ex)^m \cot^p (d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=210

$$\frac{(ex)^{m+1} (1 - e^{2iad} (cx^n)^{2ibd})^p (1 + e^{2iad} (cx^n)^{2ibd})^{-p} \left( -\frac{i(1+e^{2iad}(cx^n)^{2ibd})}{1-e^{2iad}(cx^n)^{2ibd}} \right)^p F_1 \left( -\frac{i(m+1)}{2bdn}; p, -p; 1 - \frac{i(m+1)}{2bdn}; e^{2iad} (cx^n)^{2ibd}, -e^{2iad} (cx^n)^{2ibd} \right)}{e(m+1)}$$

[Out]  $((e*x)^{(1+m)}*(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p*(((-I)*(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})))/(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p*AppellF1[(((I/2)*(1+m))/(b*d*n)), p, -p, 1 - ((I/2)*(1+m))/(b*d*n), E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}, -(E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})]/(e*(1+m)*(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p)$

**Rubi [F]** time = 0.10399, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (ex)^m \cot^p (d(a + b \log(cx^n))) dx$$

Verification is Not applicable to the result.

[In] Int[(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out] Defer[Int][(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])]^p,x]

Rubi steps

$$\int (ex)^m \cot^p (d(a + b \log(cx^n))) dx = \int (ex)^m \cot^p (d(a + b \log(cx^n))) dx$$

**Mathematica [A]** time = 1.12706, size = 205, normalized size = 0.98

$$\frac{x(ex)^m (1 - e^{2iad} (cx^n)^{2ibd})^p (1 + e^{2iad} (cx^n)^{2ibd})^{-p} \left( \frac{i(1+e^{2iad}(cx^n)^{2ibd})}{-1+e^{2iad}(cx^n)^{2ibd}} \right)^p F_1 \left( -\frac{i(m+1)}{2bdn}; p, -p; 1 - \frac{i(m+1)}{2bdn}; e^{2iad} (cx^n)^{2ibd}, -e^{2iad} (cx^n)^{2ibd} \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^m\*Cot[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out]  $(x*(e*x)^m*(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p*((I*(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})))/(-1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p*AppellF1[(((I/2)*(1+m))/(b*d*n)), p, -p, 1 - ((I/2)*(1+m))/(b*d*n), E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}, -(E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})]/((1+m)*(1 + E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p)$

**Maple [F]** time = 0.213, size = 0, normalized size = 0.

$$\int (ex)^m (\cot (d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^p,x)`

[Out] `int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \cot((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*cot((b*log(c*x^n) + a)*d)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \cot(bd \log(cx^n) + ad)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")`

[Out] `integral((e*x)^m*cot(b*d*log(c*x^n) + a*d)^p, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*cot(d*(a+b*ln(c*x**n)))**p,x)`

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.231 \quad \int \frac{\cot^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=201

$$\frac{2 \cot^2(a+b \log(cx^n))}{3bn} + \frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} - \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) - 1})}{2\sqrt{2}bn}$$

[Out]  $-(\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n)) + \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n) - (2*\text{Cot}[a + b*\text{Log}[c*x^n]]^{(3/2)})/(3*b*n) + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]/(2*\text{Sqrt}[2]*b*n) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]/(2*\text{Sqrt}[2]*b*n)$

**Rubi [A]** time = 0.139118, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3473, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2 \cot^2(a+b \log(cx^n))}{3bn} + \frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} - \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) - 1})}{2\sqrt{2}bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[a + b*\text{Log}[c*x^n]]^{(5/2)}/x, x]$

[Out]  $-(\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n)) + \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n) - (2*\text{Cot}[a + b*\text{Log}[c*x^n]]^{(3/2)})/(3*b*n) + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]/(2*\text{Sqrt}[2]*b*n) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]/(2*\text{Sqrt}[2]*b*n)$

#### Rule 3473

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n-2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1]

#### Rule 3476

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /;$  FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

#### Rule 329

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 297

$\text{Int}[x^2/((a_*) + (b_*)*(x_*)^4), x\_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*s), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \text{Dist}[1/(2*s), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x]] /;$  FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &



& AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] & EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cot^{\frac{5}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \sqrt{\cot(a + bx)} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \cot(a + b \log(cx^n))\right)}{bn} \\
&= -\frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{2 \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} \\
&= -\frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{\log\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right) - \log\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= -\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n))}{3bn}
\end{aligned}$$

**Mathematica [C]** time = 0.267729, size = 50, normalized size = 0.25

$$\frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n)) \left( \text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a + b \log(cx^n))\right) - 1 \right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b\*Log[c\*x^n]]^(5/2)/x,x]

[Out] (2\*Cot[a + b\*Log[c\*x^n]]^(3/2)\*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[a + b\*Log[c\*x^n]]^2]))/(3\*b\*n)

**Maple [A]** time = 0.062, size = 161, normalized size = 0.8

$$-\frac{2}{3bn} (\cot(a + b \ln(cx^n)))^{\frac{3}{2}} + \frac{\sqrt{2}}{2bn} \arctan\left(1 + \sqrt{2}\sqrt{\cot(a + b \ln(cx^n))}\right) + \frac{\sqrt{2}}{2bn} \arctan\left(-1 + \sqrt{2}\sqrt{\cot(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+b\*ln(c\*x^n))^(5/2)/x,x)

[Out] -2/3\*cot(a+b\*ln(c\*x^n))^(3/2)/b/n+1/2\*arctan(1+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)+1/2\*arctan(-1+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)+1/4/b/n\*2^(1/2)\*ln((1+cot(a+b\*ln(c\*x^n))-2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2))/(1+cot(a+b\*ln(c\*x^n))+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(b \log(cx^n) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(cot(b*log(c*x^n) + a)^(5/2)/x, x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.232 \quad \int \frac{\cot^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=199

$$\frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} + \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn}$$

[Out]  $-(\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n)) + \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n) - (2*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]])/(b*n) - \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]/(2*\text{Sqrt}[2]*b*n) + \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]/(2*\text{Sqrt}[2]*b*n)$

**Rubi [A]** time = 0.131385, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3473, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} + \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cot}[a + b*\text{Log}[c*x^n]]^{(3/2)}/x, x]$

[Out]  $-(\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n)) + \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n) - (2*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]])/(b*n) - \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]/(2*\text{Sqrt}[2]*b*n) + \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]/(2*\text{Sqrt}[2]*b*n)$

#### Rule 3473

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \text{Simp}[(b*(b*\tan[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\tan[c + d*x])^{(n-2)}, x], x] /;$  FreeQ[{b, c, d}, x] && GtQ[n, 1]

#### Rule 3476

$\text{Int}[(b_*)*\tan[(c_*) + (d_*)*(x_*)]^{(n_*)}, x\_Symbol] \rightarrow \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\tan[c + d*x]], x] /;$  FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

#### Rule 329

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 211

$\text{Int}[(a_*) + (b_*)*(x_*)^4)^{(-1)}, x\_Symbol] \rightarrow \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*r), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \text{Dist}[1/(2*r), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x]] /;$  FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&

AtomQ[SplitProduct[SumBaseQ, b]])

#### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \cot^{\frac{3}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\sqrt{\cot(a + b \log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cot(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\sqrt{\cot(a + b \log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \cot(a + b \log(cx^n))\right)}{bn} \\
&= -\frac{2\sqrt{\cot(a + b \log(cx^n))}}{bn} + \frac{2 \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{2\sqrt{\cot(a + b \log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{2\sqrt{\cot(a + b \log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2x+x^2}} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2x+x^2}} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} \\
&= -\frac{2\sqrt{\cot(a + b \log(cx^n))}}{bn} - \frac{\log\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} + \frac{\log\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= -\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{2\sqrt{\cot(a + b \log(cx^n))}}{bn}
\end{aligned}$$

**Mathematica [A]** time = 0.291255, size = 175, normalized size = 0.88

$$\frac{\sqrt{2} \log(\cot(a + b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + 1) - \sqrt{2} \log(\cot(a + b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))})}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b\*Log[c\*x^n]]^(3/2)/x,x]

[Out]  $-(2*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]) - 2*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]) + 8*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]] + \text{Sqrt}[2]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]] - \text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]] + \text{Cot}[a + b*\text{Log}[c*x^n]])/(4*b*n)$

**Maple [A]** time = 0.034, size = 161, normalized size = 0.8

$$-2 \frac{\sqrt{\cot(a + b \ln(cx^n))}}{bn} + \frac{\sqrt{2}}{2bn} \arctan\left(-1 + \sqrt{2}\sqrt{\cot(a + b \ln(cx^n))}\right) + \frac{\sqrt{2}}{4bn} \ln\left(\left(1 + \cot(a + b \ln(cx^n)) + \sqrt{2}\sqrt{\cot(a + b \ln(cx^n))}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a+b\*ln(c\*x^n))^(3/2)/x,x)

[Out]  $-2*\cot(a+b*\ln(c*x^n))^{(1/2)}/b/n+1/2*\arctan(-1+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/b/n*2^{(1/2)}+1/4/b/n*2^{(1/2)}*\ln((1+\cot(a+b*\ln(c*x^n))+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/(1+\cot(a+b*\ln(c*x^n))-2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)}))+1/2*\arctan(1+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/b/n*2^{(1/2)}$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate(cot(b\*log(c\*x^n) + a)^(3/2)/x, x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*ln(c\*x\*\*n))\*\*(3/2)/x,x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="giac")

[Out] Timed out

$$3.233 \quad \int \frac{\sqrt{\cot(a+b \log(cx^n))}}{x} dx$$

**Optimal.** Leaf size=176

$$\frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2bn}} + \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2bn}}$$

[Out] ArcTan[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]]/(Sqrt[2]\*b\*n) - ArcTan[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]]/(Sqrt[2]\*b\*n) - Log[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]] + Cot[a + b\*Log[c\*x^n]]]/(2\*Sqrt[2]\*b\*n) + Log[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]] + Cot[a + b\*Log[c\*x^n]]]/(2\*Sqrt[2]\*b\*n)

**Rubi [A]** time = 0.120996, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2bn}} + \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2bn}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cot[a + b\*Log[c\*x^n]]]/x,x]

[Out] ArcTan[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]]/(Sqrt[2]\*b\*n) - ArcTan[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]]/(Sqrt[2]\*b\*n) - Log[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]] + Cot[a + b\*Log[c\*x^n]]]/(2\*Sqrt[2]\*b\*n) + Log[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]] + Cot[a + b\*Log[c\*x^n]]]/(2\*Sqrt[2]\*b\*n)

#### Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r + s\*x^2)/(a + b\*x^4), x], x] - Dist[1/(2\*s), Int[(r - s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]



Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cot(a + b \log(cx^n))}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{\cot(a + bx)} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \cot(a + b \log(cx^n))\right)}{bn} \\
&= -\frac{2 \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} \\
&= -\frac{\log\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} + \frac{\log\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\log\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn}
\end{aligned}$$

**Mathematica [C]** time = 0.0984833, size = 48, normalized size = 0.27

$$\frac{2 \cot^{\frac{3}{2}}(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, -\cot^2(a + b \log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[a + b*Log[c*x^n]]]/x, x]
```

[Out]  $(-2*\cot[a + b*\log[c*x^n]]^{(3/2)}*Hypergeometric2F1[3/4, 1, 7/4, -\cot[a + b*\log[c*x^n]]^2])/(3*b*n)$

**Maple [A]** time = 0.032, size = 140, normalized size = 0.8

$$-\frac{\sqrt{2}}{4bn} \ln \left( \left( 1 + \cot(a + b \ln(cx^n)) - \sqrt{2} \sqrt{\cot(a + b \ln(cx^n))} \right) \left( 1 + \cot(a + b \ln(cx^n)) + \sqrt{2} \sqrt{\cot(a + b \ln(cx^n))} \right)^{-1} \right) - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(a+b*ln(c*x^n))^(1/2)/x,x)`

[Out]  $-1/4/b/n*2^{(1/2)}*\ln((1+\cot(a+b*\ln(c*x^n))-2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/(1+\cot(a+b*\ln(c*x^n))+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)}))-1/2*\arctan(1+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/b/n*2^{(1/2)}-1/2*\arctan(-1+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/b/n*2^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cot(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt(cot(b*log(c*x^n) + a))/x, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cot(a + b \log(cx^n))}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*ln(c*x**n))**(1/2)/x,x)`

[Out] `Integral(sqrt(cot(a + b*log(c*x**n)))/x, x)`

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")`

[Out] Timed out

$$3.234 \quad \int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}} dx$$

**Optimal.** Leaf size=176

$$\frac{\log(\cot(a+b\log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b\log(cx^n)) + 1})}{2\sqrt{2}bn} - \frac{\log(\cot(a+b\log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b\log(cx^n)) + 1})}{2\sqrt{2}bn} +$$

[Out] ArcTan[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]/(Sqrt[2]\*b\*n) - ArcTan[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]/(Sqrt[2]\*b\*n) + Log[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]] + Cot[a + b\*Log[c\*x^n]]]/(2\*Sqrt[2]\*b\*n) - Log[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]] + Cot[a + b\*Log[c\*x^n]]]/(2\*Sqrt[2]\*b\*n)

**Rubi [A]** time = 0.127702, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log(\cot(a+b\log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b\log(cx^n)) + 1})}{2\sqrt{2}bn} - \frac{\log(\cot(a+b\log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b\log(cx^n)) + 1})}{2\sqrt{2}bn} +$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[Cot[a + b\*Log[c\*x^n]]]),x]

[Out] ArcTan[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]/(Sqrt[2]\*b\*n) - ArcTan[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]/(Sqrt[2]\*b\*n) + Log[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]] + Cot[a + b\*Log[c\*x^n]]]/(2\*Sqrt[2]\*b\*n) - Log[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]] + Cot[a + b\*Log[c\*x^n]]]/(2\*Sqrt[2]\*b\*n)

#### Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := S  
 imp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,  
 e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[  
 (2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e  
 /(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &  
 & EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*S  
 imply[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b  
 ], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c))] /; Free  
 Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[  
 -a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[  
 a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cot(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \cot(a+b\log(cx^n))\right)}{bn} \\ &= \frac{2 \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} \\ &= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{bn} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{2bn} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a+b\log(cx^n))}\right)}{2bn} \\ &= \frac{\log\left(1 - \sqrt{2}\sqrt{\cot(a+b\log(cx^n))} + \cot(a+b\log(cx^n))\right)}{2\sqrt{2}bn} - \frac{\log\left(1 + \sqrt{2}\sqrt{\cot(a+b\log(cx^n))} + \cot(a+b\log(cx^n))\right)}{2\sqrt{2}bn} \\ &= \frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\log\left(\cot(a+b\log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b\log(cx^n))} + 1\right) - \log\left(\cot(a+b\log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b\log(cx^n))} + 1\right)}{2\sqrt{2}bn} \end{aligned}$$

**Mathematica [A]** time = 0.155726, size = 142, normalized size = 0.81

$$\frac{\log\left(\cot(a+b\log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b\log(cx^n))} + 1\right) - \log\left(\cot(a+b\log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b\log(cx^n))} + 1\right)}{2\sqrt{2}bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[Cot[a + b\*Log[c\*x^n]]]), x]

[Out]  $(2 \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{\cot[a + b \operatorname{Log}[c x^n]]}] - 2 \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{\cot[a + b \operatorname{Log}[c x^n]]}] + \operatorname{Log}[1 - \sqrt{2} \sqrt{\cot[a + b \operatorname{Log}[c x^n]]}] + \operatorname{Cot}[a + b \operatorname{Log}[c x^n]] - \operatorname{Log}[1 + \sqrt{2} \sqrt{\cot[a + b \operatorname{Log}[c x^n]]}] + \operatorname{Cot}[a + b \operatorname{Log}[c x^n]]) / (2 \sqrt{2} b n)$

**Maple [A]** time = 0.035, size = 140, normalized size = 0.8

$$-\frac{\sqrt{2}}{4bn} \ln \left( \left( 1 + \cot(a + b \ln(cx^n)) + \sqrt{2} \sqrt{\cot(a + b \ln(cx^n))} \right) \left( 1 + \cot(a + b \ln(cx^n)) - \sqrt{2} \sqrt{\cot(a + b \ln(cx^n))} \right)^{-1} \right) - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/cot(a+b*ln(c*x^n))^(1/2),x)`

[Out]  $-1/4/b/n*2^{(1/2)}*\ln((1+\cot(a+b*\ln(c*x^n))+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/(1+\cot(a+b*\ln(c*x^n))-2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)}))-1/2*\arctan(1+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/b/n*2^{(1/2)}-1/2*\arctan(-1+2^{(1/2)}*\cot(a+b*\ln(c*x^n))^{(1/2)})/b/n*2^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt{\cot(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(x*sqrt(cot(b*log(c*x^n) + a))), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt{\cot(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/cot(a+b*ln(c*x**n))**(1/2),x)`

[Out]  $\text{Integral}(1/(x\sqrt{\cot(a + b\log(c*x**n))}), x)$

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="giac")`

[Out] Timed out

$$3.235 \quad \int \frac{1}{x \cot^2(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=199

$$\frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} - \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} +$$

[Out]  $-(\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n)) + \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n) + 2/(b*n*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]) + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]]/(2*\text{Sqrt}[2]*b*n) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]]/(2*\text{Sqrt}[2]*b*n)$

**Rubi [A]** time = 0.131516, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3474, 3476, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} - \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} +$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{Cot}[a + b*\text{Log}[c*x^n]]^{(3/2)}), x]$

[Out]  $-(\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n)) + \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]]]/(\text{Sqrt}[2]*b*n) + 2/(b*n*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]) + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]]/(2*\text{Sqrt}[2]*b*n) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[\text{Cot}[a + b*\text{Log}[c*x^n]]] + \text{Cot}[a + b*\text{Log}[c*x^n]]]/(2*\text{Sqrt}[2]*b*n)$

#### Rule 3474

$\text{Int}(((b\_)*\text{tan}[(c\_)+(d\_)*(x\_)]^{(n\_)}, x\_Symbol] := \text{Simp}[(b*\text{Tan}[c + d*x])^{(n+1)}/(b*d*(n+1)), x] - \text{Dist}[1/b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n+2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{LtQ}[n, -1]$

#### Rule 3476

$\text{Int}(((b\_)*\text{tan}[(c\_)+(d\_)*(x\_)]^{(n\_)}, x\_Symbol] := \text{Dist}[b/d, \text{Subst}[\text{Int}[x^n/(b^2 + x^2), x], x, b*\text{Tan}[c + d*x]], x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\& \text{IntegerQ}[n]$

#### Rule 329

$\text{Int}(((c\_)*(x\_))^{(m\_)*((a\_)+(b\_)*(x\_))^{(n\_)}^{(p\_)}, x\_Symbol] := \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)*(a+(b*x^{(k*n)})/c^n)^p}, x], x, (c*x)^{(1/k)}], x]] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 297

$\text{Int}[(x_)^2/((a_)+(b_)*(x_)^4), x\_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*s), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \text{Dist}[1/(2*s), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x]] /; \text{FreeQ}\{a, b\}, x] \&\& (\text{GtQ}[a/b, 0] || (\text{PosQ}[a/b] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \&$



& AtomQ[SplitProduct[SumBaseQ, b]])

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] & EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x \cot^{\frac{3}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cot^{\frac{3}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2}{bn\sqrt{\cot(a + b \log(cx^n))}} - \frac{\text{Subst}\left(\int \sqrt{\cot(a + bx)} dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2}{bn\sqrt{\cot(a + b \log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2} dx, x, \cot(a + b \log(cx^n))\right)}{bn} \\
&= \frac{2}{bn\sqrt{\cot(a + b \log(cx^n))}} + \frac{2 \text{Subst}\left(\int \frac{x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{2}{bn\sqrt{\cot(a + b \log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{2}{bn\sqrt{\cot(a + b \log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} \\
&= \frac{2}{bn\sqrt{\cot(a + b \log(cx^n))}} + \frac{\log\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} - \frac{\log\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= -\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{1}{bn\sqrt{\cot(a + b \log(cx^n))}}
\end{aligned}$$

**Mathematica [C]** time = 0.144616, size = 46, normalized size = 0.23

$$\frac{2\text{Hypergeometric2F1}\left(-\frac{1}{4}, 1, \frac{3}{4}, -\cot^2(a + b \log(cx^n))\right)}{bn\sqrt{\cot(a + b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Cot[a + b\*Log[c\*x^n]]^(3/2)),x]

[Out] (2\*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[a + b\*Log[c\*x^n]]^2])/(b\*n\*Sqrt[Cot[a + b\*Log[c\*x^n]]])

**Maple [A]** time = 0.038, size = 161, normalized size = 0.8

$$\frac{\sqrt{2}}{2bn} \arctan\left(1 + \sqrt{2}\sqrt{\cot(a + b \ln(cx^n))}\right) + \frac{\sqrt{2}}{2bn} \arctan\left(-1 + \sqrt{2}\sqrt{\cot(a + b \ln(cx^n))}\right) + \frac{\sqrt{2}}{4bn} \ln\left(\left(1 + \cot(a + b \ln(cx^n))\right)^2 - 2\sqrt{\cot(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/cot(a+b\*ln(c\*x^n))^(3/2),x)

[Out] 1/2\*arctan(1+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)+1/2\*arctan(-1+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)+1/4/b/n\*2^(1/2)\*ln((1+cot(a+b\*ln(c\*x^n))-2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2))/(1+cot(a+b\*ln(c\*x^n))+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2)))+2/b/n/cot(a+b\*ln(c\*x^n))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \cot(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cot(a+b\*log(c\*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x\*cot(b\*log(c\*x^n) + a)^(3/2)), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cot(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cot(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cot(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] Timed out

$$3.236 \quad \int \frac{1}{x \cot^2(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=201

$$\frac{2}{3bn \cot^2(a+b \log(cx^n))} - \frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} + \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn}$$

[Out] -(ArcTan[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]/(Sqrt[2]\*b\*n)) + ArcTan[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]/(Sqrt[2]\*b\*n) + 2/(3\*b\*n\*Cot[a + b\*Log[c\*x^n]]^(3/2)) - Log[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]] + Cot[a + b\*Log[c\*x^n]]/(2\*Sqrt[2]\*b\*n) + Log[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]] + Cot[a + b\*Log[c\*x^n]]/(2\*Sqrt[2]\*b\*n)

**Rubi [A]** time = 0.134183, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {3474, 3476, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2}{3bn \cot^2(a+b \log(cx^n))} - \frac{\log(\cot(a+b \log(cx^n)) - \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn} + \frac{\log(\cot(a+b \log(cx^n)) + \sqrt{2}\sqrt{\cot(a+b \log(cx^n)) + 1})}{2\sqrt{2}bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Cot[a + b\*Log[c\*x^n]]^(5/2)),x]

[Out] -(ArcTan[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]/(Sqrt[2]\*b\*n)) + ArcTan[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]]/(Sqrt[2]\*b\*n) + 2/(3\*b\*n\*Cot[a + b\*Log[c\*x^n]]^(3/2)) - Log[1 - Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]] + Cot[a + b\*Log[c\*x^n]]/(2\*Sqrt[2]\*b\*n) + Log[1 + Sqrt[2]\*Sqrt[Cot[a + b\*Log[c\*x^n]]] + Cot[a + b\*Log[c\*x^n]]/(2\*Sqrt[2]\*b\*n)

#### Rule 3474

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*Tan[c + d\*x])^(n + 1)/(b\*d\*(n + 1)), x] - Dist[1/b^2, Int[(b\*Tan[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]

#### Rule 3476

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[b/d, Subst[Int[x^n/(b^2 + x^2), x], x, b\*Tan[c + d\*x]], x] /; FreeQ[{b, c, d, n}, x] && ! IntegerQ[n]

#### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b

}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

#### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

#### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

#### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

#### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x \cot^{\frac{5}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\cot^{\frac{5}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2}{3bn \cot^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\cot(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2}{3bn \cot^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x(1+x^2)}} dx, x, \cot(a + b \log(cx^n))\right)}{bn} \\
&= \frac{2}{3bn \cot^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{2 \text{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{2}{3bn \cot^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{bn} \\
&= \frac{2}{3bn \cot^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{\cot(a + b \log(cx^n))}\right)}{2bn} \\
&= \frac{2}{3bn \cot^{\frac{3}{2}}(a + b \log(cx^n))} - \frac{\log\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} + \frac{\log\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{2\sqrt{2}bn} \\
&= -\frac{\tan^{-1}\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\tan^{-1}\left(1 + \sqrt{2}\sqrt{\cot(a + b \log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\log\left(1 - \sqrt{2}\sqrt{\cot(a + b \log(cx^n))} + \cot(a + b \log(cx^n))\right)}{3bn \cot^{\frac{3}{2}}(a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [C]** time = 0.215727, size = 48, normalized size = 0.24

$$\frac{2 \text{Hypergeometric2F1}\left(-\frac{3}{4}, 1, \frac{1}{4}, -\cot^2(a + b \log(cx^n))\right)}{3bn \cot^{\frac{3}{2}}(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Cot[a + b\*Log[c\*x^n]]^(5/2)),x]

[Out] (2\*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[a + b\*Log[c\*x^n]]^2]/(3\*b\*n\*Cot[a + b\*Log[c\*x^n]]^(3/2)))

**Maple [A]** time = 0.038, size = 161, normalized size = 0.8

$$\frac{2}{3bn} (\cot(a + b \ln(cx^n)))^{-\frac{3}{2}} + \frac{\sqrt{2}}{2bn} \arctan\left(1 + \sqrt{2}\sqrt{\cot(a + b \ln(cx^n))}\right) + \frac{\sqrt{2}}{2bn} \arctan\left(-1 + \sqrt{2}\sqrt{\cot(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/cot(a+b\*ln(c\*x^n))^(5/2),x)

[Out] 2/3/b/n/cot(a+b\*ln(c\*x^n))^(3/2)+1/2\*arctan(1+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)+1/2\*arctan(-1+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2))/b/n\*2^(1/2)+1/4/b/n\*2^(1/2)\*ln((1+cot(a+b\*ln(c\*x^n))+2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2)))

)/(1+cot(a+b\*ln(c\*x^n))-2^(1/2)\*cot(a+b\*ln(c\*x^n))^(1/2)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \cot(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cot(a+b\*log(c\*x^n))^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x\*cot(b\*log(c\*x^n) + a)^(5/2)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cot(a+b\*log(c\*x^n))^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cot(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/cot(a+b\*log(c\*x^n))^(5/2),x, algorithm="giac")

[Out] Timed out

### 3.237 $\int x^2 \sec(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=87

$$\frac{2e^{ia}x^3 (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{3i}{bn}\right), \frac{3}{2}\left(1 - \frac{i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{3 + ibn}$$

[Out] (2\*E^(I\*a))\*x^3\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, (1 - (3\*I)/(b\*n))/2, (3\*(1 - I/(b\*n)))/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(3 + I\*b\*n)

**Rubi [A]** time = 0.0594096, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4509, 4505, 364}

$$\frac{2e^{ia}x^3 (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{3i}{bn}\right); \frac{3}{2}\left(1 - \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{3 + ibn}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sec[a + b\*Log[c\*x^n]],x]

[Out] (2\*E^(I\*a))\*x^3\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, (1 - (3\*I)/(b\*n))/2, (3\*(1 - I/(b\*n)))/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(3 + I\*b\*n)

#### Rule 4509

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x^2 \sec(a + b \log(cx^n)) dx &= \frac{(x^3 (cx^n)^{-3/n}) \operatorname{Subst}\left(\int x^{-1+\frac{3}{n}} \sec(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(2e^{ia}x^3 (cx^n)^{-3/n}) \operatorname{Subst}\left(\int \frac{x^{-1+ib+\frac{3}{n}}}{1+e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n} \\ &= \frac{2e^{ia}x^3 (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{3i}{bn}\right); \frac{3}{2}\left(1 - \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{3 + ibn} \end{aligned}$$



**Mathematica [A]** time = 0.167949, size = 86, normalized size = 0.99

$$\frac{2ie^{ia}x^3(cx^n)^{ib}\operatorname{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{3i}{2bn}, \frac{3}{2} - \frac{3i}{2bn}, -e^{2i(a+b\log(cx^n))}\right)}{bn - 3i}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sec[a + b\*Log[c\*x^n]], x]

[Out]  $((-2*I)*E^{I*a}*x^3*(c*x^n)^{I*b}*\operatorname{Hypergeometric2F1}[1, 1/2 - ((3*I)/2)/(b*n), 3/2 - ((3*I)/2)/(b*n), -E^{((2*I)*(a + b*\operatorname{Log}[c*x^n])]}])/(-3*I + b*n)$

**Maple [F]** time = 0.275, size = 0, normalized size = 0.

$$\int x^2 \sec(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sec(a+b\*ln(c\*x^n)), x)

[Out] int(x^2\*sec(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sec(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(x^2\*sec(b\*log(c\*x^n) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^2 \sec(b \log(cx^n) + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sec(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(x^2\*sec(b\*log(c\*x^n) + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sec(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(x**2*sec(a + b*log(c*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(x^2*sec(b*log(c*x^n) + a), x)
```

### 3.238 $\int x \sec(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=87

$$\frac{2e^{ia}x^2 (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{2i}{bn}\right), \frac{1}{2}\left(3 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{2 + ibn}$$

[Out]  $(2E^{(I*a)}*x^2*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 - (2*I)/(b*n))/2, (3 - (2*I)/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(2 + I*b*n)$

**Rubi [A]** time = 0.0517846, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4509, 4505, 364}

$$\frac{2e^{ia}x^2 (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{2i}{bn}\right); \frac{1}{2}\left(3 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{2 + ibn}$$

Antiderivative was successfully verified.

[In] Int[x\*Sec[a + b\*Log[c\*x^n]], x]

[Out]  $(2E^{(I*a)}*x^2*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 - (2*I)/(b*n))/2, (3 - (2*I)/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(2 + I*b*n)$

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x \sec(a + b \log(cx^n)) dx &= \frac{(x^2 (cx^n)^{-2/n}) \operatorname{Subst}\left(\int x^{-1+\frac{2}{n}} \sec(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(2e^{ia}x^2 (cx^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+ib+\frac{2}{n}}}{1+e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n} \\ &= \frac{2e^{ia}x^2 (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{2i}{bn}\right); \frac{1}{2}\left(3 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{2 + ibn} \end{aligned}$$

**Mathematica [A]** time = 0.135544, size = 82, normalized size = 0.94

$$\frac{2ie^{ia}x^2(cx^n)^{ib}\text{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{i}{bn}, \frac{3}{2} - \frac{i}{bn}, -e^{2i(a+b\log(cx^n))}\right)}{bn - 2i}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sec[a + b\*Log[c\*x^n]], x]

[Out]  $((-2I)*E^{(I*a)}*x^2*(c*x^n)^{(I*b)}*\text{Hypergeometric2F1}[1, 1/2 - I/(b*n), 3/2 - I/(b*n), -E^{((2*I)*(a + b*\text{Log}[c*x^n]))}])/(-2*I + b*n)$

**Maple [F]** time = 0.228, size = 0, normalized size = 0.

$$\int x \sec(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sec(a+b\*ln(c\*x^n)), x)

[Out] int(x\*sec(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(x\*sec(b\*log(c\*x^n) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \sec(b \log(cx^n) + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(x\*sec(b\*log(c\*x^n) + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(x*sec(a + b*log(c*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*log(c*x^n) + a), x)
```

### 3.239 $\int \sec(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=85

$$\frac{2e^{ia}x(cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{i}{bn}\right), \frac{1}{2}\left(3 - \frac{i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{1 + ibn}$$

[Out] (2\*E^(I\*a)\*x\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, (1 - I/(b\*n))/2, (3 - I/(b\*n))/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))])/(1 + I\*b\*n)

**Rubi [A]** time = 0.0500636, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {4503, 4505, 364}

$$\frac{2e^{ia}x(cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i}{bn}\right); \frac{1}{2}\left(3 - \frac{i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{1 + ibn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]], x]

[Out] (2\*E^(I\*a)\*x\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, (1 - I/(b\*n))/2, (3 - I/(b\*n))/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))])/(1 + I\*b\*n)

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sec(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sec(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(2e^{ia}x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+ib+\frac{1}{n}}}{1+e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n} \\ &= \frac{2e^{ia}x(cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i}{bn}\right); \frac{1}{2}\left(3 - \frac{i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{1 + ibn} \end{aligned}$$

**Mathematica [A]** time = 0.112316, size = 84, normalized size = 0.99

$$\frac{2ie^{ia}x(cx^n)^{ib}\text{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{i}{2bn'} - \frac{3}{2} - \frac{i}{2bn'}, -e^{2i(a+b\log(cx^n))}\right)}{bn - i}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]], x]

[Out]  $((-2*I)*E^{(I*a)}*x*(c*x^n)^{(I*b)}*\text{Hypergeometric2F1}[1, 1/2 - (I/2)/(b*n), 3/2 - (I/2)/(b*n), -E^{((2*I)*(a + b*\text{Log}[c*x^n])]})))/(-I + b*n)$

**Maple [F]** time = 0.208, size = 0, normalized size = 0.

$$\int \sec(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n)), x)

[Out] int(sec(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(sec(b\*log(c\*x^n) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sec(b \log(cx^n) + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(sec(a + b*log(c*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a), x)
```



$$3.240 \quad \int \frac{\sec(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=19

$$\frac{\tanh^{-1}(\sin(a+b \log(cx^n)))}{bn}$$

[Out] ArcTanh[Sin[a + b\*Log[c\*x^n]]]/(b\*n)

**Rubi [A]** time = 0.0163493, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {3770}

$$\frac{\tanh^{-1}(\sin(a+b \log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]/x,x]

[Out] ArcTanh[Sin[a + b\*Log[c\*x^n]]]/(b\*n)

**Rule 3770**

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\sec(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sec(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\tanh^{-1}(\sin(a+b \log(cx^n)))}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.040198, size = 19, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a+b \log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]/x,x]

[Out] ArcTanh[Sin[a + b\*Log[c\*x^n]]]/(b\*n)

**Maple [A]** time = 0.022, size = 32, normalized size = 1.7

$$\frac{\ln(\sec(a+b \ln(cx^n)) + \tan(a+b \ln(cx^n)))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(a+b*ln(c*x^n))/x,x)`

[Out] `1/n/b*ln(sec(a+b*ln(c*x^n))+tan(a+b*ln(c*x^n)))`

**Maxima [A]** time = 1.07383, size = 42, normalized size = 2.21

$$\frac{\log(\sec(b \log(cx^n) + a) + \tan(b \log(cx^n) + a))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(a+b*log(c*x^n))/x,x, algorithm="maxima")`

[Out] `log(sec(b*log(c*x^n) + a) + tan(b*log(c*x^n) + a))/(b*n)`

**Fricas [B]** time = 0.499456, size = 130, normalized size = 6.84

$$\frac{\log(\sin(bn \log(x) + b \log(c) + a) + 1) - \log(-\sin(bn \log(x) + b \log(c) + a) + 1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(a+b*log(c*x^n))/x,x, algorithm="fricas")`

[Out] `1/2*(log(sin(b*n*log(x) + b*log(c) + a) + 1) - log(-sin(b*n*log(x) + b*log(c) + a) + 1))/(b*n)`

**Sympy [A]** time = 4.00749, size = 51, normalized size = 2.68

$$-\begin{cases} -\log(x) \sec(a) & \text{for } b = 0 \\ -\log(x) \sec(a + b \log(c)) & \text{for } n = 0 \\ -\frac{\log(\tan(a + b \log(cx^n)) + \sec(a + b \log(cx^n)))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(a+b*ln(c*x**n))/x,x)`

[Out] `-Piecewise((-log(x)*sec(a), Eq(b, 0)), (-log(x)*sec(a + b*log(c)), Eq(n, 0)), (-log(tan(a + b*log(c*x**n)) + sec(a + b*log(c*x**n)))/(b*n), True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(a+b*log(c*x^n))/x,x, algorithm="giac")`

[Out] `integrate(sec(b*log(c*x^n) + a)/x, x)`

$$3.241 \quad \int \frac{\sec(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=87

$$\frac{2e^{ia} (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 + \frac{i}{bn}\right), \frac{1}{2}\left(3 + \frac{i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{x(1 - ibn)}$$

[Out]  $(-2E^{(I*a)}*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 + I/(b*n))/2, (3 + I/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})])/((1 - I*b*n)*x)$

**Rubi [A]** time = 0.0609986, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4509, 4505, 364}

$$\frac{2e^{ia} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{i}{bn}\right); \frac{1}{2}\left(3 + \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{x(1 - ibn)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]/x^2, x]

[Out]  $(-2E^{(I*a)}*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 + I/(b*n))/2, (3 + I/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})])/((1 - I*b*n)*x)$

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{\sec(a + b \log(cx^n))}{x^2} dx = \frac{(cx^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int x^{-1-\frac{1}{n}} \sec(a + b \log(x)) dx, x, cx^n\right)}{nx}$$

$$= \frac{\left(2e^{ia} (cx^n)^{\frac{1}{n}}\right) \operatorname{Subst}\left(\int \frac{x^{-1+ib-\frac{1}{n}}}{1+e^{2ia} x^{2ib}} dx, x, cx^n\right)}{nx}$$

$$= -\frac{2e^{ia} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{i}{bn}\right); \frac{1}{2}\left(3 + \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(1-ibn)x}$$

**Mathematica [A]** time = 0.142204, size = 85, normalized size = 0.98

$$\frac{2e^{ia} (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2} + \frac{i}{2bn}, \frac{3}{2} + \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right)}{x(-1+ibn)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]/x^2,x]

[Out] (2\*E^(I\*a)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, 1/2 + (I/2)/(b\*n), 3/2 + (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))])/((-1 + I\*b\*n)\*x)

**Maple [F]** time = 0.262, size = 0, normalized size = 0.

$$\int \frac{\sec(a + b \ln(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))/x^2,x)

[Out] int(sec(a+b\*ln(c\*x^n))/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))/x^2,x, algorithm="maxima")

[Out] integrate(sec(b\*log(c\*x^n) + a)/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sec(b \log(cx^n) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)/x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(a + b \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)/x^2, x)
```

$$3.242 \quad \int \frac{\sec(a+b \log(cx^n))}{x^3} dx$$

**Optimal.** Leaf size=87

$$\frac{2e^{ia} (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 + \frac{2i}{bn}\right), \frac{1}{2}\left(3 + \frac{2i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{x^2(2 - ibn)}$$

[Out]  $(-2E^{(I*a)}*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 + (2*I)/(b*n))/2, (3 + (2*I)/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})])/((2 - I*b*n)*x^2)$

**Rubi [A]** time = 0.0564884, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4509, 4505, 364}

$$\frac{2e^{ia} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{2i}{bn}\right); \frac{1}{2}\left(3 + \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{x^2(2 - ibn)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]/x^3, x]

[Out]  $(-2E^{(I*a)}*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 + (2*I)/(b*n))/2, (3 + (2*I)/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})])/((2 - I*b*n)*x^2)$

#### Rule 4509

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\sec(a + b \log(cx^n))}{x^3} dx &= \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int x^{-1-\frac{2}{n}} \sec(a + b \log(x)) dx, x, cx^n\right)}{nx^2} \\ &= \frac{(2e^{ia} (cx^n)^{2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+ib-\frac{2}{n}}}{1+e^{2ia}x^{2ib}} dx, x, cx^n\right)}{nx^2} \\ &= -\frac{2e^{ia} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{2i}{bn}\right); \frac{1}{2}\left(3 + \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(2 - ibn)x^2} \end{aligned}$$

**Mathematica [A]** time = 0.145454, size = 81, normalized size = 0.93

$$\frac{2e^{ia} (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2} + \frac{i}{bn}, \frac{3}{2} + \frac{i}{bn}, -e^{2i(a+b \log(cx^n))}\right)}{x^2(-2 + ibn)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]/x^3, x]

[Out] (2\*E^(I\*a)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, 1/2 + I/(b\*n), 3/2 + I/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))])/((-2 + I\*b\*n)\*x^2)

**Maple [F]** time = 0.313, size = 0, normalized size = 0.

$$\int \frac{\sec(a + b \ln(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))/x^3, x)

[Out] int(sec(a+b\*ln(c\*x^n))/x^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))/x^3, x, algorithm="maxima")

[Out] integrate(sec(b\*log(c\*x^n) + a)/x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sec(b \log(cx^n) + a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))/x^3,x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)/x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(a + b \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))/x\*\*3,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))/x^3,x, algorithm="giac")

[Out] integrate(sec(b\*log(c\*x^n) + a)/x^3, x)



### 3.243 $\int x^2 \sec^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=87

$$\frac{4e^{2ia}x^3(cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2 - \frac{3i}{bn}\right), \frac{1}{2}\left(4 - \frac{3i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{3 + 2ibn}$$

[Out]  $(4E^{((2*I)*a)}*x^3*(c*x^n)^{((2*I)*b)}*\operatorname{Hypergeometric2F1}[2, (2 - (3*I)/(b*n))/2, (4 - (3*I)/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(3 + (2*I)*b*n)$

**Rubi [A]** time = 0.075054, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{4e^{2ia}x^3(cx^n)^{2ib} {}_2F_1\left(2, \frac{1}{2}\left(2 - \frac{3i}{bn}\right); \frac{1}{2}\left(4 - \frac{3i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{3 + 2ibn}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sec[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(4E^{((2*I)*a)}*x^3*(c*x^n)^{((2*I)*b)}*\operatorname{Hypergeometric2F1}[2, (2 - (3*I)/(b*n))/2, (4 - (3*I)/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(3 + (2*I)*b*n)$

#### Rule 4509

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)]\*(d\_)^(p\_), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[x\_]\*(b\_)]\*(d\_)^(p\_), x\_Symbol] :> Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x^2 \sec^2(a + b \log(cx^n)) dx &= \frac{(x^3 (cx^n)^{-3/n}) \operatorname{Subst}\left(\int x^{-1+\frac{3}{n}} \sec^2(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(4e^{2ia}x^3 (cx^n)^{-3/n}) \operatorname{Subst}\left(\int \frac{x^{-1+2ib+\frac{3}{n}}}{(1+e^{2ia}x^{2ib})^2} dx, x, cx^n\right)}{n} \\ &= \frac{4e^{2ia}x^3 (cx^n)^{2ib} {}_2F_1\left(2, \frac{1}{2}\left(2 - \frac{3i}{bn}\right); \frac{1}{2}\left(4 - \frac{3i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{3 + 2ibn} \end{aligned}$$

**Mathematica [A]** time = 5.53073, size = 160, normalized size = 1.84

$$\frac{x^3 \left( 3e^{2ia} (cx^n)^{2ib} \operatorname{Hypergeometric2F1} \left( 1, 1 - \frac{3i}{2bn}, 2 - \frac{3i}{2bn}, -e^{2i(a+b \log(cx^n))} \right) + (2bn - 3i) \left( \tan(a + b \log(cx^n)) - i \operatorname{Hypergeometric2F1} \left( 1, 1 - \frac{3i}{2bn}, 2 - \frac{3i}{2bn}, -e^{2i(a+b \log(cx^n))} \right) \right) \right)}{bn(2bn - 3i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*Sec[a + b\*Log[c\*x^n]]^2,x]

[Out] (x^3\*(3\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[1, 1 - ((3\*I)/2)/(b\*n), 2 - ((3\*I)/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + (-3\*I + 2\*b\*n)\*((-I)\*Hypergeometric2F1[1, ((-3\*I)/2)/(b\*n), 1 - ((3\*I)/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + Tan[a + b\*Log[c\*x^n]])))/(b\*n\*(-3\*I + 2\*b\*n))

**Maple [F]** time = 1.485, size = 0, normalized size = 0.

$$\int x^2 (\sec(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*sec(a+b\*ln(c\*x^n))^2,x)

[Out] int(x^2\*sec(a+b\*ln(c\*x^n))^2,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sec(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^2 \sec(b \log(cx^n) + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sec(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(x^2\*sec(b\*log(c\*x^n) + a)^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sec^2(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*sec(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(x\*\*2\*sec(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sec(b \log(cx^n) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*sec(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(x^2\*sec(b\*log(c\*x^n) + a)^2, x)

### 3.244 $\int x \sec^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=79

$$\frac{2e^{2ia}x^2(cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(2, 1 - \frac{i}{bn}, 2 - \frac{i}{bn}, -e^{2ia}(cx^n)^{2ib}\right)}{1 + ibn}$$

[Out]  $(2 * E^{((2 * I) * a)} * x^2 * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, 1 - I/(b * n), 2 - I/(b * n), -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / (1 + I * b * n)$

**Rubi [A]** time = 0.0655583, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4509, 4505, 364}

$$\frac{2e^{2ia}x^2(cx^n)^{2ib} {}_2F_1\left(2, 1 - \frac{i}{bn}; 2 - \frac{i}{bn}; -e^{2ia}(cx^n)^{2ib}\right)}{1 + ibn}$$

Antiderivative was successfully verified.

[In] Int[x\*Sec[a + b\*Log[c\*x^n]]^2,x]

[Out]  $(2 * E^{((2 * I) * a)} * x^2 * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, 1 - I/(b * n), 2 - I/(b * n), -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / (1 + I * b * n)$

#### Rule 4509

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[2^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p \* (c\*x)^(m + 1) \* Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a]) / (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x \sec^2(a + b \log(cx^n)) dx &= \frac{(x^2 (cx^n)^{-2/n}) \operatorname{Subst}\left(\int x^{-1 + \frac{2}{n}} \sec^2(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(4e^{2ia}x^2 (cx^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{x^{-1 + 2ib + \frac{2}{n}}}{(1 + e^{2ia}x^{2ib})^2} dx, x, cx^n\right)}{n} \\ &= \frac{2e^{2ia}x^2 (cx^n)^{2ib} {}_2F_1\left(2, 1 - \frac{i}{bn}; 2 - \frac{i}{bn}; -e^{2ia}(cx^n)^{2ib}\right)}{1 + ibn} \end{aligned}$$

**Mathematica [A]** time = 5.34552, size = 149, normalized size = 1.89

$$\frac{x^2 \left( e^{2ia} (cx^n)^{2ib} \operatorname{Hypergeometric2F1} \left( 1, 1 - \frac{i}{bn}, 2 - \frac{i}{bn}, -e^{2i(a+b \log(cx^n))} \right) + (bn - i) \left( \tan(a + b \log(cx^n)) - i \operatorname{Hypergeometric2F1} \left( 1, 1 - \frac{i}{bn}, 2 - \frac{i}{bn}, -e^{2i(a+b \log(cx^n))} \right) \right) \right)}{bn(bn - i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sec[a + b\*Log[c\*x^n]]^2,x]

[Out] (x^2\*(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[1, 1 - I/(b\*n), 2 - I/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + (-I + b\*n)\*((-I)\*Hypergeometric2F1[1, (-I)/(b\*n), 1 - I/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + Tan[a + b\*Log[c\*x^n]])))/(b\*n\*(-I + b\*n))

**Maple [F]** time = 1.363, size = 0, normalized size = 0.

$$\int x (\sec(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sec(a+b\*ln(c\*x^n))^2,x)

[Out] int(x\*sec(a+b\*ln(c\*x^n))^2,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x \sec(b \log(cx^n) + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(x\*sec(b\*log(c\*x^n) + a)^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec^2(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(x\*sec(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(b \log(cx^n) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(x\*sec(b\*log(c\*x^n) + a)^2, x)

### 3.245 $\int \sec^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=85

$$\frac{4e^{2ia}x(cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2 - \frac{i}{bn}\right), \frac{1}{2}\left(4 - \frac{i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn}$$

[Out]  $(4 * E^{((2 * I) * a)} * x * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, (2 - I/(b * n))/2, (4 - I/(b * n))/2, -E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / (1 + (2 * I) * b * n)$

**Rubi [A]** time = 0.0625784, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4503, 4505, 364}

$$\frac{4e^{2ia}x(cx^n)^{2ib} {}_2F_1\left(2, \frac{1}{2}\left(2 - \frac{i}{bn}\right); \frac{1}{2}\left(4 - \frac{i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^2, x]

[Out]  $(4 * E^{((2 * I) * a)} * x * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, (2 - I/(b * n))/2, (4 - I/(b * n))/2, -E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / (1 + (2 * I) * b * n)$

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[2^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p)) / (1 + E^(2\*I\*a\*d) \* x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p \* (c\*x)^(m + 1) \* Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)]) / (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sec^2(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sec^2(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(4e^{2ia}x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+2ib+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^2} dx, x, cx^n\right)}{n} \\ &= \frac{4e^{2ia}x(cx^n)^{2ib} {}_2F_1\left(2, \frac{1}{2}\left(2 - \frac{i}{bn}\right); \frac{1}{2}\left(4 - \frac{i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn} \end{aligned}$$

**Mathematica [A]** time = 6.40699, size = 147, normalized size = 1.73

$$x \left( \frac{e^{2ia} (cx^n)^{2ib} \text{Hypergeometric2F1}\left(1, 1 - \frac{i}{2bn}, 2 - \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right)}{2bn-i} - i \text{Hypergeometric2F1}\left(1, -\frac{i}{2bn}, 1 - \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right) + \tan \right) / bn$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^2, x]

[Out] (x\*((E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[1, 1 - (I/2)/(b\*n), 2 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))])/(-I + 2\*b\*n) - I\*Hypergeometric2F1[1, (-I/2)/(b\*n), 1 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))]) + Tan[a + b\*Log[c\*x^n]])/(b\*n)

**Maple [F]** time = 1.22, size = 0, normalized size = 0.

$$\int (\sec(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^2, x)

[Out] int(sec(a+b\*ln(c\*x^n))^2, x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2, x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\sec(b \log(cx^n) + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2, x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^2(a + b \log(cx^n)) dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(sec(b\*log(c\*x^n) + a)^2, x)

$$3.246 \quad \int \frac{\sec^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=18

$$\frac{\tan(a + b \log(cx^n))}{bn}$$

[Out] Tan[a + b\*Log[c\*x^n]]/(b\*n)

**Rubi [A]** time = 0.027744, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3767, 8}

$$\frac{\tan(a + b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^2/x,x]

[Out] Tan[a + b\*Log[c\*x^n]]/(b\*n)

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)^(n\_)], x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\sec^2(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sec^2(a + bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\text{Subst}\left(\int 1 dx, x, -\tan(a + b \log(cx^n))\right)}{bn} \\ &= \frac{\tan(a + b \log(cx^n))}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0913382, size = 18, normalized size = 1.

$$\frac{\tan(a + b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^2/x,x]

[Out] Tan[a + b\*Log[c\*x^n]]/(b\*n)

**Maple [A]** time = 0.033, size = 19, normalized size = 1.1

$$\frac{\tan(a + b \ln(cx^n))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^2/x,x)

[Out] tan(a+b\*ln(c\*x^n))/b/n

**Maxima [B]** time = 1.16738, size = 223, normalized size = 12.39

$$\frac{2(\cos(2b \log(x^n) + 2a) \sin(2b \log(c)) + \cos(2b \log(c)) \sin(2b \log(x^n) + 2a))}{2bn \cos(2b \log(c)) \cos(2b \log(x^n) + 2a) + (b \cos(2b \log(c))^2 + b \sin(2b \log(c))^2)n \cos(2b \log(x^n) + 2a)^2 - 2bn \sin(2b \log(c)) \sin(2b \log(x^n) + 2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2/x,x, algorithm="maxima")

[Out] 2\*(cos(2\*b\*log(x^n) + 2\*a)\*sin(2\*b\*log(c)) + cos(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a))/(2\*b\*n\*cos(2\*b\*log(c))\*cos(2\*b\*log(x^n) + 2\*a) + (b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*cos(2\*b\*log(x^n) + 2\*a)^2 - 2\*b\*n\*sin(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a) + (b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*sin(2\*b\*log(x^n) + 2\*a)^2 + b\*n)

**Fricas [A]** time = 0.467164, size = 93, normalized size = 5.17

$$\frac{\sin(bn \log(x) + b \log(c) + a)}{bn \cos(bn \log(x) + b \log(c) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2/x,x, algorithm="fricas")

[Out] sin(b\*n\*log(x) + b\*log(c) + a)/(b\*n\*cos(b\*n\*log(x) + b\*log(c) + a))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(a + b \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*2/x,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*2/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^2/x, x)
```

$$3.247 \quad \int \frac{\sec^2(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=87

$$\frac{4e^{2ia} (cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2 + \frac{i}{bn}\right), \frac{1}{2}\left(4 + \frac{i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{x(1 - 2ibn)}$$

[Out]  $(-4 * E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, (2 + I / (b * n)) / 2, (4 + I / (b * n)) / 2, -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / ((1 - (2 * I) * b * n) * x)$

**Rubi [A]** time = 0.0764024, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{4e^{2ia} (cx^n)^{2ib} {}_2F_1\left(2, \frac{1}{2}\left(2 + \frac{i}{bn}\right); \frac{1}{2}\left(4 + \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{x(1 - 2ibn)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^2/x^2, x]

[Out]  $(-4 * E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, (2 + I / (b * n)) / 2, (4 + I / (b * n)) / 2, -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / ((1 - (2 * I) * b * n) * x)$

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[2^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p)) / (1 + E^(2\*I\*a\*d) \* x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p \* (c\*x)^(m + 1) \* Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)]) / (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{\sec^2(a + b \log(cx^n))}{x^2} dx = \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \sec^2(a + b \log(x)) dx, x, cx^n\right)}{nx}$$

$$= \frac{\left(4e^{2ia} (cx^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+2ib-\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^2} dx, x, cx^n\right)}{nx}$$

$$= -\frac{4e^{2ia} (cx^n)^{2ib} {}_2F_1\left(2, \frac{1}{2}\left(2 + \frac{i}{bn}\right); \frac{1}{2}\left(4 + \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(1 - 2ibn)x}$$

**Mathematica [A]** time = 3.88125, size = 160, normalized size = 1.84

$$\frac{(1 - 2ibn) \left( \text{Hypergeometric2F1}\left(1, \frac{i}{2bn}, 1 + \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right) + i \tan(a + b \log(cx^n)) \right) - e^{2ia} (cx^n)^{2ib} \text{Hypergeometric2F1}\left(1, \frac{i}{2bn}, 1 + \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right)}{bnx(2bn + i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^2/x^2, x]

[Out]  $(-E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)}*\text{Hypergeometric2F1}[1, 1 + (I/2)/(b*n), 2 + (I/2)/(b*n), -E^{((2*I)*(a + b*\text{Log}[c*x^n]))}]) + (1 - (2*I)*b*n)*(\text{Hypergeometric2F1}[1, (I/2)/(b*n), 1 + (I/2)/(b*n), -E^{((2*I)*(a + b*\text{Log}[c*x^n]))}] + I*\text{Tan}[a + b*\text{Log}[c*x^n]])/(b*n*(I + 2*b*n)*x)$

**Maple [F]** time = 1.473, size = 0, normalized size = 0.

$$\int \frac{(\sec(a + b \ln(cx^n)))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^2/x^2, x)

[Out] int(sec(a+b\*ln(c\*x^n))^2/x^2, x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2/x^2, x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(b \log(cx^n) + a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^2/x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(a + b \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**2/x**2,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**2/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^2/x^2, x)
```

$$3.248 \quad \int \frac{\sec^2(a+b \log(cx^n))}{x^3} dx$$

**Optimal.** Leaf size=79

$$\frac{2e^{2ia} (cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(2, 1 + \frac{i}{bn}, 2 + \frac{i}{bn}, -e^{2ia} (cx^n)^{2ib}\right)}{x^2(1 - ibn)}$$

[Out]  $(-2 * E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, 1 + I/(b * n), 2 + I/(b * n), -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / ((1 - I * b * n) * x^2)$

**Rubi [A]** time = 0.0706627, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{2e^{2ia} (cx^n)^{2ib} {}_2F_1\left(2, 1 + \frac{i}{bn}; 2 + \frac{i}{bn}; -e^{2ia} (cx^n)^{2ib}\right)}{x^2(1 - ibn)}$$

Antiderivative was successfully verified.

[In] `Int[Sec[a + b*Log[c*x^n]]^2/x^3, x]`

[Out]  $(-2 * E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, 1 + I/(b * n), 2 + I/(b * n), -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / ((1 - I * b * n) * x^2)$

#### Rule 4509

`Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])`

#### Rule 4505

`Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] := Dist[2^p * E^(I*a*d*p), Int[((e*x)^m * x^(I*b*d*p))/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]`

#### Rule 364

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

#### Rubi steps



$$\begin{aligned} \int \frac{\sec^2(a + b \log(cx^n))}{x^3} dx &= \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int x^{-1-\frac{2}{n}} \sec^2(a + b \log(x)) dx, x, cx^n\right)}{nx^2} \\ &= \frac{(4e^{2ia} (cx^n)^{2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+2ib-\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^2} dx, x, cx^n\right)}{nx^2} \\ &= -\frac{2e^{2ia} (cx^n)^{2ib} {}_2F_1\left(2, 1 + \frac{i}{bn}; 2 + \frac{i}{bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(1-ibn)x^2} \end{aligned}$$

**Mathematica [A]** time = 3.71769, size = 150, normalized size = 1.9

$$\frac{(bn + i) \left( \tan(a + b \log(cx^n)) - i \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bn}, 1 + \frac{i}{bn}, -e^{2i(a+b \log(cx^n))}\right) \right) - e^{2ia} (cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bn}, 1 + \frac{i}{bn}, -e^{2i(a+b \log(cx^n))}\right)}{bnx^2(bn + i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^2/x^3,x]

[Out]  $(-E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)}*\operatorname{Hypergeometric2F1}[1, 1 + I/(b*n), 2 + I/(b*n), -E^{((2*I)*(a + b*\operatorname{Log}[c*x^n]))}]) + (I + b*n)*((-I)*\operatorname{Hypergeometric2F1}[1, I/(b*n), 1 + I/(b*n), -E^{((2*I)*(a + b*\operatorname{Log}[c*x^n]))}] + \operatorname{Tan}[a + b*\operatorname{Log}[c*x^n]])/(b*n*(I + b*n)*x^2)$

**Maple [F]** time = 1.615, size = 0, normalized size = 0.

$$\int \frac{(\sec(a + b \ln(cx^n)))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^2/x^3,x)

[Out] int(sec(a+b\*ln(c\*x^n))^2/x^3,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2/x^3,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sec(b \log(cx^n) + a)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2/x^3,x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)^2/x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(a + b \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*2/x\*\*3,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*2/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^2/x^3,x, algorithm="giac")

[Out] integrate(sec(b\*log(c\*x^n) + a)^2/x^3, x)

### 3.249 $\int x \sec^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=87

$$\frac{8e^{3ia}x^2(cx^n)^{3ib} \operatorname{Hypergeometric2F1}\left(3, \frac{1}{2}\left(3 - \frac{2i}{bn}\right), \frac{1}{2}\left(5 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{2 + 3ibn}$$

[Out]  $(8E^{((3*I)*a)}*x^2*(c*x^n)^{((3*I)*b)}*\operatorname{Hypergeometric2F1}[3, (3 - (2*I)/(b*n))/2, (5 - (2*I)/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(2 + (3*I)*b*n)$

**Rubi [A]** time = 0.0661174, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4509, 4505, 364}

$$\frac{8e^{3ia}x^2(cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{2i}{bn}\right); \frac{1}{2}\left(5 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{2 + 3ibn}$$

Antiderivative was successfully verified.

[In] Int[x\*Sec[a + b\*Log[c\*x^n]]^3, x]

[Out]  $(8E^{((3*I)*a)}*x^2*(c*x^n)^{((3*I)*b)}*\operatorname{Hypergeometric2F1}[3, (3 - (2*I)/(b*n))/2, (5 - (2*I)/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(2 + (3*I)*b*n)$

#### Rule 4509

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] :> Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x \sec^3(a + b \log(cx^n)) dx &= \frac{(x^2(cx^n)^{-2/n}) \operatorname{Subst}\left(\int x^{-1+\frac{2}{n}} \sec^3(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(8e^{3ia}x^2(cx^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+3ib+\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^3} dx, x, cx^n\right)}{n} \\ &= \frac{8e^{3ia}x^2(cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{2i}{bn}\right); \frac{1}{2}\left(5 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{2 + 3ibn} \end{aligned}$$

**Mathematica [A]** time = 4.74559, size = 118, normalized size = 1.36

$$\frac{x^2 \left( (bn \tan(a + b \log(cx^n)) - 2) \sec(a + b \log(cx^n)) + 2e^{ia}(2 - ibn)(cx^n)^{ib} \operatorname{Hypergeometric2F1} \left( 1, \frac{1}{2} - \frac{i}{bn}, \frac{3}{2} - \frac{i}{bn}, -e^{2ia} \right) \right)}{2b^2n^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sec[a + b\*Log[c\*x^n]]^3,x]

[Out] (x^2\*(2\*E^(I\*a)\*(2 - I\*b\*n)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, 1/2 - I/(b\*n), 3/2 - I/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + Sec[a + b\*Log[c\*x^n]]\*(-2 + b\*n\*Tan[a + b\*Log[c\*x^n]])))/(2\*b^2\*n^2)

**Maple [F]** time = 2.13, size = 0, normalized size = 0.

$$\int x (\sec(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sec(a+b\*ln(c\*x^n))^3,x)

[Out] int(x\*sec(a+b\*ln(c\*x^n))^3,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x \sec(b \log(cx^n) + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] integral(x\*sec(b\*log(c\*x^n) + a)^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec^3(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(x*sec(a + b*log(c*x**n))**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(b \log(cx^n) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*log(c*x^n) + a)^3, x)
```

### 3.250 $\int \sec^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=85

$$\frac{8e^{3ia} x (cx^n)^{3ib} \operatorname{Hypergeometric2F1}\left(3, \frac{1}{2}\left(3 - \frac{i}{bn}\right), \frac{1}{2}\left(5 - \frac{i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{1 + 3ibn}$$

[Out]  $(8 * E^{((3 * I) * a)} * x * (c * x^n)^{((3 * I) * b)} * \operatorname{Hypergeometric2F1}[3, (3 - I/(b * n))/2, (5 - I/(b * n))/2, -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / (1 + (3 * I) * b * n)$

**Rubi [A]** time = 0.0630798, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4503, 4505, 364}

$$\frac{8e^{3ia} x (cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{i}{bn}\right); \frac{1}{2}\left(5 - \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{1 + 3ibn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^3, x]

[Out]  $(8 * E^{((3 * I) * a)} * x * (c * x^n)^{((3 * I) * b)} * \operatorname{Hypergeometric2F1}[3, (3 - I/(b * n))/2, (5 - I/(b * n))/2, -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / (1 + (3 * I) * b * n)$

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[2^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sec^3(a + b \log(cx^n)) dx &= \frac{(x (cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1 + \frac{1}{n}} \sec^3(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(8e^{3ia} x (cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1 + 3ib + \frac{1}{n}}}{(1 + e^{2ia} x^{2ib})^3} dx, x, cx^n\right)}{n} \\ &= \frac{8e^{3ia} x (cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{i}{bn}\right); \frac{1}{2}\left(5 - \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{1 + 3ibn} \end{aligned}$$

**Mathematica [A]** time = 4.41771, size = 120, normalized size = 1.41

$$x \left( (bn \tan(a + b \log(cx^n)) - 1) \sec(a + b \log(cx^n)) + 2e^{ia}(1 - ibn)(cx^n)^{ib} \operatorname{Hypergeometric2F1} \left( 1, \frac{1}{2} - \frac{i}{2bn}, \frac{3}{2} - \frac{i}{2bn}, \dots \right) \right) / (2b^2n^2)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^3,x]

[Out] (x\*(2\*E^(I\*a)\*(1 - I\*b\*n)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, 1/2 - (I/2)/(b\*n), 3/2 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + Sec[a + b\*Log[c\*x^n]]\*(-1 + b\*n\*Tan[a + b\*Log[c\*x^n]])))/(2\*b^2\*n^2)

**Maple [F]** time = 1.74, size = 0, normalized size = 0.

$$\int (\sec(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^3,x)

[Out] int(sec(a+b\*ln(c\*x^n))^3,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\sec(b \log(cx^n) + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^3(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^3, x)
```



$$3.251 \quad \int \frac{\sec^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=55

$$\frac{\tanh^{-1}(\sin(a+b \log(cx^n)))}{2bn} + \frac{\tan(a+b \log(cx^n)) \sec(a+b \log(cx^n))}{2bn}$$

[Out] ArcTanh[Sin[a + b\*Log[c\*x^n]]]/(2\*b\*n) + (Sec[a + b\*Log[c\*x^n]]\*Tan[a + b\*Log[c\*x^n]])/(2\*b\*n)

**Rubi [A]** time = 0.0384104, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3768, 3770}

$$\frac{\tanh^{-1}(\sin(a+b \log(cx^n)))}{2bn} + \frac{\tan(a+b \log(cx^n)) \sec(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^3/x, x]

[Out] ArcTanh[Sin[a + b\*Log[c\*x^n]]]/(2\*b\*n) + (Sec[a + b\*Log[c\*x^n]]\*Tan[a + b\*Log[c\*x^n]])/(2\*b\*n)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x] \* (b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\sec^3(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sec^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\sec(a+b \log(cx^n)) \tan(a+b \log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int \sec(a+bx) dx, x, \log(cx^n)\right)}{2n} \\ &= \frac{\tanh^{-1}(\sin(a+b \log(cx^n)))}{2bn} + \frac{\sec(a+b \log(cx^n)) \tan(a+b \log(cx^n))}{2bn} \end{aligned}$$

**Mathematica [A]** time = 0.0726142, size = 55, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a+b \log(cx^n)))}{2bn} + \frac{\tan(a+b \log(cx^n)) \sec(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^3/x, x]

[Out] ArcTanh[Sin[a + b\*Log[c\*x^n]]]/(2\*b\*n) + (Sec[a + b\*Log[c\*x^n]]\*Tan[a + b\*Log[c\*x^n]])/(2\*b\*n)

**Maple [A]** time = 0.039, size = 64, normalized size = 1.2

$$\frac{\sec(a + b \ln(cx^n)) \tan(a + b \ln(cx^n))}{2bn} + \frac{\ln(\sec(a + b \ln(cx^n)) + \tan(a + b \ln(cx^n)))}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^3/x,x)

[Out] 1/2\*sec(a+b\*ln(c\*x^n))\*tan(a+b\*ln(c\*x^n))/b/n+1/2/b/n\*ln(sec(a+b\*ln(c\*x^n))+tan(a+b\*ln(c\*x^n)))

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^3/x,x, algorithm="maxima")

[Out] Timed out

**Fricas [A]** time = 0.507579, size = 311, normalized size = 5.65

$$\frac{\cos(bn \log(x) + b \log(c) + a)^2 \log(\sin(bn \log(x) + b \log(c) + a) + 1) - \cos(bn \log(x) + b \log(c) + a)^2 \log(-\sin(bn \log(x) + b \log(c) + a) + 1) + 2 \sin(bn \log(x) + b \log(c) + a)}{4bn \cos(bn \log(x) + b \log(c) + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^3/x,x, algorithm="fricas")

[Out] 1/4\*(cos(b\*n\*log(x) + b\*log(c) + a)^2\*log(sin(b\*n\*log(x) + b\*log(c) + a) + 1) - cos(b\*n\*log(x) + b\*log(c) + a)^2\*log(-sin(b\*n\*log(x) + b\*log(c) + a) + 1) + 2\*sin(b\*n\*log(x) + b\*log(c) + a))/(b\*n\*cos(b\*n\*log(x) + b\*log(c) + a)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(a + b \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*3/x,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*3/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^3/x, x)
```

$$3.252 \quad \int \frac{\sec^3(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=87

$$\frac{8e^{3ia} (cx^n)^{3ib} \operatorname{Hypergeometric2F1}\left(3, \frac{1}{2}\left(3 + \frac{i}{bn}\right), \frac{1}{2}\left(5 + \frac{i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{x(1 - 3ibn)}$$

[Out]  $(-8E^{((3*I)*a)}*(c*x^n)^{((3*I)*b)}*\operatorname{Hypergeometric2F1}[3, (3 + I/(b*n))/2, (5 + I/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/((1 - (3*I)*b*n)*x)$

**Rubi [A]** time = 0.0745732, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{8e^{3ia} (cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 + \frac{i}{bn}\right); \frac{1}{2}\left(5 + \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{x(1 - 3ibn)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sec}[a + b*\operatorname{Log}[c*x^n]]^3/x^2, x]$

[Out]  $(-8E^{((3*I)*a)}*(c*x^n)^{((3*I)*b)}*\operatorname{Hypergeometric2F1}[3, (3 + I/(b*n))/2, (5 + I/(b*n))/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/((1 - (3*I)*b*n)*x)$

#### Rule 4509

$\operatorname{Int}[(e_*)*(x_*)^{(m_*)}*\operatorname{Sec}[(a_*) + \operatorname{Log}[(c_*)*(x_*)^{(n_*)}]*b_*])^{(p_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)}/(e^n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[x^{((m+1)/n-1)}*\operatorname{Sec}[d*(a+b*\operatorname{Log}[x])]^p, x], x, c*x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\operatorname{NeQ}[c, 1] \mid \mid \operatorname{NeQ}[n, 1])$

#### Rule 4505

$\operatorname{Int}[(e_*)*(x_*)^{(m_*)}*\operatorname{Sec}[(a_*) + \operatorname{Log}[x_*]*b_*])^{(p_*)}, x\_Symbol] \rightarrow \operatorname{Dist}[2^p * E^{(I*a*d*p)}, \operatorname{Int}[(e*x)^m * x^{(I*b*d*p)}]/(1 + E^{(2*I*a*d)} * x^{(2*I*b*d)})^p, x] /; \operatorname{FreeQ}\{a, b, d, e, m\}, x] \&\& \operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + b_*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a^p*(c*x)^{(m+1)}*\operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/((c*(m+1)), x] /; \operatorname{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \mid \mid \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\int \frac{\sec^3(a + b \log(cx^n))}{x^2} dx = \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \sec^3(a + b \log(x)) dx, x, cx^n\right)}{nx}$$

$$= \frac{\left(8e^{3ia} (cx^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+3ib-\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^3} dx, x, cx^n\right)}{nx}$$

$$= -\frac{8e^{3ia} (cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 + \frac{i}{bn}\right); \frac{1}{2}\left(5 + \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(1 - 3ibn)x}$$

**Mathematica [A]** time = 4.6288, size = 123, normalized size = 1.41

$$\frac{(bn \tan(a + b \log(cx^n)) + 1) \sec(a + b \log(cx^n)) - 2ie^{ia}(bn - i)(cx^n)^{ib} \text{Hypergeometric2F1}\left(1, \frac{1}{2} + \frac{i}{2bn}, \frac{3}{2} + \frac{i}{2bn}, -e^{2ia} (cx^n)^{2ib}\right)}{2b^2n^2x}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^3/x^2, x]

[Out]  $((-2*I)*E^{(I*a)*(-I + b*n)}*(c*x^n)^{(I*b)}*\text{Hypergeometric2F1}[1, 1/2 + (I/2)/(b*n), 3/2 + (I/2)/(b*n), -E^{((2*I)*(a + b*\text{Log}[c*x^n]))}] + \text{Sec}[a + b*\text{Log}[c*x^n]]*(1 + b*n*\text{Tan}[a + b*\text{Log}[c*x^n]]))/(2*b^2*n^2*x)$

**Maple [F]** time = 2.144, size = 0, normalized size = 0.

$$\int \frac{(\sec(a + b \ln(cx^n)))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^3/x^2, x)

[Out] int(sec(a+b\*ln(c\*x^n))^3/x^2, x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^3/x^2, x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(b \log(cx^n) + a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^3/x^2,x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)^3/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(a + b \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*3/x\*\*2,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*3/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^3/x^2,x, algorithm="giac")

[Out] integrate(sec(b\*log(c\*x^n) + a)^3/x^2, x)

$$3.253 \quad \int \frac{\sec^3(a+b \log(cx^n))}{x^3} dx$$

**Optimal.** Leaf size=87

$$\frac{8e^{3ia} (cx^n)^{3ib} \operatorname{Hypergeometric2F1}\left(3, \frac{1}{2}\left(3 + \frac{2i}{bn}\right), \frac{1}{2}\left(5 + \frac{2i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{x^2(2 - 3ibn)}$$

[Out]  $(-8 * E^{((3 * I) * a)} * (c * x^n)^{((3 * I) * b)} * \operatorname{Hypergeometric2F1}[3, (3 + (2 * I) / (b * n)) / 2, (5 + (2 * I) / (b * n)) / 2, -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / ((2 - (3 * I) * b * n) * x^2)$

**Rubi [A]** time = 0.0730395, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{8e^{3ia} (cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 + \frac{2i}{bn}\right); \frac{1}{2}\left(5 + \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{x^2(2 - 3ibn)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^3/x^3,x]

[Out]  $(-8 * E^{((3 * I) * a)} * (c * x^n)^{((3 * I) * b)} * \operatorname{Hypergeometric2F1}[3, (3 + (2 * I) / (b * n)) / 2, (5 + (2 * I) / (b * n)) / 2, -(E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)})]) / ((2 - (3 * I) * b * n) * x^2)$

#### Rule 4509

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)]\*(d\_)]^(p\_), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[x\_]\*(b\_)]\*(d\_)]^(p\_), x\_Symbol] :> Dist[2^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p)) / (1 + E^(2\*I\*a\*d) \* x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p \* (c\*x)^(m + 1) \* Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)]) / (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{\sec^3(a + b \log(cx^n))}{x^3} dx = \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int x^{-1-\frac{2}{n}} \sec^3(a + b \log(x)) dx, x, cx^n\right)}{nx^2}$$

$$= \frac{(8e^{3ia} (cx^n)^{2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+3ib-\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^3} dx, x, cx^n\right)}{nx^2}$$

$$= -\frac{8e^{3ia} (cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 + \frac{2i}{bn}\right); \frac{1}{2}\left(5 + \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(2 - 3ibn)x^2}$$

**Mathematica [A]** time = 4.65645, size = 119, normalized size = 1.37

$$\frac{(bn \tan(a + b \log(cx^n)) + 2) \sec(a + b \log(cx^n)) - 2ie^{ia}(bn - 2i)(cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2} + \frac{i}{bn}, \frac{3}{2} + \frac{i}{bn}, -e^{2i(a+b \log(cx^n))}\right)}{2b^2n^2x^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^3/x^3,x]

[Out]  $((-2*I)*E^{(I*a)}*(-2*I + b*n)*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, 1/2 + I/(b*n), 3/2 + I/(b*n), -E^{((2*I)*(a + b*\operatorname{Log}[c*x^n]))}] + \operatorname{Sec}[a + b*\operatorname{Log}[c*x^n]]*(2 + b*n*\operatorname{Tan}[a + b*\operatorname{Log}[c*x^n]]))/(2*b^2*n^2*x^2)$

**Maple [F]** time = 2.151, size = 0, normalized size = 0.

$$\int \frac{(\sec(a + b \ln(cx^n)))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^3/x^3,x)

[Out] int(sec(a+b\*ln(c\*x^n))^3/x^3,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^3/x^3,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sec(b \log(cx^n) + a)^3}{x^3}, x\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^3/x^3, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(a + b \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**3/x**3,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**3/x**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^3/x^3, x)
```

### 3.254 $\int x \sec^4(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=79

$$\frac{8e^{4ia}x^2(cx^n)^{4ib} \operatorname{Hypergeometric2F1}\left(4, 2 - \frac{i}{bn}, 3 - \frac{i}{bn}, -e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn}$$

[Out]  $(8E^{((4*I)*a)}*x^2*(c*x^n)^{((4*I)*b)}*\operatorname{Hypergeometric2F1}[4, 2 - I/(b*n), 3 - I/(b*n), -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(1 + (2*I)*b*n)$

**Rubi [A]** time = 0.0693265, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4509, 4505, 364}

$$\frac{8e^{4ia}x^2(cx^n)^{4ib} {}_2F_1\left(4, 2 - \frac{i}{bn}; 3 - \frac{i}{bn}; -e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn}$$

Antiderivative was successfully verified.

[In] Int[x\*Sec[a + b\*Log[c\*x^n]]^4, x]

[Out]  $(8E^{((4*I)*a)}*x^2*(c*x^n)^{((4*I)*b)}*\operatorname{Hypergeometric2F1}[4, 2 - I/(b*n), 3 - I/(b*n), -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(1 + (2*I)*b*n)$

#### Rule 4509

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] := Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x \sec^4(a + b \log(cx^n)) dx &= \frac{(x^2(cx^n)^{-2/n}) \operatorname{Subst}\left(\int x^{-1+\frac{2}{n}} \sec^4(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(16e^{4ia}x^2(cx^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+4ib+\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^4} dx, x, cx^n\right)}{n} \\ &= \frac{8e^{4ia}x^2(cx^n)^{4ib} {}_2F_1\left(4, 2 - \frac{i}{bn}; 3 - \frac{i}{bn}; -e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn} \end{aligned}$$

**Mathematica [B]** time = 12.9154, size = 204, normalized size = 2.58

$$x^2 \left( -2i (b^2 n^2 + 1) \text{Hypergeometric2F1} \left( 1, -\frac{i}{bn}, 1 - \frac{i}{bn}, -e^{2i(a+b \log(cx^n))} \right) + 2e^{2ia} (bn + i) (cx^n)^{2ib} \text{Hypergeometric2F1} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sec[a + b\*Log[c\*x^n]]^4,x]

[Out] (x^2\*(2\*E^((2\*I)\*a)\*(I + b\*n)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[1, 1 - I/(b\*n), 2 - I/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] - (2\*I)\*(1 + b^2\*n^2)\*Hypergeometric2F1[1, (-I)/(b\*n), 1 - I/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + Sec[a + b\*Log[c\*x^n]]^2\*(-(b\*n) + (1 + 2\*b^2\*n^2 + (1 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n]]))\*Tan[a + b\*Log[c\*x^n]]))/(3\*b^3\*n^3)

**Maple [F]** time = 1.415, size = 0, normalized size = 0.

$$\int x (\sec(a + b \ln(cx^n)))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sec(a+b\*ln(c\*x^n))^4,x)

[Out] int(x\*sec(a+b\*ln(c\*x^n))^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n))^4,x, algorithm="maxima")

[Out] -4/3\*(3\*(b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*x^2\*cos(4\*b\*log(x^n) + 4\*a)^2 + 3\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*x^2\*cos(2\*b\*log(x^n) + 2\*a)^2 + 3\*(b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*x^2\*sin(4\*b\*log(x^n) + 4\*a)^2 + 3\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*x^2\*sin(2\*b\*log(x^n) + 2\*a)^2 + (b\*n\*cos(2\*b\*log(c)) - sin(2\*b\*log(c)))\*x^2\*cos(2\*b\*log(x^n) + 2\*a) - (b\*n\*sin(2\*b\*log(c)) + cos(2\*b\*log(c)))\*x^2\*sin(2\*b\*log(x^n) + 2\*a) + (((b\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)))\*n - cos(4\*b\*log(c))\*sin(6\*b\*log(c)) + cos(6\*b\*log(c))\*sin(4\*b\*log(c)))\*x^2\*cos(4\*b\*log(x^n) + 4\*a) - (3\*(b^2\*cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 - (b\*cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(2\*b\*log(c)))\*n + 2\*cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - 2\*cos(6\*b\*log(c))\*sin(2\*b\*log(c)))\*x^2\*cos(2\*b\*log(x^n) + 2\*a) + ((b\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)))\*n + cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + sin(6\*b\*log(c))\*sin(4\*b\*log(c)))\*x^2\*sin(4\*b\*log(x^n) + 4\*a) + (3\*(b^2\*cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(2\*b\*log(c)))\*n + 2\*cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + 2\*sin(6\*b\*log(c))\*sin(2\*b\*log(c)))\*x^2\*sin(2\*b\*log(x^n) + 2\*a) - (b^2\*n^2\*sin(6\*b\*log(c)) + sin(6\*b\*log(c)))\*x^2\*cos(6\*b\*log(x^n) + 6\*a) - (3\*(3\*(b^2\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2

$$\begin{aligned}
& - 2*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c))) \\
& *n + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*x^2 \\
& *\cos(2*b*\log(x^n) + 2*a) - 3*(3*(b^2*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^2* \\
& \sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^2 + 2*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) \\
& - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n + \cos(4*b*\log(c))*\cos(2*b*\log(c)) + \\
& \sin(4*b*\log(c))*\sin(2*b*\log(c)))*x^2*\sin(2*b*\log(x^n) + 2*a) + (3*b^2*n^2* \\
& \sin(4*b*\log(c)) - b*n*\cos(4*b*\log(c)) + 2*\sin(4*b*\log(c)))*x^2)*\cos(4*b*\log \\
& (x^n) + 4*a) + 18*(b^8*n^8 + b^6*n^6 + ((b^8*\cos(6*b*\log(c))^2 + b^8*\sin(6* \\
& b*\log(c))^2)*n^8 + (b^6*\cos(6*b*\log(c))^2 + b^6*\sin(6*b*\log(c))^2)*n^6)*\cos \\
& (6*b*\log(x^n) + 6*a)^2 + 9*((b^8*\cos(4*b*\log(c))^2 + b^8*\sin(4*b*\log(c))^2) \\
& *n^8 + (b^6*\cos(4*b*\log(c))^2 + b^6*\sin(4*b*\log(c))^2)*n^6)*\cos(4*b*\log(x^n \\
& ) + 4*a)^2 + 9*((b^8*\cos(2*b*\log(c))^2 + b^8*\sin(2*b*\log(c))^2)*n^8 + (b^6* \\
& \cos(2*b*\log(c))^2 + b^6*\sin(2*b*\log(c))^2)*n^6)*\cos(2*b*\log(x^n) + 2*a)^2 + \\
& ((b^8*\cos(6*b*\log(c))^2 + b^8*\sin(6*b*\log(c))^2)*n^8 + (b^6*\cos(6*b*\log(c) \\
& )^2 + b^6*\sin(6*b*\log(c))^2)*n^6)*\sin(6*b*\log(x^n) + 6*a)^2 + 9*((b^8*\cos(4 \\
& *b*\log(c))^2 + b^8*\sin(4*b*\log(c))^2)*n^8 + (b^6*\cos(4*b*\log(c))^2 + b^6*si \\
& n(4*b*\log(c))^2)*n^6)*\sin(4*b*\log(x^n) + 4*a)^2 + 9*((b^8*\cos(2*b*\log(c))^2 \\
& + b^8*\sin(2*b*\log(c))^2)*n^8 + (b^6*\cos(2*b*\log(c))^2 + b^6*\sin(2*b*\log(c) \\
& )^2)*n^6)*\sin(2*b*\log(x^n) + 2*a)^2 + 2*(b^8*n^8*\cos(6*b*\log(c)) + b^6*n^6* \\
& \cos(6*b*\log(c)) + 3*((b^8*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^8*\sin(6*b*\log \\
& (c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^6*\sin( \\
& 6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*\cos(4*b*\log(x^n) + 4*a) + 3*((b^8*\cos(6*b \\
& *log(c))*\cos(2*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6* \\
& \cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^6) \\
& *\cos(2*b*\log(x^n) + 2*a) + 3*((b^8*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^8*co \\
& s(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - \\
& b^6*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*\sin(4*b*\log(x^n) + 4*a) + 3*((b^ \\
& 8*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^ \\
& 8 + (b^6*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(2*b*\log( \\
& c)))*n^6)*\sin(2*b*\log(x^n) + 2*a)*\cos(6*b*\log(x^n) + 6*a) + 6*(b^8*n^8*\cos \\
& (4*b*\log(c)) + b^6*n^6*\cos(4*b*\log(c)) + 3*((b^8*\cos(4*b*\log(c))*\cos(2*b*lo \\
& g(c)) + b^8*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c))*\cos \\
& (2*b*\log(c)) + b^6*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\cos(2*b*\log(x^n) + \\
& 2*a) + 3*((b^8*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^8*\cos(4*b*\log(c))*\sin(2 \\
& *b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^6*\cos(4*b*\log(c) \\
& )*\sin(2*b*\log(c)))*n^6)*\sin(2*b*\log(x^n) + 2*a)*\cos(4*b*\log(x^n) + 4*a) + \\
& 6*(b^8*n^8*\cos(2*b*\log(c)) + b^6*n^6*\cos(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2* \\
& a) - 2*(b^8*n^8*\sin(6*b*\log(c)) + b^6*n^6*\sin(6*b*\log(c)) + 3*((b^8*\cos(4*b \\
& *log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6* \\
& \cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^6) \\
& *\cos(4*b*\log(x^n) + 4*a) + 3*((b^8*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^8*co \\
& s(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \\
& b^6*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\cos(2*b*\log(x^n) + 2*a) - 3*((b^ \\
& 8*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^ \\
& 8 + (b^6*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(4*b*\log( \\
& c)))*n^6)*\sin(4*b*\log(x^n) + 4*a) - 3*((b^8*\cos(6*b*\log(c))*\cos(2*b*\log(c)) \\
& + b^8*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(2*b* \\
& log(c)) + b^6*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\sin(2*b*\log(x^n) + 2*a) \\
& )*\sin(6*b*\log(x^n) + 6*a) - 6*(b^8*n^8*\sin(4*b*\log(c)) + b^6*n^6*\sin(4*b*lo \\
& g(c)) + 3*((b^8*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^8*\cos(4*b*\log(c))*\sin(2 \\
& *b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^6*\cos(4*b*\log(c) \\
& )*\sin(2*b*\log(c)))*n^6)*\cos(2*b*\log(x^n) + 2*a) - 3*((b^8*\cos(4*b*\log(c))*c \\
& os(2*b*\log(c)) + b^8*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(4*b*lo \\
& g(c))*\cos(2*b*\log(c)) + b^6*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\sin(2*b*1 \\
& og(x^n) + 2*a))*\sin(4*b*\log(x^n) + 4*a) - 6*(b^8*n^8*\sin(2*b*\log(c)) + b^6* \\
& n^6*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a))*integrate(1/9*(x*\cos(2*b*\log( \\
& x^n) + 2*a)*\sin(2*b*\log(c)) + x*\cos(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a))/(2 \\
& *b^6*n^6*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) - 2*b^6*n^6*\sin(2*b*\log(c) \\
& )*\sin(2*b*\log(x^n) + 2*a) + b^6*n^6 + (b^6*\cos(2*b*\log(c))^2 + b^6*\sin(2*b*
\end{aligned}$$

$$\begin{aligned} & \log(c)^2 * n^6 * \cos(2*b*\log(x^n) + 2*a)^2 + (b^6 * \cos(2*b*\log(c))^2 + b^6 * \sin \\ & (2*b*\log(c))^2 * n^6 * \sin(2*b*\log(x^n) + 2*a)^2), x) - (((b*\cos(4*b*\log(c))*s \\ & \sin(6*b*\log(c)) - b*\cos(6*b*\log(c))*\sin(4*b*\log(c))) * n + \cos(6*b*\log(c))*\cos \\ & (4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c))) * x^2 * \cos(4*b*\log(x^n) + 4*a) \\ & + (3*(b^2*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^2*\sin(6*b*\log(c))*\sin(2*b*\log \\ & (c))) * n^2 + (b*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b*\cos(6*b*\log(c))*\sin(2*b* \\ & *\log(c))) * n + 2*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + 2*\sin(6*b*\log(c))*\sin(2*b \\ & *\log(c))) * x^2 * \cos(2*b*\log(x^n) + 2*a) - ((b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) \\ & + b*\sin(6*b*\log(c))*\sin(4*b*\log(c))) * n - \cos(4*b*\log(c))*\sin(6*b*\log(c)) + \\ & \cos(6*b*\log(c))*\sin(4*b*\log(c))) * x^2 * \sin(4*b*\log(x^n) + 4*a) + (3*(b^2*\cos \\ & (2*b*\log(c))*\sin(6*b*\log(c)) - b^2*\cos(6*b*\log(c))*\sin(2*b*\log(c))) * n^2 - ( \\ & b*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(2*b*\log(c))) * n + \\ & 2*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - 2*\cos(6*b*\log(c))*\sin(2*b*\log(c))) * x^2 * \\ & \sin(2*b*\log(x^n) + 2*a) + (b^2 * n^2 * \cos(6*b*\log(c)) + \cos(6*b*\log(c))) * x^2 * \\ & \sin(6*b*\log(x^n) + 6*a) - (3*(3*(b^2*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^2* \\ & \sin(4*b*\log(c))*\sin(2*b*\log(c))) * n^2 + 2*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) \\ & - b*\cos(4*b*\log(c))*\sin(2*b*\log(c))) * n + \cos(4*b*\log(c))*\cos(2*b*\log(c)) + \\ & \sin(4*b*\log(c))*\sin(2*b*\log(c))) * x^2 * \cos(2*b*\log(x^n) + 2*a) + 3*(3*(b^2*\cos \\ & (2*b*\log(c))*\sin(4*b*\log(c)) - b^2*\cos(4*b*\log(c))*\sin(2*b*\log(c))) * n^2 - \\ & 2*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c))) * \\ & n + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c))) * x^2 * \\ & \sin(2*b*\log(x^n) + 2*a) + (3*b^2 * n^2 * \cos(4*b*\log(c)) + b * n * \sin(4*b*\log(c)) \\ & + 2*\cos(4*b*\log(c))) * x^2 * \sin(4*b*\log(x^n) + 4*a) / (6*b^3 * n^3 * \cos(2*b*\log(c)) \\ & ) * \cos(2*b*\log(x^n) + 2*a) - 6*b^3 * n^3 * \sin(2*b*\log(c)) * \sin(2*b*\log(x^n) + 2 \\ & *a) + b^3 * n^3 + (b^3 * \cos(6*b*\log(c))^2 + b^3 * \sin(6*b*\log(c))^2) * n^3 * \cos(6*b \\ & *\log(x^n) + 6*a)^2 + 9*(b^3 * \cos(4*b*\log(c))^2 + b^3 * \sin(4*b*\log(c))^2) * n^3 * \\ & \cos(4*b*\log(x^n) + 4*a)^2 + 9*(b^3 * \cos(2*b*\log(c))^2 + b^3 * \sin(2*b*\log(c))^2) \\ & * n^3 * \cos(2*b*\log(x^n) + 2*a)^2 + (b^3 * \cos(6*b*\log(c))^2 + b^3 * \sin(6*b*\log \\ & (c))^2) * n^3 * \sin(6*b*\log(x^n) + 6*a)^2 + 9*(b^3 * \cos(4*b*\log(c))^2 + b^3 * \sin( \\ & 4*b*\log(c))^2) * n^3 * \sin(4*b*\log(x^n) + 4*a)^2 + 9*(b^3 * \cos(2*b*\log(c))^2 + b \\ & ^3 * \sin(2*b*\log(c))^2) * n^3 * \sin(2*b*\log(x^n) + 2*a)^2 + 2*(b^3 * n^3 * \cos(6*b*\log \\ & (c)) + 3*(b^3 * \cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^3 * \sin(6*b*\log(c))*\sin(4* \\ & b*\log(c))) * n^3 * \cos(4*b*\log(x^n) + 4*a) + 3*(b^3 * \cos(6*b*\log(c))*\cos(2*b*\log \\ & (c)) + b^3 * \sin(6*b*\log(c))*\sin(2*b*\log(c))) * n^3 * \cos(2*b*\log(x^n) + 2*a) + 3 \\ & *(b^3 * \cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^3 * \cos(6*b*\log(c))*\sin(4*b*\log(c)) \\ & ) * n^3 * \sin(4*b*\log(x^n) + 4*a) + 3*(b^3 * \cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^3 \\ & * \cos(6*b*\log(c))*\sin(2*b*\log(c))) * n^3 * \sin(2*b*\log(x^n) + 2*a) * \cos(6*b*\log \\ & (x^n) + 6*a) + 6*(b^3 * n^3 * \cos(4*b*\log(c)) + 3*(b^3 * \cos(4*b*\log(c))*\cos(2*b* \\ & \log(c)) + b^3 * \sin(4*b*\log(c))*\sin(2*b*\log(c))) * n^3 * \cos(2*b*\log(x^n) + 2*a) \\ & + 3*(b^3 * \cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^3 * \cos(4*b*\log(c))*\sin(2*b*\log \\ & (c))) * n^3 * \sin(2*b*\log(x^n) + 2*a) * \cos(4*b*\log(x^n) + 4*a) - 2*(b^3 * n^3 * \sin( \\ & 6*b*\log(c)) + 3*(b^3 * \cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^3 * \cos(6*b*\log(c))* \\ & \sin(4*b*\log(c))) * n^3 * \cos(4*b*\log(x^n) + 4*a) + 3*(b^3 * \cos(2*b*\log(c))*\sin(6 \\ & *b*\log(c)) - b^3 * \cos(6*b*\log(c))*\sin(2*b*\log(c))) * n^3 * \cos(2*b*\log(x^n) + 2* \\ & a) - 3*(b^3 * \cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^3 * \sin(6*b*\log(c))*\sin(4*b*\log \\ & (c))) * n^3 * \sin(4*b*\log(x^n) + 4*a) - 3*(b^3 * \cos(6*b*\log(c))*\cos(2*b*\log(c)) \\ & ) + b^3 * \sin(6*b*\log(c))*\sin(2*b*\log(c))) * n^3 * \sin(2*b*\log(x^n) + 2*a) * \sin(6 \\ & *b*\log(x^n) + 6*a) - 6*(b^3 * n^3 * \sin(4*b*\log(c)) + 3*(b^3 * \cos(2*b*\log(c))*\sin \\ & (4*b*\log(c)) - b^3 * \cos(4*b*\log(c))*\sin(2*b*\log(c))) * n^3 * \cos(2*b*\log(x^n) + \\ & 2*a) - 3*(b^3 * \cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^3 * \sin(4*b*\log(c))*\sin(2* \\ & b*\log(c))) * n^3 * \sin(2*b*\log(x^n) + 2*a) * \sin(4*b*\log(x^n) + 4*a)) \end{aligned}$$


---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x \sec(b \log(cx^n) + a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(x*sec(b*log(c*x^n) + a)^4, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n))**4,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(b \log(cx^n) + a)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*log(c*x^n) + a)^4, x)
```

### 3.255 $\int \sec^4(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=85

$$\frac{16e^{4ia}x(cx^n)^{4ib} \operatorname{Hypergeometric2F1}\left(4, \frac{1}{2}\left(4 - \frac{i}{bn}\right), \frac{1}{2}\left(6 - \frac{i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{1 + 4ibn}$$

[Out] (16\*E^((4\*I)\*a))\*x\*(c\*x^n)^((4\*I)\*b)\*Hypergeometric2F1[4, (4 - I/(b\*n))/2, (6 - I/(b\*n))/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(1 + (4\*I)\*b\*n)

**Rubi [A]** time = 0.0625623, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4503, 4505, 364}

$$\frac{16e^{4ia}x(cx^n)^{4ib} {}_2F_1\left(4, \frac{1}{2}\left(4 - \frac{i}{bn}\right); \frac{1}{2}\left(6 - \frac{i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{1 + 4ibn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^4, x]

[Out] (16\*E^((4\*I)\*a))\*x\*(c\*x^n)^((4\*I)\*b)\*Hypergeometric2F1[4, (4 - I/(b\*n))/2, (6 - I/(b\*n))/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(1 + (4\*I)\*b\*n)

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/ (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sec^4(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sec^4(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(16e^{4ia}x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+4ib+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^4} dx, x, cx^n\right)}{n} \\ &= \frac{16e^{4ia}x(cx^n)^{4ib} {}_2F_1\left(4, \frac{1}{2}\left(4 - \frac{i}{bn}\right); \frac{1}{2}\left(6 - \frac{i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{1 + 4ibn} \end{aligned}$$

**Mathematica [B]** time = 10.7975, size = 213, normalized size = 2.51

$$x \left( -2i(4b^2n^2 + 1) \text{Hypergeometric2F1} \left( 1, -\frac{i}{2bn}, 1 - \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))} \right) + 2e^{2ia}(2bn + i)(cx^n)^{2ib} \text{Hypergeometric2F1} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^4,x]

[Out] (x\*(2\*E^((2\*I)\*a)\*(I + 2\*b\*n)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[1, 1 - (I/2)/(b\*n), 2 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] - (2\*I)\*(1 + 4\*b^2\*n^2)\*Hypergeometric2F1[1, (-I/2)/(b\*n), 1 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + Sec[a + b\*Log[c\*x^n]]^2\*(-2\*b\*n + (1 + 8\*b^2\*n^2 + (1 + 4\*b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n]])\*Tan[a + b\*Log[c\*x^n]])))/(12\*b^3\*n^3)

**Maple [F]** time = 1.325, size = 0, normalized size = 0.

$$\int (\sec(a + b \ln(cx^n)))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^4,x)

[Out] int(sec(a+b\*ln(c\*x^n))^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4,x, algorithm="maxima")

[Out] -1/3\*(6\*(b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*x\*cos(4\*b\*log(x^n) + 4\*a)^2 + 6\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*x\*cos(2\*b\*log(x^n) + 2\*a)^2 + 6\*(b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*x\*sin(4\*b\*log(x^n) + 4\*a)^2 + 6\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*x\*sin(2\*b\*log(x^n) + 2\*a)^2 + (2\*b\*n\*cos(2\*b\*log(c)) - sin(2\*b\*log(c)))\*x\*cos(2\*b\*log(x^n) + 2\*a) - (2\*b\*n\*sin(2\*b\*log(c)) + cos(2\*b\*log(c)))\*x\*sin(2\*b\*log(x^n) + 2\*a) + ((2\*(b\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(4\*b\*log(c)))\*n - cos(4\*b\*log(c))\*sin(6\*b\*log(c)) + cos(6\*b\*log(c))\*sin(4\*b\*log(c)))\*x\*cos(4\*b\*log(x^n) + 4\*a) - 2\*(6\*(b^2\*cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - b^2\*cos(6\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 - (b\*cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(2\*b\*log(c)))\*n + cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - cos(6\*b\*log(c))\*sin(2\*b\*log(c)))\*x\*cos(2\*b\*log(x^n) + 2\*a) + (2\*(b\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(4\*b\*log(c)))\*n + cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + sin(6\*b\*log(c))\*sin(4\*b\*log(c)))\*x\*sin(4\*b\*log(x^n) + 4\*a) + 2\*(6\*(b^2\*cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + b^2\*sin(6\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 + (b\*cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(2\*b\*log(c)))\*n + cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + sin(6\*b\*log(c))\*sin(2\*b\*log(c)))\*x\*sin(2\*b\*log(x^n) + 2\*a) - (4\*b^2\*n^2\*sin(6\*b\*log(c)) + sin(6\*b\*log(c)))\*x\*cos(6\*b\*log(x^n) + 6\*a) - (3\*(12\*(b^2\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b^2\*cos(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n^2 - 4\*(b\*cos(4\*b\*log(c))



$$\begin{aligned}
& \log(c)) \cos(2b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c)) \cdot n + \cos(2b \log(c)) \sin(4b \log(c)) - \cos(4b \log(c)) \sin(2b \log(c)) \cdot x \cos(2b \log(x^n) \\
& + 2a) - 3(12(b^2 \cos(4b \log(c)) \cos(2b \log(c)) + b^2 \sin(4b \log(c)) \sin(2b \log(c))) \cdot n^2 + 4(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(2b \log(c))) \cdot n + \cos(4b \log(c)) \cos(2b \log(c)) + \sin(4b \log(c)) \\
& \sin(2b \log(c)) \cdot x \sin(2b \log(x^n) + 2a) + 2(6b^2 n^2 \sin(4b \log(c)) - b n \cos(4b \log(c)) + \sin(4b \log(c))) \cdot x \cos(4b \log(x^n) + 4a) + 9(4b^8 n^8 + b^6 n^6 + (4(b^8 \cos(6b \log(c))^2 + b^8 \sin(6b \log(c))^2) \cdot n^8 \\
& + (b^6 \cos(6b \log(c))^2 + b^6 \sin(6b \log(c))^2) \cdot n^6) \cos(6b \log(x^n) + 6a)^2 + 9(4(b^8 \cos(4b \log(c))^2 + b^8 \sin(4b \log(c))^2) \cdot n^8 + (b^6 \cos(4b \log(c))^2 + b^6 \sin(4b \log(c))^2) \cdot n^6) \cos(4b \log(x^n) + 4a)^2 + 9(4(b^8 \cos(2b \log(c))^2 + b^8 \sin(2b \log(c))^2) \cdot n^8 + (b^6 \cos(2b \log(c))^2 + b^6 \sin(2b \log(c))^2) \cdot n^6) \cos(2b \log(x^n) + 2a)^2 + (4(b^8 \cos(6b \log(c))^2 + b^8 \sin(6b \log(c))^2) \cdot n^8 + (b^6 \cos(6b \log(c))^2 + b^6 \sin(6b \log(c))^2) \cdot n^6) \sin(6b \log(x^n) + 6a)^2 + 9(4(b^8 \cos(4b \log(c))^2 + b^8 \sin(4b \log(c))^2) \cdot n^8 + (b^6 \cos(4b \log(c))^2 + b^6 \sin(4b \log(c))^2) \cdot n^6) \sin(4b \log(x^n) + 4a)^2 + 9(4(b^8 \cos(2b \log(c))^2 + b^8 \sin(2b \log(c))^2) \cdot n^8 + (b^6 \cos(2b \log(c))^2 + b^6 \sin(2b \log(c))^2) \cdot n^6) \sin(2b \log(x^n) + 2a)^2 + 2(4b^8 n^8 \cos(6b \log(c)) + b^6 n^6 \cos(6b \log(c)) + 3(4(b^8 \cos(6b \log(c)) \cos(4b \log(c)) + b^8 \sin(6b \log(c)) \sin(4b \log(c))) \cdot n^8 + (b^6 \cos(6b \log(c)) \cos(4b \log(c)) + b^6 \sin(6b \log(c)) \sin(4b \log(c))) \cdot n^6) \cos(4b \log(x^n) + 4a) + 3(4(b^8 \cos(6b \log(c)) \cos(2b \log(c)) + b^8 \sin(6b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(6b \log(c)) \cos(2b \log(c)) + b^6 \sin(6b \log(c)) \sin(2b \log(c))) \cdot n^6) \cos(2b \log(x^n) + 2a) + 3(4(b^8 \cos(4b \log(c)) \sin(6b \log(c)) - b^8 \cos(6b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(4b \log(c)) \sin(6b \log(c)) - b^6 \cos(6b \log(c)) \sin(2b \log(c))) \cdot n^6) \sin(4b \log(x^n) + 4a) + 3(4(b^8 \cos(2b \log(c)) \sin(6b \log(c)) - b^8 \cos(6b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(2b \log(c)) \sin(6b \log(c)) - b^6 \cos(6b \log(c)) \sin(2b \log(c))) \cdot n^6) \sin(2b \log(x^n) + 2a)) \cos(6b \log(x^n) + 6a) + 6(4b^8 n^8 \cos(4b \log(c)) + b^6 n^6 \cos(4b \log(c)) + 3(4(b^8 \cos(4b \log(c)) \cos(2b \log(c)) + b^8 \sin(4b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(4b \log(c)) \cos(2b \log(c)) + b^6 \sin(4b \log(c)) \sin(2b \log(c))) \cdot n^6) \cos(2b \log(x^n) + 2a) + 3(4(b^8 \cos(2b \log(c)) \sin(4b \log(c)) - b^8 \cos(4b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(2b \log(c)) \sin(4b \log(c)) - b^6 \cos(4b \log(c)) \sin(2b \log(c))) \cdot n^6) \sin(2b \log(x^n) + 2a)) \cos(4b \log(x^n) + 4a) + 6(4b^8 n^8 \cos(2b \log(c)) + b^6 n^6 \cos(2b \log(c))) \cos(2b \log(x^n) + 2a) - 2(4b^8 n^8 \sin(6b \log(c)) + b^6 n^6 \sin(6b \log(c)) + 3(4(b^8 \cos(4b \log(c)) \sin(6b \log(c)) - b^8 \cos(6b \log(c)) \sin(4b \log(c))) \cdot n^8 + (b^6 \cos(4b \log(c)) \sin(6b \log(c)) - b^6 \cos(6b \log(c)) \sin(4b \log(c))) \cdot n^6) \cos(4b \log(x^n) + 4a) + 3(4(b^8 \cos(2b \log(c)) \sin(6b \log(c)) - b^8 \cos(6b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(2b \log(c)) \sin(6b \log(c)) - b^6 \cos(6b \log(c)) \sin(2b \log(c))) \cdot n^6) \cos(2b \log(x^n) + 2a) - 3(4(b^8 \cos(6b \log(c)) \cos(4b \log(c)) + b^8 \sin(6b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(6b \log(c)) \cos(4b \log(c)) + b^6 \sin(6b \log(c)) \sin(2b \log(c))) \cdot n^6) \sin(2b \log(x^n) + 2a)) \sin(6b \log(x^n) + 6a) - 6(4b^8 n^8 \sin(4b \log(c)) + b^6 n^6 \sin(4b \log(c)) + 3(4(b^8 \cos(2b \log(c)) \sin(4b \log(c)) - b^8 \cos(4b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(2b \log(c)) \sin(4b \log(c)) - b^6 \cos(4b \log(c)) \sin(2b \log(c))) \cdot n^6) \cos(2b \log(x^n) + 2a) - 3(4(b^8 \cos(4b \log(c)) \cos(2b \log(c)) + b^8 \sin(4b \log(c)) \sin(2b \log(c))) \cdot n^8 + (b^6 \cos(4b \log(c)) \cos(2b \log(c)) + b^6 \sin(4b \log(c)) \sin(2b \log(c))) \cdot n^6) \sin(2b \log(x^n) + 2a)) \sin(4b \log(x^n) + 4a) - 6(4b^8 n^8 \sin(2b \log(c)) + b^6 n^6 \sin(2b \log(c))) \sin(2b \log(x^n) + 2a)) \int (1/9(\cos(2b \log(x^n) + 2a) \sin(2b \log(c)) + \cos(2b \log(c)) \sin(2b \log(x^n) + 2a)) / (2b^6 n^6 \cos(2b \log(c)) \cos(2b \log(x^n) + 2a) - 2b^6 n^6 \sin(2b \log(c)) \sin(2b \log(x^n) + 2a) + b^6 n^6 + (b^6 \cos(2b
\end{aligned}$$

```

*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^6*cos(2*b*log(x^n) + 2*a)^2 + (b^6*cos
s(2*b*log(c))^2 + b^6*sin(2*b*log(c))^2)*n^6*sin(2*b*log(x^n) + 2*a)^2), x)
- ((2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)
))) *n + cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c))) *
x*cos(4*b*log(x^n) + 4*a) + 2*(6*(b^2*cos(6*b*log(c))*cos(2*b*log(c)) + b^2
*sin(6*b*log(c))*sin(2*b*log(c)))*n^2 + (b*cos(2*b*log(c))*sin(6*b*log(c))
- b*cos(6*b*log(c))*sin(2*b*log(c)))*n + cos(6*b*log(c))*cos(2*b*log(c)) +
sin(6*b*log(c))*sin(2*b*log(c))) *x*cos(2*b*log(x^n) + 2*a) - (2*(b*cos(6*b*
log(c))*cos(4*b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n - cos(4*b*lo
g(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4*b*log(c))) *x*sin(4*b*log(x^n)
+ 4*a) + 2*(6*(b^2*cos(2*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*s
in(2*b*log(c)))*n^2 - (b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c)
)*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*si
n(2*b*log(c))) *x*sin(2*b*log(x^n) + 2*a) + (4*b^2*n^2*cos(6*b*log(c)) + cos
(6*b*log(c))) *x*sin(6*b*log(x^n) + 6*a) - (3*(12*(b^2*cos(4*b*log(c))*cos(
2*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n^2 + 4*(b*cos(2*b*log(c)
))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n + cos(4*b*log(c))
*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c))) *x*cos(2*b*log(x^n) + 2*
a) + 3*(12*(b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2
*b*log(c)))*n^2 - 4*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*
sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(
2*b*log(c))) *x*sin(2*b*log(x^n) + 2*a) + 2*(6*b^2*n^2*cos(4*b*log(c)) + b*n
*sin(4*b*log(c)) + cos(4*b*log(c))) *x*sin(4*b*log(x^n) + 4*a))/(6*b^3*n^3*
cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 6*b^3*n^3*sin(2*b*log(c))*sin(2*b
*log(x^n) + 2*a) + b^3*n^3 + (b^3*cos(6*b*log(c))^2 + b^3*sin(6*b*log(c))^2
)*n^3*cos(6*b*log(x^n) + 6*a)^2 + 9*(b^3*cos(4*b*log(c))^2 + b^3*sin(4*b*lo
g(c))^2)*n^3*cos(4*b*log(x^n) + 4*a)^2 + 9*(b^3*cos(2*b*log(c))^2 + b^3*sin
(2*b*log(c))^2)*n^3*cos(2*b*log(x^n) + 2*a)^2 + (b^3*cos(6*b*log(c))^2 + b^
3*sin(6*b*log(c))^2)*n^3*sin(6*b*log(x^n) + 6*a)^2 + 9*(b^3*cos(4*b*log(c)
)^2 + b^3*sin(4*b*log(c))^2)*n^3*sin(4*b*log(x^n) + 4*a)^2 + 9*(b^3*cos(2*b*
log(c))^2 + b^3*sin(2*b*log(c))^2)*n^3*sin(2*b*log(x^n) + 2*a)^2 + 2*(b^3*n
^3*cos(6*b*log(c)) + 3*(b^3*cos(6*b*log(c))*cos(4*b*log(c)) + b^3*sin(6*b*l
og(c))*sin(4*b*log(c)))*n^3*cos(4*b*log(x^n) + 4*a) + 3*(b^3*cos(6*b*log(c)
))*cos(2*b*log(c)) + b^3*sin(6*b*log(c))*sin(2*b*log(c)))*n^3*cos(2*b*log(x^
n) + 2*a) + 3*(b^3*cos(4*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*si
n(4*b*log(c)))*n^3*sin(4*b*log(x^n) + 4*a) + 3*(b^3*cos(2*b*log(c))*sin(6*b
*log(c)) - b^3*cos(6*b*log(c))*sin(2*b*log(c)))*n^3*sin(2*b*log(x^n) + 2*a)
)*cos(6*b*log(x^n) + 6*a) + 6*(b^3*n^3*cos(4*b*log(c)) + 3*(b^3*cos(4*b*log
(c))*cos(2*b*log(c)) + b^3*sin(4*b*log(c))*sin(2*b*log(c)))*n^3*cos(2*b*log
(x^n) + 2*a) + 3*(b^3*cos(2*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))
*sin(2*b*log(c)))*n^3*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 2*
(b^3*n^3*sin(6*b*log(c)) + 3*(b^3*cos(4*b*log(c))*sin(6*b*log(c)) - b^3*cos
(6*b*log(c))*sin(4*b*log(c)))*n^3*cos(4*b*log(x^n) + 4*a) + 3*(b^3*cos(2*b*
log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(2*b*log(c)))*n^3*cos(2*b*
log(x^n) + 2*a) - 3*(b^3*cos(6*b*log(c))*cos(4*b*log(c)) + b^3*sin(6*b*log(
c))*sin(4*b*log(c)))*n^3*sin(4*b*log(x^n) + 4*a) - 3*(b^3*cos(6*b*log(c))*c
os(2*b*log(c)) + b^3*sin(6*b*log(c))*sin(2*b*log(c)))*n^3*sin(2*b*log(x^n)
+ 2*a))*sin(6*b*log(x^n) + 6*a) - 6*(b^3*n^3*sin(4*b*log(c)) + 3*(b^3*cos(2
*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3*cos(2
*b*log(x^n) + 2*a) - 3*(b^3*cos(4*b*log(c))*cos(2*b*log(c)) + b^3*sin(4*b*l
og(c))*sin(2*b*log(c)))*n^3*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4*a
))

```

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**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec(b \log(cx^n) + a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^4, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^4(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**4,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**4, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^4, x)
```

$$3.256 \quad \int \frac{\sec^4(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=42

$$\frac{\tan^3(a+b \log(cx^n))}{3bn} + \frac{\tan(a+b \log(cx^n))}{bn}$$

[Out] Tan[a + b\*Log[c\*x^n]]/(b\*n) + Tan[a + b\*Log[c\*x^n]]^3/(3\*b\*n)

**Rubi [A]** time = 0.034284, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {3767}

$$\frac{\tan^3(a+b \log(cx^n))}{3bn} + \frac{\tan(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^4/x, x]

[Out] Tan[a + b\*Log[c\*x^n]]/(b\*n) + Tan[a + b\*Log[c\*x^n]]^3/(3\*b\*n)

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :-> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{\sec^4(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sec^4(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\text{Subst}\left(\int (1+x^2) dx, x, -\tan(a+b \log(cx^n))\right)}{bn} \\ &= \frac{\tan(a+b \log(cx^n))}{bn} + \frac{\tan^3(a+b \log(cx^n))}{3bn} \end{aligned}$$

**Mathematica [A]** time = 0.115666, size = 36, normalized size = 0.86

$$\frac{\frac{1}{3} \tan^3(a+b \log(cx^n)) + \tan(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^4/x, x]

[Out] (Tan[a + b\*Log[c\*x^n]] + Tan[a + b\*Log[c\*x^n]]^3/3)/(b\*n)

**Maple [A]** time = 0.043, size = 37, normalized size = 0.9

$$-\frac{\tan(a+b \ln(cx^n))}{bn} \left( \frac{2}{3} - \frac{(\sec(a+b \ln(cx^n)))^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(a+b*ln(c*x^n))^4/x,x)
```

```
[Out] -1/n/b*(-2/3-1/3*sec(a+b*ln(c*x^n))^2)*tan(a+b*ln(c*x^n))
```

**Maxima [B]** time = 1.16045, size = 1786, normalized size = 42.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 4/3*((3*(cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))
*cos(2*b*log(x^n) + 2*a) - 3*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log
(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(6*b*log(c))*cos(6*b*lo
g(x^n) + 6*a) + 3*(3*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin
(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 3*(cos(4*b*log(c))*cos(2*b*log(c))
+ sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(4*b*log(c)
))*cos(4*b*log(x^n) + 4*a) + (3*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*
log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(cos(2*b*log(c))*sin(6
*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + cos
(6*b*log(c))*sin(6*b*log(x^n) + 6*a) + 3*(3*(cos(4*b*log(c))*cos(2*b*log(c)
)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(cos(2*b*
log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n)
+ 2*a) + cos(4*b*log(c))*sin(4*b*log(x^n) + 4*a))/((b*cos(6*b*log(c))^2 +
b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2
+ b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 + 6*b*n*cos(2*b*log(c))*
cos(2*b*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*c
os(2*b*log(x^n) + 2*a)^2 + (b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*si
n(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*si
n(4*b*log(x^n) + 4*a)^2 - 6*b*n*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) +
9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) + 2*a)^2 +
b*n + 2*(b*n*cos(6*b*log(c)) + 3*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*si
n(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) + 3*(b*cos(6*b*log
(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n)
+ 2*a) + 3*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b
*log(c)))*n*sin(4*b*log(x^n) + 4*a) + 3*(b*cos(2*b*log(c))*sin(6*b*log(c))
- b*cos(6*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*cos(6*b*log
(x^n) + 6*a) + 6*(b*n*cos(4*b*log(c)) + 3*(b*cos(4*b*log(c))*cos(2*b*log(c)
) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a) + 3*(b*cos
(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*sin(2*b
*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 2*(3*(b*cos(4*b*log(c))*sin(6*b
*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n*cos(4*b*log(x^n) + 4*a) + 3
*(b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n*
cos(2*b*log(x^n) + 2*a) + b*n*sin(6*b*log(c)) - 3*(b*cos(6*b*log(c))*cos(4*
b*log(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*sin(4*b*log(x^n) + 4*a) -
3*(b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n
*sin(2*b*log(x^n) + 2*a))*sin(6*b*log(x^n) + 6*a) - 6*(3*(b*cos(2*b*log(c)
))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) +
2*a) + b*n*sin(4*b*log(c)) - 3*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4
*b*log(c))*sin(2*b*log(c)))*n*sin(2*b*log(x^n) + 2*a))*sin(4*b*log(x^n) + 4
*a))
```

**Fricas [A]** time = 0.47226, size = 157, normalized size = 3.74

$$\frac{(2 \cos(bn \log(x) + b \log(c) + a)^2 + 1) \sin(bn \log(x) + b \log(c) + a)}{3bn \cos(bn \log(x) + b \log(c) + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4/x,x, algorithm="fricas")

[Out] 1/3\*(2\*cos(b\*n\*log(x) + b\*log(c) + a)^2 + 1)\*sin(b\*n\*log(x) + b\*log(c) + a) / (b\*n\*cos(b\*n\*log(x) + b\*log(c) + a)^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(a + b \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*4/x,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*4/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4/x,x, algorithm="giac")

[Out] integrate(sec(b\*log(c\*x^n) + a)^4/x, x)

$$3.257 \quad \int \frac{\sec^4(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=87

$$\frac{16e^{4ia} (cx^n)^{4ib} \operatorname{Hypergeometric2F1}\left(4, \frac{1}{2}\left(4 + \frac{i}{bn}\right), \frac{1}{2}\left(6 + \frac{i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{x(1 - 4ibn)}$$

[Out] (-16\*E^((4\*I)\*a)\*(c\*x^n)^((4\*I)\*b)\*Hypergeometric2F1[4, (4 + I/(b\*n))/2, (6 + I/(b\*n))/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))])/((1 - (4\*I)\*b\*n)\*x)

**Rubi [A]** time = 0.0753623, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{16e^{4ia} (cx^n)^{4ib} {}_2F_1\left(4, \frac{1}{2}\left(4 + \frac{i}{bn}\right); \frac{1}{2}\left(6 + \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{x(1 - 4ibn)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^4/x^2, x]

[Out] (-16\*E^((4\*I)\*a)\*(c\*x^n)^((4\*I)\*b)\*Hypergeometric2F1[4, (4 + I/(b\*n))/2, (6 + I/(b\*n))/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))])/((1 - (4\*I)\*b\*n)\*x)

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\sec^4(a + b \log(cx^n))}{x^2} dx &= \frac{(cx^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int x^{-1-\frac{1}{n}} \sec^4(a + b \log(x)) dx, x, cx^n\right)}{nx} \\ &= \frac{\left(16e^{4ia} (cx^n)^{\frac{1}{n}}\right) \operatorname{Subst}\left(\int \frac{x^{-1+4ib-\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^4} dx, x, cx^n\right)}{nx} \\ &= \frac{16e^{4ia} (cx^n)^{4ib} {}_2F_1\left(4, \frac{1}{2}\left(4 + \frac{i}{bn}\right); \frac{1}{2}\left(6 + \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(1 - 4ibn)x} \end{aligned}$$

**Mathematica [B]** time = 9.45248, size = 215, normalized size = 2.47

$$\frac{-2i(4b^2n^2 + 1) \operatorname{Hypergeometric2F1}\left(1, \frac{i}{2bn}, 1 + \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right) - 2e^{2ia}(2bn - i)(cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(1,$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^4/x^2, x]

[Out]  $(-2E^{((2*I)*a)}*(-I + 2*b*n)*(c*x^n)^{((2*I)*b)}\operatorname{Hypergeometric2F1}[1, 1 + (I/2)/(b*n), 2 + (I/2)/(b*n), -E^{((2*I)*(a + b*\operatorname{Log}[c*x^n]))}] - (2*I)*(1 + 4*b^2*n^2)*\operatorname{Hypergeometric2F1}[1, (I/2)/(b*n), 1 + (I/2)/(b*n), -E^{((2*I)*(a + b*\operatorname{Log}[c*x^n]))}] + \operatorname{Sec}[a + b*\operatorname{Log}[c*x^n]]^2*(2*b*n + (1 + 8*b^2*n^2 + (1 + 4*b^2*n^2)*\operatorname{Cos}[2*(a + b*\operatorname{Log}[c*x^n])])*\operatorname{Tan}[a + b*\operatorname{Log}[c*x^n]])/(12*b^3*n^3*x)$

**Maple [F]** time = 1.551, size = 0, normalized size = 0.

$$\int \frac{(\sec(a + b \ln(cx^n)))^4}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^4/x^2, x)

[Out] int(sec(a+b\*ln(c\*x^n))^4/x^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4/x^2, x, algorithm="maxima")

[Out]  $\frac{1}{3}(6*(b*\cos(4*b*\log(c)))^2 + b*\sin(4*b*\log(c))^2)*n*\cos(4*b*\log(x^n) + 4*a)^2 + 6*(b*\cos(2*b*\log(c)))^2 + b*\sin(2*b*\log(c))^2)*n*\cos(2*b*\log(x^n) + 2*a)^2 + 6*(b*\cos(4*b*\log(c)))^2 + b*\sin(4*b*\log(c))^2)*n*\sin(4*b*\log(x^n) + 4*a)^2 + 6*(b*\cos(2*b*\log(c)))^2 + b*\sin(2*b*\log(c))^2)*n*\sin(2*b*\log(x^n) + 2*a)^2 + (4*b^2*n^2*\sin(6*b*\log(c)) + (2*(b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(4*b*\log(c))))*n + \cos(4*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) + 2*(6*(b^2*\cos(2$



$$\begin{aligned}
& *b*\log(c))*\sin(6*b*\log(c)) - b^2*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^2 + (b* \\
& \cos(6*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n + \cos \\
& (2*b*\log(c))*\sin(6*b*\log(c)) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log \\
& (x^n) + 2*a) + (2*(b*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b*\cos(6*b*\log(c))*\sin \\
& (4*b*\log(c)))*n - \cos(6*b*\log(c))*\cos(4*b*\log(c)) - \sin(6*b*\log(c))*\sin(4 \\
& *b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) - 2*(6*(b^2*\cos(6*b*\log(c))*\cos(2*b*\log \\
& (c)) + b^2*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^2 - (b*\cos(2*b*\log(c))*\sin(6* \\
& b*\log(c)) - b*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n + \cos(6*b*\log(c))*\cos(2*b* \\
& \log(c)) + \sin(6*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) + \sin(6* \\
& b*\log(c))*\cos(6*b*\log(x^n) + 6*a) + (12*b^2*n^2*\sin(4*b*\log(c)) + 2*b*n*\cos \\
& (4*b*\log(c)) + 3*(12*(b^2*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^2*\cos(4*b*\log \\
& (c))*\sin(2*b*\log(c)))*n^2 + 4*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4 \\
& *b*\log(c))*\sin(2*b*\log(c)))*n + \cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log \\
& (c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 3*(12*(b^2*\cos(4*b*\log(c) \\
& )*\cos(2*b*\log(c)) + b^2*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^2 - 4*(b*\cos(2*b \\
& *log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n + \cos(4*b*\log \\
& (c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) \\
& + 2*a) + 2*\sin(4*b*\log(c))*\cos(4*b*\log(x^n) + 4*a) + (2*b*n*\cos(2*b*\log(c) \\
& ) + \sin(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 9*((4*(b^8*\cos(6*b*\log(c))^2 \\
& + b^8*\sin(6*b*\log(c))^2)*n^8 + (b^6*\cos(6*b*\log(c))^2 + b^6*\sin(6*b*\log(c) \\
& )^2)*n^6)*x*\cos(6*b*\log(x^n) + 6*a)^2 + 9*(4*(b^8*\cos(4*b*\log(c))^2 + b^8*\sin \\
& (4*b*\log(c))^2)*n^8 + (b^6*\cos(4*b*\log(c))^2 + b^6*\sin(4*b*\log(c))^2)*n^6 \\
& )*x*\cos(4*b*\log(x^n) + 4*a)^2 + 9*(4*(b^8*\cos(2*b*\log(c))^2 + b^8*\sin(2*b*\log \\
& (c))^2)*n^8 + (b^6*\cos(2*b*\log(c))^2 + b^6*\sin(2*b*\log(c))^2)*n^6)*x*\cos( \\
& 2*b*\log(x^n) + 2*a)^2 + (4*(b^8*\cos(6*b*\log(c))^2 + b^8*\sin(6*b*\log(c))^2)* \\
& n^8 + (b^6*\cos(6*b*\log(c))^2 + b^6*\sin(6*b*\log(c))^2)*n^6)*x*\sin(6*b*\log(x^n) \\
& + 6*a)^2 + 9*(4*(b^8*\cos(4*b*\log(c))^2 + b^8*\sin(4*b*\log(c))^2)*n^8 + (b \\
& ^6*\cos(4*b*\log(c))^2 + b^6*\sin(4*b*\log(c))^2)*n^6)*x*\sin(4*b*\log(x^n) + 4*a \\
& )^2 + 9*(4*(b^8*\cos(2*b*\log(c))^2 + b^8*\sin(2*b*\log(c))^2)*n^8 + (b^6*\cos(2 \\
& *b*\log(c))^2 + b^6*\sin(2*b*\log(c))^2)*n^6)*x*\sin(2*b*\log(x^n) + 2*a)^2 + 6* \\
& (4*b^8*n^8*\cos(2*b*\log(c)) + b^6*n^6*\cos(2*b*\log(c)))*x*\cos(2*b*\log(x^n) + \\
& 2*a) - 6*(4*b^8*n^8*\sin(2*b*\log(c)) + b^6*n^6*\sin(2*b*\log(c)))*x*\sin(2*b*\log \\
& (x^n) + 2*a) + (4*b^8*n^8 + b^6*n^6)*x + 2*(3*(4*(b^8*\cos(6*b*\log(c))*\cos( \\
& 4*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c) \\
& )*\cos(4*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*x*\cos(4*b*\log \\
& (x^n) + 4*a) + 3*(4*(b^8*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^8*\sin(6*b*\log \\
& (c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin( \\
& 6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*x*\cos(2*b*\log(x^n) + 2*a) + 3*(4*(b^8*\cos \\
& (4*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + ( \\
& b^6*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(4*b*\log(c)))* \\
& n^6)*x*\sin(4*b*\log(x^n) + 4*a) + 3*(4*(b^8*\cos(2*b*\log(c))*\sin(6*b*\log(c)) \\
& - b^8*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(6*b*\log \\
& (c)) - b^6*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*x*\sin(2*b*\log(x^n) + 2*a) \\
& + (4*b^8*n^8*\cos(6*b*\log(c)) + b^6*n^6*\cos(6*b*\log(c)))*x*\cos(6*b*\log(x^n) \\
& + 6*a) + 6*(3*(4*(b^8*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^8*\sin(4*b*\log \\
& (c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin(4 \\
& *b*\log(c))*\sin(2*b*\log(c)))*n^6)*x*\cos(2*b*\log(x^n) + 2*a) + 3*(4*(b^8*\cos( \\
& 2*b*\log(c))*\sin(4*b*\log(c)) - b^8*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b \\
& ^6*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^6*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n \\
& ^6)*x*\sin(2*b*\log(x^n) + 2*a) + (4*b^8*n^8*\cos(4*b*\log(c)) + b^6*n^6*\cos(4* \\
& b*\log(c)))*x*\cos(4*b*\log(x^n) + 4*a) - 2*(3*(4*(b^8*\cos(4*b*\log(c))*\sin(6* \\
& b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c)) \\
& )*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*x*\cos(4*b*\log( \\
& x^n) + 4*a) + 3*(4*(b^8*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c) \\
& )*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6* \\
& b*\log(c))*\sin(2*b*\log(c)))*n^6)*x*\cos(2*b*\log(x^n) + 2*a) - 3*(4*(b^8*\cos(6 \\
& *b*\log(c))*\cos(4*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^ \\
& 6*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^ \\
& 6)*x*\sin(4*b*\log(x^n) + 4*a) - 3*(4*(b^8*\cos(6*b*\log(c))*\cos(2*b*\log(c)) +
\end{aligned}$$



) \* sin(4 \* b \* log(c)) \* n^3 \* x \* sin(4 \* b \* log(x^n) + 4 \* a) - 3 \* (b^3 \* cos(6 \* b \* log(c)) \* cos(2 \* b \* log(c)) + b^3 \* sin(6 \* b \* log(c)) \* sin(2 \* b \* log(c))) \* n^3 \* x \* sin(2 \* b \* log(x^n) + 2 \* a) \* sin(6 \* b \* log(x^n) + 6 \* a) - 6 \* (b^3 \* n^3 \* x \* sin(4 \* b \* log(c)) + 3 \* (b^3 \* cos(2 \* b \* log(c)) \* sin(4 \* b \* log(c)) - b^3 \* cos(4 \* b \* log(c)) \* sin(2 \* b \* log(c)))) \* n^3 \* x \* cos(2 \* b \* log(x^n) + 2 \* a) - 3 \* (b^3 \* cos(4 \* b \* log(c)) \* cos(2 \* b \* log(c)) + b^3 \* sin(4 \* b \* log(c)) \* sin(2 \* b \* log(c))) \* n^3 \* x \* sin(2 \* b \* log(x^n) + 2 \* a) \* sin(4 \* b \* log(x^n) + 4 \* a))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(b \log(cx^n) + a)^4}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4/x^2,x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)^4/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(a + b \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*4/x\*\*2,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*4/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^4}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4/x^2,x, algorithm="giac")

[Out] integrate(sec(b\*log(c\*x^n) + a)^4/x^2, x)

$$3.258 \quad \int \frac{\sec^4(a+b \log(cx^n))}{x^3} dx$$

**Optimal.** Leaf size=79

$$\frac{8e^{4ia} (cx^n)^{4ib} \operatorname{Hypergeometric2F1}\left(4, 2 + \frac{i}{bn}, 3 + \frac{i}{bn}, -e^{2ia} (cx^n)^{2ib}\right)}{x^2(1 - 2ibn)}$$

[Out]  $(-8E^{(4*I)*a}*(c*x^n)^{(4*I)*b}*\operatorname{Hypergeometric2F1}[4, 2 + I/(b*n), 3 + I/(b*n), -(E^{(2*I)*a}*(c*x^n)^{(2*I)*b})])/((1 - (2*I)*b*n)*x^2)$

**Rubi [A]** time = 0.0723831, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{8e^{4ia} (cx^n)^{4ib} {}_2F_1\left(4, 2 + \frac{i}{bn}; 3 + \frac{i}{bn}; -e^{2ia} (cx^n)^{2ib}\right)}{x^2(1 - 2ibn)}$$

Antiderivative was successfully verified.

[In] `Int[Sec[a + b*Log[c*x^n]]^4/x^3, x]`

[Out]  $(-8E^{(4*I)*a}*(c*x^n)^{(4*I)*b}*\operatorname{Hypergeometric2F1}[4, 2 + I/(b*n), 3 + I/(b*n), -(E^{(2*I)*a}*(c*x^n)^{(2*I)*b})])/((1 - (2*I)*b*n)*x^2)$

#### Rule 4509

`Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])`

#### Rule 4505

`Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] := Dist[2^p*E^(I*a*d*p), Int[((e*x)^m*x^(I*b*d*p))/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]`

#### Rule 364

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

#### Rubi steps

$$\int \frac{\sec^4(a + b \log(cx^n))}{x^3} dx = \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int x^{-1-\frac{2}{n}} \sec^4(a + b \log(x)) dx, x, cx^n\right)}{nx^2}$$

$$= \frac{(16e^{4ia} (cx^n)^{2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+4ib-\frac{2}{n}}}{(1+e^{2ia}x^{2ib})^4} dx, x, cx^n\right)}{nx^2}$$

$$= -\frac{8e^{4ia} (cx^n)^{4ib} {}_2F_1\left(4, 2 + \frac{i}{bn}; 3 + \frac{i}{bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(1 - 2ibn)x^2}$$

**Mathematica [B]** time = 9.37856, size = 203, normalized size = 2.57

$$-2i(b^2n^2 + 1) \operatorname{Hypergeometric2F1}\left(1, \frac{i}{bn}, 1 + \frac{i}{bn}, -e^{2i(a+b \log(cx^n))}\right) - 2e^{2ia}(bn - i)(cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(1, 1 + \frac{i}{bn}, 2 + \frac{i}{bn}, -e^{2i(a+b \log(cx^n))}\right) + \sec[a + b \log(cx^n)]^2 (bn + (1 + 2b^2n^2 + (1 + b^2n^2)\cos[2(a + b \log(cx^n))])\tan[a + b \log(cx^n)]) / (3b^3n^3x^2)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^4/x^3,x]

[Out] (-2\*E^((2\*I)\*a)\*(-I + b\*n)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[1, 1 + I/(b\*n), 2 + I/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] - (2\*I)\*(1 + b^2\*n^2)\*Hypergeometric2F1[1, I/(b\*n), 1 + I/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + Sec[a + b\*Log[c\*x^n]]^2\*(b\*n + (1 + 2\*b^2\*n^2 + (1 + b^2\*n^2)\*Cos[2\*(a + b\*Log[c\*x^n])])\*Tan[a + b\*Log[c\*x^n]]))/(3\*b^3\*n^3\*x^2)

**Maple [F]** time = 1.699, size = 0, normalized size = 0.

$$\int \frac{(\sec(a + b \ln(cx^n)))^4}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^4/x^3,x)

[Out] int(sec(a+b\*ln(c\*x^n))^4/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4/x^3,x, algorithm="maxima")

[Out] 4/3\*(3\*(b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*cos(4\*b\*log(x^n) + 4\*a)^2 + 3\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*cos(2\*b\*log(x^n) + 2\*a)^2 + 3\*(b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*sin(4\*b\*log(x^n) + 4\*a)^2 + 3\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*sin(2\*b\*log(x^n) + 2\*a)^2 + (b^2\*n^2\*sin(6\*b\*log(c)) + ((b\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(4\*b\*log(c))))\*n + cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - cos(6\*b\*log(c))\*sin(4\*b\*log(c)))\*cos(4\*b\*log(x^n) + 4\*a) + (3\*(b^2\*cos(2\*b\*log(c))

$$\begin{aligned}
& (c) * \sin(6 * b * \log(c)) - b^2 * \cos(6 * b * \log(c)) * \sin(2 * b * \log(c)) * n^2 + (b * \cos(6 * \\
& b * \log(c)) * \cos(2 * b * \log(c)) + b * \sin(6 * b * \log(c)) * \sin(2 * b * \log(c))) * n + 2 * \cos(2 * \\
& b * \log(c)) * \sin(6 * b * \log(c)) - 2 * \cos(6 * b * \log(c)) * \sin(2 * b * \log(c)) * \cos(2 * b * \log( \\
& x^n) + 2 * a) + ((b * \cos(4 * b * \log(c)) * \sin(6 * b * \log(c)) - b * \cos(6 * b * \log(c)) * \sin(4 \\
& * b * \log(c))) * n - \cos(6 * b * \log(c)) * \cos(4 * b * \log(c)) - \sin(6 * b * \log(c)) * \sin(4 * b * l \\
& \log(c))) * \sin(4 * b * \log(x^n) + 4 * a) - (3 * (b^2 * \cos(6 * b * \log(c)) * \cos(2 * b * \log(c)) + \\
& b^2 * \sin(6 * b * \log(c)) * \sin(2 * b * \log(c))) * n^2 - (b * \cos(2 * b * \log(c)) * \sin(6 * b * \log( \\
& c)) - b * \cos(6 * b * \log(c)) * \sin(2 * b * \log(c))) * n + 2 * \cos(6 * b * \log(c)) * \cos(2 * b * \log( \\
& c)) + 2 * \sin(6 * b * \log(c)) * \sin(2 * b * \log(c))) * \sin(2 * b * \log(x^n) + 2 * a) + \sin(6 * b * \\
& \log(c)) * \cos(6 * b * \log(x^n) + 6 * a) + (3 * b^2 * n^2 * \sin(4 * b * \log(c)) + b * n * \cos(4 * b \\
& * \log(c)) + 3 * (3 * (b^2 * \cos(2 * b * \log(c)) * \sin(4 * b * \log(c)) - b^2 * \cos(4 * b * \log(c)) * \\
& \sin(2 * b * \log(c))) * n^2 + 2 * (b * \cos(4 * b * \log(c)) * \cos(2 * b * \log(c)) + b * \sin(4 * b * \log \\
& (c)) * \sin(2 * b * \log(c))) * n + \cos(2 * b * \log(c)) * \sin(4 * b * \log(c)) - \cos(4 * b * \log(c)) \\
& * \sin(2 * b * \log(c))) * \cos(2 * b * \log(x^n) + 2 * a) - 3 * (3 * (b^2 * \cos(4 * b * \log(c)) * \cos(2 \\
& * b * \log(c)) + b^2 * \sin(4 * b * \log(c)) * \sin(2 * b * \log(c))) * n^2 - 2 * (b * \cos(2 * b * \log(c) \\
& ) * \sin(4 * b * \log(c)) - b * \cos(4 * b * \log(c)) * \sin(2 * b * \log(c))) * n + \cos(4 * b * \log(c)) * \\
& \cos(2 * b * \log(c)) + \sin(4 * b * \log(c)) * \sin(2 * b * \log(c))) * \sin(2 * b * \log(x^n) + 2 * a) \\
& + 2 * \sin(4 * b * \log(c)) * \cos(4 * b * \log(x^n) + 4 * a) + (b * n * \cos(2 * b * \log(c)) + \sin(2 \\
& * b * \log(c))) * \cos(2 * b * \log(x^n) + 2 * a) + 18 * (((b^8 * \cos(6 * b * \log(c))^2 + b^8 * \sin \\
& (6 * b * \log(c))^2) * n^8 + (b^6 * \cos(6 * b * \log(c))^2 + b^6 * \sin(6 * b * \log(c))^2) * n^6) * \\
& x^2 * \cos(6 * b * \log(x^n) + 6 * a)^2 + 9 * ((b^8 * \cos(4 * b * \log(c))^2 + b^8 * \sin(4 * b * \log \\
& (c))^2) * n^8 + (b^6 * \cos(4 * b * \log(c))^2 + b^6 * \sin(4 * b * \log(c))^2) * n^6) * x^2 * \cos( \\
& 4 * b * \log(x^n) + 4 * a)^2 + 9 * ((b^8 * \cos(2 * b * \log(c))^2 + b^8 * \sin(2 * b * \log(c))^2) * \\
& n^8 + (b^6 * \cos(2 * b * \log(c))^2 + b^6 * \sin(2 * b * \log(c))^2) * n^6) * x^2 * \cos(2 * b * \log( \\
& x^n) + 2 * a)^2 + ((b^8 * \cos(6 * b * \log(c))^2 + b^8 * \sin(6 * b * \log(c))^2) * n^8 + (b^6 \\
& * \cos(6 * b * \log(c))^2 + b^6 * \sin(6 * b * \log(c))^2) * n^6) * x^2 * \sin(6 * b * \log(x^n) + 6 * a \\
& )^2 + 9 * ((b^8 * \cos(4 * b * \log(c))^2 + b^8 * \sin(4 * b * \log(c))^2) * n^8 + (b^6 * \cos(4 * b \\
& * \log(c))^2 + b^6 * \sin(4 * b * \log(c))^2) * n^6) * x^2 * \sin(4 * b * \log(x^n) + 4 * a)^2 + 9 * \\
& ((b^8 * \cos(2 * b * \log(c))^2 + b^8 * \sin(2 * b * \log(c))^2) * n^8 + (b^6 * \cos(2 * b * \log(c)) \\
& )^2 + b^6 * \sin(2 * b * \log(c))^2) * n^6) * x^2 * \sin(2 * b * \log(x^n) + 2 * a)^2 + 6 * (b^8 * n^8 \\
& * \cos(2 * b * \log(c)) + b^6 * n^6 * \cos(2 * b * \log(c))) * x^2 * \cos(2 * b * \log(x^n) + 2 * a) - 6 \\
& * (b^8 * n^8 * \sin(2 * b * \log(c)) + b^6 * n^6 * \sin(2 * b * \log(c))) * x^2 * \sin(2 * b * \log(x^n) + \\
& 2 * a) + (b^8 * n^8 + b^6 * n^6) * x^2 + 2 * (3 * ((b^8 * \cos(6 * b * \log(c)) * \cos(4 * b * \log(c) \\
& ) + b^8 * \sin(6 * b * \log(c)) * \sin(4 * b * \log(c))) * n^8 + (b^6 * \cos(6 * b * \log(c)) * \cos(4 * b \\
& * \log(c)) + b^6 * \sin(6 * b * \log(c)) * \sin(4 * b * \log(c))) * n^6) * x^2 * \cos(4 * b * \log(x^n) + \\
& 4 * a) + 3 * ((b^8 * \cos(6 * b * \log(c)) * \cos(2 * b * \log(c)) + b^8 * \sin(6 * b * \log(c)) * \sin(2 \\
& * b * \log(c))) * n^8 + (b^6 * \cos(6 * b * \log(c)) * \cos(2 * b * \log(c)) + b^6 * \sin(6 * b * \log(c) \\
& ) * \sin(2 * b * \log(c))) * n^6) * x^2 * \cos(2 * b * \log(x^n) + 2 * a) + 3 * ((b^8 * \cos(4 * b * \log(c) \\
& )) * \sin(6 * b * \log(c)) - b^8 * \cos(6 * b * \log(c)) * \sin(4 * b * \log(c))) * n^8 + (b^6 * \cos(4 * \\
& b * \log(c)) * \sin(6 * b * \log(c)) - b^6 * \cos(6 * b * \log(c)) * \sin(4 * b * \log(c))) * n^6) * x^2 * s \\
& \sin(4 * b * \log(x^n) + 4 * a) + 3 * ((b^8 * \cos(2 * b * \log(c)) * \sin(6 * b * \log(c)) - b^8 * \cos( \\
& 6 * b * \log(c)) * \sin(2 * b * \log(c))) * n^8 + (b^6 * \cos(2 * b * \log(c)) * \sin(6 * b * \log(c)) - b \\
& ^6 * \cos(6 * b * \log(c)) * \sin(2 * b * \log(c))) * n^6) * x^2 * \sin(2 * b * \log(x^n) + 2 * a) + (b^8 \\
& * n^8 * \cos(6 * b * \log(c)) + b^6 * n^6 * \cos(6 * b * \log(c))) * x^2 * \cos(6 * b * \log(x^n) + 6 * a \\
& ) + 6 * (3 * ((b^8 * \cos(4 * b * \log(c)) * \cos(2 * b * \log(c)) + b^8 * \sin(4 * b * \log(c)) * \sin(2 * \\
& b * \log(c))) * n^8 + (b^6 * \cos(4 * b * \log(c)) * \cos(2 * b * \log(c)) + b^6 * \sin(4 * b * \log(c)) \\
& * \sin(2 * b * \log(c))) * n^6) * x^2 * \cos(2 * b * \log(x^n) + 2 * a) + 3 * ((b^8 * \cos(2 * b * \log(c) \\
& ) * \sin(4 * b * \log(c)) - b^8 * \cos(4 * b * \log(c)) * \sin(2 * b * \log(c))) * n^8 + (b^6 * \cos(2 * b \\
& * \log(c)) * \sin(4 * b * \log(c)) - b^6 * \cos(4 * b * \log(c)) * \sin(2 * b * \log(c))) * n^6) * x^2 * si \\
& \sin(2 * b * \log(x^n) + 2 * a) + (b^8 * n^8 * \cos(4 * b * \log(c)) + b^6 * n^6 * \cos(4 * b * \log(c))) \\
& * x^2 * \cos(4 * b * \log(x^n) + 4 * a) - 2 * (3 * ((b^8 * \cos(4 * b * \log(c)) * \sin(6 * b * \log(c)) \\
& - b^8 * \cos(6 * b * \log(c)) * \sin(4 * b * \log(c))) * n^8 + (b^6 * \cos(4 * b * \log(c)) * \sin(6 * b * l \\
& \log(c)) - b^6 * \cos(6 * b * \log(c)) * \sin(4 * b * \log(c))) * n^6) * x^2 * \cos(4 * b * \log(x^n) + 4 \\
& * a) + 3 * ((b^8 * \cos(2 * b * \log(c)) * \sin(6 * b * \log(c)) - b^8 * \cos(6 * b * \log(c)) * \sin(2 * b \\
& * \log(c))) * n^8 + (b^6 * \cos(2 * b * \log(c)) * \sin(6 * b * \log(c)) - b^6 * \cos(6 * b * \log(c)) * \\
& \sin(2 * b * \log(c))) * n^6) * x^2 * \cos(2 * b * \log(x^n) + 2 * a) - 3 * ((b^8 * \cos(6 * b * \log(c)) \\
& * \cos(4 * b * \log(c)) + b^8 * \sin(6 * b * \log(c)) * \sin(4 * b * \log(c))) * n^8 + (b^6 * \cos(6 * b * \\
& \log(c)) * \cos(4 * b * \log(c)) + b^6 * \sin(6 * b * \log(c)) * \sin(4 * b * \log(c))) * n^6) * x^2 * \sin \\
& (4 * b * \log(x^n) + 4 * a) - 3 * ((b^8 * \cos(6 * b * \log(c)) * \cos(2 * b * \log(c)) + b^8 * \sin(6 *
\end{aligned}$$

$$\begin{aligned}
& b \log(c) \sin(2b \log(c)) n^8 + (b^6 \cos(6b \log(c)) \cos(2b \log(c)) + b^6 \\
& \sin(6b \log(c)) \sin(2b \log(c))) n^6 x^2 \sin(2b \log(x^n) + 2a) + (b^8 n^8 \sin(6b \log(c)) + b^6 n^6 \sin(6b \log(c))) x^2 \sin(6b \log(x^n) + 6a) \\
& - 6(3((b^8 \cos(2b \log(c)) \sin(4b \log(c)) - b^8 \cos(4b \log(c)) \sin(2b \log(c))) n^8 + (b^6 \cos(2b \log(c)) \sin(4b \log(c)) - b^6 \cos(4b \log(c)) \sin(2b \log(c))) n^6) x^2 \cos(2b \log(x^n) + 2a) - 3((b^8 \cos(4b \log(c)) \cos(2b \log(c)) + b^8 \sin(4b \log(c)) \sin(2b \log(c))) n^8 + (b^6 \cos(4b \log(c)) \cos(2b \log(c)) + b^6 \sin(4b \log(c)) \sin(2b \log(c))) n^6) x^2 \sin(2b \log(x^n) + 2a) + (b^8 n^8 \sin(4b \log(c)) + b^6 n^6 \sin(4b \log(c))) x^2 \sin(4b \log(x^n) + 4a) \int \frac{1}{9} (\cos(2b \log(x^n) + 2a) \sin(2b \log(c)) + \cos(2b \log(c)) \sin(2b \log(x^n) + 2a)) / (2b^6 n^6 x^3 \cos(2b \log(c)) \cos(2b \log(x^n) + 2a) - 2b^6 n^6 x^3 \sin(2b \log(c)) \sin(2b \log(x^n) + 2a) + b^6 n^6 x^3 + (b^6 \cos(2b \log(c))^2 + b^6 \sin(2b \log(c))^2) n^6 x^3 \cos(2b \log(x^n) + 2a)^2 + (b^6 \cos(2b \log(c))^2 + b^6 \sin(2b \log(c))^2) n^6 x^3 \sin(2b \log(x^n) + 2a)^2, x) + (b^2 n^2 \cos(6b \log(c)) - ((b \cos(4b \log(c)) \sin(6b \log(c)) - b \cos(6b \log(c)) \sin(4b \log(c))) n - \cos(6b \log(c)) \cos(4b \log(c)) - \sin(6b \log(c)) \sin(4b \log(c))) \cos(4b \log(x^n) + 4a) + (3(b^2 \cos(6b \log(c)) \cos(2b \log(c)) + b^2 \sin(6b \log(c)) \sin(2b \log(c))) n^2 - (b \cos(2b \log(c)) \sin(6b \log(c)) - b \cos(6b \log(c)) \sin(2b \log(c))) n + 2 \cos(6b \log(c)) \cos(2b \log(c)) + 2 \sin(6b \log(c)) \sin(2b \log(c))) \cos(2b \log(x^n) + 2a) + ((b \cos(6b \log(c)) \cos(4b \log(c)) + b \sin(6b \log(c)) \sin(4b \log(c))) n + \cos(4b \log(c)) \sin(6b \log(c)) - \cos(6b \log(c)) \sin(4b \log(c))) \sin(4b \log(x^n) + 4a) + (3(b^2 \cos(2b \log(c)) \sin(6b \log(c)) - b^2 \cos(6b \log(c)) \sin(2b \log(c))) n^2 + (b \cos(6b \log(c)) \cos(2b \log(c)) + b \sin(6b \log(c)) \sin(2b \log(c))) n + 2 \cos(2b \log(c)) \sin(6b \log(c)) - 2 \cos(6b \log(c)) \sin(2b \log(c))) \sin(2b \log(x^n) + 2a) + \cos(6b \log(c)) \sin(6b \log(x^n) + 6a) + (3b^2 n^2 \cos(4b \log(c)) - b n \sin(4b \log(c)) + 3(3(b^2 \cos(4b \log(c)) \cos(2b \log(c)) + b^2 \sin(4b \log(c)) \sin(2b \log(c))) n^2 - 2(b \cos(2b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(2b \log(c))) n + \cos(4b \log(c)) \cos(2b \log(c)) + \sin(4b \log(c)) \sin(2b \log(c))) \cos(2b \log(x^n) + 2a) + 3(3(b^2 \cos(2b \log(c)) \sin(4b \log(c)) - b^2 \cos(4b \log(c)) \sin(2b \log(c))) n^2 + 2(b \cos(4b \log(c)) \cos(2b \log(c)) + b \sin(4b \log(c)) \sin(2b \log(c))) n + \cos(2b \log(c)) \sin(4b \log(c)) - \cos(4b \log(c)) \sin(2b \log(c))) \sin(2b \log(x^n) + 2a) + 2 \cos(4b \log(c)) \sin(4b \log(x^n) + 4a) - (b n \sin(2b \log(c)) - \cos(2b \log(c))) \sin(2b \log(x^n) + 2a) / (6b^3 n^3 x^2 \cos(2b \log(c)) \cos(2b \log(x^n) + 2a) - 6b^3 n^3 x^2 \sin(2b \log(c)) \sin(2b \log(x^n) + 2a) + b^3 n^3 x^2 + (b^3 \cos(6b \log(c))^2 + b^3 \sin(6b \log(c))^2) n^3 x^2 \cos(6b \log(x^n) + 6a)^2 + 9(b^3 \cos(4b \log(c))^2 + b^3 \sin(4b \log(c))^2) n^3 x^2 \cos(4b \log(x^n) + 4a)^2 + 9(b^3 \cos(2b \log(c))^2 + b^3 \sin(2b \log(c))^2) n^3 x^2 \cos(2b \log(x^n) + 2a)^2 + (b^3 \cos(6b \log(c))^2 + b^3 \sin(6b \log(c))^2) n^3 x^2 \sin(6b \log(x^n) + 6a)^2 + 9(b^3 \cos(4b \log(c))^2 + b^3 \sin(4b \log(c))^2) n^3 x^2 \sin(4b \log(x^n) + 4a)^2 + 9(b^3 \cos(2b \log(c))^2 + b^3 \sin(2b \log(c))^2) n^3 x^2 \sin(2b \log(x^n) + 2a)^2 + 2(b^3 n^3 x^2 \cos(6b \log(c)) + 3(b^3 \cos(6b \log(c)) \cos(4b \log(c)) + b^3 \sin(6b \log(c)) \sin(4b \log(c))) n^3 x^2 \cos(4b \log(x^n) + 4a) + 3(b^3 \cos(6b \log(c)) \cos(2b \log(c)) + b^3 \sin(6b \log(c)) \sin(2b \log(c))) n^3 x^2 \cos(2b \log(x^n) + 2a) + 3(b^3 \cos(4b \log(c)) \sin(6b \log(c)) - b^3 \cos(6b \log(c)) \sin(4b \log(c))) n^3 x^2 \sin(4b \log(x^n) + 4a) + 3(b^3 \cos(2b \log(c)) \sin(6b \log(c)) - b^3 \cos(6b \log(c)) \sin(2b \log(c))) n^3 x^2 \sin(2b \log(x^n) + 2a) \cos(6b \log(x^n) + 6a) + 6(b^3 n^3 x^2 \cos(4b \log(c)) + 3(b^3 \cos(4b \log(c)) \cos(2b \log(c)) + b^3 \sin(4b \log(c)) \sin(2b \log(c))) n^3 x^2 \cos(2b \log(x^n) + 2a) + 3(b^3 \cos(2b \log(c)) \sin(4b \log(c)) - b^3 \cos(4b \log(c)) \sin(2b \log(c))) n^3 x^2 \sin(2b \log(x^n) + 2a) \cos(4b \log(x^n) + 4a) - 2(b^3 n^3 x^2 \sin(6b \log(c)) + 3(b^3 \cos(4b \log(c)) \sin(6b \log(c)) - b^3 \cos(6b \log(c)) \sin(4b \log(c))) n^3 x^2 \cos(4b \log(x^n) + 4a) + 3(b^3 \cos(2b \log(c)) \sin(6b \log(c)) - b^3 \cos(6b \log(c)) \sin(2b \log(c))) n^3 x^2 \cos(2b \log(x^n) + 2a) - 3(b^3 \cos(6b \log(c)) \cos(4b \log(c))
\end{aligned}$$

$$\begin{aligned} &)) + b^3 \sin(6b \log(c)) \sin(4b \log(c)) n^3 x^2 \sin(4b \log(x^n) + 4a) - \\ &3(b^3 \cos(6b \log(c)) \cos(2b \log(c)) + b^3 \sin(6b \log(c)) \sin(2b \log(c))) \\ &n^3 x^2 \sin(2b \log(x^n) + 2a) \sin(6b \log(x^n) + 6a) - 6(b^3 n^3 x^2 \sin(4b \log(c)) \\ &+ 3(b^3 \cos(2b \log(c)) \sin(4b \log(c)) - b^3 \cos(4b \log(c)) \sin(2b \log(c))) \\ &n^3 x^2 \cos(2b \log(x^n) + 2a) - 3(b^3 \cos(4b \log(c)) \cos(2b \log(c)) + b^3 \sin(4b \log(c)) \sin(2b \log(c))) \\ &n^3 x^2 \sin(2b \log(x^n) + 2a) \sin(4b \log(x^n) + 4a) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(b \log(cx^n) + a)^4}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4/x^3,x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)^4/x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(a + b \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*4/x\*\*3,x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*4/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^4}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^4/x^3,x, algorithm="giac")

[Out] integrate(sec(b\*log(c\*x^n) + a)^4/x^3, x)



### 3.259 $\int \left( - \left( 1 + b^2 n^2 \right) \sec \left( a + b \log \left( c x^n \right) \right) + 2 b^2 n^2 \sec^3 \left( a + b \log \left( c x^n \right) \right) \right) dx$

**Optimal.** Leaf size=41

$$bnx \tan(a + b \log(cx^n)) \sec(a + b \log(cx^n)) - x \sec(a + b \log(cx^n))$$

[Out]  $-(x \operatorname{Sec}[a + b \operatorname{Log}[c x^n]]) + b n x \operatorname{Sec}[a + b \operatorname{Log}[c x^n]] \operatorname{Tan}[a + b \operatorname{Log}[c x^n]]$

**Rubi [C]** time = 0.132896, antiderivative size = 175, normalized size of antiderivative = 4.27, number of steps used = 7, number of rules used = 3, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$ , Rules used = {4503, 4505, 364}

$$\frac{16e^{3ia} b^2 n^2 x (cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{i}{bn}\right); \frac{1}{2}\left(5 - \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{1 + 3ibn} - 2e^{ia} x (1 - ibn) (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i}{bn}\right); \frac{1}{2}\left(3 - \frac{i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)$$

Warning: Unable to verify antiderivative.

[In]  $\operatorname{Int}[-((1 + b^2 n^2) \operatorname{Sec}[a + b \operatorname{Log}[c x^n]]) + 2 b^2 n^2 \operatorname{Sec}[a + b \operatorname{Log}[c x^n]]^3, x]$

[Out]  $-2E^{(I*a)}*(1 - I*b*n)*x*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}\left[1, \left(1 - \frac{I}{(b*n)}\right)/2, \left(3 - \frac{I}{(b*n)}\right)/2, -\left(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)}\right)\right] + (16*b^2*E^{((3*I)*a)}*n^2*x*(c*x^n)^{((3*I)*b)}*\operatorname{Hypergeometric2F1}\left[3, \left(3 - \frac{I}{(b*n)}\right)/2, \left(5 - \frac{I}{(b*n)}\right)/2, -\left(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)}\right)\right]/(1 + (3*I)*b*n)$

#### Rule 4503

$\operatorname{Int}[\operatorname{Sec}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[x/(n*(c*x^n)^{(1/n)}), \operatorname{Subst}[\operatorname{Int}[x^{(1/n - 1)}*\operatorname{Sec}[d*(a + b*\operatorname{Log}[x])]^p, x], x, c*x^n], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n, p\}, x \&\& (\operatorname{NeQ}[c, 1] \mid \mid \operatorname{NeQ}[n, 1])$

#### Rule 4505

$\operatorname{Int}[(e_.)*(x_.)^{(m_.)}*\operatorname{Sec}[(a_.) + \operatorname{Log}[x_*](b_.)]*(d_.)]^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[2^p*E^{(I*a*d*p)}, \operatorname{Int}[(e*x)^m*x^{(I*b*d*p)}]/(1 + E^{(2*I*a*d)}*x^{(2*I*b*d)})^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, d, e, m\}, x \&\& \operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a^p*(c*x)^{(m+1)}*\operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -(b*x^n)/a])/ (c*(m+1)), x] /;$   $\operatorname{FreeQ}\{a, b, c, m, n, p\}, x \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \mid \mid \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\begin{aligned}
\int \left( -(1 + b^2 n^2) \sec(a + b \log(cx^n)) + 2b^2 n^2 \sec^3(a + b \log(cx^n)) \right) dx &= (2b^2 n^2) \int \sec^3(a + b \log(cx^n)) dx + (-1 - b^2 n^2) \int \sec(a + b \log(cx^n)) dx \\
&= (2b^2 n x (cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1 + \frac{1}{n}} \sec^3(a + b \log(x)) dx \right) \\
&= (16b^2 e^{3ia} n x (cx^n)^{-1/n}) \text{Subst} \left( \int \frac{x^{-1 + 3ib + \frac{1}{n}}}{(1 + e^{2ia} x^{2ib})^3} dx \right) \\
&= -2e^{ia} (1 - ibn)x (cx^n)^{ib} {}_2F_1 \left( 1, \frac{1}{2} \left( 1 - \frac{i}{bn} \right); \frac{1}{2} \left( 3 - \frac{i}{bn} \right); -\frac{e^{2ia} x^{2ib}}{1 + e^{2ia} x^{2ib}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.655003, size = 29, normalized size = 0.71

$$x(bn \tan(a + b \log(cx^n)) - 1) \sec(a + b \log(cx^n))$$

Antiderivative was successfully verified.

[In] Integrate[-((1 + b^2\*n^2)\*Sec[a + b\*Log[c\*x^n]]) + 2\*b^2\*n^2\*Sec[a + b\*Log[c\*x^n]]^3,x]

[Out] x\*Sec[a + b\*Log[c\*x^n]]\*(-1 + b\*n\*Tan[a + b\*Log[c\*x^n]])

**Maple [C]** time = 0.48, size = 537, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(b^2\*n^2+1)\*sec(a+b\*ln(c\*x^n))+2\*b^2\*n^2\*sec(a+b\*ln(c\*x^n))^3,x)

[Out] 
$$\begin{aligned}
& -2Ix / (\exp(I*(-I*b*Pi*csgn(I*c*x^n)^3 + I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c) + I*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n) - I*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n) + 2*b*ln(x^n) + 2*b*ln(c) + 2*a)) + 1)^2 * ((x^n)^{(I*b)})^3 * (c^{(I*b)})^3 * b*n * \exp(3/2*b*Pi*csgn(I*c*x^n)^3) * \exp(-3/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)) * \exp(-3/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n)) * \exp(3/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)) * \exp(3*I*a) - (x^n)^{(I*b)} * c^{(I*b)} * b*n * \exp(1/2*b*Pi*csgn(I*c*x^n)^3) * \exp(-1/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)) * \exp(-1/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n)) * \exp(1/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)) * \exp(I*a) - I * ((x^n)^{(I*b)})^3 * (c^{(I*b)})^3 * \exp(3/2*b*Pi*csgn(I*c*x^n)^3) * \exp(-3/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)) * \exp(-3/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n)) * \exp(3/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)) * \exp(3*I*a) - I * (x^n)^{(I*b)} * c^{(I*b)} * \exp(1/2*b*Pi*csgn(I*c*x^n)^3) * \exp(-1/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)) * \exp(-1/2*b*Pi*csgn(I*c*x^n)^2*csgn(I*x^n)) * \exp(1/2*b*Pi*csgn(I*c*x^n)*csgn(I*c)*csgn(I*x^n)) * \exp(I*a)
\end{aligned}$$

**Maxima [B]** time = 2.87452, size = 2290, normalized size = 55.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b^2\*n^2+1)\*sec(a+b\*log(c\*x^n))+2\*b^2\*n^2\*sec(a+b\*log(c\*x^n))^3,  
x, algorithm="maxima")

[Out] 
$$-2*((b*n*\sin(b*\log(c)) + \cos(b*\log(c)))x*\cos(b*\log(x^n) + a) + (b*n*\cos(b*\log(c)) - \sin(b*\log(c)))x*\sin(b*\log(x^n) + a) + (((b*\cos(3*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(3*b*\log(c)))n + \cos(4*b*\log(c))*\cos(3*b*\log(c)) + \sin(4*b*\log(c))*\sin(3*b*\log(c)))x*\cos(3*b*\log(x^n) + 3*a) - ((b*\cos(b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(b*\log(c)))n - \cos(4*b*\log(c))*\cos(b*\log(c)) - \sin(4*b*\log(c))*\sin(b*\log(c)))x*\cos(b*\log(x^n) + a) - ((b*\cos(4*b*\log(c))*\cos(3*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(3*b*\log(c)))n - \cos(3*b*\log(c))*\sin(4*b*\log(c)) + \cos(4*b*\log(c))*\sin(3*b*\log(c)))x*\sin(3*b*\log(x^n) + 3*a) + ((b*\cos(4*b*\log(c))*\cos(b*\log(c)) + b*\sin(4*b*\log(c))*\sin(b*\log(c)))n + \cos(b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(b*\log(c)))x*\sin(b*\log(x^n) + a))*\cos(4*b*\log(x^n) + 4*a) - (2*((b*\cos(2*b*\log(c))*\sin(3*b*\log(c)) - b*\cos(3*b*\log(c))*\sin(2*b*\log(c)))n - \cos(3*b*\log(c))*\cos(2*b*\log(c)) - \sin(3*b*\log(c))*\sin(2*b*\log(c)))x*\cos(2*b*\log(x^n) + 2*a) - 2*((b*\cos(3*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(3*b*\log(c))*\sin(2*b*\log(c)))n + \cos(2*b*\log(c))*\sin(3*b*\log(c)) - \cos(3*b*\log(c))*\sin(2*b*\log(c)))x*\sin(2*b*\log(x^n) + 2*a) + (b*n*\sin(3*b*\log(c)) - \cos(3*b*\log(c)))x*\cos(3*b*\log(x^n) + 3*a) - 2*((b*\cos(b*\log(c))*\sin(2*b*\log(c)) - b*\cos(2*b*\log(c))*\sin(b*\log(c)))n - \cos(2*b*\log(c))*\cos(b*\log(c)) - \sin(2*b*\log(c))*\sin(b*\log(c)))x*\cos(b*\log(x^n) + a) - ((b*\cos(2*b*\log(c))*\cos(b*\log(c)) + b*\sin(2*b*\log(c))*\sin(b*\log(c)))n + \cos(b*\log(c))*\sin(2*b*\log(c)) - \cos(2*b*\log(c))*\sin(b*\log(c)))x*\sin(b*\log(x^n) + a))*\cos(2*b*\log(x^n) + 2*a) + (((b*\cos(4*b*\log(c))*\cos(3*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(3*b*\log(c)))n - \cos(3*b*\log(c))*\sin(4*b*\log(c)) + \cos(4*b*\log(c))*\sin(3*b*\log(c)))x*\cos(3*b*\log(x^n) + 3*a) - ((b*\cos(4*b*\log(c))*\cos(b*\log(c)) + b*\sin(4*b*\log(c))*\sin(b*\log(c)))n + \cos(b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(b*\log(c)))x*\cos(b*\log(x^n) + a) + ((b*\cos(3*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(3*b*\log(c)))n + \cos(4*b*\log(c))*\cos(3*b*\log(c)) + \sin(4*b*\log(c))*\sin(3*b*\log(c)))x*\sin(3*b*\log(x^n) + 3*a) - ((b*\cos(b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(b*\log(c)))n - \cos(4*b*\log(c))*\cos(b*\log(c)) - \sin(4*b*\log(c))*\sin(b*\log(c)))x*\sin(b*\log(x^n) + a))*\sin(4*b*\log(x^n) + 4*a) - (2*((b*\cos(3*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(3*b*\log(c))*\sin(2*b*\log(c)))n + \cos(2*b*\log(c))*\sin(3*b*\log(c)) - \cos(3*b*\log(c))*\sin(2*b*\log(c)))x*\cos(2*b*\log(x^n) + 2*a) + 2*((b*\cos(2*b*\log(c))*\sin(3*b*\log(c)) - b*\cos(3*b*\log(c))*\sin(2*b*\log(c)))n - \cos(3*b*\log(c))*\cos(2*b*\log(c)) - \sin(3*b*\log(c))*\sin(2*b*\log(c)))x*\sin(2*b*\log(x^n) + 2*a) + (b*n*\cos(3*b*\log(c)) + \sin(3*b*\log(c)))x*\sin(3*b*\log(x^n) + 3*a) - 2*((b*\cos(2*b*\log(c))*\cos(b*\log(c)) + b*\sin(2*b*\log(c))*\sin(b*\log(c)))n + \cos(b*\log(c))*\sin(2*b*\log(c)) - \cos(2*b*\log(c))*\sin(b*\log(c)))x*\cos(b*\log(x^n) + a) + ((b*\cos(b*\log(c))*\sin(2*b*\log(c)) - b*\cos(2*b*\log(c))*\sin(b*\log(c)))n - \cos(2*b*\log(c))*\cos(b*\log(c)) - \sin(2*b*\log(c))*\sin(b*\log(c)))x*\sin(b*\log(x^n) + a))*\sin(2*b*\log(x^n) + 2*a))/((\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\cos(4*b*\log(x^n) + 4*a)^2 + 4*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\cos(2*b*\log(x^n) + 2*a)^2 + (\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\sin(4*b*\log(x^n) + 4*a)^2 + 4*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\sin(2*b*\log(x^n) + 2*a)^2 + 2*(2*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))\cos(2*b*\log(x^n) + 2*a) + 2*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))\sin(2*b*\log(x^n) + 2*a) + \cos(4*b*\log(c))\cos(4*b*\log(x^n) + 4*a) + 4*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) - 2*(2*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))\cos(2*b*\log(x^n) + 2*a) - 2*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))\sin(2*b*\log(x^n) + 2*a) + \sin(4*b*\log(c))*\sin(4*b*\log(x^n) + 4*a) - 4*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 1)$$

**Fricas [A]** time = 0.474285, size = 146, normalized size = 3.56

$$\frac{bnx \sin(bn \log(x) + b \log(c) + a) - x \cos(bn \log(x) + b \log(c) + a)}{\cos(bn \log(x) + b \log(c) + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b^2\*n^2+1)\*sec(a+b\*log(c\*x^n))+2\*b^2\*n^2\*sec(a+b\*log(c\*x^n))^3, x, algorithm="fricas")

[Out] (b\*n\*x\*sin(b\*n\*log(x) + b\*log(c) + a) - x\*cos(b\*n\*log(x) + b\*log(c) + a))/cos(b\*n\*log(x) + b\*log(c) + a)^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (2b^2n^2 \sec^2(a + b \log(cx^n)) - b^2n^2 - 1) \sec(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b\*\*2\*n\*\*2+1)\*sec(a+b\*ln(c\*x\*\*n))+2\*b\*\*2\*n\*\*2\*sec(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral((2\*b\*\*2\*n\*\*2\*sec(a + b\*log(c\*x\*\*n))\*\*2 - b\*\*2\*n\*\*2 - 1)\*sec(a + b\*log(c\*x\*\*n)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int 2b^2n^2 \sec(b \log(cx^n) + a)^3 - (b^2n^2 + 1) \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b^2\*n^2+1)\*sec(a+b\*log(c\*x^n))+2\*b^2\*n^2\*sec(a+b\*log(c\*x^n))^3, x, algorithm="giac")

[Out] integrate(2\*b^2\*n^2\*sec(b\*log(c\*x^n) + a)^3 - (b^2\*n^2 + 1)\*sec(b\*log(c\*x^n) + a), x)

$$3.260 \quad \int x^m \sec^3 \left( a + 2 \log \left( cx^{\frac{1}{2}\sqrt{-(1+m)^2}} \right) \right) dx$$

**Optimal.** Leaf size=110

$$\frac{x^{m+1} \sec \left( a + 2 \log \left( cx^{\frac{1}{2}\sqrt{-(m+1)^2}} \right) \right)}{2(m+1)} + \frac{x^{m+1} \tan \left( a + 2 \log \left( cx^{\frac{1}{2}\sqrt{-(m+1)^2}} \right) \right) \sec \left( a + 2 \log \left( cx^{\frac{1}{2}\sqrt{-(m+1)^2}} \right) \right)}{2\sqrt{-(m+1)^2}}$$

[Out] (x^(1 + m)\*Sec[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]])/(2\*(1 + m)) + (x^(1 + m)\*Sec[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]]\*Tan[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]])/(2\*Sqrt[-(1 + m)^2])

**Rubi [C]** time = 0.217478, antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {4509, 4505, 364}

$$\frac{8e^{3ia}x^{m+1} \left( cx^{\frac{1}{2}\sqrt{-(m+1)^2}} \right)^{6i} {}_2F_1 \left( 3, \frac{1}{2} \left( 3 - \frac{i(m+1)}{\sqrt{-(m+1)^2}} \right); \frac{1}{2} \left( 5 - \frac{i(m+1)}{\sqrt{-(m+1)^2}} \right); -e^{2ia} \left( cx^{\frac{1}{2}\sqrt{-(m+1)^2}} \right)^{4i} \right)}{1 - i \left( -3\sqrt{-(m+1)^2} + im \right)}$$

Warning: Unable to verify antiderivative.

[In] Int[x^m\*Sec[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]

[Out] (8\*E^((3\*I)\*a)\*x^(1 + m)\*(c\*x^(Sqrt[-(1 + m)^2]/2))^(6\*I)\*Hypergeometric2F1[3, (3 - (I\*(1 + m))/Sqrt[-(1 + m)^2])/2, (5 - (I\*(1 + m))/Sqrt[-(1 + m)^2])/2, -(E^((2\*I)\*a)\*(c\*x^(Sqrt[-(1 + m)^2]/2))^(4\*I))]/(1 - I\*(I\*m - 3\*Sqrt[-(1 + m)^2])))

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \sec^3\left(a + 2 \log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right) dx = \frac{\left(2x^{1+m} \left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{-\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right) \text{Subst}\left(\int x^{-1+\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\sec^3(a + 2 \log(x)) dx, x, \right)}{\sqrt{-(1+m)^2}}$$

$$= \frac{\left(16e^{3ia}x^{1+m} \left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{-\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right) \text{Subst}\left(\int \frac{x^{(-1+6i)+\frac{2(1+m)}{\sqrt{-(1+m)^2}}}}{(1+e^{2ia}x^{4i})^3} dx, x, cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)}{\sqrt{-(1+m)^2}}$$

$$= \frac{8e^{3ia}x^{1+m} \left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{6i} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{i(1+m)}{\sqrt{-(1+m)^2}}\right); \frac{1}{2}\left(5 - \frac{i(1+m)}{\sqrt{-(1+m)^2}}\right); -e^{2ia}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{1 - i\left(im - 3\sqrt{-(1+m)^2}\right)}$$

**Mathematica [A]** time = 2.11106, size = 198, normalized size = 1.8

$$\frac{x^{m+1} \left( (m+1) \cos\left(a + 2 \log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right) - \sqrt{-(m+1)^2} \sin\left(a + 2 \log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right) \right)}{2(m+1)^2 \left( \cos\left(\frac{a}{2} + \log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right) - \sin\left(\frac{a}{2} + \log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right) \right)^2 \left( \sin\left(\frac{a}{2} + \log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right) + \cos\left(\frac{a}{2} + \log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Sec[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]

[Out] (x^(1 + m)\*((1 + m)\*Cos[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]] - Sqrt[-(1 + m)^2]\*Sin[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]]))/(2\*(1 + m)^2\*(Cos[a/2 + Log[c\*x^(Sqrt[-(1 + m)^2]/2)]] - Sin[a/2 + Log[c\*x^(Sqrt[-(1 + m)^2]/2)]])^2\*(Cos[a/2 + Log[c\*x^(Sqrt[-(1 + m)^2]/2)]] + Sin[a/2 + Log[c\*x^(Sqrt[-(1 + m)^2]/2)]])^2)

**Maple [F]** time = 0.228, size = 0, normalized size = 0.

$$\int x^m \left( \sec\left(a + 2 \ln\left(cx^{1/2\sqrt{-(1+m)^2}}\right)\right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sec(a+2\*ln(c\*x^(1/2\*(-(1+m)^2)^(1/2))))^3,x)

[Out] int(x^m\*sec(a+2\*ln(c\*x^(1/2\*(-(1+m)^2)^(1/2))))^3,x)

**Maxima [B]** time = 1.42587, size = 1318, normalized size = 11.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+2\*log(c\*x^(1/2\*(-(1+m)^2)^(1/2))))^3,x, algorithm="maxima")

```
[Out] 2*((cos(a)*cos(2*log(c)) - sin(a)*sin(2*log(c)))*x*e^(m*log(x) + 14*arctan2(
sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 14*arctan2(sin(1/2*log(x)), cos(1/
2*log(x)))) + 2*(((cos(2*a)*cos(a) + sin(2*a)*sin(a))*cos(2*log(c)) + (cos(
a)*sin(2*a) - cos(2*a)*sin(a))*sin(2*log(c)))*cos(4*log(c)) - ((cos(a)*sin(
2*a) - cos(2*a)*sin(a))*cos(2*log(c)) - (cos(2*a)*cos(a) + sin(2*a)*sin(a)
)*sin(2*log(c)))*sin(4*log(c)))*x*e^(m*log(x) + 10*arctan2(sin(1/2*m*log(x))
, cos(1/2*m*log(x))) + 10*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))) + (((c
os(4*a)*cos(a) + sin(4*a)*sin(a))*cos(2*log(c)) + (cos(a)*sin(4*a) - cos(4*
a)*sin(a))*sin(2*log(c)))*cos(8*log(c)) - ((cos(a)*sin(4*a) - cos(4*a)*sin(
a))*cos(2*log(c)) - (cos(4*a)*cos(a) + sin(4*a)*sin(a))*sin(2*log(c)))*sin(
8*log(c)))*x*e^(m*log(x) + 6*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x)))
+ 6*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))))/((cos(4*a)^2 + sin(4*a)^2)*
cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin(8*log(c))^2 + ((cos(4*a)^2
+ sin(4*a)^2)*cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin(8*log(c))^2)*
m + (m + 1)*e^(16*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 16*arctan
2(sin(1/2*log(x)), cos(1/2*log(x)))) + 4*(((cos(2*a)*cos(4*log(c)) - sin(2*a
)*sin(4*log(c)))*m + cos(2*a)*cos(4*log(c)) - sin(2*a)*sin(4*log(c)))*e^(12
*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 12*arctan2(sin(1/2*log(x))
, cos(1/2*log(x)))) + 2*(2*(cos(2*a)^2 + sin(2*a)^2)*cos(4*log(c))^2 + 2*(c
os(2*a)^2 + sin(2*a)^2)*sin(4*log(c))^2 + (2*(cos(2*a)^2 + sin(2*a)^2)*cos(
4*log(c))^2 + 2*(cos(2*a)^2 + sin(2*a)^2)*sin(4*log(c))^2 + cos(4*a)*cos(8*
log(c)) - sin(4*a)*sin(8*log(c)))*m + cos(4*a)*cos(8*log(c)) - sin(4*a)*sin
(8*log(c)))*e^(8*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 8*arctan2(
sin(1/2*log(x)), cos(1/2*log(x)))) + 4*(((cos(4*a)*cos(2*a) + sin(4*a)*sin
(2*a))*cos(4*log(c)) + (cos(2*a)*sin(4*a) - cos(4*a)*sin(2*a))*sin(4*log(c)
))*cos(8*log(c)) - ((cos(2*a)*sin(4*a) - cos(4*a)*sin(2*a))*cos(4*log(c)) -
(cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*sin(4*log(c)))*sin(8*log(c)))*m +
((cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*cos(4*log(c)) + (cos(2*a)*sin(4*a)
- cos(4*a)*sin(2*a))*sin(4*log(c)))*cos(8*log(c)) - ((cos(2*a)*sin(4*a) -
cos(4*a)*sin(2*a))*cos(4*log(c)) - (cos(4*a)*cos(2*a) + sin(4*a)*sin(2*a))*
sin(4*log(c)))*sin(8*log(c)))*e^(4*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log
(x))) + 4*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))
```

**Fricas [C]** time = 0.476942, size = 231, normalized size = 2.1

$$\frac{2 \left( 2x^2x^{2m}e^{(3ia+6i \log(c))} + e^{(5ia+10i \log(c))} \right)}{(m+1)x^4x^{4m} + 2(m+1)x^2x^{2m}e^{(2ia+4i \log(c))} + (m+1)e^{(4ia+8i \log(c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="fric
as")
```

```
[Out] -2*(2*x^2*x^(2*m)*e^(3*I*a + 6*I*log(c)) + e^(5*I*a + 10*I*log(c)))/((m + 1
)*x^4*x^(4*m) + 2*(m + 1)*x^2*x^(2*m)*e^(2*I*a + 4*I*log(c)) + (m + 1)*e^(4
*I*a + 8*I*log(c)))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+2*ln(c*x**(1/2*(-(1+m)**2)**(1/2))))**3,x)
```

[Out] Timed out

**Giac [C]** time = 15.4405, size = 1126, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+2\*log(c\*x^(1/2\*(-(1+m)^2)^(1/2))))^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & c^{(6I)} * m * x * x^m * x^{\text{abs}(m+1)} * e^{(3I*a)} / (c^{(8I)} * m^2 * e^{(4I*a)} + 2 * c^{(8I)} * m * e^{(4I*a)} + c^{(8I)} * e^{(4I*a)} + 2 * c^{(4I)} * m^2 * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 4 * c^{(4I)} * m * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 2 * c^{(4I)} * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + m^2 * x^{(4*\text{abs}(m+1))} + 2 * m * x^{(4*\text{abs}(m+1))} + x^{(4*\text{abs}(m+1))}) - c^{(6I)} * x * x^m * x^{\text{abs}(m+1)} * \text{abs}(m+1) * e^{(3I*a)} / (c^{(8I)} * m^2 * e^{(4I*a)} + 2 * c^{(8I)} * m * e^{(4I*a)} + c^{(8I)} * e^{(4I*a)} + 2 * c^{(4I)} * m^2 * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 4 * c^{(4I)} * m * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 2 * c^{(4I)} * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + m^2 * x^{(4*\text{abs}(m+1))} + 2 * m * x^{(4*\text{abs}(m+1))} + x^{(4*\text{abs}(m+1))}) \\ & + c^{(6I)} * x * x^m * x^{\text{abs}(m+1)} * e^{(3I*a)} / (c^{(8I)} * m^2 * e^{(4I*a)} + 2 * c^{(8I)} * m * e^{(4I*a)} + c^{(8I)} * e^{(4I*a)} + 2 * c^{(4I)} * m^2 * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 4 * c^{(4I)} * m * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 2 * c^{(4I)} * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + m^2 * x^{(4*\text{abs}(m+1))} + 2 * m * x^{(4*\text{abs}(m+1))} + x^{(4*\text{abs}(m+1))}) + c^{(2I)} * x * x^m * x^{(3*\text{abs}(m+1))} * e^{(I*a)} / (c^{(8I)} * m^2 * e^{(4I*a)} + 2 * c^{(8I)} * m * e^{(4I*a)} + c^{(8I)} * e^{(4I*a)} + 2 * c^{(4I)} * m^2 * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 4 * c^{(4I)} * m * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 2 * c^{(4I)} * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + m^2 * x^{(4*\text{abs}(m+1))} + 2 * m * x^{(4*\text{abs}(m+1))} + x^{(4*\text{abs}(m+1))}) + c^{(2I)} * x * x^m * x^{(3*\text{abs}(m+1))} * e^{(I*a)} / (c^{(8I)} * m^2 * e^{(4I*a)} + 2 * c^{(8I)} * m * e^{(4I*a)} + c^{(8I)} * e^{(4I*a)} + 2 * c^{(4I)} * m^2 * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 4 * c^{(4I)} * m * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 2 * c^{(4I)} * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + m^2 * x^{(4*\text{abs}(m+1))} + 2 * m * x^{(4*\text{abs}(m+1))} + x^{(4*\text{abs}(m+1))}) + c^{(2I)} * x * x^m * x^{(3*\text{abs}(m+1))} * e^{(I*a)} / (c^{(8I)} * m^2 * e^{(4I*a)} + 2 * c^{(8I)} * m * e^{(4I*a)} + c^{(8I)} * e^{(4I*a)} + 2 * c^{(4I)} * m^2 * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 4 * c^{(4I)} * m * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + 2 * c^{(4I)} * x^{(2*\text{abs}(m+1))} * e^{(2I*a)} + m^2 * x^{(4*\text{abs}(m+1))} + 2 * m * x^{(4*\text{abs}(m+1))} + x^{(4*\text{abs}(m+1))}) \end{aligned}$$



### 3.261 $\int x \sec^3(a + 2 \log(cx^i)) dx$

**Optimal.** Leaf size=45

$$\frac{e^{ia} x^2 (cx^i)^{2i}}{(1 + e^{2ia} (cx^i)^{4i})^2}$$

[Out]  $(E^{(I*a)*(c*x^I)^{(2*I)*x^2})/(1 + E^{((2*I)*a)*(c*x^I)^{(4*I)})^2}$

**Rubi [A]** time = 0.0427162, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 261}

$$\frac{e^{ia} x^2 (cx^i)^{2i}}{(1 + e^{2ia} (cx^i)^{4i})^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*\text{Sec}[a + 2*\text{Log}[c*x^I]]^3, x]$

[Out]  $(E^{(I*a)*(c*x^I)^{(2*I)*x^2})/(1 + E^{((2*I)*a)*(c*x^I)^{(4*I)})^2}$

#### Rule 4509

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sec}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}]* (b_{.})*(d_{.})]^{(p_{.})}, x\_Symbol] \rightarrow \text{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \text{Subst}[\text{Int}[x^{((m+1)/n-1)*\text{Sec}[d*(a+b*\text{Log}[x])]}^p, x], x, c*x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\text{NeQ}[c, 1] \parallel \text{NeQ}[n, 1])$

#### Rule 4505

$\text{Int}[(e_{.})*(x_{.})^{(m_{.})}*\text{Sec}[(a_{.}) + \text{Log}[x_{.}]* (b_{.})*(d_{.})]^{(p_{.})}, x\_Symbol] \rightarrow \text{Dist}[2^p * E^{(I*a*d*p)}, \text{Int}[(e*x)^m * x^{(I*b*d*p)} / (1 + E^{(2*I*a*d)*x^{(2*I*b*d)})}^p, x], x] /; \text{FreeQ}\{a, b, d, e, m\}, x] \&\& \text{IntegerQ}[p]$

#### Rule 261

$\text{Int}[(x_{.})^{(m_{.})}*((a_{.}) + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p+1)}/(b*n*(p+1)), x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{EqQ}[m, n-1] \&\& \text{NeQ}[p, -1]$

#### Rubi steps

$$\begin{aligned} \int x \sec^3(a + 2 \log(cx^i)) dx &= -\left( (i (cx^i)^{2i} x^2) \text{Subst} \left( \int x^{-1-2i} \sec^3(a + 2 \log(x)) dx, x, cx^i \right) \right) \\ &= -\left( (8ie^{3ia} (cx^i)^{2i} x^2) \text{Subst} \left( \int \frac{x^{-1+4i}}{(1 + e^{2ia} x^{4i})^3} dx, x, cx^i \right) \right) \\ &= \frac{e^{ia} (cx^i)^{2i} x^2}{(1 + e^{2ia} (cx^i)^{4i})^2} \end{aligned}$$



```
[In] integrate(x*sec(a+2*log(c*x^I))^3,x, algorithm="fricas")
```

```
[Out] x^2*e^(I*a + 2*I*log(c*x^I))/(e^(4*I*a + 8*I*log(c*x^I)) + 2*e^(2*I*a + 4*I*log(c*x^I)) + 1)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec^3(a + 2 \log(cx^i)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+2*ln(c*x**I))**3,x)
```

```
[Out] Integral(x*sec(a + 2*log(c*x**I))**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(a + 2 \log(cx^i))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+2*log(c*x^I))^3,x, algorithm="giac")
```

```
[Out] integrate(x*sec(a + 2*log(c*x^I))^3, x)
```

### 3.262 $\int \sec^3 \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) dx$

**Optimal.** Leaf size=58

$$\frac{1}{2}x \sec \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) - \frac{1}{2}ix \tan \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) \sec \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right)$$

[Out] (x\*Sec[a + 2\*Log[c\*x^(I/2)]])/2 - (I/2)\*x\*Sec[a + 2\*Log[c\*x^(I/2)]]\*Tan[a + 2\*Log[c\*x^(I/2)]]

**Rubi [A]** time = 0.0349121, antiderivative size = 48, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4503, 4505, 261}

$$\frac{2e^{ia}x \left( cx^{\frac{i}{2}} \right)^{2i}}{\left( 1 + e^{2ia} \left( cx^{\frac{i}{2}} \right)^{4i} \right)^2}$$

Warning: Unable to verify antiderivative.

[In] Int[Sec[a + 2\*Log[c\*x^(I/2)]]^3,x]

[Out] (2\*E^(I\*a)\*(c\*x^(I/2))^(2\*I)\*x)/(1 + E^((2\*I)\*a)\*(c\*x^(I/2))^(4\*I))^2

#### Rule 4503

Int[Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \sec^3 \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) dx &= - \left( \left( 2i \left( cx^{\frac{i}{2}} \right)^{2i} x \right) \text{Subst} \left( \int x^{-1-2i} \sec^3(a + 2 \log(x)) dx, x, cx^{\frac{i}{2}} \right) \right) \\ &= - \left( \left( 16ie^{3ia} \left( cx^{\frac{i}{2}} \right)^{2i} x \right) \text{Subst} \left( \int \frac{x^{-1+4i}}{\left( 1 + e^{2ia}x^{4i} \right)^3} dx, x, cx^{\frac{i}{2}} \right) \right) \\ &= \frac{2e^{ia} \left( cx^{\frac{i}{2}} \right)^{2i} x}{\left( 1 + e^{2ia} \left( cx^{\frac{i}{2}} \right)^{4i} \right)^2} \end{aligned}$$

**Mathematica [B]** time = 0.113546, size = 137, normalized size = 2.36

$$\frac{\sec^2\left(a + 2 \log\left(cx^{\frac{i}{2}}\right)\right)\left(i(1 - 2x^2) \sin\left(a + 2 \log\left(cx^{\frac{i}{2}}\right) - i \log(x)\right) + (2x^2 + 1) \cos\left(a + 2 \log\left(cx^{\frac{i}{2}}\right) - i \log(x)\right)\right)\left(i \sin\left(a + 2 \log\left(cx^{\frac{i}{2}}\right) - i \log(x)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + 2\*Log[c\*x^(I/2)]]^3,x]

[Out] -(Sec[a + 2\*Log[c\*x^(I/2)]]^2\*((1 + 2\*x^2)\*Cos[a + 2\*Log[c\*x^(I/2)]] - I\*Log[x]) + I\*(1 - 2\*x^2)\*Sin[a + 2\*Log[c\*x^(I/2)]] - I\*Log[x])\*(Cos[2\*(a + 2\*Log[c\*x^(I/2)]] - I\*Log[x]) + I\*Sine[2\*(a + 2\*Log[c\*x^(I/2)]] - I\*Log[x]))/(2\*x^2)

**Maple [C]** time = 0.204, size = 214, normalized size = 3.7

$$2 \frac{x e^{-i\left(\pi \operatorname{csgn}(icx^{i/2})\right)^3 - i\pi \left(\operatorname{csgn}(icx^{i/2})\right)^2 \operatorname{csgn}(ic) - i\pi \left(\operatorname{csgn}(icx^{i/2})\right)^2 \operatorname{csgn}(ix^{i/2}) + i\pi \operatorname{csgn}(icx^{i/2}) \operatorname{csgn}(ic) \operatorname{csgn}(ix^{i/2}) - 2 \ln(c) - 2 \ln(x^{i/2}) - a}}{\left(e^{-2i\left(\pi \operatorname{csgn}(icx^{i/2})\right)^3 - i\pi \left(\operatorname{csgn}(icx^{i/2})\right)^2 \operatorname{csgn}(ic) - i\pi \left(\operatorname{csgn}(icx^{i/2})\right)^2 \operatorname{csgn}(ix^{i/2}) + i\pi \operatorname{csgn}(icx^{i/2}) \operatorname{csgn}(ic) \operatorname{csgn}(ix^{i/2}) - 2 \ln(c) - 2 \ln(x^{i/2}) - a} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+2\*ln(c\*x^(1/2\*I)))^3,x)

[Out] 2\*x\*exp(-I\*(I\*Pi\*csgn(I\*c\*x^(1/2\*I)))^3-I\*Pi\*csgn(I\*c\*x^(1/2\*I))^2\*csgn(I\*c)-I\*Pi\*csgn(I\*c\*x^(1/2\*I))^2\*csgn(I\*x^(1/2\*I))+I\*Pi\*csgn(I\*c\*x^(1/2\*I))\*csgn(I\*c)\*csgn(I\*x^(1/2\*I))-2\*ln(c)-2\*ln(x^(1/2\*I))-a)/(exp(-2\*I\*(I\*Pi\*csgn(I\*c\*x^(1/2\*I)))^3-I\*Pi\*csgn(I\*c\*x^(1/2\*I))^2\*csgn(I\*c)-I\*Pi\*csgn(I\*c\*x^(1/2\*I))^2\*csgn(I\*x^(1/2\*I))+I\*Pi\*csgn(I\*c\*x^(1/2\*I))\*csgn(I\*c)\*csgn(I\*x^(1/2\*I))-2\*ln(c)-2\*ln(x^(1/2\*I))-a))+1)^2

**Maxima [B]** time = 1.24468, size = 208, normalized size = 3.59

$$\frac{((2 \cos(a) + 2i \sin(a)) \cos(2 \log(c)) + 2(i \cos(a) - \sin(a)) \sin(2 \log(c)))}{(\cos(4a) + i \sin(4a)) \cos(8 \log(c)) + ((2 \cos(2a) + 2i \sin(2a)) \cos(4 \log(c)) - 2(-i \cos(2a) + \sin(2a)) \sin(4 \log(c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+2\*log(c\*x^(1/2\*I)))^3,x, algorithm="maxima")

[Out] ((2\*cos(a) + 2\*I\*sin(a))\*cos(2\*log(c)) + 2\*(I\*cos(a) - sin(a))\*sin(2\*log(c)))\*x\*e^(6\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x))))/((cos(4\*a) + I\*sin(4\*a))\*cos(8\*log(c)) + ((2\*cos(2\*a) + 2\*I\*sin(2\*a))\*cos(4\*log(c)) - 2\*(-I\*cos(2\*a) + sin(2\*a))\*sin(4\*log(c))))\*e^(4\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x)))) + (I\*cos(4\*a) - sin(4\*a))\*sin(8\*log(c)) + e^(8\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x))))))

**Fricas [A]** time = 0.449053, size = 151, normalized size = 2.6

$$\frac{2xe^{\left(ia+2i\log\left(cx^{\frac{1}{2}i}\right)\right)}}{e^{\left(4ia+8i\log\left(cx^{\frac{1}{2}i}\right)\right)} + 2e^{\left(2ia+4i\log\left(cx^{\frac{1}{2}i}\right)\right)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+2\*log(c\*x^(1/2\*I)))^3,x, algorithm="fricas")

[Out] 2\*x\*e^(I\*a + 2\*I\*log(c\*x^(1/2\*I)))/(e^(4\*I\*a + 8\*I\*log(c\*x^(1/2\*I))) + 2\*e^(2\*I\*a + 4\*I\*log(c\*x^(1/2\*I))) + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^3\left(a + 2\log\left(cx^{\frac{i}{2}}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+2\*ln(c\*x\*\*(1/2\*I)))\*\*3,x)

[Out] Integral(sec(a + 2\*log(c\*x\*\*(I/2)))\*\*3, x)

**Giac [A]** time = 4.75822, size = 100, normalized size = 1.72

$$-\frac{2c^{10i}e^{(5ia)}}{c^{8i}e^{(4ia)} + 2c^{4i}x^2e^{(2ia)} + x^4} - \frac{4c^{6i}x^2e^{(3ia)}}{c^{8i}e^{(4ia)} + 2c^{4i}x^2e^{(2ia)} + x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+2\*log(c\*x^(1/2\*I)))^3,x, algorithm="giac")

[Out] -2\*c^(10\*I)\*e^(5\*I\*a)/(c^(8\*I)\*e^(4\*I\*a) + 2\*c^(4\*I)\*x^2\*e^(2\*I\*a) + x^4) - 4\*c^(6\*I)\*x^2\*e^(3\*I\*a)/(c^(8\*I)\*e^(4\*I\*a) + 2\*c^(4\*I)\*x^2\*e^(2\*I\*a) + x^4)

$$3.263 \quad \int \sec^3 \left( a + 2 \log \left( cx^{-\frac{i}{2}} \right) \right) dx$$

**Optimal.** Leaf size=48

$$\frac{2e^{3ia}x \left( cx^{-\frac{i}{2}} \right)^{6i}}{\left( 1 + e^{2ia} \left( cx^{-\frac{i}{2}} \right)^{4i} \right)^2}$$

[Out]  $(2 * E^{((3 * I) * a)} * (c / x^{(I / 2)})^{(6 * I)} * x) / (1 + E^{((2 * I) * a)} * (c / x^{(I / 2)})^{(4 * I)})^2$

**Rubi [A]** time = 0.0406423, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4503, 4505, 264}

$$\frac{2e^{3ia}x \left( cx^{-\frac{i}{2}} \right)^{6i}}{\left( 1 + e^{2ia} \left( cx^{-\frac{i}{2}} \right)^{4i} \right)^2}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + 2\*Log[c/x^(I/2)]]^3,x]

[Out]  $(2 * E^{((3 * I) * a)} * (c / x^{(I / 2)})^{(6 * I)} * x) / (1 + E^{((2 * I) * a)} * (c / x^{(I / 2)})^{(4 * I)})^2$

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[2^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p)) / (1 + E^(2\*I\*a\*d) \* x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)) / (a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \sec^3\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right)\right) dx &= \left(2i\left(cx^{-\frac{i}{2}}\right)^{-2i} x\right) \text{Subst}\left(\int x^{-1+2i} \sec^3(a + 2 \log(x)) dx, x, cx^{-\frac{i}{2}}\right) \\ &= \left(16ie^{3ia}\left(cx^{-\frac{i}{2}}\right)^{-2i} x\right) \text{Subst}\left(\int \frac{x^{-1+8i}}{\left(1 + e^{2ia}x^{4i}\right)^3} dx, x, cx^{-\frac{i}{2}}\right) \\ &= \frac{2e^{3ia}\left(cx^{-\frac{i}{2}}\right)^{6i} x}{\left(1 + e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2} \end{aligned}$$

**Mathematica [B]** time = 0.139061, size = 139, normalized size = 2.9

$$\frac{\sec^2\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right)\right)\left(i(2x^2 - 1) \sin\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right) + i \log(x)\right) + (2x^2 + 1) \cos\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right) + i \log(x)\right)\right)\left(2i \sin\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right) + i \log(x)\right)\right)}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + 2\*Log[c/x^(I/2)]]^3,x]

[Out] (Sec[a + 2\*Log[c/x^(I/2)]]^2\*((1 + 2\*x^2)\*Cos[a + 2\*Log[c/x^(I/2)] + I\*Log[x]] + I\*(-1 + 2\*x^2)\*Sin[a + 2\*Log[c/x^(I/2)] + I\*Log[x]])\*(-2\*Cos[2\*(a + 2\*Log[c/x^(I/2)] + I\*Log[x])] + (2\*I)\*Sin[2\*(a + 2\*Log[c/x^(I/2)] + I\*Log[x])])/ (4\*x^2)

**Maple [C]** time = 0.204, size = 238, normalized size = 5.

$$2xe^{-3i\left(i\pi\left(\operatorname{csgn}\left(\frac{ic}{x^{i/2}}\right)\right)^3 - i\pi\left(\operatorname{csgn}\left(\frac{ic}{x^{i/2}}\right)\right)^2 \operatorname{csgn}(ic) - i\pi\left(\operatorname{csgn}\left(\frac{ic}{x^{i/2}}\right)\right) \operatorname{csgn}\left(\frac{i}{x^{i/2}}\right) + i\pi \operatorname{csgn}\left(\frac{ic}{x^{i/2}}\right) \operatorname{csgn}(ic) \operatorname{csgn}\left(\frac{i}{x^{i/2}}\right) - 2 \ln(c) + 2 \ln(x^{i/2}) - a\right)} \left( e^{-2i\left(i\pi\left(\operatorname{csgn}\left(\frac{i}{x^{i/2}}\right)\right)\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+2\*ln(c/(x^(1/2\*I))))^3,x)

[Out] 2\*x\*exp(-3\*I\*(I\*Pi\*csgn(I\*c/(x^(1/2\*I))))^3 - I\*Pi\*csgn(I\*c/(x^(1/2\*I)))^2\*csgn(I\*c) - I\*Pi\*csgn(I\*c/(x^(1/2\*I)))^2\*csgn(I/(x^(1/2\*I))) + I\*Pi\*csgn(I\*c/(x^(1/2\*I))) \*csgn(I\*c)\*csgn(I/(x^(1/2\*I))) - 2\*ln(c) + 2\*ln(x^(1/2\*I)) - a) / (exp(-2\*I\*(I\*Pi\*csgn(I\*c/(x^(1/2\*I))))^3 - I\*Pi\*csgn(I\*c/(x^(1/2\*I)))^2\*csgn(I\*c) - I\*Pi\*csgn(I\*c/(x^(1/2\*I)))^2\*csgn(I/(x^(1/2\*I))) + I\*Pi\*csgn(I\*c/(x^(1/2\*I))) \*csgn(I\*c)\*csgn(I/(x^(1/2\*I))) - 2\*ln(c) + 2\*ln(x^(1/2\*I)) - a) + 1)^2

**Maxima [B]** time = 1.23325, size = 224, normalized size = 4.67

$$\frac{((2 \cos(3a) + 2i \sin(3a)) \cos(6 \log(c)) + 2(i \cos(3a) - \sin(3a)) \sin(6 \log(c))) \left( (\cos(4a) + i \sin(4a)) \cos(8 \log(c)) - (-i \cos(4a) + \sin(4a)) \sin(8 \log(c)) \right) e^{(8 \arctan\left(\sin\left(\frac{1}{2} \log(x)\right), \cos\left(\frac{1}{2} \log(x)\right)\right))}}{((2 \cos(3a) + 2i \sin(3a)) \cos(6 \log(c)) + 2(i \cos(3a) - \sin(3a)) \sin(6 \log(c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+2\*log(c/(x^(1/2\*I))))^3,x, algorithm="maxima")



```
[Out] ((2*cos(3*a) + 2*I*sin(3*a))*cos(6*log(c)) + 2*(I*cos(3*a) - sin(3*a))*sin(
6*log(c)))*x*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/(((cos(4*a) +
I*sin(4*a))*cos(8*log(c)) - (-I*cos(4*a) + sin(4*a))*sin(8*log(c)))*e^(8*ar
ctan2(sin(1/2*log(x)), cos(1/2*log(x)))) + ((2*cos(2*a) + 2*I*sin(2*a))*cos
(4*log(c)) + 2*(I*cos(2*a) - sin(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*
log(x)), cos(1/2*log(x)))) + 1)
```

**Fricas [A]** time = 0.449647, size = 158, normalized size = 3.29

$$\frac{2xe^{\left(3ia+6i\log\left(cx^{-\frac{1}{2}i}\right)\right)}}{e^{\left(4ia+8i\log\left(cx^{-\frac{1}{2}i}\right)\right)} + 2e^{\left(2ia+4i\log\left(cx^{-\frac{1}{2}i}\right)\right)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="fricas")
```

```
[Out] 2*x*e^(3*I*a + 6*I*log(c*x^(-1/2*I)))/(e^(4*I*a + 8*I*log(c*x^(-1/2*I))) +
2*e^(2*I*a + 4*I*log(c*x^(-1/2*I))) + 1)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^3\left(a + 2\log\left(cx^{-\frac{i}{2}}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*ln(c/(x**(1/2*I))))**3,x)
```

```
[Out] Integral(sec(a + 2*log(c*x**(-I/2)))**3, x)
```

**Giac [B]** time = 4.55561, size = 112, normalized size = 2.33

$$\frac{4c^{4i}x^2e^{2ia}}{c^{10i}x^4e^{5ia} + 2c^{6i}x^2e^{3ia} + c^{2i}e^{ia}} - \frac{2}{c^{10i}x^4e^{5ia} + 2c^{6i}x^2e^{3ia} + c^{2i}e^{ia}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="giac")
```

```
[Out] -4*c^(4*I)*x^2*e^(2*I*a)/(c^(10*I)*x^4*e^(5*I*a) + 2*c^(6*I)*x^2*e^(3*I*a)
+ c^(2*I)*e^(I*a)) - 2/(c^(10*I)*x^4*e^(5*I*a) + 2*c^(6*I)*x^2*e^(3*I*a) +
c^(2*I)*e^(I*a))
```

$$3.264 \quad \int \sec^p \left( a + \frac{i \log(cx^n)}{n(-2+p)} \right) dx$$

**Optimal.** Leaf size=95

$$\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}} \left( 1 + e^{2ia}(cx^n)^{\frac{2}{n(2-p)}} \right) \sec^p \left( a - \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)}$$

[Out] ((2 - p)\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^(2/(n\*(2 - p))))\*Sec[a - (I\*Log[c\*x^n])/(n\*(2 - p))]^p)/(2\*E^((2\*I)\*a)\*(1 - p)\*(c\*x^n)^(2/(n\*(2 - p))))

**Rubi [A]** time = 0.0910376, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4503, 4507, 261}

$$\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}} \left( 1 + e^{2ia}(cx^n)^{\frac{2}{n(2-p)}} \right) \sec^p \left( a - \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + (I\*Log[c\*x^n])/(n\*(-2 + p))]^p, x]

[Out] ((2 - p)\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^(2/(n\*(2 - p))))\*Sec[a - (I\*Log[c\*x^n])/(n\*(2 - p))]^p)/(2\*E^((2\*I)\*a)\*(1 - p)\*(c\*x^n)^(2/(n\*(2 - p))))

#### Rule 4503

Int[Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_)^(m\_.))\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \sec^p \left( a + \frac{i \log(cx^n)}{n(-2+p)} \right) dx &= \frac{(x(cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1+\frac{1}{n}} \sec^p \left( a + \frac{i \log(x)}{n(-2+p)} \right) dx, x, cx^n \right)}{n} \\ &= \frac{\left( x(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}} \left( 1 + e^{2ia} (cx^n)^{-\frac{2}{n(-2+p)}} \right)^p \sec^p \left( a + \frac{i \log(cx^n)}{n(-2+p)} \right) \right) \text{Subst} \left( \int x^{-1+\frac{1}{n}-\frac{p}{n(-2+p)}} \left( 1 + e^{2ia} x^{\frac{2}{n(-2+p)}} \right)^p dx, x, cx^n \right)}{n} \\ &= \frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}} \left( 1 + e^{2ia} (cx^n)^{\frac{2}{n(2-p)}} \right) \sec^p \left( a - \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)} \end{aligned}$$

**Mathematica [A]** time = 0.858004, size = 67, normalized size = 0.71

$$\frac{e^{-2ia}(p-2)x \left( (cx^n)^{\frac{2}{n(p-2)}} + e^{2ia} \right) \sec^p \left( a + \frac{i \log(cx^n)}{n(p-2)} \right)}{2(p-1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + (I\*Log[c\*x^n])/(n\*(-2 + p))]^p, x]

[Out] ((-2 + p)\*x\*(E^((2\*I)\*a) + (c\*x^n)^(2/(n\*(-2 + p))))\*Sec[a + (I\*Log[c\*x^n])/(n\*(-2 + p))]^p)/(2\*E^((2\*I)\*a)\*(-1 + p))

**Maple [F]** time = 0.372, size = 0, normalized size = 0.

$$\int \left( \sec \left( a + \frac{i \ln(cx^n)}{n(p-2)} \right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+I\*ln(c\*x^n)/n/(p-2))^p, x)

[Out] int(sec(a+I\*ln(c\*x^n)/n/(p-2))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec \left( a + \frac{i \log(cx^n)}{n(p-2)} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+I\*log(c\*x^n)/n/(-2+p))^p, x, algorithm="maxima")

[Out] integrate(sec(a + I\*log(c\*x^n)/(n\*(p - 2)))^p, x)

**Fricas [A]** time = 0.505553, size = 340, normalized size = 3.58

$$\frac{\left( (p-2)xe^{\frac{2(ianp-2ian-\log(cx^n))}{np-2n}} + (p-2)x \right) \left( \frac{2e^{\frac{(ianp-2ian-\log(cx^n))}{np-2n}}}{e^{\frac{2(ianp-2ian-\log(cx^n))}{np-2n}} + 1} \right)^p e^{\frac{-2(ianp-2ian-\log(cx^n))}{np-2n}}}{2(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="fricas")

[Out] 1/2\*((p - 2)\*x\*e^(2\*(I\*a\*n\*p - 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n)) + (p - 2)\*x)\*(2\*e^((I\*a\*n\*p - 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n))/(e^(2\*(I\*a\*n\*p - 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n)) + 1))^p\*e^(-2\*(I\*a\*n\*p - 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n))/(p - 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^p \left( a + \frac{i \log(cx^n)}{n(p-2)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+I\*ln(c\*x\*\*n)/n/(-2+p))\*\*p,x)

[Out] Integral(sec(a + I\*log(c\*x\*\*n)/(n\*(p - 2)))\*\*p, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec \left( a + \frac{i \log(cx^n)}{n(p-2)} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="giac")

[Out] integrate(sec(a + I\*log(c\*x^n)/(n\*(p - 2)))^p, x)

$$3.265 \quad \int \sec^p \left( a - \frac{i \log(cx^n)}{n(-2+p)} \right) dx$$

**Optimal.** Leaf size=70

$$\frac{(2-p)x \left( 1 + e^{2ia} (cx^n)^{-\frac{2}{n(2-p)}} \right) \sec^p \left( a + \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)}$$

[Out] ((2 - p)\*x\*(1 + E^((2\*I)\*a)/(c\*x^n)^(2/(n\*(2 - p))))\*Sec[a + (I\*Log[c\*x^n])/(n\*(2 - p))]^p)/(2\*(1 - p))

**Rubi [A]** time = 0.0751668, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4503, 4507, 264}

$$\frac{(2-p)x \left( 1 + e^{2ia} (cx^n)^{-\frac{2}{n(2-p)}} \right) \sec^p \left( a + \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)}$$

Antiderivative was successfully verified.

[In] Int[Sec[a - (I\*Log[c\*x^n])/(n\*(-2 + p))]^p, x]

[Out] ((2 - p)\*x\*(1 + E^((2\*I)\*a)/(c\*x^n)^(2/(n\*(2 - p))))\*Sec[a + (I\*Log[c\*x^n])/(n\*(2 - p))]^p)/(2\*(1 - p))

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[(e\_.)\*(x\_)^(m\_.)\*Sec[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[(e\*x)^m\*x^(I\*b\*d\*p)/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 264

Int[(c\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))]^(p\_.), x\_Symbol] := Simp[(c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \sec^p \left( a - \frac{i \log(cx^n)}{n(-2+p)} \right) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst} \left( \int x^{-1+\frac{1}{n}} \sec^p \left( a - \frac{i \log(x)}{n(-2+p)} \right) dx, x, cx^n \right)}{n} \\ &= \frac{\left( x(cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}} \left( 1 + e^{2ia} (cx^n)^{\frac{2}{n(-2+p)}} \right)^p \sec^p \left( a - \frac{i \log(cx^n)}{n(-2+p)} \right) \right)}{n} \operatorname{Subst} \left( \int x^{-1+\frac{1}{n}+\frac{p}{n(-2+p)}} \left( 1 + e^{2ia} (cx^n)^{\frac{2}{n(-2+p)}} \right)^p \sec^p \left( a - \frac{i \log(x)}{n(-2+p)} \right) dx, x, cx^n \right) \\ &= \frac{(2-p)x \left( 1 + e^{2ia} (cx^n)^{-\frac{2}{n(2-p)}} \right) \sec^p \left( a + \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)} \end{aligned}$$

**Mathematica [A]** time = 0.791994, size = 62, normalized size = 0.89

$$\frac{(p-2)x \left( 1 + e^{2ia} (cx^n)^{\frac{2}{n(p-2)}} \right) \sec^p \left( a - \frac{i \log(cx^n)}{n(p-2)} \right)}{2(p-1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a - (I\*Log[c\*x^n])/(n\*(-2 + p))]^p, x]

[Out] ((-2 + p)\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^(2/(n\*(-2 + p))))\*Sec[a - (I\*Log[c\*x^n])/(n\*(-2 + p))]^p)/(2\*(-1 + p))

**Maple [F]** time = 0.322, size = 0, normalized size = 0.

$$\int \left( \sec \left( a - \frac{i \ln(cx^n)}{n(p-2)} \right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a-I\*ln(c\*x^n)/n/(p-2))^p, x)

[Out] int(sec(a-I\*ln(c\*x^n)/n/(p-2))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec \left( -a + \frac{i \log(cx^n)}{n(p-2)} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a-I\*log(c\*x^n)/n/(-2+p))^p, x, algorithm="maxima")

[Out] integrate(sec(-a + I\*log(c\*x^n)/(n\*(p - 2)))^p, x)

**Fricas [B]** time = 0.50292, size = 346, normalized size = 4.94

$$\frac{\left( (p-2)xe^{\left(\frac{2(-ianp+2ian-\log(cx^n))}{np-2n}\right)} + (p-2)x \right) \left( \frac{2e^{\left(\frac{-ianp+2ian-\log(cx^n)}{np-2n}\right)}}{\left( \frac{2(-ianp+2ian-\log(cx^n))}{np-2n} \right) + 1} \right)^p e^{\left(\frac{-2(-ianp+2ian-\log(cx^n))}{np-2n}\right)}}{2(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a-I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="fricas")

[Out] 1/2\*((p - 2)\*x\*e^(2\*(-I\*a\*n\*p + 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n)) + (p - 2)\*x)\*(2\*e^((-I\*a\*n\*p + 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n))/(e^(2\*(-I\*a\*n\*p + 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n)) + 1))^p\*e^(-2\*(-I\*a\*n\*p + 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n))/(p - 1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a-I\*ln(c\*x\*\*n)/n/(-2+p))\*\*p,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec \left( a - \frac{i \log (cx^n)}{n(p-2)} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a-I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="giac")

[Out] integrate(sec(a - I\*log(c\*x^n)/(n\*(p - 2)))^p, x)

### 3.266 $\int \sqrt{\sec(a + b \log(cx^n))} dx$

**Optimal.** Leaf size=109

$$\frac{2x\sqrt{1 + e^{2ia}(cx^n)^{2ib}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right), \frac{1}{4}\left(5 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sec(a + b \log(cx^n))}}{2 + ibn}$$

[Out] (2\*x\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, (1 - (2\*I)/(b\*n))/4, (5 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(2 + I\*b\*n)

**Rubi [A]** time = 0.0704075, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4503, 4507, 364}

$$\frac{2x\sqrt{1 + e^{2ia}(cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right); \frac{1}{4}\left(5 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sec(a + b \log(cx^n))}}{2 + ibn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, (1 - (2\*I)/(b\*n))/4, (5 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(2 + I\*b\*n)

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))]^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\begin{aligned} \int \sqrt{\sec(a + b \log(cx^n))} dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sqrt{\sec(a + b \log(x))} dx, x, cx^n\right)}{n} \\ &= \frac{\left(x(cx^n)^{-\frac{ib}{2}-\frac{1}{n}} \sqrt{1 + e^{2ia}(cx^n)^{2ib}} \sqrt{\sec(a + b \log(cx^n))}\right) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1}{n}}}{\sqrt{1+e^{2ia}x^{2ib}}} dx, x, cx^n\right)}{n} \\ &= \frac{2x \sqrt{1 + e^{2ia}(cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right); \frac{1}{4}\left(5 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right) \sqrt{\sec(a + b \log(cx^n))}}{2 + ibn} \end{aligned}$$

**Mathematica [A]** time = 0.481745, size = 99, normalized size = 0.91

$$\frac{2ix \left(1 + e^{2i(a+b \log(cx^n))}\right) \sqrt{\sec(a + b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right)}{bn - 2i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Sec[a + b\*Log[c\*x^n]]], x]

[Out]  $((-2*I)*(1 + E^{((2*I)*(a + b*Log[c*x^n])})) * x * \operatorname{Hypergeometric2F1}[1, 3/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^{((2*I)*(a + b*Log[c*x^n])}]] * \operatorname{Sqrt}[\operatorname{Sec}[a + b*Log[c*x^n]]]) / (-2*I + b*n)$

**Maple [F]** time = 0.357, size = 0, normalized size = 0.

$$\int \sqrt{\sec(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(sec(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(sec(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sec(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(a + b*log(c*x**n))), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.267 \quad \int \frac{\sqrt{\sec(a+b \log(cx^n))}}{x} dx$$

**Optimal.** Leaf size=54

$$\frac{2\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}\text{EllipticF}\left(\frac{1}{2}(a+b \log(cx^n)), 2\right)}{bn}$$

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*EllipticF[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]])]/(b\*n)

**Rubi [A]** time = 0.0431732, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3771, 2641}

$$\frac{2\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}F\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[a + b\*Log[c\*x^n]]]/x, x]

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*EllipticF[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]])]/(b\*n)

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\sec(a+b \log(cx^n))}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{\sec(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\left(\sqrt{\cos(a+b \log(cx^n))}\sqrt{\sec(a+b \log(cx^n))}\right) \text{Subst}\left(\int \frac{1}{\sqrt{\cos(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2\sqrt{\cos(a+b \log(cx^n))}F\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)\sqrt{\sec(a+b \log(cx^n))}}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.115889, size = 54, normalized size = 1.

$$\frac{2\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}\text{EllipticF}\left(\frac{1}{2}(a+b \log(cx^n)), 2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[a + b\*Log[c\*x^n]]]/x,x]

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticF[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(b\*n)

**Maple [B]** time = 1.984, size = 181, normalized size = 3.4

$$-2 \frac{\sqrt{(2 (\cos(a/2 + 1/2 b \ln(cx^n)))^2 - 1) (\sin(a/2 + 1/2 b \ln(cx^n)))^2} \sqrt{(\sin(a/2 + 1/2 b \ln(cx^n)))^2} \sqrt{-2 (\cos(a/2 + 1/2 b \ln(cx^n)))^2}}{n \sqrt{-2 (\sin(a/2 + 1/2 b \ln(cx^n)))^4 + (\sin(a/2 + 1/2 b \ln(cx^n)))^2 \sin(a/2 + 1/2 b \ln(cx^n))} \sqrt{2 (\cos(a/2 + 1/2 b \ln(cx^n)))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^(1/2)/x,x)

[Out] -2/n\*((2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(-2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2+1)^(1/2)/(-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*EllipticF(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(sec(b\*log(c\*x^n) + a))/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec(b \log(cx^n) + a)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(sec(b\*log(c\*x^n) + a))/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(a + b \log(cx^n))}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sec(a + b*log(c*x**n)))/x, x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.268 $\int \sec^{\frac{3}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=109

$$\frac{2x(1 + e^{2ia}(cx^n)^{2ib})^{3/2} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), \frac{1}{4}\left(7 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right) \sec^{\frac{3}{2}}(a + b \log(cx^n))}{2 + 3ibn}$$

[Out] (2\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Hypergeometric2F1[3/2, (3 - (2\*I)/(b\*n))/4, (7 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^(3/2)/(2 + (3\*I)\*b\*n)

**Rubi [A]** time = 0.0724252, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4503, 4507, 364}

$$\frac{2x(1 + e^{2ia}(cx^n)^{2ib})^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right); \frac{1}{4}\left(7 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right) \sec^{\frac{3}{2}}(a + b \log(cx^n))}{2 + 3ibn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Hypergeometric2F1[3/2, (3 - (2\*I)/(b\*n))/4, (7 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^(3/2)/(2 + (3\*I)\*b\*n)

#### Rule 4503

Int[Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \sec^{\frac{3}{2}}(a + b \log(cx^n)) dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sec^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{3ib}{2}-\frac{1}{n}} (1 + e^{2ia}(cx^n)^{2ib})\right)^{3/2} \sec^{\frac{3}{2}}(a + b \log(cx^n)) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^{3/2}} dx, x, cx^n\right)}{n}$$

$$= \frac{2x(1 + e^{2ia}(cx^n)^{2ib})^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right); \frac{1}{4}\left(7 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right) \sec^{\frac{3}{2}}(a + b \log(cx^n))}{2 + 3ibn}$$

**Mathematica [B]** time = 5.86212, size = 415, normalized size = 3.81

$$\frac{\sqrt{2}x^{1-ibn} \left( (3bn - 2i) \left( -bn + 2i \right) \sqrt{\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}} \sqrt{1 + e^{2ia}(cx^n)^{2ib}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{bn+2i}{4bn}, \frac{3}{4} - \frac{i}{2bn}, -e^{2ia}(cx^n)^{2ib}\right) \right)}{bn(3bn - 2i)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (Sqrt[2]\*x^(1 - I\*b\*n)\*(-(4 + b^2\*n^2)\*x^((2\*I)\*b\*n)\*Sqrt[(E^(I\*a)\*(c\*x^n)^(I\*b))/(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]]\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]) + (-2\*I + 3\*b\*n)\*((2\*I - b\*n)\*Sqrt[(E^(I\*a)\*(c\*x^n)^(I\*b))/(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]]\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, -(2\*I + b\*n)/(4\*b\*n), 3/4 - (I/2)/(b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]) + Sqrt[2]\*x^(I\*b\*n)\*Sqrt[Sec[a + b\*Log[c\*x^n]]\*(b\*n\*Cos[b\*n\*Log[x]] - 2\*Sin[b\*n\*Log[x]])])/(b\*n\*(-2\*I + 3\*b\*n)\*(-2\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]] + b\*n\*Sin[a - b\*n\*Log[x] + b\*Log[c\*x^n]]))

**Maple [F]** time = 0.273, size = 0, normalized size = 0.

$$\int (\sec(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(sec(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] `integrate(sec(b*log(c*x^n) + a)^(3/2), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(a+b*ln(c*x**n))**(3/2),x)`

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")`

[Out] Timed out



$$3.269 \quad \int \frac{\sec^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=89

$$\frac{2 \sin(a + b \log(cx^n)) \sqrt{\sec(a + b \log(cx^n))}}{bn} - \frac{2 \sqrt{\sec(a + b \log(cx^n))} \sqrt{\cos(a + b \log(cx^n))} E\left(\frac{1}{2}(a + b \log(cx^n)) \middle| 2\right)}{bn}$$

[Out] (-2\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticE[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(b\*n) + (2\*Sqrt[Sec[a + b\*Log[c\*x^n]]]\*Sin[a + b\*Log[c\*x^n]])/(b\*n)

**Rubi [A]** time = 0.0609058, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3768, 3771, 2639}

$$\frac{2 \sin(a + b \log(cx^n)) \sqrt{\sec(a + b \log(cx^n))}}{bn} - \frac{2 \sqrt{\sec(a + b \log(cx^n))} \sqrt{\cos(a + b \log(cx^n))} E\left(\frac{1}{2}(a + b \log(cx^n)) \middle| 2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^(3/2)/x, x]

[Out] (-2\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticE[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(b\*n) + (2\*Sqrt[Sec[a + b\*Log[c\*x^n]]]\*Sin[a + b\*Log[c\*x^n]])/(b\*n)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx = \frac{\text{Subst}\left(\int \sec^{\frac{3}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\sec(a + b \log(cx^n))} \sin(a + b \log(cx^n))}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sec(a+bx)}} dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sqrt{\sec(a + b \log(cx^n))} \sin(a + b \log(cx^n))}{bn} - \frac{(\sqrt{\cos(a + b \log(cx^n))} \sqrt{\sec(a + b \log(cx^n))})}{n}$$

$$= -\frac{2\sqrt{\cos(a + b \log(cx^n))} E\left(\frac{1}{2}(a + b \log(cx^n)) \middle| 2\right) \sqrt{\sec(a + b \log(cx^n))}}{bn} + \frac{2\sqrt{\sec(a + b \log(cx^n))}}{bn}$$

**Mathematica [A]** time = 0.138382, size = 68, normalized size = 0.76

$$\frac{2\sqrt{\sec(a + b \log(cx^n))} \left( \sin(a + b \log(cx^n)) - \sqrt{\cos(a + b \log(cx^n))} E\left(\frac{1}{2}(a + b \log(cx^n)) \middle| 2\right) \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^(3/2)/x,x]

[Out] (2\*Sqrt[Sec[a + b\*Log[c\*x^n]]]\*(-(Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticE[(a + b\*Log[c\*x^n])/2, 2]) + Sin[a + b\*Log[c\*x^n]]))/(b\*n)

**Maple [A]** time = 2.643, size = 139, normalized size = 1.6

$$-2 \frac{\sqrt{2 (\sin(a/2 + 1/2 b \ln(cx^n)))^2 - 1} \sqrt{(\sin(a/2 + 1/2 b \ln(cx^n)))^2} \text{EllipticE}\left(\cos(a/2 + 1/2 b \ln(cx^n)), \sqrt{2}\right) - 2 (\sin(a/2 + 1/2 b \ln(cx^n)))^2}{n \sin(a/2 + 1/2 b \ln(cx^n)) \sqrt{2 (\cos(a/2 + 1/2 b \ln(cx^n)))^2 - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^(3/2)/x,x)

[Out] -2/n\*((2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*EllipticE(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n)))/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate(sec(b\*log(c\*x^n) + a)^(3/2)/x, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)^(3/2)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*(3/2)/x,x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="giac")

[Out] Timed out

### 3.270 $\int \sec^{\frac{5}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=109

$$\frac{2x(1 + e^{2ia}(cx^n)^{2ib})^{5/2} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right), \frac{1}{4}\left(9 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right) \sec^{\frac{5}{2}}(a + b \log(cx^n))}{2 + 5ibn}$$

[Out] (2\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Hypergeometric2F1[5/2, (5 - (2\*I)/(b\*n))/4, (9 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^(5/2)/(2 + (5\*I)\*b\*n)

**Rubi [A]** time = 0.0722975, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4503, 4507, 364}

$$\frac{2x(1 + e^{2ia}(cx^n)^{2ib})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right); \frac{1}{4}\left(9 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right) \sec^{\frac{5}{2}}(a + b \log(cx^n))}{2 + 5ibn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Hypergeometric2F1[5/2, (5 - (2\*I)/(b\*n))/4, (9 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^(5/2)/(2 + (5\*I)\*b\*n)

#### Rule 4503

Int[Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \sec^{\frac{5}{2}}(a + b \log(cx^n)) dx = \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int x^{-1+\frac{1}{n}} \sec^{\frac{5}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{5ib}{2}-\frac{1}{n}} (1 + e^{2ia}(cx^n)^{2ib})^{5/2} \sec^{\frac{5}{2}}(a + b \log(cx^n))\right) \text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1}{n}}}{(1+e^{2ia}x^{2ib})^{5/2}} dx, x, cx^n\right)}{n}$$

$$= \frac{2x(1 + e^{2ia}(cx^n)^{2ib})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right); \frac{1}{4}\left(9 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right) \sec^{\frac{5}{2}}(a + b \log(cx^n))}{2 + 5ibn}$$

**Mathematica [A]** time = 1.35077, size = 124, normalized size = 1.14

$$\frac{2x\sqrt{\sec(a + b \log(cx^n))} \left( (2 - ibn) (1 + e^{2ia}(cx^n)^{2ib}) \text{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right) + bn \right)}{3b^2n^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x\*Sqrt[Sec[a + b\*Log[c\*x^n]]]\*(-2 + (2 - I\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^(2\*I\*b)))\*Hypergeometric2F1[1, 3/4 - (I/2)/(b\*n), 5/4 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] + b\*n\*Tan[a + b\*Log[c\*x^n]])/(3\*b^2\*n^2)

**Maple [F]** time = 0.275, size = 0, normalized size = 0.

$$\int (\sec(a + b \ln(cx^n)))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^(5/2), x)

[Out] int(sec(a+b\*ln(c\*x^n))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(5/2), x, algorithm="maxima")

[Out] integrate(sec(b\*log(c\*x^n) + a)^(5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.271 \quad \int \frac{\sec^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=93

$$\frac{2\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}\text{EllipticF}\left(\frac{1}{2}(a+b \log(cx^n)), 2\right)}{3bn} + \frac{2 \sin(a+b \log(cx^n)) \sec^{\frac{3}{2}}(a+b \log(cx^n))}{3bn}$$

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*EllipticF[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(3\*b\*n) + (2\*Sec[a + b\*Log[c\*x^n]]^(3/2)\*Sin[a + b\*Log[c\*x^n]])/(3\*b\*n)

**Rubi [A]** time = 0.0610006, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3768, 3771, 2641}

$$\frac{2 \sin(a+b \log(cx^n)) \sec^{\frac{3}{2}}(a+b \log(cx^n))}{3bn} + \frac{2\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}F\left(\frac{1}{2}(a+b \log(cx^n))\middle|2\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^(5/2)/x, x]

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*EllipticF[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(3\*b\*n) + (2\*Sec[a + b\*Log[c\*x^n]]^(3/2)\*Sin[a + b\*Log[c\*x^n]])/(3\*b\*n)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Csc[c + d\*x]^(n-1))/(d\*(n-1)), x] + Dist[(b^2\*(n-2))/(n-1), Int[(b\*Csc[c + d\*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n \* Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \sec^{\frac{5}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2 \sec^{\frac{3}{2}}(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\sec(a + bx)} dx, x, \log(cx^n)\right)}{3n} \\
&= \frac{2 \sec^{\frac{3}{2}}(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{3bn} + \frac{(\sqrt{\cos(a + b \log(cx^n))} \sqrt{\sec(a + b \log(cx^n))})}{3n} \\
&= \frac{2 \sqrt{\cos(a + b \log(cx^n))} F\left(\frac{1}{2}(a + b \log(cx^n)) \middle| 2\right) \sqrt{\sec(a + b \log(cx^n))}}{3bn} + \frac{2 \sec^{\frac{3}{2}}(a + b \log(cx^n)) \sin(a + b \log(cx^n))}{3bn}
\end{aligned}$$

**Mathematica [A]** time = 0.15626, size = 69, normalized size = 0.74

$$\frac{2 \sec^{\frac{3}{2}}(a + b \log(cx^n)) \left( \cos^{\frac{3}{2}}(a + b \log(cx^n)) \text{EllipticF}\left(\frac{1}{2}(a + b \log(cx^n)), 2\right) + \sin(a + b \log(cx^n)) \right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^(5/2)/x,x]

[Out] (2\*Sec[a + b\*Log[c\*x^n]]^(3/2)\*(Cos[a + b\*Log[c\*x^n]]^(3/2)\*EllipticF[(a + b\*Log[c\*x^n])/2, 2] + Sin[a + b\*Log[c\*x^n]]))/(3\*b\*n)

**Maple [B]** time = 2.697, size = 291, normalized size = 3.1

$$-\frac{2}{3bn} \left( -2 \sqrt{(\sin(a/2 + 1/2 b \ln(cx^n)))^2} \sqrt{2 (\sin(a/2 + 1/2 b \ln(cx^n)))^2 - 1} \text{EllipticF}\left(\cos(a/2 + 1/2 b \ln(cx^n)), \sqrt{2}\right) (\sin(a/2 + 1/2 b \ln(cx^n))) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^(5/2)/x,x)

[Out] -2/3/n\*(-2\*(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*EllipticF(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2+(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*EllipticF(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n)))\*((2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)/(-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(3/2)/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(b \log(cx^n) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="maxima")



[Out] integrate(sec(b\*log(c\*x^n) + a)^(5/2)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="fricas")

[Out] integral(sec(b\*log(c\*x^n) + a)^(5/2)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*ln(c\*x\*\*n))\*\*(5/2)/x,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.272 \quad \int \frac{1}{\sqrt{\sec(a+b \log(cx^n))}} dx$$

**Optimal.** Leaf size=110

$$\frac{{}_2x\text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{bn+2i}{4bn}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b \log(cx^n))}}$$

[Out] (2\*x\*Hypergeometric2F1[-1/2, -(2\*I + b\*n)/(4\*b\*n), (3 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 - I\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])

**Rubi [A]** time = 0.0693818, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4503, 4507, 364}

$$\frac{{}_2x_2F_1\left(-\frac{1}{2}, -\frac{bn+2i}{4bn}; \frac{1}{4}\left(3 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Sec[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x\*Hypergeometric2F1[-1/2, -(2\*I + b\*n)/(4\*b\*n), (3 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 - I\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{\sqrt{\sec(a + b \log(cx^n))}} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sqrt{\sec(a+b \log(x))}} dx, x, cx^n\right)}{n}$$

$$= \frac{(x(cx^n)^{\frac{ib}{2}-\frac{1}{n}}) \operatorname{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1}{n}} \sqrt{1 + e^{2ia} x^{2ib}} dx, x, cx^n\right)}{n \sqrt{1 + e^{2ia} (cx^n)^{2ib}} \sqrt{\sec(a + b \log(cx^n))}}$$

$$= \frac{2x {}_2F_1\left(-\frac{1}{2}, -\frac{2i+bn}{4bn}; \frac{1}{4}\left(3 - \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(2 - ibn) \sqrt{1 + e^{2ia} (cx^n)^{2ib}} \sqrt{\sec(a + b \log(cx^n))}}$$

**Mathematica [B]** time = 4.31546, size = 380, normalized size = 3.45

$$\frac{2x \cos(a + b \log(cx^n) - bn \log(x))}{\sqrt{\sec(a + b \log(cx^n))} (bn \sin(a + b \log(cx^n) - bn \log(x)) - 2 \cos(a + b \log(cx^n) - bn \log(x)))} + \frac{2e^{2ia} b n x (cx^n)^{2ib}}{(bn \sin(a + b \log(cx^n) - bn \log(x)) - 2 \cos(a + b \log(cx^n) - bn \log(x)))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[Sec[a + b\*Log[c\*x^n]]], x]

[Out] (2\*b\*E^((2\*I)\*a)\*n\*x\*(c\*x^n)^((2\*I)\*b)\*((2\*I + b\*n)\*x^((2\*I)\*b\*n)\*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))] + (-2\*I + 3\*b\*n)\*Hypergeometric2F1[1/2, -(2\*I + b\*n)/(4\*b\*n), 3/4 - (I/2)/(b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))])/(2\*I + b\*n)\*(-2\*I + 3\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[(E^(I\*a)\*(c\*x^n)^(I\*b))/(2 + 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*((-2 + I\*b\*n)\*x^((2\*I)\*b\*n) - I\*E^((2\*I)\*a)\*(-2\*I + b\*n)\*(c\*x^n)^((2\*I)\*b)) - (2\*x\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]])/(Sqrt[Sec[a + b\*Log[c\*x^n]]]\*(-2\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]] + b\*n\*Sin[a - b\*n\*Log[x] + b\*Log[c\*x^n]]))

**Maple [F]** time = 0.283, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(a + b \ln(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(1/sec(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(1/sqrt(sec(b\*log(c\*x^n) + a)), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(a+b\*log(c\*x^n))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(a+b\*ln(c\*x\*\*n))\*\*(1/2),x)

[Out] Integral(1/sqrt(sec(a + b\*log(c\*x\*\*n))), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(a+b\*log(c\*x^n))^(1/2),x, algorithm="giac")

[Out] Timed out

$$3.273 \quad \int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}} dx$$

**Optimal.** Leaf size=54

$$\frac{2\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*EllipticE[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(b\*n)

**Rubi [A]** time = 0.0440065, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3771, 2639}

$$\frac{2\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[Sec[a + b\*Log[c\*x^n]]]), x]

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]])\*EllipticE[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(b\*n)

**Rule 3771**

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

**Rule 2639**

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sec(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\left(\sqrt{\cos(a+b\log(cx^n))}\sqrt{\sec(a+b\log(cx^n))}\right)\text{Subst}\left(\int \sqrt{\cos(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)\sqrt{\sec(a+b\log(cx^n))}}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.104799, size = 54, normalized size = 1.

$$\frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[Sec[a + b\*Log[c\*x^n]]]),x]

[Out] (2\*EllipticE[(a + b\*Log[c\*x^n])/2, 2])/(b\*n\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])

**Maple [B]** time = 1.849, size = 181, normalized size = 3.4

$$2 \frac{\sqrt{(2(\cos(a/2 + 1/2 b \ln(cx^n)))^2 - 1)(\sin(a/2 + 1/2 b \ln(cx^n)))^2} \sqrt{(\sin(a/2 + 1/2 b \ln(cx^n)))^2} \sqrt{-2(\cos(a/2 + 1/2 b \ln(cx^n)))^2}}{n \sqrt{-2(\sin(a/2 + 1/2 b \ln(cx^n)))^4 + (\sin(a/2 + 1/2 b \ln(cx^n)))^2} \sin(a/2 + 1/2 b \ln(cx^n)) \sqrt{2(\cos(a/2 + 1/2 b \ln(cx^n)))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sec(a+b\*ln(c\*x^n))^(1/2),x)

[Out] 2/n\*((2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(-2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2+1)^(1/2)\*EllipticE(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))/(-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt{\sec(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*log(c\*x^n))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(x\*sqrt(sec(b\*log(c\*x^n) + a))), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \sqrt{\sec(b \log(cx^n) + a)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*log(c\*x^n))^(1/2),x, algorithm="fricas")

[Out] integral(1/(x\*sqrt(sec(b\*log(c\*x^n) + a))), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt{\sec(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(sec(a + b*log(c*x**n))))), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.274 \quad \int \frac{1}{\sec^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=109

$$\frac{2x \operatorname{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right), \frac{1}{4}\left(1 - \frac{2i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1 + e^{2ia}(cx^n)^{2ib}\right)^{3/2} \sec^{\frac{3}{2}}(a+b \log(cx^n))}$$

[Out] (2\*x\*Hypergeometric2F1[-3/2, (-3 - (2\*I)/(b\*n))/4, (1 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 - (3\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Sec[a + b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.0702157, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4503, 4507, 364}

$$\frac{2x {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right); \frac{1}{4}\left(1 - \frac{2i}{bn}\right); -e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1 + e^{2ia}(cx^n)^{2ib}\right)^{3/2} \sec^{\frac{3}{2}}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^(-3/2), x]

[Out] (2\*x\*Hypergeometric2F1[-3/2, (-3 - (2\*I)/(b\*n))/4, (1 - (2\*I)/(b\*n))/4, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 - (3\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Sec[a + b\*Log[c\*x^n]]^(3/2))

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\int \frac{1}{\sec^{\frac{3}{2}}(a + b \log(cx^n))} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sec^{\frac{3}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{(x(cx^n)^{\frac{3ib}{2}-\frac{1}{n}}) \operatorname{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1}{n}} (1 + e^{2ia} x^{2ib})^{3/2} dx, x, cx^n\right)}{n(1 + e^{2ia} (cx^n)^{2ib})^{3/2} \sec^{\frac{3}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right); \frac{1}{4}\left(1 - \frac{2i}{bn}\right); -e^{2ia} (cx^n)^{2ib}\right)}{(2 - 3ibn) (1 + e^{2ia} (cx^n)^{2ib})^{3/2} \sec^{\frac{3}{2}}(a + b \log(cx^n))}$$

**Mathematica [A]** time = 1.59463, size = 168, normalized size = 1.54

$$\frac{2x \left(3b^2 n^2 (1 + e^{2ia} (cx^n)^{2ib}) \sec^2(a + b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} - \frac{i}{2bn}, \frac{5}{4} - \frac{i}{2bn}, -e^{2i(a+b \log(cx^n))}\right) + (2 + ibn)\right)}{(2 + 3ibn)(bn - 2i)(3bn + 2i) \sec^{\frac{3}{2}}(a + b \log(cx^n))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^(-3/2), x]

[Out] (2\*x\*(3\*b^2\*n^2\*(1 + E^((2\*I)\*a))\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, 3/4 - (I/2)/(b\*n), 5/4 - (I/2)/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))]\*Sec[a + b\*Log[c\*x^n]]^2 + (2 + I\*b\*n)\*(2 + 3\*b\*n\*Tan[a + b\*Log[c\*x^n]]))/((2 + (3\*I)\*b\*n)\*(-2\*I + b\*n)\*(2\*I + 3\*b\*n)\*Sec[a + b\*Log[c\*x^n]]^(3/2))

**Maple [F]** time = 0.283, size = 0, normalized size = 0.

$$\int (\sec(a + b \ln(cx^n)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(1/sec(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(sec(b\*log(c\*x^n) + a)^(-3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Integral(sec(a + b\*log(c\*x\*\*n))\*\*(-3/2), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] Timed out

$$3.275 \quad \int \frac{1}{x \sec^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=93

$$\frac{2\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}\text{EllipticF}\left(\frac{1}{2}(a+b \log(cx^n)), 2\right)}{3bn} + \frac{2 \sin(a+b \log(cx^n))}{3bn\sqrt{\sec(a+b \log(cx^n))}}$$

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticF[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(3\*b\*n) + (2\*Sin[a + b\*Log[c\*x^n]])/(3\*b\*n\*Sqrt[Sec[a + b\*Log[c\*x^n]]])

**Rubi [A]** time = 0.0603745, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3769, 3771, 2641}

$$\frac{2 \sin(a+b \log(cx^n))}{3bn\sqrt{\sec(a+b \log(cx^n))}} + \frac{2\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}F\left(\frac{1}{2}(a+b \log(cx^n))\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sec[a + b\*Log[c\*x^n]]^(3/2)), x]

[Out] (2\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticF[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(3\*b\*n) + (2\*Sin[a + b\*Log[c\*x^n]])/(3\*b\*n\*Sqrt[Sec[a + b\*Log[c\*x^n]]])

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x \sec^{\frac{3}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2 \sin(a + b \log(cx^n))}{3bn \sqrt{\sec(a + b \log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\sec(a + bx)} dx, x, \log(cx^n)\right)}{3n} \\
&= \frac{2 \sin(a + b \log(cx^n))}{3bn \sqrt{\sec(a + b \log(cx^n))}} + \frac{(\sqrt{\cos(a + b \log(cx^n))} \sqrt{\sec(a + b \log(cx^n))}) \text{Subst}\left(\int \frac{1}{\sqrt{\cos}}\right)}{3n} \\
&= \frac{2 \sqrt{\cos(a + b \log(cx^n))} F\left(\frac{1}{2}(a + b \log(cx^n)) \middle| 2\right) \sqrt{\sec(a + b \log(cx^n))}}{3bn} + \frac{2 \sin(a + b \log(cx^n))}{3bn \sqrt{\sec(a + b \log(cx^n))}}
\end{aligned}$$

**Mathematica [A]** time = 0.133149, size = 72, normalized size = 0.77

$$\frac{\sqrt{\sec(a + b \log(cx^n))} \left(2 \sqrt{\cos(a + b \log(cx^n))} \text{EllipticF}\left(\frac{1}{2}(a + b \log(cx^n)), 2\right) + \sin(2(a + b \log(cx^n)))\right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sec[a + b\*Log[c\*x^n]]^(3/2)),x]

[Out] (Sqrt[Sec[a + b\*Log[c\*x^n]]]\*(2\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticF[(a + b\*Log[c\*x^n])/2, 2] + Sin[2\*(a + b\*Log[c\*x^n])]))/(3\*b\*n)

**Maple [B]** time = 2.337, size = 247, normalized size = 2.7

$$-\frac{2}{3bn} \sqrt{\left(2 \left(\cos\left(\frac{a}{2} + \frac{1}{2} b \ln(cx^n)\right)\right)^2 - 1\right) \left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right)\right)^2} \left(4 \cos\left(\frac{a}{2} + \frac{1}{2} b \ln(cx^n)\right) \left(\sin\left(\frac{a}{2} + \frac{1}{2} b \ln(cx^n)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sec(a+b\*ln(c\*x^n))^(3/2),x)

[Out] -2/3/n\*((2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(4\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*EllipticF(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n)))/(-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sec(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*log(c\*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x\*sec(b\*log(c\*x^n) + a)^(3/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \sec(b \log(cx^n) + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] integral(1/(x\*sec(b\*log(c\*x^n) + a)^(3/2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] Timed out

$$3.276 \quad \int \frac{1}{\sec^{\frac{5}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=110

$$\frac{2x \operatorname{Hypergeometric2F1}\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right), -\frac{bn+2i}{4bn}, -e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2} \sec^{\frac{5}{2}}(a+b \log(cx^n))}$$

[Out] (2\*x\*Hypergeometric2F1[-5/2, (-5 - (2\*I)/(b\*n))/4, -(2\*I + b\*n)/(4\*b\*n), -E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/(2 - (5\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(5/2)\*Sec[a + b\*Log[c\*x^n]]^(5/2)

**Rubi [A]** time = 0.0724982, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4503, 4507, 364}

$$\frac{2x {}_2F_1\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right); -\frac{bn+2i}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2} \sec^{\frac{5}{2}}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^(-5/2), x]

[Out] (2\*x\*Hypergeometric2F1[-5/2, (-5 - (2\*I)/(b\*n))/4, -(2\*I + b\*n)/(4\*b\*n), -E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/(2 - (5\*I)\*b\*n)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(5/2)\*Sec[a + b\*Log[c\*x^n]]^(5/2)

#### Rule 4503

Int[Sec[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(a + b \log(cx^n))} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sec^{\frac{5}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{(x(cx^n)^{\frac{5ib}{2}-\frac{1}{n}}) \operatorname{Subst}\left(\int x^{-1-\frac{5ib}{2}+\frac{1}{n}} (1 + e^{2ia}x^{2ib})^{5/2} dx, x, cx^n\right)}{n(1 + e^{2ia}(cx^n)^{2ib})^{5/2} \sec^{\frac{5}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x {}_2F_1\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right); -\frac{2i+bn}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(2 - 5ibn)(1 + e^{2ia}(cx^n)^{2ib})^{5/2} \sec^{\frac{5}{2}}(a + b \log(cx^n))}$$

**Mathematica [B]** time = 8.65317, size = 867, normalized size = 7.88

$$\frac{30b^3 e^{2i(a+b(\log(cx^n)-n \log(x)))} x \left( (bn+2i) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4} - \frac{i}{2bn}, \frac{7}{4} - \frac{i}{2bn}, -e^{2i(a+b(\log(cx^n)-n \log(x)))} x^{2ibn}\right) x^{2ibn} + \right)}{(2 - 5ibn)(bn+2i)(3bn-2i)(5bn-2i)(-bn + e^{2i(a+b(\log(cx^n)-n \log(x)))}(bn-2i) - 2i) \sqrt{e^{2i(a+b(\log(cx^n)-n \log(x)))} x^{2ibn}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^(-5/2), x]

[Out] (30\*b^3\*E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* n^3 \* x^((2\*I + b\*n)\*x^((2\*I)\*b\*n)\*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), -(E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n))] + (-2\*I + 3\*b\*n)\*Hypergeometric2F1[1/2, -(2\*I + b\*n)/(4\*b\*n), 3/4 - (I/2)/(b\*n), -(E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n))])]) / ((2 - (5\*I)\*b\*n)\*(2\*I + b\*n)\*(-2\*I + 3\*b\*n)\*(-2\*I + 5\*b\*n)\*(-2\*I - b\*n + E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* (-2\*I + b\*n))\*Sqrt[1 + E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n)]\*Sqrt[(E^(I\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^(I\*b\*n)) / (2 + 2\*E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n))] + Sqrt[Sec[a + b\*n\*Log[x] + b\*(-(n\*Log[x]) + Log[c\*x^n])] \* (-x\*Cos[b\*n\*Log[x]]\*(12 + 55\*b^2\*n^2 + 12\*Cos[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 65\*b^2\*n^2\*Cos[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 4\*b\*n\*Sin[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))])]) / (4\*(-2\*I + 5\*b\*n)\*(2\*I + 5\*b\*n)\*(-2\*Cos[a + b\*(-(n\*Log[x]) + Log[c\*x^n])] + b\*n\*Sin[a + b\*(-(n\*Log[x]) + Log[c\*x^n])])]) + (x\*Sin[b\*n\*Log[x]]\*(-16\*b\*n - 4\*b\*n\*Cos[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 12\*Sin[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 65\*b^2\*n^2\*Sin[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))]) / (4\*(-2\*I + 5\*b\*n)\*(2\*I + 5\*b\*n)\*(-2\*Cos[a + b\*(-(n\*Log[x]) + Log[c\*x^n])] + b\*n\*Sin[a + b\*(-(n\*Log[x]) + Log[c\*x^n])])]) + (x\*Sin[3\*b\*n\*Log[x]]\*(5\*b\*n\*Cos[3\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] - 2\*Sin[3\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))]) / (2\*(-2\*I + 5\*b\*n)\*(2\*I + 5\*b\*n)) + (x\*Cos[3\*b\*n\*Log[x]]\*(2\*Cos[3\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 5\*b\*n\*Sin[3\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))]) / (2\*(-2\*I + 5\*b\*n)\*(2\*I + 5\*b\*n)))

**Maple [F]** time = 0.289, size = 0, normalized size = 0.

$$\int (\sec(a + b \ln(cx^n)))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(a+b\*ln(c\*x^n))^(5/2), x)

[Out] `int(1/sec(a+b*ln(c*x^n))^(5/2),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sec(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sec(b*log(c*x^n) + a)^(-5/2), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(a+b*ln(c*x**n))**(5/2),x)`

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")`

[Out] Timed out



$$3.277 \quad \int \frac{1}{x \sec^{\frac{5}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=93

$$\frac{2 \sin(a+b \log(cx^n))}{5bn \sec^{\frac{3}{2}}(a+b \log(cx^n))} + \frac{6\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}E\left(\frac{1}{2}(a+b \log(cx^n))\right)\left|2\right.}{5bn}$$

[Out] (6\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticE[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(5\*b\*n) + (2\*Sin[a + b\*Log[c\*x^n]])/(5\*b\*n\*Sec[a + b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.061194, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3769, 3771, 2639}

$$\frac{2 \sin(a+b \log(cx^n))}{5bn \sec^{\frac{3}{2}}(a+b \log(cx^n))} + \frac{6\sqrt{\sec(a+b \log(cx^n))}\sqrt{\cos(a+b \log(cx^n))}E\left(\frac{1}{2}(a+b \log(cx^n))\right)\left|2\right.}{5bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sec[a + b\*Log[c\*x^n]]^(5/2)), x]

[Out] (6\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticE[(a + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])/(5\*b\*n) + (2\*Sin[a + b\*Log[c\*x^n]])/(5\*b\*n\*Sec[a + b\*Log[c\*x^n]]^(3/2))

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x \sec^{\frac{5}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= \frac{2 \sin(a + b \log(cx^n))}{5bn \sec^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{3 \text{Subst}\left(\int \frac{1}{\sqrt{\sec(a+bx)}} dx, x, \log(cx^n)\right)}{5n} \\
&= \frac{2 \sin(a + b \log(cx^n))}{5bn \sec^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{(3\sqrt{\cos(a + b \log(cx^n))}\sqrt{\sec(a + b \log(cx^n))}) \text{Subst}\left(\int \sqrt{\cos}\right)}{5n} \\
&= \frac{6\sqrt{\cos(a + b \log(cx^n))}E\left(\frac{1}{2}(a + b \log(cx^n))\middle|2\right)\sqrt{\sec(a + b \log(cx^n))}}{5bn} + \frac{2 \sin(a + b \log(cx^n))}{5bn \sec^{\frac{3}{2}}(a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.175315, size = 83, normalized size = 0.89

$$\frac{\sqrt{\sec(a + b \log(cx^n))} \left( \sin(a + b \log(cx^n)) + \sin(3(a + b \log(cx^n))) + 12\sqrt{\cos(a + b \log(cx^n))}E\left(\frac{1}{2}(a + b \log(cx^n))\middle|2\right) \right)}{10bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sec[a + b\*Log[c\*x^n]]^(5/2)),x]

[Out] (Sqrt[Sec[a + b\*Log[c\*x^n]]]\*(12\*Sqrt[Cos[a + b\*Log[c\*x^n]]]\*EllipticE[(a + b\*Log[c\*x^n])/2, 2] + Sin[a + b\*Log[c\*x^n]] + Sin[3\*(a + b\*Log[c\*x^n])]))/(10\*b\*n)

**Maple [B]** time = 2.312, size = 280, normalized size = 3.

$$-\frac{2}{5bn} \sqrt{\left(2 \left(\cos\left(\frac{a}{2} + \frac{1}{2}b \ln(cx^n)\right)\right)^2 - 1\right) \left(\sin\left(\frac{a}{2} + \frac{b \ln(cx^n)}{2}\right)\right)^2} \left(-8 \cos\left(\frac{a}{2} + \frac{1}{2}b \ln(cx^n)\right) \left(\sin\left(\frac{a}{2} + \frac{1}{2}b \ln(cx^n)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/sec(a+b\*ln(c\*x^n))^(5/2),x)

[Out] -2/5/n\*((2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*(-8\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^6+8\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4-3\*(2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)\*(sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)\*EllipticE(cos(1/2\*a+1/2\*b\*ln(c\*x^n)),2^(1/2))-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n)))/(-2\*sin(1/2\*a+1/2\*b\*ln(c\*x^n))^4+sin(1/2\*a+1/2\*b\*ln(c\*x^n))^2)^(1/2)/sin(1/2\*a+1/2\*b\*ln(c\*x^n))/(2\*cos(1/2\*a+1/2\*b\*ln(c\*x^n))^2-1)^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \sec(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*log(c\*x^n))^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x\*sec(b\*log(c\*x^n) + a)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \sec(b \log(cx^n) + a)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*log(c\*x^n))^(5/2),x, algorithm="fricas")

[Out] integral(1/(x\*sec(b\*log(c\*x^n) + a)^(5/2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/sec(a+b\*log(c\*x^n))^(5/2),x, algorithm="giac")

[Out] Timed out

### 3.278 $\int x^m \sec^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=102

$$\frac{8e^{3ia} x^{m+1} (cx^n)^{3ib} \operatorname{Hypergeometric2F1}\left(3, -\frac{-3bn+i(m+1)}{2bn}, -\frac{-5bn+i(m+1)}{2bn}, -e^{2ia} (cx^n)^{2ib}\right)}{3ibn + m + 1}$$

[Out] (8\*E^((3\*I)\*a)\*x^(1 + m)\*(c\*x^n)^((3\*I)\*b)\*Hypergeometric2F1[3, -(I\*(1 + m) - 3\*b\*n)/(2\*b\*n), -(I\*(1 + m) - 5\*b\*n)/(2\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/(1 + m + (3\*I)\*b\*n)

**Rubi [A]** time = 0.0886081, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{8e^{3ia} x^{m+1} (cx^n)^{3ib} {}_2F_1\left(3, -\frac{i(m+1)-3bn}{2bn}; -\frac{i(m+1)-5bn}{2bn}; -e^{2ia} (cx^n)^{2ib}\right)}{3ibn + m + 1}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sec[a + b\*Log[c\*x^n]]^3,x]

[Out] (8\*E^((3\*I)\*a)\*x^(1 + m)\*(c\*x^n)^((3\*I)\*b)\*Hypergeometric2F1[3, -(I\*(1 + m) - 3\*b\*n)/(2\*b\*n), -(I\*(1 + m) - 5\*b\*n)/(2\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/(1 + m + (3\*I)\*b\*n)

#### Rule 4509

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_)\*(x\_))^(m\_)\*Sec[((a\_) + Log[x\_]\*(b\_))\*(d\_)]^(p\_), x\_Symbol] :> Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \sec^3(a + b \log(cx^n)) dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sec^3(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(8e^{3ia} x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+3ib+\frac{1+m}{n}}}{(1+e^{2ia}x^{2ib})^3} dx, x, cx^n\right)}{n}$$

$$= \frac{8e^{3ia} x^{1+m} (cx^n)^{3ib} {}_2F_1\left(3, -\frac{i(1+m)-3bn}{2bn}; -\frac{i(1+m)-5bn}{2bn}; -e^{2ia} (cx^n)^{2ib}\right)}{1+m+3ibn}$$

**Mathematica [A]** time = 5.65811, size = 134, normalized size = 1.31

$$\frac{x^{m+1} \left(-2 \sec(a + b \log(cx^n)) (-bn \tan(a + b \log(cx^n)) + m + 1) + 4e^{ia} (-ibn + m + 1) (cx^n)^{ib} \text{Hypergeometric2F1}\left(\dots\right)\right)}{4b^2n^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Sec[a + b\*Log[c\*x^n]]^3,x]

[Out] (x^(1+m)\*(4\*E^(I\*a)\*(1+m-I\*b\*n)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, 1/2 - ((I/2)\*(1+m))/(b\*n), 3/2 - ((I/2)\*(1+m))/(b\*n), -E^((2\*I)\*(a+b\*Log[c\*x^n]))] - 2\*Sec[a + b\*Log[c\*x^n]]\*(1+m-b\*n\*Tan[a + b\*Log[c\*x^n]])))/(4\*b^2\*n^2)

**Maple [F]** time = 2.368, size = 0, normalized size = 0.

$$\int x^m (\sec(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sec(a+b\*ln(c\*x^n))^3,x)

[Out] int(x^m\*sec(a+b\*ln(c\*x^n))^3,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \sec(b \log(cx^n) + a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*sec(a+b\*log(c\*x<sup>n</sup>))<sup>3</sup>,x, algorithm="fricas")

[Out] integral(x<sup>m</sup>\*sec(b\*log(c\*x<sup>n</sup>) + a)<sup>3</sup>, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*sec(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sec(b \log(cx^n) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*sec(a+b\*log(c\*x<sup>n</sup>))<sup>3</sup>,x, algorithm="giac")

[Out] integrate(x<sup>m</sup>\*sec(b\*log(c\*x<sup>n</sup>) + a)<sup>3</sup>, x)

### 3.279 $\int x^m \sec^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=102

$$\frac{4e^{2ia}x^{m+1}(cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(2, -\frac{-2bn+i(m+1)}{2bn}, -\frac{-4bn+i(m+1)}{2bn}, -e^{2ia}(cx^n)^{2ib}\right)}{2ibn + m + 1}$$

[Out] (4\*E^((2\*I)\*a)\*x^(1 + m)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[2, -(I\*(1 + m) - 2\*b\*n)/(2\*b\*n), -(I\*(1 + m) - 4\*b\*n)/(2\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/(1 + m + (2\*I)\*b\*n)

**Rubi [A]** time = 0.0846519, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4509, 4505, 364}

$$\frac{4e^{2ia}x^{m+1}(cx^n)^{2ib} {}_2F_1\left(2, -\frac{i(m+1)-2bn}{2bn}; -\frac{i(m+1)-4bn}{2bn}; -e^{2ia}(cx^n)^{2ib}\right)}{2ibn + m + 1}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sec[a + b\*Log[c\*x^n]]^2,x]

[Out] (4\*E^((2\*I)\*a)\*x^(1 + m)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[2, -(I\*(1 + m) - 2\*b\*n)/(2\*b\*n), -(I\*(1 + m) - 4\*b\*n)/(2\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/(1 + m + (2\*I)\*b\*n)

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \sec^2(a + b \log(cx^n)) dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sec^2(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(4e^{2ia} x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+2ib+\frac{1+m}{n}}}{(1+e^{2ia} x^{2ib})^2} dx, x, cx^n\right)}{n}$$

$$= \frac{4e^{2ia} x^{1+m} (cx^n)^{2ib} {}_2F_1\left(2, -\frac{i(1+m)-2bn}{2bn}; -\frac{i(1+m)-4bn}{2bn}; -e^{2ia} (cx^n)^{2ib}\right)}{1+m+2ibn}$$

**Mathematica [B]** time = 17.5407, size = 482, normalized size = 4.73

$$\frac{x^{m+1} \sin(bn \log(x)) \sec(a + b(\log(cx^n) - n \log(x))) \sec(a + b(\log(cx^n) - n \log(x)) + bn \log(x))}{bn} - \frac{(m+1) \sec(a + b(\log(cx^n) - n \log(x)))}{bn}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Sec[a + b\*Log[c\*x^n]]^2,x]

[Out] (x^(1+m)\*Sec[a + b\*(-(n\*Log[x]) + Log[c\*x^n])]\*Sec[a + b\*n\*Log[x] + b\*(-(n\*Log[x]) + Log[c\*x^n])]\*Sin[b\*n\*Log[x]])/(b\*n) - ((1+m)\*Sec[a + b\*(-(n\*Log[x]) + Log[c\*x^n])]\*((x^(1+m)\*Sec[a + b\*Log[c\*x^n]]\*Sin[b\*n\*Log[x]])/(1+m) - (I\*Cos[a + b\*(-(n\*Log[x]) + Log[c\*x^n])]\*(-E^((a + 2\*a\*m + b\*(1+m)\*n\*Log[x] + b\*(1+2\*m)\*(-(n\*Log[x]) + Log[c\*x^n])))/(b\*n))\*((1+m + (2\*I)\*b\*n)\*Hypergeometric2F1[1, ((-I/2)\*(1+m))/(b\*n), 1 - ((I/2)\*(1+m))/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))]) + E^((a\*(1+2\*m + (2\*I)\*b\*n))/(b\*n) + (1+m + (2\*I)\*b\*n)\*Log[x] + ((1+2\*m + (2\*I)\*b\*n)\*(-(n\*Log[x]) + Log[c\*x^n])))/n)\*(1+m)\*Hypergeometric2F1[1, ((-I/2)\*(1+m + (2\*I)\*b\*n))/(b\*n), ((-I/2)\*(1+m + (4\*I)\*b\*n))/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))]) - I\*E^((a + 2\*a\*m + b\*(1+m)\*n\*Log[x] + b\*(1+2\*m)\*(-(n\*Log[x]) + Log[c\*x^n])))/(b\*n))\*((1+m + (2\*I)\*b\*n)\*Tan[a + b\*Log[c\*x^n]]))/(E^(((1+2\*m)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n])))/(b\*n))\*((1+m)\*(1+m + (2\*I)\*b\*n))))/(b\*n)

**Maple [F]** time = 1.779, size = 0, normalized size = 0.

$$\int x^m (\sec(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sec(a+b\*ln(c\*x^n))^2,x)

[Out] int(x^m\*sec(a+b\*ln(c\*x^n))^2,x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x^m\*sec(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^m \sec(b \log(cx^n) + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(x^m\*sec(b\*log(c\*x^n) + a)^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sec^2(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*sec(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(x\*\*m\*sec(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sec(b \log(cx^n) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(x^m\*sec(b\*log(c\*x^n) + a)^2, x)

### 3.280 $\int x^m \sec(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=103

$$\frac{2e^{ia}x^{m+1}(cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, -\frac{-bn+im+i}{2bn}, -\frac{-3bn+i(m+1)}{2bn}, -e^{2ia}(cx^n)^{2ib}\right)}{ibn + m + 1}$$

[Out] (2\*E^(I\*a)\*x^(1 + m)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, -(I + I\*m - b\*n)/(2\*b\*n), -(I\*(1 + m) - 3\*b\*n)/(2\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/(1 + m + I\*b\*n)

**Rubi [A]** time = 0.0686398, antiderivative size = 99, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4509, 4505, 364}

$$\frac{2e^{ia}x^{m+1}(cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i(m+1)}{bn}\right); -\frac{i(m+1)-3bn}{2bn}; -e^{2ia}(cx^n)^{2ib}\right)}{ibn + m + 1}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sec[a + b\*Log[c\*x^n]],x]

[Out] (2\*E^(I\*a)\*x^(1 + m)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, (1 - (I\*(1 + m))/(b\*n))/2, -(I\*(1 + m) - 3\*b\*n)/(2\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/(1 + m + I\*b\*n)

#### Rule 4509

Int[((e.\_)\*(x.\_))^(m.\_)\*Sec[((a.\_) + Log[(c.\_)\*(x.\_)^(n.\_)]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4505

Int[((e.\_)\*(x.\_))^(m.\_)\*Sec[((a.\_) + Log[x\_]\*(b.\_))\*(d.\_)]^(p.\_), x\_Symbol] := Dist[2^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c.\_)\*(x.\_))^(m.\_)\*((a.\_) + (b.\_)\*(x.\_)^(n.\_))^(p.\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x^m \sec(a + b \log(cx^n)) dx &= \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sec(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{\left(2e^{ia} x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+ib+\frac{1+m}{n}}}{1+e^{2ia} x^{2ib}} dx, x, cx^n\right)}{n} \\ &= \frac{2e^{ia} x^{1+m} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i(1+m)}{bn}\right); -\frac{i(1+m)-3bn}{2bn}; -e^{2ia} (cx^n)^{2ib}\right)}{1+m+ibn} \end{aligned}$$

**Mathematica [A]** time = 0.212423, size = 94, normalized size = 0.91

$$\frac{2e^{ia} x^{m+1} (cx^n)^{ib} \text{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{i(m+1)}{2bn}, \frac{3}{2} - \frac{i(m+1)}{2bn}, -e^{2i(a+b \log(cx^n))}\right)}{ibn + m + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Sec[a + b\*Log[c\*x^n]], x]

[Out] (2\*E^(I\*a)\*x^(1+m)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, 1/2 - ((I/2)\*(1+m))/(b\*n), 3/2 - ((I/2)\*(1+m))/(b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))])/(1+m + I\*b\*n)

**Maple [F]** time = 0.349, size = 0, normalized size = 0.

$$\int x^m \sec(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sec(a+b\*ln(c\*x^n)), x)

[Out] int(x^m\*sec(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(x^m\*sec(b\*log(c\*x^n) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^m \sec(b \log(cx^n) + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] integral(x^m*sec(b*log(c*x^n) + a), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sec(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(x**m*sec(a + b*log(c*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sec(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(x^m*sec(b*log(c*x^n) + a), x)
```

$$3.281 \quad \int x^m \sec^{\frac{5}{2}}(a + b \log(cx^n)) dx$$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} (1 + e^{2ia} (cx^n)^{2ib})^{5/2} \sec^{\frac{5}{2}}(a + b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, -\frac{-5bn+2im+2i}{4bn}, -\frac{-9bn+2im+2i}{4bn}, -e^{2ia} (cx^n)^{2ib}\right)}{5ibn + 2m + 2}$$

[Out] (2\*x^(1 + m)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Hypergeometric2F1[5/2, -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 9\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^(5/2)/(2 + 2\*m + (5\*I)\*b\*n)

**Rubi [A]** time = 0.0991482, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4509, 4507, 364}

$$\frac{2x^{m+1} (1 + e^{2ia} (cx^n)^{2ib})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4} \left(5 - \frac{2i(m+1)}{bn}\right); -\frac{2im-9bn+2i}{4bn}; -e^{2ia} (cx^n)^{2ib}\right) \sec^{\frac{5}{2}}(a + b \log(cx^n))}{5ibn + 2m + 2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sec[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x^(1 + m)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Hypergeometric2F1[5/2, (5 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 9\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^(5/2)/(2 + 2\*m + (5\*I)\*b\*n)

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/ (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \sec^{\frac{5}{2}}(a + b \log(cx^n)) dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sec^{\frac{5}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{5ib}{2}-\frac{1+m}{n}} \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{5/2} \sec^{\frac{5}{2}}(a + b \log(cx^n))\right) \text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1+m}{n}}}{(1+e^{2ia}x^{2ib})^{5/2}} dx\right)}{n}$$

$$= \frac{2x^{1+m} \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4} \left(5 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-9bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right) \sec^{\frac{5}{2}}(a + b \log(cx^n))}{2 + 2m + 5ibn}$$

**Mathematica [A]** time = 2.1366, size = 182, normalized size = 1.4

$$\frac{2x^{m+1} \sqrt{\sec(a + b \log(cx^n))} \left( (b^2 n^2 + 4m^2 + 8m + 4) \left(1 + e^{2ia} (cx^n)^{2ib}\right) \text{Hypergeometric2F1}\left(1, -\frac{-3bn+2im+2i}{4bn}, -\frac{-5bn+2im}{4bn}, \frac{-3b^2 n^2 (ibn + 2m + 2)}{4bn}\right) \right)}{3b^2 n^2 (ibn + 2m + 2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Sec[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x^(1 + m)\*Sqrt[Sec[a + b\*Log[c\*x^n]]]\*((4 + 8\*m + 4\*m^2 + b^2\*n^2)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))] - (2 + 2\*m + I\*b\*n)\*(2 + 2\*m - b\*n\*Tan[a + b\*Log[c\*x^n]])))/(3\*b^2\*n^2\*(2 + 2\*m + I\*b\*n))

**Maple [F]** time = 0.287, size = 0, normalized size = 0.

$$\int x^m (\sec(a + b \ln(cx^n)))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sec(a+b\*ln(c\*x^n))^(5/2), x)

[Out] int(x^m\*sec(a+b\*ln(c\*x^n))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sec(b \log(cx^n) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+b\*log(c\*x^n))^(5/2), x, algorithm="maxima")

[Out] integrate(x^m\*sec(b\*log(c\*x^n) + a)^(5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.282 $\int x^m \sec^{\frac{3}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} (1 + e^{2ia} (cx^n)^{2ib})^{3/2} \sec^{\frac{3}{2}}(a + b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, -\frac{-3bn+2im+2i}{4bn}, -\frac{-7bn+2im+2i}{4bn}, -e^{2ia} (cx^n)^{2ib}\right)}{3ibn + 2m + 2}$$

[Out] (2\*x^(1 + m)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Hypergeometric2F1[3/2, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^(3/2)/(2 + 2\*m + (3\*I)\*b\*n)

**Rubi [A]** time = 0.0955526, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4509, 4507, 364}

$$\frac{2x^{m+1} (1 + e^{2ia} (cx^n)^{2ib})^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i(m+1)}{bn}\right); -\frac{2im-7bn+2i}{4bn}; -e^{2ia} (cx^n)^{2ib}\right) \sec^{\frac{3}{2}}(a + b \log(cx^n))}{3ibn + 2m + 2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sec[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^(1 + m)\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Hypergeometric2F1[3/2, (3 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^(3/2)/(2 + 2\*m + (3\*I)\*b\*n)

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\int x^m \sec^{\frac{3}{2}}(a + b \log(cx^n)) dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sec^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{3ib}{2}-\frac{1+m}{n}} (1 + e^{2ia} (cx^n)^{2ib})^{3/2} \sec^{\frac{3}{2}}(a + b \log(cx^n))\right) \text{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1+m}{n}}}{(1+e^{2ia}x^{2ib})^{3/2}}\right)}{n}$$

$$= \frac{2x^{1+m} (1 + e^{2ia} (cx^n)^{2ib})^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4} \left(3 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-7bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right) \sec^{\frac{3}{2}}(a)}{2 + 2m + 3ibn}$$

**Mathematica [B]** time = 9.5809, size = 470, normalized size = 3.62

$$\sqrt{2}x^{-ibn+m+1} \left( (3ibn + 2m + 2) \left( ibn + 2m + 2 \right) \sqrt{\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}} \sqrt{1 + e^{2ia} (cx^n)^{2ib}} \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{bn+2im+2i}{4bn}, -\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Sec[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (Sqrt[2]\*x^(1 + m - I\*b\*n))\*(-(4 + 8\*m + 4\*m^2 + b^2\*n^2)\*x^((2\*I)\*b\*n)\*Sqrt[(E^(I\*a)\*(c\*x^n)^(I\*b))/(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, ((-I/2)\*(1 + m + ((3\*I)/2)\*b\*n))/(b\*n), -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]) + (2 + 2\*m + (3\*I)\*b\*n)\*((2 + 2\*m + I\*b\*n)\*Sqrt[(E^(I\*a)\*(c\*x^n)^(I\*b))/(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m + b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]) - I\*Sqrt[2]\*x^(I\*b\*n)\*Sqrt[Sec[a + b\*Log[c\*x^n]]\*(b\*n\*Cos[b\*n\*Log[x]] - 2\*(1 + m)\*Sin[b\*n\*Log[x]])]/(b\*n\*(-2\*I - (2\*I)\*m + 3\*b\*n)\*(-2\*(1 + m)\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]] + b\*n\*Sin[a - b\*n\*Log[x] + b\*Log[c\*x^n]]))

**Maple [F]** time = 0.295, size = 0, normalized size = 0.

$$\int x^m (\sec(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sec(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(x^m\*sec(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sec(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] `integrate(x^m*sec(b*log(c*x^n) + a)^(3/2), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*sec(a+b*ln(c*x**n))**(3/2),x)`

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")`

[Out] Timed out

### 3.283 $\int x^m \sqrt{\sec(a + b \log(cx^n))} dx$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \sqrt{1 + e^{2ia} (cx^n)^{2ib}} \sqrt{\sec(a + b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{-bn+2im+2i}{4bn}, -\frac{-5bn+2im+2i}{4bn}, -e^{2ia} (cx^n)^{2ib}\right)}{ibn + 2m + 2}$$

[Out] (2\*x^(1 + m)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))] \* Sqrt[Sec[a + b\*Log[c\*x^n]]])/(2 + 2\*m + I\*b\*n)

**Rubi [A]** time = 0.0914406, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4509, 4507, 364}

$$\frac{2x^{m+1} \sqrt{1 + e^{2ia} (cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, -\frac{2im-bn+2i}{4bn}; -\frac{2im-5bn+2i}{4bn}; -e^{2ia} (cx^n)^{2ib}\right) \sqrt{\sec(a + b \log(cx^n))}}{ibn + 2m + 2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sqrt[Sec[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x^(1 + m)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))] \* Sqrt[Sec[a + b\*Log[c\*x^n]]])/(2 + 2\*m + I\*b\*n)

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/ (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \sqrt{\sec(a + b \log(cx^n))} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sqrt{\sec(a + b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{ib}{2}-\frac{1+m}{n}} \sqrt{1 + e^{2ia} (cx^n)^{2ib}} \sqrt{\sec(a + b \log(cx^n))}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1+m}{n}}}{\sqrt{1+e^{2ia}x^{2ib}}} dx, x, c\right)}{n}$$

$$= \frac{2x^{1+m} \sqrt{1 + e^{2ia} (cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, -\frac{2i+2im-bn}{4bn}; -\frac{2i+2im-5bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right) \sqrt{\sec(a + b \log(cx^n))}}{2 + 2m + ibn}$$

**Mathematica [A]** time = 0.784376, size = 119, normalized size = 0.92

$$\frac{2x^{m+1} \left(1 + e^{2i(a+b \log(cx^n))}\right) \sqrt{\sec(a + b \log(cx^n))} \text{Hypergeometric2F1}\left(1, -\frac{-3bn+2im+2i}{4bn}, -\frac{-5bn+2im+2i}{4bn}, -e^{2i(a+b \log(cx^n))}\right)}{ibn + 2m + 2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Sqrt[Sec[a + b\*Log[c\*x^n]]], x]

[Out] (2\*(1 + E^((2\*I)\*(a + b\*Log[c\*x^n]))))\*x^(1 + m)\*Hypergeometric2F1[1, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))]\*Sqrt[Sec[a + b\*Log[c\*x^n]]]/(2 + 2\*m + I\*b\*n)

**Maple [F]** time = 0.306, size = 0, normalized size = 0.

$$\int x^m \sqrt{\sec(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sec(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(x^m\*sec(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\sec(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sec(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(x^m\*sqrt(sec(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\sec(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*sec(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(sec(a + b*log(c*x**n))), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.284 \quad \int \frac{x^m}{\sqrt{\sec(a+b \log(cx^n))}} dx$$

**Optimal.** Leaf size=129

$$\frac{2x^{m+1} \text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{bn+2im+2i}{4bn}, -\frac{-3bn+2im+2i}{4bn}, -e^{2ia} (cx^n)^{2ib}\right)}{(-ibn + 2m + 2)\sqrt{1 + e^{2ia} (cx^n)^{2ib}} \sqrt{\sec(a + b \log(cx^n))}}$$

[Out] (2\*x^(1 + m)\*Hypergeometric2F1[-1/2, -(2\*I + (2\*I)\*m + b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m - I\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])

**Rubi [A]** time = 0.0919304, antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4509, 4507, 364}

$$\frac{2x^{m+1} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-\frac{2i(m+1)}{bn} - 1\right); -\frac{2im-3bn+2i}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(-ibn + 2m + 2)\sqrt{1 + e^{2ia} (cx^n)^{2ib}} \sqrt{\sec(a + b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[x^m/Sqrt[Sec[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x^(1 + m)\*Hypergeometric2F1[-1/2, (-1 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/((2 + 2\*m - I\*b\*n)\*Sqrt[1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Sec[a + b\*Log[c\*x^n]]])

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[(e\*x)^m\*x^(I\*b\*d\*p)/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{x^m}{\sqrt{\sec(a + b \log(cx^n))}} dx &= \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sqrt{\sec(a+b \log(x))}} dx, x, cx^n\right)}{n} \\
&= \frac{\left(x^{1+m} (cx^n)^{\frac{ib}{2}-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1+m}{n}} \sqrt{1+e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b \log(cx^n))}} \\
&= \frac{2x^{1+m} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-1-\frac{2i(1+m)}{bn}\right); -\frac{2i+2im-3bn}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b \log(cx^n))}}
\end{aligned}$$

**Mathematica [B]** time = 6.98673, size = 437, normalized size = 3.39

$$\frac{2bx^{m+1}e^{2i(a+b \log(cx^n)-bn \log(x))}}{\sqrt{\sec(a+b \log(cx^n))(2(m+1)\cos(a+b \log(cx^n)-bn \log(x))-bn \sin(a+b \log(cx^n)-bn \log(x)))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m/Sqrt[Sec[a + b\*Log[c\*x^n]]], x]

[Out]  $(-2*b*E^{((2*I)*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n]))}*n*x^{(1 + m)*((2*I) + (2*I)*m + b*n)}*x^{((2*I)*b*n)}*\text{Hypergeometric2F1}[1/2, ((-I/2)*(1 + m + ((3*I)/2)*b*n))/((b*n), -(2*I + (2*I)*m - 7*b*n)/(4*b*n), -(E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}})] + (-2*I - (2*I)*m + 3*b*n)*\text{Hypergeometric2F1}[1/2, -(2*I + (2*I)*m + b*n)/(4*b*n), -(2*I + (2*I)*m - 3*b*n)/(4*b*n), -(E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}})])))/((2 + 2*m - I*b*n)*(2 + 2*m + (3*I)*b*n)*(2 + 2*m - I*b*n + E^{((2*I)*(a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n]))*(2 + 2*m + I*b*n)}*\text{Sqrt}[1 + E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}}]*\text{Sqrt}[(E^{(I*a)*(c*x^n)^{(I*b)}})/(2 + 2*E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}})]) + (2*x^{(1 + m)}*\text{Cos}[a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n]])/(\text{Sqrt}[\text{Sec}[a + b*\text{Log}[c*x^n]])*(2*(1 + m)*\text{Cos}[a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n]] - b*n*\text{Sin}[a - b*n*\text{Log}[x] + b*\text{Log}[c*x^n]]))$

**Maple [F]** time = 0.268, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{\sec(a + b \ln(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/sec(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(x^m/sec(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\sec(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/sec(a+b\*log(c\*x^n))^(1/2),x, algorithm="maxima")

[Out] integrate(x^m/sqrt(sec(b\*log(c\*x^n) + a)), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/sec(a+b\*log(c\*x^n))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\sec(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/sec(a+b\*ln(c\*x\*\*n))\*\*(1/2),x)

[Out] Integral(x\*\*m/sqrt(sec(a + b\*log(c\*x\*\*n))), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/sec(a+b\*log(c\*x^n))^(1/2),x, algorithm="giac")

[Out] Timed out



$$3.285 \quad \int \frac{x^m}{\sec^2(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \text{Hypergeometric2F1}\left(-\frac{3}{2}, -\frac{3bn+2im+2i}{4bn}, -\frac{-bn+2im+2i}{4bn}, -e^{2ia}(cx^n)^{2ib}\right)}{(-3ibn+2m+2)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2} \sec^{\frac{3}{2}}(a+b \log(cx^n))}$$

[Out] (2\*x^(1+m)\*Hypergeometric2F1[-3/2, -(2\*I+(2\*I)\*m+3\*b\*n)/(4\*b\*n), -(2\*I+(2\*I)\*m-b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/((2+2\*m-(3\*I)\*b\*n)\*(1+E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(3/2)\*Sec[a+b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.0959615, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4509, 4507, 364}

$$\frac{2x^{m+1} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-\frac{2i(m+1)}{bn}-3\right); -\frac{2im-bn+2i}{4bn}; -e^{2ia}(cx^n)^{2ib}\right)}{(-3ibn+2m+2)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2} \sec^{\frac{3}{2}}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x^m/Sec[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^(1+m)\*Hypergeometric2F1[-3/2, (-3-((2\*I)\*(1+m))/(b\*n))/4, -(2\*I+(2\*I)\*m-b\*n)/(4\*b\*n), -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))]/((2+2\*m-(3\*I)\*b\*n)\*(1+E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^(3/2)\*Sec[a+b\*Log[c\*x^n]]^(3/2))

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.)+Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[x^((m+1)/n-1)\*Sec[d\*(a+b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.)+Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] :> Dist[(Sec[d\*(a+b\*Log[x])]^p\*(1+E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[(e\*x)^m\*x^(I\*b\*d\*p)/(1+E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.)+(b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/((c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a + b \log(cx^n))} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sec^{\frac{3}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{\frac{3ib}{2}-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1+m}{n}} (1 + e^{2ia}x^{2ib})^{3/2} dx, x, cx^n\right)}{n \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{3/2} \sec^{\frac{3}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x^{1+m} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-bn}{4bn}; -e^{2ia} (cx^n)^{2ib}\right)}{(2 + 2m - 3ibn) \left(1 + e^{2ia} (cx^n)^{2ib}\right)^{3/2} \sec^{\frac{3}{2}}(a + b \log(cx^n))}$$

**Mathematica [A]** time = 2.51573, size = 202, normalized size = 1.55

$$\frac{2x^{m+1} \left(3b^2n^2 (1 + e^{2ia} (cx^n)^{2ib}) \sec^2(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, -\frac{-3bn+2im+2i}{4bn}, -\frac{-5bn+2im+2i}{4bn}, -e^{2i(a+b \log(cx^n))}\right)\right)}{(ibn + 2m + 2)(-3ibn + 2m + 2)(3ibn + 2m + 2) \sec^{\frac{3}{2}}(a + b \log(cx^n))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m/Sec[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^(1 + m)\*(3\*b^2\*n^2\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -E^((2\*I)\*(a + b\*Log[c\*x^n]))]\*Sec[a + b\*Log[c\*x^n]]^2 + (2 + 2\*m + I\*b\*n)\*(2 + 2\*m + 3\*b\*n\*Tan[a + b\*Log[c\*x^n]])))/((2 + 2\*m + I\*b\*n)\*(2 + 2\*m - (3\*I)\*b\*n)\*(2 + 2\*m + (3\*I)\*b\*n)\*Sec[a + b\*Log[c\*x^n]]^(3/2))

**Maple [F]** time = 0.288, size = 0, normalized size = 0.

$$\int x^m (\sec(a + b \ln(cx^n)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/sec(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(x^m/sec(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sec(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/sec(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(x^m/sec(b\*log(c\*x^n) + a)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/sec(a+b\*log(c\*x<sup>n</sup>))<sup>(3/2)</sup>,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/sec(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/sec(a+b\*log(c\*x<sup>n</sup>))<sup>(3/2)</sup>,x, algorithm="giac")

[Out] Timed out

### 3.286 $\int (ex)^m \sec^p(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=139

$$\frac{(ex)^{m+1} \left(1 + e^{2iad} (cx^n)^{2ibd}\right)^p \sec^p(d(a + b \log(cx^n))) \operatorname{Hypergeometric2F1}\left(p, -\frac{bdnp+im+i}{2bdn}, \frac{1}{2}\left(-\frac{i(m+1)}{bdn} + p + 2\right), -e^{2iad} (cx^n)^{2ibd}\right)}{e(ibdnp + m + 1)}$$

[Out] ((e\*x)^(1 + m)\*(1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^p\*Hypergeometric2F1[p, -(I + I\*m - b\*d\*n\*p)/(2\*b\*d\*n), (2 - (I\*(1 + m)))/(b\*d\*n) + p)/2, -(E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))]\*Sec[d\*(a + b\*Log[c\*x^n])]^p/(e\*(1 + m + I\*b\*d\*n\*p))

**Rubi [A]** time = 0.121491, antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4509, 4507, 364}

$$\frac{(ex)^{m+1} \left(1 + e^{2iad} (cx^n)^{2ibd}\right)^p {}_2F_1\left(p, \frac{1}{2}\left(p - \frac{i(m+1)}{bdn}\right); \frac{1}{2}\left(-\frac{i(m+1)}{bdn} + p + 2\right); -e^{2iad} (cx^n)^{2ibd}\right) \sec^p(d(a + b \log(cx^n)))}{e(ibdnp + m + 1)}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^m\*Sec[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out] ((e\*x)^(1 + m)\*(1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^p\*Hypergeometric2F1[p, (((-I)\*(1 + m)))/(b\*d\*n) + p)/2, (2 - (I\*(1 + m)))/(b\*d\*n) + p)/2, -(E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))]\*Sec[d\*(a + b\*Log[c\*x^n])]^p/(e\*(1 + m + I\*b\*d\*n\*p))

#### Rule 4509

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int (ex)^m \sec^p(d(a + b \log(cx^n))) dx &= \frac{\left( (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{n}} \sec^p(d(a + b \log(x))) dx, x, cx^n \right)}{en} \\ &= \frac{\left( (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}-ibdp} \left( 1 + e^{2iad} (cx^n)^{2ibd} \right)^p \sec^p(d(a + b \log(cx^n))) \right) \text{Subst} \left( \int \right)}{en} \\ &= \frac{(ex)^{1+m} \left( 1 + e^{2iad} (cx^n)^{2ibd} \right)^p {}_2F_1 \left( p, \frac{1}{2} \left( -\frac{i(1+m)}{bdn} + p \right); \frac{1}{2} \left( 2 - \frac{i(1+m)}{bdn} + p \right); -e^{2iad} (cx^n)^{2ibd} \right)}{e(1+m+ibdn)} \end{aligned}$$

**Mathematica [A]** time = 1.55888, size = 169, normalized size = 1.22

$$\frac{2^p x (ex)^m \left( \frac{e^{iad} (cx^n)^{ibd}}{1 + e^{2iad} (cx^n)^{2ibd}} \right)^p \left( 1 + e^{2iad} (cx^n)^{2ibd} \right)^p \text{Hypergeometric2F1} \left( p, -\frac{i(bdn+p+1)}{2bdn}, \frac{1}{2} \left( -\frac{i(m+1)}{bdn} + p + 2 \right), -e^{2iad} (cx^n)^{2ibd} \right)}{ibdn + m + 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m\*Sec[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out] (2^p\*x\*(e\*x)^m\*((E^(I\*a\*d)\*(c\*x^n)^(I\*b\*d))/(1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))^p\*(1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^p\*Hypergeometric2F1[p, ((-I/2)\*(1 + m + I\*b\*d\*n\*p))/(b\*d\*n), (2 - (I\*(1 + m))/(b\*d\*n) + p)/2, -(E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))]/(1 + m + I\*b\*d\*n\*p)

**Maple [F]** time = 0.27, size = 0, normalized size = 0.

$$\int (ex)^m (\sec(d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*sec(d\*(a+b\*ln(c\*x^n)))^p,x)

[Out] int((e\*x)^m\*sec(d\*(a+b\*ln(c\*x^n)))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sec((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sec(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="maxima")

[Out] integrate((e\*x)^m\*sec((b\*log(c\*x^n) + a)\*d)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (ex)^m \sec(bd \log(cx^n) + ad)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sec(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="fricas")

[Out] integral((e\*x)^m\*sec(b\*d\*log(c\*x^n) + a\*d)^p, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*sec(d\*(a+b\*ln(c\*x\*\*n))))\*\*p,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \sec((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*sec(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="giac")

[Out] integrate((e\*x)^m\*sec((b\*log(c\*x^n) + a)\*d)^p, x)

### 3.287 $\int x \sec^p(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=106

$$\frac{x^2 (1 + e^{2ia} (cx^n)^{2ib})^p \operatorname{Hypergeometric2F1}\left(p, \frac{1}{2}\left(p - \frac{2i}{bn}\right), \frac{1}{2}\left(-\frac{2i}{bn} + p + 2\right), -e^{2ia} (cx^n)^{2ib}\right) \sec^p(a + b \log(cx^n))}{2 + ibnp}$$

[Out]  $(x^2*(1 + E^{((2*I)*a)*(c*x^n)^{(2*I)*b})}^p*\operatorname{Hypergeometric2F1}[p, ((-2*I)/(b*n) + p)/2, (2 - (2*I)/(b*n) + p)/2, -(E^{((2*I)*a)*(c*x^n)^{(2*I)*b})}])*\operatorname{Sec}[a + b*\operatorname{Log}[c*x^n]]^p)/(2 + I*b*n*p)$

**Rubi [A]** time = 0.0827361, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4509, 4507, 364}

$$\frac{x^2 (1 + e^{2ia} (cx^n)^{2ib})^p {}_2F_1\left(p, \frac{1}{2}\left(p - \frac{2i}{bn}\right); \frac{1}{2}\left(p - \frac{2i}{bn} + 2\right); -e^{2ia} (cx^n)^{2ib}\right) \sec^p(a + b \log(cx^n))}{2 + ibnp}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{Sec}[a + b*\operatorname{Log}[c*x^n]]^p, x]$

[Out]  $(x^2*(1 + E^{((2*I)*a)*(c*x^n)^{(2*I)*b})}^p*\operatorname{Hypergeometric2F1}[p, ((-2*I)/(b*n) + p)/2, (2 - (2*I)/(b*n) + p)/2, -(E^{((2*I)*a)*(c*x^n)^{(2*I)*b})}])*\operatorname{Sec}[a + b*\operatorname{Log}[c*x^n]]^p)/(2 + I*b*n*p)$

#### Rule 4509

$\operatorname{Int}[(e_{.})*(x_{.})^{(m_{.})}*\operatorname{Sec}[(a_{.}) + \operatorname{Log}[(c_{.})*(x_{.})^{(n_{.})}]]*(b_{.})*(d_{.})]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[x^{((m+1)/n-1)*\operatorname{Sec}[d*(a+b*\operatorname{Log}[x])]}^p, x], x, c*x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\operatorname{NeQ}[c, 1] \parallel \operatorname{NeQ}[n, 1])$

#### Rule 4507

$\operatorname{Int}[(e_{.})*(x_{.})^{(m_{.})}*\operatorname{Sec}[(a_{.}) + \operatorname{Log}[x_{.}]]*(b_{.})*(d_{.})]^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(\operatorname{Sec}[d*(a+b*\operatorname{Log}[x])]}^p*(1 + E^{(2*I*a*d)*x^{(2*I*b*d)}})^p/x^{(I*b*d*p)}, \operatorname{Int}[(e*x)^{m*x^{(I*b*d*p)}}/(1 + E^{(2*I*a*d)*x^{(2*I*b*d)}})^p, x], x] /; \operatorname{FreeQ}\{a, b, d, e, m, p\}, x] \&\& !\operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c_{.})*(x_{.})^{(m_{.})}*((a_{.}) + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(a^p*(c*x)^{(m+1)}*\operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/(c*(m+1)), x] /; \operatorname{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \parallel \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\int x \sec^p(a + b \log(cx^n)) dx = \frac{(x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int x^{-1+\frac{2}{n}} \sec^p(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^2 (cx^n)^{-\frac{2}{n}-ibp} (1 + e^{2ia} (cx^n)^{2ib})^p \sec^p(a + b \log(cx^n))\right) \text{Subst}\left(\int x^{-1+\frac{2}{n}+ibp} (1 + e^{2ia} x^{2ib})\right)}{n}$$

$$= \frac{x^2 (1 + e^{2ia} (cx^n)^{2ib})^p {}_2F_1\left(p, \frac{1}{2}\left(-\frac{2i}{bn} + p\right); \frac{1}{2}\left(2 - \frac{2i}{bn} + p\right); -e^{2ia} (cx^n)^{2ib}\right) \sec^p(a + b \log(cx^n))}{2 + ibnp}$$

**Mathematica [A]** time = 0.962605, size = 142, normalized size = 1.34

$$\frac{i2^p x^2 \left(\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}\right)^p (1 + e^{2ia} (cx^n)^{2ib})^p \text{Hypergeometric2F1}\left(\frac{p}{2} - \frac{i}{bn}, p, -\frac{i}{bn} + \frac{p}{2} + 1, -e^{2ia} (cx^n)^{2ib}\right)}{bnp - 2i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Sec[a + b\*Log[c\*x^n]]^p, x]

[Out] ((-I)\*2^p\*x^2\*((E^(I\*a)\*(c\*x^n)^(I\*b))/(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))^p\*Hypergeometric2F1[(-I)/(b\*n) + p/2, p, 1 - I/(b\*n) + p/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(-2\*I + b\*n\*p)

**Maple [F]** time = 0.212, size = 0, normalized size = 0.

$$\int x (\sec(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sec(a+b\*ln(c\*x^n))^p, x)

[Out] int(x\*sec(a+b\*ln(c\*x^n))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sec(a+b\*log(c\*x^n))^p, x, algorithm="maxima")

[Out] integrate(x\*sec(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \sec(b \log(cx^n) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x*sec(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(x*sec(b*log(c*x^n) + a)^p, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec^p(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(x*sec(a + b*log(c*x**n))**p, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sec(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*log(c*x^n) + a)^p, x)
```

### 3.288 $\int \sec^p (a + b \log (cx^n)) dx$

**Optimal.** Leaf size=107

$$\frac{x \left(1 + e^{2ia} (cx^n)^{2ib}\right)^p \operatorname{Hypergeometric2F1}\left(p, -\frac{-bnp+i}{2bn}, \frac{1}{2}\left(-\frac{i}{bn} + p + 2\right), -e^{2ia} (cx^n)^{2ib}\right) \sec^p (a + b \log (cx^n))}{1 + ibnp}$$

[Out] (x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*Hypergeometric2F1[p, -(I - b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) + p)/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^p)/(1 + I\*b\*n\*p)

**Rubi [A]** time = 0.0694251, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4503, 4507, 364}

$$\frac{x \left(1 + e^{2ia} (cx^n)^{2ib}\right)^p {}_2F_1\left(p, -\frac{i-bnp}{2bn}; \frac{1}{2}\left(p - \frac{i}{bn} + 2\right); -e^{2ia} (cx^n)^{2ib}\right) \sec^p (a + b \log (cx^n))}{1 + ibnp}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b\*Log[c\*x^n]]^p, x]

[Out] (x\*(1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*Hypergeometric2F1[p, -(I - b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) + p)/2, -(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Sec[a + b\*Log[c\*x^n]]^p)/(1 + I\*b\*n\*p)

#### Rule 4503

Int[Sec[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Sec[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4507

Int[((e\_.)\*(x\_))^(m\_.)\*Sec[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[(Sec[d\*(a + b\*Log[x])]^p\*(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 + E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sec^p(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sec^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{\left(x(cx^n)^{-\frac{1}{n}-ibp} (1 + e^{2ia} (cx^n)^{2ib})^p \sec^p(a + b \log(cx^n))\right) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}+ibp} (1 + e^{2ia} x^{2ib})\right)}{n} \\ &= \frac{x(1 + e^{2ia} (cx^n)^{2ib})^p {}_2F_1\left(p, -\frac{i-bnp}{2bn}; \frac{1}{2}\left(2 - \frac{i}{bn} + p\right); -e^{2ia} (cx^n)^{2ib}\right) \sec^p(a + b \log(cx^n))}{1 + ibnp} \end{aligned}$$

**Mathematica [A]** time = 0.781166, size = 142, normalized size = 1.33

$$\frac{i2^p x \left(\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}\right)^p (1 + e^{2ia} (cx^n)^{2ib})^p \operatorname{Hypergeometric2F1}\left(p, \frac{bnp-i}{2bn}, \frac{1}{2}\left(-\frac{i}{bn} + p + 2\right), -e^{2ia} (cx^n)^{2ib}\right)}{bnp - i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[a + b\*Log[c\*x^n]]^p, x]

[Out]  $((-1)*2^p*x*((E^{(I*a)}*(c*x^n)^{(I*b)})/(1 + E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)}))^p * (1 + E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})^p * \operatorname{Hypergeometric2F1}[p, (-I + b*n*p)/(2*b*n), (2 - I/(b*n) + p)/2, -(E^{((2*I)*a)}*(c*x^n)^{((2*I)*b)})]/(-I + b*n*p)$

**Maple [F]** time = 0.178, size = 0, normalized size = 0.

$$\int (\sec(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(a+b\*ln(c\*x^n))^p, x)

[Out] int(sec(a+b\*ln(c\*x^n))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(a+b\*log(c\*x^n))^p, x, algorithm="maxima")

[Out] integrate(sec(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sec(b \log(cx^n) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(sec(b*log(c*x^n) + a)^p, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec^p(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(sec(a + b*log(c*x**n))**p, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sec(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(sec(b*log(c*x^n) + a)^p, x)
```

### 3.289 $\int x^2 \csc(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=86

$$\frac{2e^{ia}x^3 (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{3i}{bn}\right), \frac{3}{2}\left(1 - \frac{i}{bn}\right), e^{2ia} (cx^n)^{2ib}\right)}{-bn + 3i}$$

[Out]  $(2E^{(I*a)}*x^3*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 - (3*I)/(b*n))/2, (3*(1 - I/(b*n)))/2, E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}}]/(3*I - b*n)$

**Rubi [A]** time = 0.0618108, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4510, 4506, 364}

$$\frac{2e^{ia}x^3 (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{3i}{bn}\right); \frac{3}{2}\left(1 - \frac{i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{-bn + 3i}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Csc[a + b\*Log[c\*x^n]], x]

[Out]  $(2E^{(I*a)}*x^3*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 - (3*I)/(b*n))/2, (3*(1 - I/(b*n)))/2, E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}}]/(3*I - b*n)$

#### Rule 4510

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(-2\*I)^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d))\*x^(2\*I\*b\*d)^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x^2 \csc(a + b \log(cx^n)) dx &= \frac{(x^3 (cx^n)^{-3/n}) \operatorname{Subst}\left(\int x^{-1+\frac{3}{n}} \csc(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= -\frac{(2ie^{ia}x^3 (cx^n)^{-3/n}) \operatorname{Subst}\left(\int \frac{x^{-1+ib+\frac{3}{n}}}{1-e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n} \\ &= \frac{2e^{ia}x^3 (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{3i}{bn}\right); \frac{3}{2}\left(1 - \frac{i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{3i - bn} \end{aligned}$$

**Mathematica [A]** time = 1.49831, size = 82, normalized size = 0.95

$$\frac{2e^{ia}x^3(cx^n)^{ib}\operatorname{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{3i}{2bn}, \frac{3}{2} - \frac{3i}{2bn}, e^{2i(a+b\log(cx^n))}\right)}{bn - 3i}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Csc[a + b\*Log[c\*x^n]], x]

[Out]  $(-2E^{(I*a)}x^3(cx^n)^{(I*b)}\operatorname{Hypergeometric2F1}[1, 1/2 - ((3*I)/2)/(b*n), 3/2 - ((3*I)/2)/(b*n), E^{((2*I)*(a + b*\operatorname{Log}[c*x^n])]})]/(-3*I + b*n)$

**Maple [F]** time = 0.384, size = 0, normalized size = 0.

$$\int x^2 \csc(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*csc(a+b\*ln(c\*x^n)), x)

[Out] int(x^2\*csc(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \csc(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*csc(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(x^2\*csc(b\*log(c\*x^n) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^2 \csc(b \log(cx^n) + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*csc(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(x^2\*csc(b\*log(c\*x^n) + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \csc(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(x**2*csc(a + b*log(c*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \csc(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(x^2*csc(b*log(c*x^n) + a), x)
```

### 3.290 $\int x \csc(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=86

$$\frac{2e^{ia}x^2 (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{2i}{bn}\right), \frac{1}{2}\left(3 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{-bn + 2i}$$

[Out]  $(2E^{(I*a)}*x^2*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 - (2*I)/(b*n))/2, (3 - (2*I)/(b*n))/2, E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}])/(2*I - b*n)$

**Rubi [A]** time = 0.0557441, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4510, 4506, 364}

$$\frac{2e^{ia}x^2 (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{2i}{bn}\right); \frac{1}{2}\left(3 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{-bn + 2i}$$

Antiderivative was successfully verified.

[In] `Int[x*Csc[a + b*Log[c*x^n]], x]`

[Out]  $(2E^{(I*a)}*x^2*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, (1 - (2*I)/(b*n))/2, (3 - (2*I)/(b*n))/2, E^{((2*I)*a)*(c*x^n)^{(2*I)*b}}])/(2*I - b*n)$

#### Rule 4510

`Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])`

#### Rule 4506

`Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] := Dist[(-2*I)^p*E^(I*a*d*p), Int[((e*x)^m*x^(I*b*d*p))/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]`

#### Rule 364

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

#### Rubi steps

$$\begin{aligned} \int x \csc(a + b \log(cx^n)) dx &= \frac{(x^2 (cx^n)^{-2/n}) \operatorname{Subst}\left(\int x^{-1+\frac{2}{n}} \csc(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= -\frac{(2ie^{ia}x^2 (cx^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+ib+\frac{2}{n}}}{1-e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n} \\ &= \frac{2e^{ia}x^2 (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{2i}{bn}\right); \frac{1}{2}\left(3 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{2i - bn} \end{aligned}$$



**Mathematica [A]** time = 1.47282, size = 78, normalized size = 0.91

$$\frac{2e^{ia}x^2(cx^n)^{ib}\text{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{i}{bn}, \frac{3}{2} - \frac{i}{bn}, e^{2i(a+b\log(cx^n))}\right)}{bn - 2i}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Csc[a + b\*Log[c\*x^n]], x]

[Out]  $(-2E^{Ia}x^2(c*x^n)^{Ib}\text{Hypergeometric2F1}[1, 1/2 - I/(b*n), 3/2 - I/(b*n), E^{((2*I)*(a + b*\text{Log}[c*x^n]))}])/(-2*I + b*n)$

**Maple [F]** time = 0.31, size = 0, normalized size = 0.

$$\int x \csc(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*csc(a+b\*ln(c\*x^n)), x)

[Out] int(x\*csc(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \csc(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*csc(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(x\*csc(b\*log(c\*x^n) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \csc(b \log(cx^n) + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*csc(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(x\*csc(b\*log(c\*x^n) + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \csc(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(x*csc(a + b*log(c*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \csc(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(x*csc(b*log(c*x^n) + a), x)
```

### 3.291 $\int \csc(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=84

$$\frac{2e^{ia}x(cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{i}{bn}\right), \frac{1}{2}\left(3 - \frac{i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{-bn + i}$$

[Out] (2\*E^(I\*a)\*x\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, (1 - I/(b\*n))/2, (3 - I/(b\*n))/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/(I - b\*n)

**Rubi [A]** time = 0.0507095, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {4504, 4506, 364}

$$\frac{2e^{ia}x(cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i}{bn}\right); \frac{1}{2}\left(3 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{-bn + i}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]], x]

[Out] (2\*E^(I\*a)\*x\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, (1 - I/(b\*n))/2, (3 - I/(b\*n))/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/(I - b\*n)

#### Rule 4504

Int[Csc[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)]\*(d\_)^(p\_), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[((a\_) + Log[x\_]\*(b\_)]\*(d\_)^(p\_)\*((e\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(-2\*I)^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \csc(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \csc(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= -\frac{(2ie^{ia}x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+ib+\frac{1}{n}}}{1-e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n} \\ &= \frac{2e^{ia}x(cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i}{bn}\right); \frac{1}{2}\left(3 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{i - bn} \end{aligned}$$

**Mathematica [A]** time = 1.27051, size = 80, normalized size = 0.95

$$\frac{2e^{ia}x(cx^n)^{ib}\operatorname{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{i}{2bn}, \frac{3}{2} - \frac{i}{2bn}, e^{2i(a+b\log(cx^n))}\right)}{bn - i}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b\*Log[c\*x^n]], x]

[Out]  $(-2E^{(I*a)}*x*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, 1/2 - (I/2)/(b*n), 3/2 - (I/2)/(b*n), E^{((2*I)*(a + b*\operatorname{Log}[c*x^n]))}])/(-I + b*n)$

**Maple [F]** time = 0.259, size = 0, normalized size = 0.

$$\int \csc(a + b \ln(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n)), x)

[Out] int(csc(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\csc(b \log(cx^n) + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(csc(b\*log(c\*x^n) + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a), x)
```

$$3.292 \quad \int \frac{\csc(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=20

$$-\frac{\tanh^{-1}(\cos(a+b \log(cx^n)))}{bn}$$

[Out] -(ArcTanh[Cos[a + b\*Log[c\*x^n]])/(b\*n))

**Rubi [A]** time = 0.015723, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {3770}

$$-\frac{\tanh^{-1}(\cos(a+b \log(cx^n)))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]/x,x]

[Out] -(ArcTanh[Cos[a + b\*Log[c\*x^n]])/(b\*n))

**Rule 3770**

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned} \int \frac{\csc(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \csc(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\tanh^{-1}(\cos(a+b \log(cx^n)))}{bn} \end{aligned}$$

**Mathematica [B]** time = 0.0571344, size = 54, normalized size = 2.7

$$\frac{\log\left(\sin\left(\frac{a}{2} + \frac{1}{2}b \log(cx^n)\right)\right)}{bn} - \frac{\log\left(\cos\left(\frac{a}{2} + \frac{1}{2}b \log(cx^n)\right)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]/x,x]

[Out] -(Log[Cos[a/2 + (b\*Log[c\*x^n])/2]]/(b\*n)) + Log[Sin[a/2 + (b\*Log[c\*x^n])/2]]/(b\*n)

**Maple [A]** time = 0.027, size = 33, normalized size = 1.7

$$-\frac{\ln(\csc(a+b \ln(cx^n)) + \cot(a+b \ln(cx^n)))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(a+b*ln(c*x^n))/x,x)`

[Out] `-1/n/b*ln(csc(a+b*ln(c*x^n))+cot(a+b*ln(c*x^n)))`

**Maxima [A]** time = 0.995549, size = 43, normalized size = 2.15

$$\frac{\log(\cot(b \log(cx^n) + a) + \csc(b \log(cx^n) + a))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(a+b*log(c*x^n))/x,x, algorithm="maxima")`

[Out] `-log(cot(b*log(c*x^n) + a) + csc(b*log(c*x^n) + a))/(b*n)`

**Fricas [B]** time = 0.497139, size = 147, normalized size = 7.35

$$\frac{\log\left(\frac{1}{2} \cos(bn \log(x) + b \log(c) + a) + \frac{1}{2}\right) - \log\left(-\frac{1}{2} \cos(bn \log(x) + b \log(c) + a) + \frac{1}{2}\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(a+b*log(c*x^n))/x,x, algorithm="fricas")`

[Out] `-1/2*(log(1/2*cos(b*n*log(x) + b*log(c) + a) + 1/2) - log(-1/2*cos(b*n*log(x) + b*log(c) + a) + 1/2))/(b*n)`

**Sympy [A]** time = 2.88101, size = 49, normalized size = 2.45

$$-\begin{cases} -\log(x) \csc(a) & \text{for } b = 0 \\ -\log(x) \csc(a + b \log(c)) & \text{for } n = 0 \\ \frac{\log(\cot(a + b \log(cx^n)) + \csc(a + b \log(cx^n)))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(a+b*ln(c*x**n))/x,x)`

[Out] `-Piecewise((-log(x)*csc(a), Eq(b, 0)), (-log(x)*csc(a + b*log(c)), Eq(n, 0)), (log(cot(a + b*log(c*x**n)) + csc(a + b*log(c*x**n)))/(b*n), True))`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)/x, x)
```



$$3.293 \quad \int \frac{\csc(a+b \log(cx^n))}{x^2} dx$$

**Optimal.** Leaf size=85

$$\frac{2e^{ia} (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 + \frac{i}{bn}\right), \frac{1}{2}\left(3 + \frac{i}{bn}\right), e^{2ia} (cx^n)^{2ib}\right)}{x(bn + i)}$$

[Out]  $(-2 * E^{(I * a)} * (c * x^n)^{(I * b)} * \operatorname{Hypergeometric2F1}[1, (1 + I/(b * n))/2, (3 + I/(b * n))/2, E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / ((I + b * n) * x)$

**Rubi [A]** time = 0.059023, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4510, 4506, 364}

$$\frac{2e^{ia} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{i}{bn}\right); \frac{1}{2}\left(3 + \frac{i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{x(bn + i)}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]/x^2,x]

[Out]  $(-2 * E^{(I * a)} * (c * x^n)^{(I * b)} * \operatorname{Hypergeometric2F1}[1, (1 + I/(b * n))/2, (3 + I/(b * n))/2, E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / ((I + b * n) * x)$

#### Rule 4510

Int[Csc[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(-2\*I)^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p \* (c\*x)^(m + 1) \* Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)]) / (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\csc(a + b \log(cx^n))}{x^2} dx &= \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int x^{-1-\frac{1}{n}} \csc(a + b \log(x)) dx, x, cx^n\right)}{nx} \\ &= -\frac{\left(2ie^{ia} (cx^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+ib-\frac{1}{n}}}{1-e^{2ia}x^{2ib}} dx, x, cx^n\right)}{nx} \\ &= -\frac{2e^{ia} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{i}{bn}\right); \frac{1}{2}\left(3 + \frac{i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{(i + bn)x} \end{aligned}$$

**Mathematica [A]** time = 1.07642, size = 82, normalized size = 0.96

$$-\frac{2e^{ia} (cx^n)^{ib} \text{Hypergeometric2F1}\left(1, \frac{1}{2} + \frac{i}{2bn}, \frac{3}{2} + \frac{i}{2bn}, e^{2i(a+b \log(cx^n))}\right)}{x(bn + i)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]/x^2,x]

[Out]  $(-2E^{(I*a)}*(c*x^n)^{(I*b)}*\text{Hypergeometric2F1}[1, 1/2 + (I/2)/(b*n), 3/2 + (I/2)/(b*n), E^{((2*I)*(a + b*\text{Log}[c*x^n]))}])/(I + b*n)*x$

**Maple [F]** time = 0.377, size = 0, normalized size = 0.

$$\int \frac{\csc(a + b \ln(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))/x^2,x)

[Out] int(csc(a+b\*ln(c\*x^n))/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))/x^2,x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a)/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(b \log(cx^n) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*log(c*x^n) + a)/x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a + b \log(cx^n))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))/x**2,x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)/x^2, x)
```

$$3.294 \quad \int \frac{\csc(a+b \log(cx^n))}{x^3} dx$$

**Optimal.** Leaf size=85

$$\frac{2e^{ia} (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 + \frac{2i}{bn}\right), \frac{1}{2}\left(3 + \frac{2i}{bn}\right), e^{2ia} (cx^n)^{2ib}\right)}{x^2(bn + 2i)}$$

[Out]  $(-2 * E^{(I * a)} * (c * x^n)^{(I * b)} * \operatorname{Hypergeometric2F1}[1, (1 + (2 * I) / (b * n)) / 2, (3 + (2 * I) / (b * n)) / 2, E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / ((2 * I + b * n) * x^2)$

**Rubi [A]** time = 0.0585942, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4510, 4506, 364}

$$\frac{2e^{ia} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{2i}{bn}\right); \frac{1}{2}\left(3 + \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{x^2(bn + 2i)}$$

Antiderivative was successfully verified.

[In] `Int[Csc[a + b*Log[c*x^n]]/x^3,x]`

[Out]  $(-2 * E^{(I * a)} * (c * x^n)^{(I * b)} * \operatorname{Hypergeometric2F1}[1, (1 + (2 * I) / (b * n)) / 2, (3 + (2 * I) / (b * n)) / 2, E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / ((2 * I + b * n) * x^2)$

#### Rule 4510

`Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_)^(m_.), x_Symbol] := Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])`

#### Rule 4506

`Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_)^(m_.), x_Symbol] := Dist[(-2*I)^p * E^(I*a*d*p), Int[((e*x)^m * x^(I*b*d*p))/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]`

#### Rule 364

`Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

#### Rubi steps

$$\begin{aligned} \int \frac{\csc(a + b \log(cx^n))}{x^3} dx &= \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int x^{-1-\frac{2}{n}} \csc(a + b \log(x)) dx, x, cx^n\right)}{nx^2} \\ &= -\frac{(2ie^{ia} (cx^n)^{2/n}) \operatorname{Subst}\left(\int \frac{x^{-1+ib-\frac{2}{n}}}{1-e^{2ia}x^{2ib}} dx, x, cx^n\right)}{nx^2} \\ &= -\frac{2e^{ia} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 + \frac{2i}{bn}\right); \frac{1}{2}\left(3 + \frac{2i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{(2i + bn)x^2} \end{aligned}$$

**Mathematica [A]** time = 1.09152, size = 78, normalized size = 0.92

$$\frac{2e^{ia} (cx^n)^{ib} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2} + \frac{i}{bn}, \frac{3}{2} + \frac{i}{bn}, e^{2i(a+b \log(cx^n))}\right)}{x^2(bn + 2i)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]/x^3, x]

[Out] (-2\*E^(I\*a)\*(c\*x^n)^(I\*b)\*Hypergeometric2F1[1, 1/2 + I/(b\*n), 3/2 + I/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))])/((2\*I + b\*n)\*x^2)

**Maple [F]** time = 0.454, size = 0, normalized size = 0.

$$\int \frac{\csc(a + b \ln(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))/x^3, x)

[Out] int(csc(a+b\*ln(c\*x^n))/x^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))/x^3, x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a)/x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\csc(b \log(cx^n) + a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))/x^3,x, algorithm="fricas")

[Out] integral(csc(b\*log(c\*x^n) + a)/x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a + b \log(cx^n))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))/x\*\*3,x)

[Out] Integral(csc(a + b\*log(c\*x\*\*n))/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))/x^3,x, algorithm="giac")

[Out] integrate(csc(b\*log(c\*x^n) + a)/x^3, x)

### 3.295 $\int \csc^2(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=84

$$\frac{4e^{2ia}x(cx^n)^{2ib} \operatorname{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2 - \frac{i}{bn}\right), \frac{1}{2}\left(4 - \frac{i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn}$$

[Out]  $(-4 * E^{((2 * I) * a)} * x * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, (2 - I/(b * n))/2, (4 - I/(b * n))/2, E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / (1 + (2 * I) * b * n)$

**Rubi [A]** time = 0.0599466, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4504, 4506, 364}

$$\frac{4e^{2ia}x(cx^n)^{2ib} {}_2F_1\left(2, \frac{1}{2}\left(2 - \frac{i}{bn}\right); \frac{1}{2}\left(4 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^2, x]

[Out]  $(-4 * E^{((2 * I) * a)} * x * (c * x^n)^{((2 * I) * b)} * \operatorname{Hypergeometric2F1}[2, (2 - I/(b * n))/2, (4 - I/(b * n))/2, E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / (1 + (2 * I) * b * n)$

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[(-2\*I)^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p)) / (1 - E^(2\*I\*a\*d)) \* x^(2\*I\*b\*d)^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p \* (c\*x)^(m + 1) \* Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)]) / (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \csc^2(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \csc^2(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= -\frac{(4e^{2ia}x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+2ib+\frac{1}{n}}}{(1-e^{2ia}x^{2ib})^2} dx, x, cx^n\right)}{n} \\ &= -\frac{4e^{2ia}x(cx^n)^{2ib} {}_2F_1\left(2, \frac{1}{2}\left(2 - \frac{i}{bn}\right); \frac{1}{2}\left(4 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{1 + 2ibn} \end{aligned}$$

**Mathematica [A]** time = 5.20936, size = 146, normalized size = 1.74

$$x \left( \frac{e^{2ia} (cx^n)^{2ib} \text{Hypergeometric2F1} \left( 1, 1 - \frac{i}{2bn}, 2 - \frac{i}{2bn}, e^{2i(a+b \log(cx^n))} \right)}{2bn-i} - i \text{Hypergeometric2F1} \left( 1, -\frac{i}{2bn}, 1 - \frac{i}{2bn}, e^{2i(a+b \log(cx^n))} \right) - \cot(a) \right) / bn$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^2, x]

[Out] (x\*(-Cot[a + b\*Log[c\*x^n]] - (E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)\*Hypergeometric2F1[1, 1 - (I/2)/(b\*n), 2 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))])/(-I + 2\*b\*n) - I\*Hypergeometric2F1[1, (-I/2)/(b\*n), 1 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*Log[c\*x^n]))]))/(b\*n)

**Maple [F]** time = 1.417, size = 0, normalized size = 0.

$$\int (\csc(a + b \ln(cx^n)))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^2, x)

[Out] int(csc(a+b\*ln(c\*x^n))^2, x)

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^2, x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\csc(b \log(cx^n) + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^2, x, algorithm="fricas")

[Out] integral(csc(b\*log(c\*x^n) + a)^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^2(a + b \log(cx^n)) dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(csc(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(csc(b\*log(c\*x^n) + a)^2, x)

$$3.296 \quad \int \frac{\csc^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=19

$$-\frac{\cot(a+b \log(cx^n))}{bn}$$

[Out] -(Cot[a + b\*Log[c\*x^n]]/(b\*n))

**Rubi [A]** time = 0.0279832, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3767, 8}

$$-\frac{\cot(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^2/x,x]

[Out] -(Cot[a + b\*Log[c\*x^n]]/(b\*n))

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\csc^2(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \csc^2(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\text{Subst}\left(\int 1 dx, x, \cot(a+b \log(cx^n))\right)}{bn} \\ &= -\frac{\cot(a+b \log(cx^n))}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.096024, size = 19, normalized size = 1.

$$-\frac{\cot(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^2/x,x]

[Out] -(Cot[a + b\*Log[c\*x^n]]/(b\*n))

**Maple [A]** time = 0.034, size = 20, normalized size = 1.1

$$\frac{\cot(a + b \ln(cx^n))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^2/x,x)

[Out] -cot(a+b\*ln(c\*x^n))/b/n

**Maxima [B]** time = 1.20872, size = 227, normalized size = 11.95

$$\frac{2(\cos(2b \log(x^n) + 2a) \sin(2b \log(c)) + \cos(2b \log(c)) \sin(2b \log(x^n) + 2a))}{2bn \cos(2b \log(c)) \cos(2b \log(x^n) + 2a) - (b \cos(2b \log(c))^2 + b \sin(2b \log(c))^2)n \cos(2b \log(x^n) + 2a)^2 - 2bn \sin(2b \log(c)) \sin(2b \log(x^n) + 2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^2/x,x, algorithm="maxima")

[Out] 2\*(cos(2\*b\*log(x^n) + 2\*a)\*sin(2\*b\*log(c)) + cos(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a))/(2\*b\*n\*cos(2\*b\*log(c))\*cos(2\*b\*log(x^n) + 2\*a) - (b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*cos(2\*b\*log(x^n) + 2\*a)^2 - 2\*b\*n\*sin(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a) - (b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*sin(2\*b\*log(x^n) + 2\*a)^2 - b\*n)

**Fricas [A]** time = 0.467328, size = 95, normalized size = 5.

$$\frac{\cos(bn \log(x) + b \log(c) + a)}{bn \sin(bn \log(x) + b \log(c) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^2/x,x, algorithm="fricas")

[Out] -cos(b\*n\*log(x) + b\*log(c) + a)/(b\*n\*sin(b\*n\*log(x) + b\*log(c) + a))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a + b \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*2/x,x)

[Out] Integral(csc(a + b\*log(c\*x\*\*n))\*\*2/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^2/x, x)
```

### 3.297 $\int \csc^3(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=84

$$\frac{8e^{3ia}x(cx^n)^{3ib} \operatorname{Hypergeometric2F1}\left(3, \frac{1}{2}\left(3 - \frac{i}{bn}\right), \frac{1}{2}\left(5 - \frac{i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{-3bn + i}$$

[Out]  $(-8 * E^{((3 * I) * a)} * x * (c * x^n)^{((3 * I) * b)} * \operatorname{Hypergeometric2F1}[3, (3 - I / (b * n)) / 2, (5 - I / (b * n)) / 2, E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / (I - 3 * b * n)$

**Rubi [A]** time = 0.0622205, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4504, 4506, 364}

$$\frac{8e^{3ia}x(cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{i}{bn}\right); \frac{1}{2}\left(5 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{-3bn + i}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^3, x]

[Out]  $(-8 * E^{((3 * I) * a)} * x * (c * x^n)^{((3 * I) * b)} * \operatorname{Hypergeometric2F1}[3, (3 - I / (b * n)) / 2, (5 - I / (b * n)) / 2, E^{((2 * I) * a)} * (c * x^n)^{((2 * I) * b)}]) / (I - 3 * b * n)$

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(-2\*I)^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p)) / (1 - E^(2\*I\*a\*d)) \* x^(2\*I\*b\*d)]^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))]^(p\_.), x\_Symbol] := Simp[(a^p \* (c\*x)^(m + 1) \* Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)]) / (c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \csc^3(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \csc^3(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(8ie^{3ia}x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+3ib+\frac{1}{n}}}{(1-e^{2ia}x^{2ib})^3} dx, x, cx^n\right)}{n} \\ &= \frac{8e^{3ia}x(cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{i}{bn}\right); \frac{1}{2}\left(5 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{i - 3bn} \end{aligned}$$

**Mathematica [A]** time = 5.6157, size = 117, normalized size = 1.39

$$x \left( (bn \cot(a + b \log(cx^n)) + 1) \csc(a + b \log(cx^n)) + 2e^{ia}(bn + i)(cx^n)^{ib} \operatorname{Hypergeometric2F1} \left( 1, \frac{1}{2} - \frac{i}{2bn}, \frac{3}{2} - \frac{i}{2bn}, e^{2i(a + b \log(cx^n))} \right) \right) / (2b^2n^2)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^3,x]

[Out]  $-(x*((1 + b*n*\cot[a + b*\log[c*x^n]])*\csc[a + b*\log[c*x^n]] + 2E^{(I*a)}*(I + b*n)*(c*x^n)^{(I*b)}*\operatorname{Hypergeometric2F1}[1, 1/2 - (I/2)/(b*n), 3/2 - (I/2)/(b*n), E^{((2*I)*(a + b*\log[c*x^n]))}])))/(2*b^2*n^2)$

**Maple [F]** time = 2.277, size = 0, normalized size = 0.

$$\int (\csc(a + b \ln(cx^n)))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^3,x)

[Out] int(csc(a+b\*ln(c\*x^n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out]  $-\left( (b*n*\cos(b*\log(c)) - \sin(b*\log(c))) * x * \cos(b*\log(x^n) + a) - (b*n*\sin(b*\log(c)) + \cos(b*\log(c))) * x * \sin(b*\log(x^n) + a) + \left( (b*\cos(4*b*\log(c))*\cos(3*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(3*b*\log(c))) * n - \cos(3*b*\log(c))*\sin(4*b*\log(c)) + \cos(4*b*\log(c))*\sin(3*b*\log(c)) \right) * x * \cos(3*b*\log(x^n) + 3*a) + \left( (b*\cos(4*b*\log(c))*\cos(b*\log(c)) + b*\sin(4*b*\log(c))*\sin(b*\log(c))) * n + \cos(b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(b*\log(c)) \right) * x * \cos(b*\log(x^n) + a) + \left( (b*\cos(3*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(3*b*\log(c))) * n + \cos(4*b*\log(c))*\cos(3*b*\log(c)) + \sin(4*b*\log(c))*\sin(3*b*\log(c)) \right) * x * \sin(3*b*\log(x^n) + 3*a) + \left( (b*\cos(b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(b*\log(c))) * n - \cos(4*b*\log(c))*\cos(b*\log(c)) - \sin(4*b*\log(c))*\sin(b*\log(c)) \right) * x * \sin(b*\log(x^n) + a) * \cos(4*b*\log(x^n) + 4*a) - (2*((b*\cos(3*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(3*b*\log(c))*\sin(2*b*\log(c))) * n + \cos(2*b*\log(c))*\sin(3*b*\log(c)) - \cos(3*b*\log(c))*\sin(2*b*\log(c))) * x * \cos(2*b*\log(x^n) + 2*a) + 2*((b*\cos(2*b*\log(c))*\sin(3*b*\log(c)) - b*\cos(3*b*\log(c))*\sin(2*b*\log(c))) * n - \cos(3*b*\log(c))*\cos(2*b*\log(c)) - \sin(3*b*\log(c))*\sin(2*b*\log(c))) * x * \sin(2*b*\log(x^n) + 2*a) - (b*n*\cos(3*b*\log(c)) + \sin(3*b*\log(c))) * x * \cos(3*b*\log(x^n) + 3*a) - 2*((b*\cos(2*b*\log(c))*\cos(b*\log(c)) + b*\sin(2*b*\log(c))*\sin(b*\log(c))) * n + \cos(b*\log(c))*\sin(2*b*\log(c)) - \cos(2*b*\log(c))*\sin(b*\log(c))) * x * \cos(b*\log(x^n) + a) + \left( (b*\cos(b*\log(c))*\sin(2*b*\log(c)) - b*\cos(2*b*\log(c))*\sin(b*\log(c))) * n - \cos(2*b*\log(c))*\cos(b*\log(c)) - \sin(2*b*\log(c))*\sin(b*\log(c)) \right) * x * \sin(b*\log(x^n) + a) * \cos(2*b*\log(x^n) + 2*a) + 2*(b^6*n^6 + b^4*n^4 + ((b^6*\cos(4*b*\log(c))^2 + b^6*\sin(4*b*\log(c))^2) * \sin(4*b*\log(c)) - 2*b^6*\cos(4*b*\log(c))*\sin(4*b*\log(c))) * \cos(4*b*\log(c)) - 2*b^6*\sin(4*b*\log(c))*\cos(4*b*\log(c))$

$$\begin{aligned}
& n^6 + (b^4 \cos(4b \log(c))^2 + b^4 \sin(4b \log(c))^2) n^4 \cos(4b \log(x^n) + 4a)^2 + 4((b^6 \cos(2b \log(c))^2 + b^6 \sin(2b \log(c))^2) n^6 + (b^4 \cos(2b \log(c))^2 + b^4 \sin(2b \log(c))^2) n^4) \cos(2b \log(x^n) + 2a)^2 + ((b^6 \cos(4b \log(c))^2 + b^6 \sin(4b \log(c))^2) n^6 + (b^4 \cos(4b \log(c))^2 + b^4 \sin(4b \log(c))^2) n^4) \sin(4b \log(x^n) + 4a)^2 + 4((b^6 \cos(2b \log(c))^2 + b^6 \sin(2b \log(c))^2) n^6 + (b^4 \cos(2b \log(c))^2 + b^4 \sin(2b \log(c))^2) n^4) \sin(2b \log(x^n) + 2a)^2 + 2(b^6 n^6 \cos(4b \log(c)) + b^4 n^4 \cos(4b \log(c)) - 2((b^6 \cos(4b \log(c)) \cos(2b \log(c)) + b^6 \sin(4b \log(c)) \sin(2b \log(c))) n^6 + (b^4 \cos(4b \log(c)) \cos(2b \log(c)) + b^4 \sin(4b \log(c)) \sin(2b \log(c))) n^4) \cos(2b \log(x^n) + 2a) - 2((b^6 \cos(2b \log(c)) \sin(4b \log(c)) - b^6 \cos(4b \log(c)) \sin(2b \log(c))) n^6 + (b^4 \cos(2b \log(c)) \sin(4b \log(c)) - b^4 \cos(4b \log(c)) \sin(2b \log(c))) n^4) \cos(2b \log(x^n) + 2a) - 4(b^6 n^6 \cos(2b \log(c)) + b^4 n^4 \cos(2b \log(c))) \cos(2b \log(x^n) + 2a) - 2(b^6 n^6 \sin(4b \log(c)) + b^4 n^4 \sin(4b \log(c)) - 2((b^6 \cos(2b \log(c)) \sin(4b \log(c)) - b^6 \cos(4b \log(c)) \sin(2b \log(c))) n^6 + (b^4 \cos(2b \log(c)) \sin(4b \log(c)) - b^4 \cos(4b \log(c)) \sin(2b \log(c))) n^4) \cos(2b \log(x^n) + 2a) + 2((b^6 \cos(4b \log(c)) \cos(2b \log(c)) + b^6 \sin(4b \log(c)) \sin(2b \log(c))) n^6 + (b^4 \cos(4b \log(c)) \cos(2b \log(c)) + b^4 \sin(4b \log(c)) \sin(2b \log(c))) n^4) \sin(2b \log(x^n) + 2a) \sin(4b \log(x^n) + 4a) + 4(b^6 n^6 \sin(2b \log(c)) + b^4 n^4 \sin(2b \log(c))) \sin(2b \log(x^n) + 2a) \int (1/4(\cos(b \log(x^n) + a) \sin(b \log(c)) + \cos(b \log(c)) \sin(b \log(x^n) + a)) / (2b^4 n^4 \cos(b \log(c)) \cos(b \log(x^n) + a) - 2b^4 n^4 \sin(b \log(c)) \sin(b \log(x^n) + a) + b^4 n^4 + (b^4 \cos(b \log(c))^2 + b^4 \sin(b \log(c))^2) n^4 \cos(b \log(x^n) + a)^2 + (b^4 \cos(b \log(c))^2 + b^4 \sin(b \log(c))^2) n^4 \sin(b \log(x^n) + a)^2), x) + 2(b^6 n^6 + b^4 n^4 + ((b^6 \cos(4b \log(c))^2 + b^6 \sin(4b \log(c))^2) n^6 + (b^4 \cos(4b \log(c))^2 + b^4 \sin(4b \log(c))^2) n^4) \cos(4b \log(x^n) + 4a)^2 + 4((b^6 \cos(2b \log(c))^2 + b^6 \sin(2b \log(c))^2) n^6 + (b^4 \cos(2b \log(c))^2 + b^4 \sin(2b \log(c))^2) n^4) \cos(2b \log(x^n) + 2a)^2 + ((b^6 \cos(4b \log(c))^2 + b^6 \sin(4b \log(c))^2) n^6 + (b^4 \cos(4b \log(c))^2 + b^4 \sin(4b \log(c))^2) n^4) \sin(4b \log(x^n) + 4a)^2 + 4((b^6 \cos(2b \log(c))^2 + b^6 \sin(2b \log(c))^2) n^6 + (b^4 \cos(2b \log(c))^2 + b^4 \sin(2b \log(c))^2) n^4) \sin(2b \log(x^n) + 2a)^2 + 2(b^6 n^6 \cos(4b \log(c)) + b^4 n^4 \cos(4b \log(c)) - 2((b^6 \cos(4b \log(c)) \cos(2b \log(c)) + b^6 \sin(4b \log(c)) \sin(2b \log(c))) n^6 + (b^4 \cos(4b \log(c)) \cos(2b \log(c)) + b^4 \sin(4b \log(c)) \sin(2b \log(c))) n^4) \cos(2b \log(x^n) + 2a) - 2((b^6 \cos(2b \log(c)) \sin(4b \log(c)) - b^6 \cos(4b \log(c)) \sin(2b \log(c))) n^6 + (b^4 \cos(2b \log(c)) \sin(4b \log(c)) - b^4 \cos(4b \log(c)) \sin(2b \log(c))) n^4) \sin(2b \log(x^n) + 2a) \cos(4b \log(x^n) + 4a) - 4(b^6 n^6 \cos(2b \log(c)) + b^4 n^4 \cos(2b \log(c))) \cos(2b \log(x^n) + 2a) - 2(b^6 n^6 \sin(4b \log(c)) + b^4 n^4 \sin(4b \log(c)) - 2((b^6 \cos(2b \log(c)) \sin(4b \log(c)) - b^6 \cos(4b \log(c)) \sin(2b \log(c))) n^6 + (b^4 \cos(2b \log(c)) \sin(4b \log(c)) - b^4 \cos(4b \log(c)) \sin(2b \log(c))) n^4) \cos(2b \log(x^n) + 2a) + 2((b^6 \cos(4b \log(c)) \cos(2b \log(c)) + b^6 \sin(4b \log(c)) \sin(2b \log(c))) n^6 + (b^4 \cos(4b \log(c)) \cos(2b \log(c)) + b^4 \sin(4b \log(c)) \sin(2b \log(c))) n^4) \sin(2b \log(x^n) + 2a) \sin(4b \log(x^n) + 4a) + 4(b^6 n^6 \sin(2b \log(c)) + b^4 n^4 \sin(2b \log(c))) \sin(2b \log(x^n) + 2a) \int (-1/4(\cos(b \log(x^n) + a) \sin(b \log(c)) + \cos(b \log(c)) \sin(b \log(x^n) + a)) / (2b^4 n^4 \cos(b \log(c)) \cos(b \log(x^n) + a) - 2b^4 n^4 \sin(b \log(c)) \sin(b \log(x^n) + a) - b^4 n^4 - (b^4 \cos(b \log(c))^2 + b^4 \sin(b \log(c))^2) n^4 \cos(b \log(x^n) + a)^2 - (b^4 \cos(b \log(c))^2 + b^4 \sin(b \log(c))^2) n^4 \sin(b \log(x^n) + a)^2), x) - (((b \cos(3b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(3b \log(c))) n + \cos(4b \log(c)) \cos(3b \log(c)) + \sin(4b \log(c)) \sin(3b \log(c))) x \cos(3b \log(x^n) + 3a) + ((b \cos(b \log(c)) \sin(4b \log(c)) - b \cos(4b \log(c)) \sin(b \log(c))) n - \cos(4b \log(c)) \cos(b \log(c)) - \sin(4b \log(c)) \sin(b \log(c))) x \cos(b \log(x^n) + a) - ((b \cos(4b \log(c)) \cos(3b \log(c)) + b \sin(4b \log(c)) \sin(3b \log(c))) n - \cos(3b \log(c)) \sin(4b \log(c)) + \cos(4b \log(c)) \sin(3b \log(c))) x \sin(3b \log(x^n) + 3a) -
\end{aligned}$$

```

((b*cos(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c))) * n +
cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c))) * x * sin(b*log(x
^n) + a) * sin(4*b*log(x^n) + 4*a) + (2*((b*cos(2*b*log(c))*sin(3*b*log(c))
- b*cos(3*b*log(c))*sin(2*b*log(c))) * n - cos(3*b*log(c))*cos(2*b*log(c)) -
sin(3*b*log(c))*sin(2*b*log(c))) * x * cos(2*b*log(x^n) + 2*a) - 2*((b*cos(3*b*
log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c))) * n + cos(2*b*lo
g(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c))) * x * sin(2*b*log(x^n)
+ 2*a) - (b*n*sin(3*b*log(c)) - cos(3*b*log(c))) * x * sin(3*b*log(x^n) + 3*a
) + 2*((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))
*n - cos(2*b*log(c))*cos(b*log(c)) - sin(2*b*log(c))*sin(b*log(c))) * x * cos(b
*log(x^n) + a) - ((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(
b*log(c))) * n + cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)
)) * x * sin(b*log(x^n) + a) * sin(2*b*log(x^n) + 2*a)) / (4*b^2*n^2*cos(2*b*log(c)
))*cos(2*b*log(x^n) + 2*a) - 4*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2
*a) - (b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2) * n^2*cos(4*b*log(x^n)
+ 4*a)^2 - 4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2) * n^2*cos(2*b*lo
g(x^n) + 2*a)^2 - (b^2*cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2) * n^2*sin(4
*b*log(x^n) + 4*a)^2 - 4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2) * n^
2*sin(2*b*log(x^n) + 2*a)^2 - b^2*n^2 - 2*(b^2*n^2*cos(4*b*log(c)) - 2*(b^2
*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c))) * n^2
*cos(2*b*log(x^n) + 2*a) - 2*(b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos
(4*b*log(c))*sin(2*b*log(c))) * n^2*sin(2*b*log(x^n) + 2*a)) * cos(4*b*log(x^n)
+ 4*a) + 2*(b^2*n^2*sin(4*b*log(c)) - 2*(b^2*cos(2*b*log(c))*sin(4*b*log(c)
)) - b^2*cos(4*b*log(c))*sin(2*b*log(c))) * n^2*cos(2*b*log(x^n) + 2*a) + 2*(
b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c))) *
n^2*sin(2*b*log(x^n) + 2*a)) * sin(4*b*log(x^n) + 4*a)

```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\csc(b \log(cx^n) + a)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] integral(csc(b\*log(c\*x^n) + a)^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^3(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral(csc(a + b\*log(c\*x\*\*n))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(csc(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^3, x)
```

$$3.298 \quad \int \frac{\csc^3(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=55

$$-\frac{\tanh^{-1}(\cos(a+b \log(cx^n)))}{2bn} - \frac{\cot(a+b \log(cx^n)) \csc(a+b \log(cx^n))}{2bn}$$

[Out]  $-\text{ArcTanh}[\text{Cos}[a + b \cdot \text{Log}[c \cdot x^n]]]/(2 \cdot b \cdot n) - (\text{Cot}[a + b \cdot \text{Log}[c \cdot x^n]] \cdot \text{Csc}[a + b \cdot \text{Log}[c \cdot x^n]])/(2 \cdot b \cdot n)$

**Rubi [A]** time = 0.0395494, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {3768, 3770}

$$-\frac{\tanh^{-1}(\cos(a+b \log(cx^n)))}{2bn} - \frac{\cot(a+b \log(cx^n)) \csc(a+b \log(cx^n))}{2bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Csc}[a + b \cdot \text{Log}[c \cdot x^n]]^3/x, x]$

[Out]  $-\text{ArcTanh}[\text{Cos}[a + b \cdot \text{Log}[c \cdot x^n]]]/(2 \cdot b \cdot n) - (\text{Cot}[a + b \cdot \text{Log}[c \cdot x^n]] \cdot \text{Csc}[a + b \cdot \text{Log}[c \cdot x^n]])/(2 \cdot b \cdot n)$

#### Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow -\text{Simp}[(b \cdot \text{Cos}[c + d \cdot x] \cdot (b \cdot \text{Csc}[c + d \cdot x])^{(n-1)})/(d \cdot (n-1)), x] + \text{Dist}[(b^{2 \cdot (n-2)})/(n-1), \text{Int}[(b \cdot \text{Csc}[c + d \cdot x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2 \cdot n]$

#### Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d \cdot x]]/d, x] /; \text{FreeQ}\{c, d, x\}$

#### Rubi steps

$$\begin{aligned} \int \frac{\csc^3(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \csc^3(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\cot(a+b \log(cx^n)) \csc(a+b \log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int \csc(a+bx) dx, x, \log(cx^n)\right)}{2n} \\ &= -\frac{\tanh^{-1}(\cos(a+b \log(cx^n)))}{2bn} - \frac{\cot(a+b \log(cx^n)) \csc(a+b \log(cx^n))}{2bn} \end{aligned}$$

**Mathematica [A]** time = 0.0891395, size = 107, normalized size = 1.95

$$\frac{\log\left(\sin\left(\frac{1}{2}(a+b \log(cx^n))\right)\right)}{2bn} + \frac{\sec^2\left(\frac{1}{2}(a+b \log(cx^n))\right)}{8bn} - \frac{\log\left(\cos\left(\frac{1}{2}(a+b \log(cx^n))\right)\right)}{2bn} - \frac{\csc^2\left(\frac{1}{2}(a+b \log(cx^n))\right)}{8bn}$$

Antiderivative was successfully verified.



$$\begin{aligned}
& 2*a^2 + (\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\sin(4*b*\log(x^n) + 4*a)^2 \\
& + 4*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\sin(2*b*\log(x^n) + 2*a)^2 - 2*( \\
& 2*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2 \\
& *b*\log(x^n) + 2*a) + 2*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*s \\
& \sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \cos(4*b*\log(c))*\cos(4*b*\log(x^n) \\
& + 4*a) - 4*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 2*(2*(\cos(2*b*\log(c)) \\
& *\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) \\
& - 2*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\sin \\
& (2*b*\log(x^n) + 2*a) - \sin(4*b*\log(c))*\sin(4*b*\log(x^n) + 4*a) + 4*\sin(2* \\
& b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 1)*\log((\cos(a)^2 + \sin(a)^2)*\cos(b*\log(c) \\
& )^2 + (\cos(a)^2 + \sin(a)^2)*\sin(b*\log(c))^2 - 2*(\cos(b*\log(c))*\cos(a) - \sin \\
& (b*\log(c))*\sin(a))*\cos(b*\log(x^n)) + \cos(b*\log(x^n))^2 + 2*(\cos(a)*\sin(b* \\
& \log(c)) + \cos(b*\log(c))*\sin(a))*\sin(b*\log(x^n)) + \sin(b*\log(x^n))^2) - 4*(( \\
& \cos(3*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(3*b*\log(c)))*\cos(3*b* \\
& \log(x^n) + 3*a) + (\cos(b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(b*\log \\
& (c)))*\cos(b*\log(x^n) + a) - (\cos(4*b*\log(c))*\cos(3*b*\log(c)) + \sin(4*b*\log \\
& (c))*\sin(3*b*\log(c)))*\sin(3*b*\log(x^n) + 3*a) - (\cos(4*b*\log(c))*\cos(b*\log \\
& (c)) + \sin(4*b*\log(c))*\sin(b*\log(c)))*\sin(b*\log(x^n) + a))*\sin(4*b*\log(x^n) \\
& + 4*a) + 4*(2*(\cos(2*b*\log(c))*\sin(3*b*\log(c)) - \cos(3*b*\log(c))*\sin(2*b*\log \\
& (c)))*\cos(2*b*\log(x^n) + 2*a) - 2*(\cos(3*b*\log(c))*\cos(2*b*\log(c)) + \sin(3 \\
& *b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \sin(3*b*\log(c))*\sin( \\
& 3*b*\log(x^n) + 3*a) + 8*((\cos(b*\log(c))*\sin(2*b*\log(c)) - \cos(2*b*\log(c))*\sin \\
& (b*\log(c)))*\cos(b*\log(x^n) + a) - (\cos(2*b*\log(c))*\cos(b*\log(c)) + \sin(2* \\
& b*\log(c))*\sin(b*\log(c)))*\sin(b*\log(x^n) + a))*\sin(2*b*\log(x^n) + 2*a) - 4*\sin \\
& (b*\log(c))*\sin(b*\log(x^n) + a))/((b*\cos(4*b*\log(c))^2 + b*\sin(4*b*\log(c)) \\
& ^2)*n*\cos(4*b*\log(x^n) + 4*a)^2 - 4*b*n*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + \\
& 2*a) + 4*(b*\cos(2*b*\log(c))^2 + b*\sin(2*b*\log(c))^2)*n*\cos(2*b*\log(x^n) + 2 \\
& *a)^2 + (b*\cos(4*b*\log(c))^2 + b*\sin(4*b*\log(c))^2)*n*\sin(4*b*\log(x^n) + 4* \\
& a)^2 + 4*b*n*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 4*(b*\cos(2*b*\log(c)) \\
& ^2 + b*\sin(2*b*\log(c))^2)*n*\sin(2*b*\log(x^n) + 2*a)^2 + b*n + 2*(b*n*\cos(4* \\
& b*\log(c)) - 2*(b*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2* \\
& b*\log(c)))*n*\cos(2*b*\log(x^n) + 2*a) - 2*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) \\
& - b*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n*\sin(2*b*\log(x^n) + 2*a))*\cos(4*b*\log \\
& (x^n) + 4*a) + 2*(2*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c)) \\
& *\sin(2*b*\log(c)))*n*\cos(2*b*\log(x^n) + 2*a) - b*n*\sin(4*b*\log(c)) - 2*(b*\cos \\
& (4*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n*\sin(2* \\
& b*\log(x^n) + 2*a))*\sin(4*b*\log(x^n) + 4*a))
\end{aligned}$$

**Fricas [B]** time = 0.503356, size = 352, normalized size = 6.4

$$\frac{(\cos(bn \log(x) + b \log(c) + a)^2 - 1) \log\left(\frac{1}{2} \cos(bn \log(x) + b \log(c) + a) + \frac{1}{2}\right) - (\cos(bn \log(x) + b \log(c) + a)^2 - 1)}{4(bn \cos(bn \log(x) + b \log(c) + a)^2 - bn)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^3/x,x, algorithm="fricas")

[Out] -1/4\*((cos(b\*n\*log(x) + b\*log(c) + a)^2 - 1)\*log(1/2\*cos(b\*n\*log(x) + b\*log(c) + a) + 1/2) - (cos(b\*n\*log(x) + b\*log(c) + a)^2 - 1)\*log(-1/2\*cos(b\*n\*log(x) + b\*log(c) + a) + 1/2) - 2\*cos(b\*n\*log(x) + b\*log(c) + a))/(b\*n\*cos(b\*n\*log(x) + b\*log(c) + a)^2 - b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(a + b \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*3/x,x)

[Out] Integral(csc(a + b\*log(c\*x\*\*n))\*\*3/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^3/x,x, algorithm="giac")

[Out] integrate(csc(b\*log(c\*x^n) + a)^3/x, x)

### 3.299 $\int \csc^4(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=84

$$\frac{16e^{4ia}x(cx^n)^{4ib} \operatorname{Hypergeometric2F1}\left(4, \frac{1}{2}\left(4 - \frac{i}{bn}\right), \frac{1}{2}\left(6 - \frac{i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{1 + 4ibn}$$

[Out] (16\*E^((4\*I)\*a)\*x\*(c\*x^n)^((4\*I)\*b)\*Hypergeometric2F1[4, (4 - I/(b\*n))/2, (6 - I/(b\*n))/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/(1 + (4\*I)\*b\*n)

**Rubi [A]** time = 0.0610124, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4504, 4506, 364}

$$\frac{16e^{4ia}x(cx^n)^{4ib} {}_2F_1\left(4, \frac{1}{2}\left(4 - \frac{i}{bn}\right); \frac{1}{2}\left(6 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{1 + 4ibn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^4, x]

[Out] (16\*E^((4\*I)\*a)\*x\*(c\*x^n)^((4\*I)\*b)\*Hypergeometric2F1[4, (4 - I/(b\*n))/2, (6 - I/(b\*n))/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/(1 + (4\*I)\*b\*n)

#### Rule 4504

Int[Csc[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(-2\*I)^p\*E^(I\*a\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \csc^4(a + b \log(cx^n)) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \csc^4(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(16e^{4ia}x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+4ib+\frac{1}{n}}}{(1-e^{2ia}x^{2ib})^4} dx, x, cx^n\right)}{n} \\ &= \frac{16e^{4ia}x(cx^n)^{4ib} {}_2F_1\left(4, \frac{1}{2}\left(4 - \frac{i}{bn}\right); \frac{1}{2}\left(6 - \frac{i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{1 + 4ibn} \end{aligned}$$

**Mathematica [B]** time = 13.4751, size = 782, normalized size = 9.31

$$e^{-\frac{a+b(\log(cx^n)-n\log(x))}{bn}} \csc(a+b(\log(cx^n)-n\log(x))) \left( x(2bn-i)e^{\frac{a}{bn}+\frac{\log(cx^n)-n\log(x)}{n}} \left( \cos(a+b(\log(cx^n)-n\log(x))) + \dots \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^4,x]

[Out]  $((1 + 4*b^2*n^2)*x*Csc[a + b*(-(n*Log[x]) + Log[c*x^n])] * Csc[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])] * Sin[b*n*Log[x]]) / (6*b^3*n^3) + (x*Csc[a + b*(-(n*Log[x]) + Log[c*x^n])] * Csc[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])] ^3 * Sin[b*n*Log[x]]) / (3*b*n) - (x*Csc[a + b*(-(n*Log[x]) + Log[c*x^n])] * Csc[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])] ^2 * (2*b*n*Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + Sin[a + b*(-(n*Log[x]) + Log[c*x^n])])) / (6*b^2*n^2) - (Csc[a + b*(-(n*Log[x]) + Log[c*x^n])] * (E^((2*I + 1/(b*n))*(a + b*Log[c*x^n])) * Hypergeometric2F1[1, 1 - (I/2)/(b*n), 2 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))] * Sin[a + b*(-(n*Log[x]) + Log[c*x^n])] + E^(a/(b*n) + (-(n*Log[x]) + Log[c*x^n])/n) * (-I + 2*b*n) * x * (Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + I * Hypergeometric2F1[1, (-I/2)/(b*n), 1 - (I/2)/(b*n), E^((2*I)*(a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])))] * Sin[a + b*(-(n*Log[x]) + Log[c*x^n])])))) / (6*b^3*E^((a + b*(-(n*Log[x]) + Log[c*x^n]))/(b*n)) * n^3 * (-I + 2*b*n)) - (2*Csc[a + b*(-(n*Log[x]) + Log[c*x^n])] * (E^((2*I + 1/(b*n))*(a + b*Log[c*x^n])) * Hypergeometric2F1[1, 1 - (I/2)/(b*n), 2 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))] * Sin[a + b*(-(n*Log[x]) + Log[c*x^n])] + E^(a/(b*n) + (-(n*Log[x]) + Log[c*x^n])/n) * (-I + 2*b*n) * x * (Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + I * Hypergeometric2F1[1, (-I/2)/(b*n), 1 - (I/2)/(b*n), E^((2*I)*(a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])))] * Sin[a + b*(-(n*Log[x]) + Log[c*x^n])])))) / (3*b*E^((a + b*(-(n*Log[x]) + Log[c*x^n]))/(b*n)) * n * (-I + 2*b*n))$

**Maple [F]** time = 1.441, size = 0, normalized size = 0.

$$\int (\csc(a + b \ln(cx^n)))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^4,x)

[Out] int(csc(a+b\*ln(c\*x^n))^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^4,x, algorithm="maxima")

[Out]  $1/3*(6*(b*\cos(4*b*\log(c)))^2 + b*\sin(4*b*\log(c))^2)*n*x*\cos(4*b*\log(x^n)) + 4*a^2 + 6*(b*\cos(2*b*\log(c)))^2 + b*\sin(2*b*\log(c))^2)*n*x*\cos(2*b*\log(x^n)) + 2*a^2 + 6*(b*\cos(4*b*\log(c)))^2 + b*\sin(4*b*\log(c))^2)*n*x*\sin(4*b*\log(x^n))$

$$\begin{aligned}
& n) + 4*a)^2 + 6*(b*\cos(2*b*\log(c))^2 + b*\sin(2*b*\log(c))^2)*n*x*\sin(2*b*\log \\
& (x^n) + 2*a)^2 - (2*b*n*\cos(2*b*\log(c)) - \sin(2*b*\log(c)))*x*\cos(2*b*\log(x^n) \\
& + 2*a) + (2*b*n*\sin(2*b*\log(c)) + \cos(2*b*\log(c)))*x*\sin(2*b*\log(x^n) + \\
& 2*a) - ((2*(b*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b*\sin(6*b*\log(c))*\sin(4*b*\log \\
& (c)))*n - \cos(4*b*\log(c))*\sin(6*b*\log(c)) + \cos(6*b*\log(c))*\sin(4*b*\log(c) \\
& ))*x*\cos(4*b*\log(x^n) + 4*a) + 2*(6*(b^2*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - \\
& b^2*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^2 - (b*\cos(6*b*\log(c))*\cos(2*b*\log( \\
& c)) + b*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n + \cos(2*b*\log(c))*\sin(6*b*\log(c) \\
& ) - \cos(6*b*\log(c))*\sin(2*b*\log(c)))*x*\cos(2*b*\log(x^n) + 2*a) + (2*(b*\cos( \\
& 4*b*\log(c))*\sin(6*b*\log(c)) - b*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n + \cos(6* \\
& b*\log(c))*\cos(4*b*\log(c)) + \sin(6*b*\log(c))*\sin(4*b*\log(c)))*x*\sin(4*b*\log( \\
& x^n) + 4*a) - 2*(6*(b^2*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^2*\sin(6*b*\log(c) \\
& ))*\sin(2*b*\log(c)))*n^2 + (b*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b*\cos(6*b*\log \\
& (c))*\sin(2*b*\log(c)))*n + \cos(6*b*\log(c))*\cos(2*b*\log(c)) + \sin(6*b*\log(c) \\
& )*\sin(2*b*\log(c)))*x*\sin(2*b*\log(x^n) + 2*a) - (4*b^2*n^2*\sin(6*b*\log(c)) + \\
& \sin(6*b*\log(c)))*x*\cos(6*b*\log(x^n) + 6*a) + (3*(12*(b^2*\cos(2*b*\log(c))* \\
& \sin(4*b*\log(c)) - b^2*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n^2 - 4*(b*\cos(4*b*\log \\
& (c))*\cos(2*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n + \cos(2*b*\log \\
& (c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*x*\cos(2*b*\log(x^n) \\
& + 2*a) - 3*(12*(b^2*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^2*\sin(4*b*\log(c))*\sin \\
& (2*b*\log(c)))*n^2 + 4*(b*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log( \\
& c))*\sin(2*b*\log(c)))*n + \cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))* \\
& \sin(2*b*\log(c)))*x*\sin(2*b*\log(x^n) + 2*a) - 2*(6*b^2*n^2*\sin(4*b*\log(c)) - \\
& b*n*\cos(4*b*\log(c)) + \sin(4*b*\log(c)))*x*\cos(4*b*\log(x^n) + 4*a) + 18*(4* \\
& b^8*n^8 + b^6*n^6 + (4*(b^8*\cos(6*b*\log(c))^2 + b^8*\sin(6*b*\log(c))^2)*n^8 \\
& + (b^6*\cos(6*b*\log(c))^2 + b^6*\sin(6*b*\log(c))^2)*n^6)*\cos(6*b*\log(x^n) + 6 \\
& *a)^2 + 9*(4*(b^8*\cos(4*b*\log(c))^2 + b^8*\sin(4*b*\log(c))^2)*n^8 + (b^6*\cos \\
& (4*b*\log(c))^2 + b^6*\sin(4*b*\log(c))^2)*n^6)*\cos(4*b*\log(x^n) + 4*a)^2 + 9* \\
& (4*(b^8*\cos(2*b*\log(c))^2 + b^8*\sin(2*b*\log(c))^2)*n^8 + (b^6*\cos(2*b*\log(c) \\
& ))^2 + b^6*\sin(2*b*\log(c))^2)*n^6)*\cos(2*b*\log(x^n) + 2*a)^2 + (4*(b^8*\cos( \\
& 6*b*\log(c))^2 + b^8*\sin(6*b*\log(c))^2)*n^8 + (b^6*\cos(6*b*\log(c))^2 + b^6*\sin \\
& (6*b*\log(c))^2)*n^6)*\sin(6*b*\log(x^n) + 6*a)^2 + 9*(4*(b^8*\cos(4*b*\log(c) \\
& ))^2 + b^8*\sin(4*b*\log(c))^2)*n^8 + (b^6*\cos(4*b*\log(c))^2 + b^6*\sin(4*b*\log \\
& (c))^2)*n^6)*\sin(4*b*\log(x^n) + 4*a)^2 + 9*(4*(b^8*\cos(2*b*\log(c))^2 + b^8* \\
& \sin(2*b*\log(c))^2)*n^8 + (b^6*\cos(2*b*\log(c))^2 + b^6*\sin(2*b*\log(c))^2)*n^6 \\
& )*\sin(2*b*\log(x^n) + 2*a)^2 - 2*(4*b^8*n^8*\cos(6*b*\log(c)) + b^6*n^6*\cos(6 \\
& *b*\log(c)) + 3*(4*(b^8*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^8*\sin(6*b*\log(c) \\
& )*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^6*\sin(6*b \\
& *\log(c))*\sin(4*b*\log(c)))*n^6)*\cos(4*b*\log(x^n) + 4*a) - 3*(4*(b^8*\cos(6*b* \\
& \log(c))*\cos(2*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*c \\
& \cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)* \\
& \cos(2*b*\log(x^n) + 2*a) + 3*(4*(b^8*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^8*c \\
& \cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c))*\sin(6*b*\log(c)) \\
& - b^6*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*\sin(4*b*\log(x^n) + 4*a) - 3*(4* \\
& (b^8*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(2*b*\log(c)) \\
& )*n^8 + (b^6*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(2*b*\log \\
& (c)))*n^6)*\sin(2*b*\log(x^n) + 2*a))*\cos(6*b*\log(x^n) + 6*a) + 6*(4*b^8*n^8 \\
& * \cos(4*b*\log(c)) + b^6*n^6*\cos(4*b*\log(c)) - 3*(4*(b^8*\cos(4*b*\log(c))*\cos \\
& (2*b*\log(c)) + b^8*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log( \\
& c))*\cos(2*b*\log(c)) + b^6*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\cos(2*b*\log \\
& (x^n) + 2*a) - 3*(4*(b^8*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^8*\cos(4*b*\log( \\
& c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^6*\cos(4 \\
& *b*\log(c))*\sin(2*b*\log(c)))*n^6)*\sin(2*b*\log(x^n) + 2*a))*\cos(4*b*\log(x^n) \\
& + 4*a) - 6*(4*b^8*n^8*\cos(2*b*\log(c)) + b^6*n^6*\cos(2*b*\log(c)))*\cos(2*b*\log \\
& (x^n) + 2*a) + 2*(4*b^8*n^8*\sin(6*b*\log(c)) + b^6*n^6*\sin(6*b*\log(c)) + 3* \\
& (4*(b^8*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(4*b*\log(c) \\
& ))*n^8 + (b^6*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(4* \\
& b*\log(c)))*n^6)*\cos(4*b*\log(x^n) + 4*a) - 3*(4*(b^8*\cos(2*b*\log(c))*\sin(6*b \\
& *\log(c)) - b^8*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*
\end{aligned}$$



$$\begin{aligned}
& \sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^6*\cos(2*b*\log(x^n \\
& ) + 2*a) - 3*(4*(b^8*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^8*\sin(6*b*\log(c))* \\
& \sin(4*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^6*\sin(6*b*\log(c))* \\
& \sin(4*b*\log(c)))*n^6)*\sin(4*b*\log(x^n) + 4*a) + 3*(4*(b^8*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\sin(2*b*\log(x^n) + 2*a))*\sin(6*b*\log(x^n) + 6*a) - 6*(4*b^8*n^8*\sin(4*b*\log(c)) + b^6*n^6*\sin(4*b*\log(c)) - 3*(4*(b^8*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^8*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^6*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\cos(2*b*\log(x^n) + 2*a) + 3*(4*(b^8*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^8*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\sin(2*b*\log(x^n) + 2*a))*\sin(4*b*\log(x^n) + 4*a) + 6*(4*b^8*n^8*\sin(2*b*\log(c)) + b^6*n^6*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a))* \\
& \text{integrate}(1/36*(\cos(b*\log(x^n) + a)*\sin(b*\log(c)) + \cos(b*\log(c))*\sin(b*\log(x^n) + a))/(2*b^6*n^6*\cos(b*\log(c))*\cos(b*\log(x^n) + a) - 2*b^6*n^6*\sin(b*\log(c))*\sin(b*\log(x^n) + a) + b^6*n^6 + (b^6*\cos(b*\log(c))^2 + b^6*\sin(b*\log(c))^2)*n^6*\cos(b*\log(x^n) + a)^2 + (b^6*\cos(b*\log(c))^2 + b^6*\sin(b*\log(c))^2)*n^6*\sin(b*\log(x^n) + a)^2), x) - 18*(4*b^8*n^8 + b^6*n^6 + (4*(b^8*\cos(6*b*\log(c))^2 + b^8*\sin(6*b*\log(c))^2)*n^8 + (b^6*\cos(6*b*\log(c))^2 + b^6*\sin(6*b*\log(c))^2)*n^6)*\cos(6*b*\log(x^n) + 6*a)^2 + 9*(4*(b^8*\cos(4*b*\log(c))^2 + b^8*\sin(4*b*\log(c))^2)*n^8 + (b^6*\cos(4*b*\log(c))^2 + b^6*\sin(4*b*\log(c))^2)*n^6)*\cos(4*b*\log(x^n) + 4*a)^2 + 9*(4*(b^8*\cos(2*b*\log(c))^2 + b^8*\sin(2*b*\log(c))^2)*n^8 + (b^6*\cos(2*b*\log(c))^2 + b^6*\sin(2*b*\log(c))^2)*n^6)*\cos(2*b*\log(x^n) + 2*a)^2 + (4*(b^8*\cos(6*b*\log(c))^2 + b^8*\sin(6*b*\log(c))^2)*n^8 + (b^6*\cos(6*b*\log(c))^2 + b^6*\sin(6*b*\log(c))^2)*n^6)*\sin(6*b*\log(x^n) + 6*a)^2 + 9*(4*(b^8*\cos(4*b*\log(c))^2 + b^8*\sin(4*b*\log(c))^2)*n^8 + (b^6*\cos(4*b*\log(c))^2 + b^6*\sin(4*b*\log(c))^2)*n^6)*\sin(4*b*\log(x^n) + 4*a)^2 + 9*(4*(b^8*\cos(2*b*\log(c))^2 + b^8*\sin(2*b*\log(c))^2)*n^8 + (b^6*\cos(2*b*\log(c))^2 + b^6*\sin(2*b*\log(c))^2)*n^6)*\sin(2*b*\log(x^n) + 2*a)^2 - 2*(4*b^8*n^8*\cos(6*b*\log(c)) + b^6*n^6*\cos(6*b*\log(c)) + 3*(4*(b^8*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*\cos(4*b*\log(x^n) + 4*a) - 3*(4*(b^8*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\cos(2*b*\log(x^n) + 2*a) + 3*(4*(b^8*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*\sin(4*b*\log(x^n) + 4*a) - 3*(4*(b^8*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\sin(2*b*\log(x^n) + 2*a))*\cos(6*b*\log(x^n) + 6*a) + 6*(4*b^8*n^8*\cos(4*b*\log(c)) + b^6*n^6*\cos(4*b*\log(c)) - 3*(4*(b^8*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^8*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\cos(2*b*\log(x^n) + 2*a) - 3*(4*(b^8*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^8*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^6*\cos(4*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\sin(2*b*\log(x^n) + 2*a))*\cos(4*b*\log(x^n) + 4*a) - 6*(4*b^8*n^8*\cos(2*b*\log(c)) + b^6*n^6*\cos(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 2*(4*b^8*n^8*\sin(6*b*\log(c)) + b^6*n^6*\sin(6*b*\log(c)) + 3*(4*(b^8*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(4*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*\cos(4*b*\log(x^n) + 4*a) - 3*(4*(b^8*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^8*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(2*b*\log(c))*\sin(6*b*\log(c)) - b^6*\cos(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\cos(2*b*\log(x^n) + 2*a) - 3*(4*(b^8*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(4*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(4*b*\log(c)))*n^6)*\sin(4*b*\log(x^n) + 4*a) + 3*(4*(b^8*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^8*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^8 + (b^6*\cos(6*b*\log(c))*\cos(2*b*\log(c)) + b^6*\sin(6*b*\log(c))*\sin(2*b*\log(c)))*n^6)*\sin(2*b*\log(c)) + (b^6*\cos(6*b*\log(c))*\cos(2*b*\log(c)) +
\end{aligned}$$



$$3*\cos(2*b*\log(c))*\sin(4*b*\log(c)) - b^3*\cos(4*b*\log(c))*\sin(2*b*\log(c))*n^3*\cos(2*b*\log(x^n) + 2*a) + 3*(b^3*\cos(4*b*\log(c))*\cos(2*b*\log(c)) + b^3*\sin(4*b*\log(c))*\sin(2*b*\log(c)))*n^3*\sin(2*b*\log(x^n) + 2*a))*\sin(4*b*\log(x^n) + 4*a)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\csc(b \log(cx^n) + a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^4,x, algorithm="fricas")

[Out] integral(csc(b\*log(c\*x^n) + a)^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^4(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*4,x)

[Out] Integral(csc(a + b\*log(c\*x\*\*n))\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^4,x, algorithm="giac")

[Out] integrate(csc(b\*log(c\*x^n) + a)^4, x)

$$3.300 \quad \int \frac{\csc^4(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=43

$$-\frac{\cot^3(a+b \log(cx^n))}{3bn} - \frac{\cot(a+b \log(cx^n))}{bn}$$

[Out]  $-(\text{Cot}[a + b*\text{Log}[c*x^n]]/(b*n)) - \text{Cot}[a + b*\text{Log}[c*x^n]]^3/(3*b*n)$

**Rubi [A]** time = 0.0340999, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {3767}

$$-\frac{\cot^3(a+b \log(cx^n))}{3bn} - \frac{\cot(a+b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^4/x,x]

[Out]  $-(\text{Cot}[a + b*\text{Log}[c*x^n]]/(b*n)) - \text{Cot}[a + b*\text{Log}[c*x^n]]^3/(3*b*n)$

**Rule 3767**

Int[csc[(c\_.) + (d\_.)\*(x\_.)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{\csc^4(a+b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \csc^4(a+bx) dx, x, \log(cx^n)\right)}{n} \\ &= -\frac{\text{Subst}\left(\int (1+x^2) dx, x, \cot(a+b \log(cx^n))\right)}{bn} \\ &= -\frac{\cot(a+b \log(cx^n))}{bn} - \frac{\cot^3(a+b \log(cx^n))}{3bn} \end{aligned}$$

**Mathematica [A]** time = 0.0875734, size = 56, normalized size = 1.3

$$-\frac{2 \cot(a+b \log(cx^n))}{3bn} - \frac{\cot(a+b \log(cx^n)) \csc^2(a+b \log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^4/x,x]

[Out]  $(-2*\text{Cot}[a + b*\text{Log}[c*x^n]])/(3*b*n) - (\text{Cot}[a + b*\text{Log}[c*x^n]]*\text{Csc}[a + b*\text{Log}[c*x^n]]^2)/(3*b*n)$

**Maple [A]** time = 0.049, size = 36, normalized size = 0.8

$$\frac{\cot(a + b \ln(cx^n))}{bn} \left( -\frac{2}{3} - \frac{(\csc(a + b \ln(cx^n)))^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^4/x,x)

[Out] 1/n/b\*(-2/3-1/3\*csc(a+b\*ln(c\*x^n))^2)\*cot(a+b\*ln(c\*x^n))

**Maxima [B]** time = 1.18059, size = 1798, normalized size = 41.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^4/x,x, algorithm="maxima")

[Out] 4/3\*((3\*(cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - cos(6\*b\*log(c))\*sin(2\*b\*log(c))) \*cos(2\*b\*log(x^n) + 2\*a) - 3\*(cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + sin(6\*b\*log(c))\*sin(2\*b\*log(c))) \*sin(2\*b\*log(x^n) + 2\*a) - sin(6\*b\*log(c))\*cos(6\*b\*log(x^n) + 6\*a) - 3\*(3\*(cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c))) \*cos(2\*b\*log(x^n) + 2\*a) - 3\*(cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c))) \*sin(2\*b\*log(x^n) + 2\*a) - sin(4\*b\*log(c)) \*cos(4\*b\*log(x^n) + 4\*a) + (3\*(cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + sin(6\*b\*log(c))\*sin(2\*b\*log(c))) \*cos(2\*b\*log(x^n) + 2\*a) + 3\*(cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - cos(6\*b\*log(c))\*sin(2\*b\*log(c))) \*sin(2\*b\*log(x^n) + 2\*a) - cos(6\*b\*log(c)) \*sin(6\*b\*log(x^n) + 6\*a) - 3\*(3\*(cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + sin(4\*b\*log(c))\*sin(2\*b\*log(c))) \*cos(2\*b\*log(x^n) + 2\*a) + 3\*(cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - cos(4\*b\*log(c))\*sin(2\*b\*log(c))) \*sin(2\*b\*log(x^n) + 2\*a) - cos(4\*b\*log(c)) \*sin(4\*b\*log(x^n) + 4\*a))/((b\*cos(6\*b\*log(c))^2 + b\*sin(6\*b\*log(c))^2)\*n\*cos(6\*b\*log(x^n) + 6\*a)^2 + 9\*(b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*cos(4\*b\*log(x^n) + 4\*a)^2 - 6\*b\*n\*cos(2\*b\*log(c))\*cos(2\*b\*log(x^n) + 2\*a) + 9\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*cos(2\*b\*log(x^n) + 2\*a)^2 + (b\*cos(6\*b\*log(c))^2 + b\*sin(6\*b\*log(c))^2)\*n\*sin(6\*b\*log(x^n) + 6\*a)^2 + 9\*(b\*cos(4\*b\*log(c))^2 + b\*sin(4\*b\*log(c))^2)\*n\*sin(4\*b\*log(x^n) + 4\*a)^2 + 6\*b\*n\*sin(2\*b\*log(c))\*sin(2\*b\*log(x^n) + 2\*a) + 9\*(b\*cos(2\*b\*log(c))^2 + b\*sin(2\*b\*log(c))^2)\*n\*sin(2\*b\*log(x^n) + 2\*a)^2 + b\*n - 2\*(b\*n\*cos(6\*b\*log(c)) + 3\*(b\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(4\*b\*log(c))) \*n\*cos(4\*b\*log(x^n) + 4\*a) - 3\*(b\*cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(2\*b\*log(c))) \*n\*cos(2\*b\*log(x^n) + 2\*a) + 3\*(b\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(4\*b\*log(c))) \*n\*sin(4\*b\*log(x^n) + 4\*a) - 3\*(b\*cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(2\*b\*log(c))) \*n\*sin(2\*b\*log(x^n) + 2\*a)) \*cos(6\*b\*log(x^n) + 6\*a) + 6\*(b\*n\*cos(4\*b\*log(c)) - 3\*(b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c))) \*n\*cos(2\*b\*log(x^n) + 2\*a) - 3\*(b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c))) \*n\*sin(2\*b\*log(x^n) + 2\*a)) \*cos(4\*b\*log(x^n) + 4\*a) + 2\*(3\*(b\*cos(4\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(4\*b\*log(c))) \*n\*cos(4\*b\*log(x^n) + 4\*a) - 3\*(b\*cos(2\*b\*log(c))\*sin(6\*b\*log(c)) - b\*cos(6\*b\*log(c))\*sin(2\*b\*log(c))) \*n\*cos(2\*b\*log(x^n) + 2\*a) + b\*n\*sin(6\*b\*log(c)) - 3\*(b\*cos(6\*b\*log(c))\*cos(4\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(4\*b\*log(c))) \*n\*sin(4\*b\*log(x^n) + 4\*a) + 3\*(b\*cos(6\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(6\*b\*log(c))\*sin(2\*b\*log(c))) \*n\*sin(2\*b\*log(x^n) + 2\*a)) \*sin(6\*b\*log(x^n) + 6\*a) + 6\*(3\*(b\*cos(2\*b\*log(c))\*sin(4\*b\*log(c)) - b\*cos(4\*b\*log(c))\*sin(2\*b\*log(c))) \*n\*cos(2\*b\*log(x^n) +

2\*a) - b\*n\*sin(4\*b\*log(c)) - 3\*(b\*cos(4\*b\*log(c))\*cos(2\*b\*log(c)) + b\*sin(4\*b\*log(c))\*sin(2\*b\*log(c)))\*n\*sin(2\*b\*log(x^n) + 2\*a))\*sin(4\*b\*log(x^n) + 4\*a))

**Fricas [A]** time = 0.472507, size = 211, normalized size = 4.91

$$\frac{2 \cos(bn \log(x) + b \log(c) + a)^3 - 3 \cos(bn \log(x) + b \log(c) + a)}{3 (bn \cos(bn \log(x) + b \log(c) + a)^2 - bn) \sin(bn \log(x) + b \log(c) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^4/x,x, algorithm="fricas")

[Out] -1/3\*(2\*cos(b\*n\*log(x) + b\*log(c) + a)^3 - 3\*cos(b\*n\*log(x) + b\*log(c) + a) )/((b\*n\*cos(b\*n\*log(x) + b\*log(c) + a)^2 - b\*n)\*sin(b\*n\*log(x) + b\*log(c) + a))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^4(a + b \log(cx^n))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*4/x,x)

[Out] Integral(csc(a + b\*log(c\*x\*\*n))\*\*4/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)^4}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^4/x,x, algorithm="giac")

[Out] integrate(csc(b\*log(c\*x^n) + a)^4/x, x)

### 3.301 $\int \left( - \left( 1 + b^2 n^2 \right) \csc \left( a + b \log \left( c x^n \right) \right) + 2 b^2 n^2 \csc^3 \left( a + b \log \left( c x^n \right) \right) \right) dx$

**Optimal.** Leaf size=42

$$-x \csc(a + b \log(cx^n)) - bnx \cot(a + b \log(cx^n)) \csc(a + b \log(cx^n))$$

[Out]  $-(x \text{Csc}[a + b \text{Log}[c x^n]]) - b n x \text{Cot}[a + b \text{Log}[c x^n]] \text{Csc}[a + b \text{Log}[c x^n]]$

**Rubi [C]** time = 0.126581, antiderivative size = 172, normalized size of antiderivative = 4.1, number of steps used = 7, number of rules used = 3, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$ , Rules used = {4504, 4506, 364}

$$2e^{ia} x^{(bn+i)} (cx^n)^{ib} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i}{bn}\right); \frac{1}{2}\left(3 - \frac{i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right) - \frac{16e^{3ia} b^2 n^2 x (cx^n)^{3ib} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{i}{bn}\right); \frac{1}{2}\left(5 - \frac{i}{bn}\right); e^{2ia} (cx^n)^{2ib}\right)}{-3bn + i}$$

Warning: Unable to verify antiderivative.

[In]  $\text{Int}[-((1 + b^2 n^2) \text{Csc}[a + b \text{Log}[c x^n]]) + 2 b^2 n^2 \text{Csc}[a + b \text{Log}[c x^n]]^3, x]$

[Out]  $2 E^{(I a)} (I + b n) x (c x^n)^{(I b)} \text{Hypergeometric2F1}\left[1, \frac{(1 - I/(b n))}{2}, \frac{(3 - I/(b n))}{2}, E^{((2 I) a) (c x^n)^{(2 I) b}}\right] - (16 b^2 n^2 E^{((3 I) a) n^2 x} (c x^n)^{(3 I) b} \text{Hypergeometric2F1}\left[3, \frac{(3 - I/(b n))}{2}, \frac{(5 - I/(b n))}{2}, E^{((2 I) a) (c x^n)^{(2 I) b}}\right]) / (I - 3 b n)$

#### Rule 4504

$\text{Int}[\text{Csc}[(a + \text{Log}[c x^n]) (b x^n)^p], x\_Symbol] \rightarrow \text{Dist}[x / (n (c x^n)^{1/n}), \text{Subst}[\text{Int}[x^{1/n - 1} \text{Csc}[d (a + b \text{Log}[x])]^p, x], x, c x^n], x] /;$  FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

$\text{Int}[\text{Csc}[(a + \text{Log}[x] (b x))^p] (e x)^m, x\_Symbol] \rightarrow \text{Dist}[(-2 I)^p E^{(I a d p)}, \text{Int}[(e x)^m x^{(I b d p)} / (1 - E^{(2 I a d) x} (2 I b d)^p), x], x] /;$  FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

$\text{Int}[(c x)^m (a + (b x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(a^p (c x)^{m+1} \text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -(b x^n)/a]) / (c (m+1)), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \left( -(1 + b^2 n^2) \csc(a + b \log(cx^n)) + 2b^2 n^2 \csc^3(a + b \log(cx^n)) \right) dx &= (2b^2 n^2) \int \csc^3(a + b \log(cx^n)) dx + (-1 - b^2 n^2) \int \csc(a + b \log(cx^n)) dx \\
&= (2b^2 n x (cx^n)^{-1/n}) \text{Subst} \left( \int x^{-1 + \frac{1}{n}} \csc^3(a + b \log(x)) dx \right) \\
&= (16ib^2 e^{3ia} n x (cx^n)^{-1/n}) \text{Subst} \left( \int \frac{x^{-1 + 3ib + \frac{1}{n}}}{(1 - e^{2ia} x^{2ib})^3} dx \right) \\
&= 2e^{ia} (i + bn) x (cx^n)^{ib} {}_2F_1 \left( 1, \frac{1}{2} \left( 1 - \frac{i}{bn} \right); \frac{1}{2} \left( 3 - \frac{i}{bn} \right); -\frac{e^{2ia} x^{2ib}}{1 - e^{2ia} x^{2ib}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.41807, size = 30, normalized size = 0.71

$$-x (bn \cot(a + b \log(cx^n)) + 1) \csc(a + b \log(cx^n))$$

Antiderivative was successfully verified.

[In] Integrate[-((1 + b^2\*n^2)\*Csc[a + b\*Log[c\*x^n]]) + 2\*b^2\*n^2\*Csc[a + b\*Log[c\*x^n]]^3,x]

[Out] -(x\*(1 + b\*n\*Cot[a + b\*Log[c\*x^n]])\*Csc[a + b\*Log[c\*x^n]])

**Maple [C]** time = 0.479, size = 535, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(b^2\*n^2+1)\*csc(a+b\*ln(c\*x^n))+2\*b^2\*n^2\*csc(a+b\*ln(c\*x^n))^3,x)

[Out] 2\*x/(exp(I\*(-I\*b\*Pi\*csgn(I\*c\*x^n)^3+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*x^n)-I\*b\*Pi\*csgn(I\*c\*x^n)\*csgn(I\*c)\*csgn(I\*x^n)+2\*b\*ln(x^n)+2\*b\*ln(c)+2\*a))-1)^2\*((x^n)^(I\*b))^3\*(c^(I\*b))^3\*b\*n\*exp(3/2\*b\*Pi\*csgn(I\*c\*x^n)^3)\*exp(-3/2\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c))\*exp(-3/2\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*x^n))\*exp(3/2\*b\*Pi\*csgn(I\*c\*x^n)\*csgn(I\*c)\*csgn(I\*x^n))\*exp(3\*I\*a)+(x^n)^(I\*b)\*c^(I\*b)\*b\*n\*exp(1/2\*b\*Pi\*csgn(I\*c\*x^n)^3)\*exp(-1/2\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c))\*exp(-1/2\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*x^n))\*exp(1/2\*b\*Pi\*csgn(I\*c\*x^n)\*csgn(I\*c)\*csgn(I\*x^n))\*exp(I\*a)-I\*((x^n)^(I\*b))^3\*(c^(I\*b))^3\*exp(3/2\*b\*Pi\*csgn(I\*c\*x^n)^3)\*exp(-3/2\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c))\*exp(-3/2\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*x^n))\*exp(3/2\*b\*Pi\*csgn(I\*c\*x^n)\*csgn(I\*c)\*csgn(I\*x^n))\*exp(3\*I\*a)+I\*c^(I\*b)\*(x^n)^(I\*b)\*exp(1/2\*b\*Pi\*csgn(I\*c\*x^n)^3)\*exp(-1/2\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c))\*exp(-1/2\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*x^n))\*exp(1/2\*b\*Pi\*csgn(I\*c\*x^n)\*csgn(I\*c)\*csgn(I\*x^n))\*exp(I\*a))

**Maxima [B]** time = 2.03531, size = 2296, normalized size = 54.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(-(b^2\*n^2+1)\*csc(a+b\*log(c\*x^n))+2\*b^2\*n^2\*csc(a+b\*log(c\*x^n))^3, x, algorithm="maxima")

[Out]  $2*((b*n*\cos(b*\log(c)) - \sin(b*\log(c)))*x*\cos(b*\log(x^n) + a) - (b*n*\sin(b*\log(c)) + \cos(b*\log(c)))*x*\sin(b*\log(x^n) + a) + (((b*\cos(4*b*\log(c))*\cos(3*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(3*b*\log(c)))*n - \cos(3*b*\log(c))*\sin(4*b*\log(c)) + \cos(4*b*\log(c))*\sin(3*b*\log(c)))*x*\cos(3*b*\log(x^n) + 3*a) + ((b*\cos(4*b*\log(c))*\cos(b*\log(c)) + b*\sin(4*b*\log(c))*\sin(b*\log(c)))*n + \cos(b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(b*\log(c)))*x*\cos(b*\log(x^n) + a) + ((b*\cos(3*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(3*b*\log(c)))*n + \cos(4*b*\log(c))*\cos(3*b*\log(c)) + \sin(4*b*\log(c))*\sin(3*b*\log(c)))*x*\sin(3*b*\log(x^n) + 3*a) + ((b*\cos(b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(b*\log(c)))*n - \cos(4*b*\log(c))*\cos(b*\log(c)) - \sin(4*b*\log(c))*\sin(b*\log(c)))*x*\sin(b*\log(x^n) + a))*\cos(4*b*\log(x^n) + 4*a) - (2*((b*\cos(3*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(3*b*\log(c))*\sin(2*b*\log(c)))*n + \cos(2*b*\log(c))*\sin(3*b*\log(c)) - \cos(3*b*\log(c))*\sin(2*b*\log(c)))*x*\cos(2*b*\log(x^n) + 2*a) + 2*((b*\cos(2*b*\log(c))*\sin(3*b*\log(c)) - b*\cos(3*b*\log(c))*\sin(2*b*\log(c)))*n - \cos(3*b*\log(c))*\cos(2*b*\log(c)) - \sin(3*b*\log(c))*\sin(2*b*\log(c)))*x*\sin(2*b*\log(x^n) + 2*a) - (b*n*\cos(3*b*\log(c)) + \sin(3*b*\log(c)))*x)*\cos(3*b*\log(x^n) + 3*a) - 2*((b*\cos(2*b*\log(c))*\cos(b*\log(c)) + b*\sin(2*b*\log(c))*\sin(b*\log(c)))*n + \cos(b*\log(c))*\sin(2*b*\log(c)) - \cos(2*b*\log(c))*\sin(b*\log(c)))*x*\cos(b*\log(x^n) + a) + ((b*\cos(b*\log(c))*\sin(2*b*\log(c)) - b*\cos(2*b*\log(c))*\sin(b*\log(c)))*n - \cos(2*b*\log(c))*\cos(b*\log(c)) - \sin(2*b*\log(c))*\sin(b*\log(c)))*x*\sin(b*\log(x^n) + a))*\cos(2*b*\log(x^n) + 2*a) - (((b*\cos(3*b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(3*b*\log(c)))*n + \cos(4*b*\log(c))*\cos(3*b*\log(c)) + \sin(4*b*\log(c))*\sin(3*b*\log(c)))*x*\cos(3*b*\log(x^n) + 3*a) + ((b*\cos(b*\log(c))*\sin(4*b*\log(c)) - b*\cos(4*b*\log(c))*\sin(b*\log(c)))*n - \cos(4*b*\log(c))*\cos(b*\log(c)) - \sin(4*b*\log(c))*\sin(b*\log(c)))*x*\cos(b*\log(x^n) + a) - ((b*\cos(4*b*\log(c))*\cos(3*b*\log(c)) + b*\sin(4*b*\log(c))*\sin(3*b*\log(c)))*n - \cos(3*b*\log(c))*\sin(4*b*\log(c)) + \cos(4*b*\log(c))*\sin(3*b*\log(c)))*x*\sin(3*b*\log(x^n) + 3*a) - ((b*\cos(4*b*\log(c))*\cos(b*\log(c)) + b*\sin(4*b*\log(c))*\sin(b*\log(c)))*n + \cos(b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(b*\log(c)))*x*\sin(b*\log(x^n) + a))*\sin(4*b*\log(x^n) + 4*a) + (2*((b*\cos(2*b*\log(c))*\sin(3*b*\log(c)) - b*\cos(3*b*\log(c))*\sin(2*b*\log(c)))*n - \cos(3*b*\log(c))*\cos(2*b*\log(c)) - \sin(3*b*\log(c))*\sin(2*b*\log(c)))*x*\cos(2*b*\log(x^n) + 2*a) - 2*((b*\cos(3*b*\log(c))*\cos(2*b*\log(c)) + b*\sin(3*b*\log(c))*\sin(2*b*\log(c)))*n + \cos(2*b*\log(c))*\sin(3*b*\log(c)) - \cos(3*b*\log(c))*\sin(2*b*\log(c)))*x*\sin(2*b*\log(x^n) + 2*a) - (b*n*\sin(3*b*\log(c)) - \cos(3*b*\log(c)))*x)*\sin(3*b*\log(x^n) + 3*a) + 2*((b*\cos(b*\log(c))*\sin(2*b*\log(c)) - b*\cos(2*b*\log(c))*\sin(b*\log(c)))*n - \cos(2*b*\log(c))*\cos(b*\log(c)) - \sin(2*b*\log(c))*\sin(b*\log(c)))*x*\cos(b*\log(x^n) + a) - ((b*\cos(2*b*\log(c))*\cos(b*\log(c)) + b*\sin(2*b*\log(c))*\sin(b*\log(c)))*n + \cos(b*\log(c))*\sin(2*b*\log(c)) - \cos(2*b*\log(c))*\sin(b*\log(c)))*x*\sin(b*\log(x^n) + a))*\sin(2*b*\log(x^n) + 2*a))/((\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\cos(4*b*\log(x^n) + 4*a)^2 + 4*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\cos(2*b*\log(x^n) + 2*a)^2 + (\cos(4*b*\log(c))^2 + \sin(4*b*\log(c))^2)*\sin(4*b*\log(x^n) + 4*a)^2 + 4*(\cos(2*b*\log(c))^2 + \sin(2*b*\log(c))^2)*\sin(2*b*\log(x^n) + 2*a)^2 - 2*(2*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) + 2*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \cos(4*b*\log(c)))*\cos(4*b*\log(x^n) + 4*a) - 4*\cos(2*b*\log(c))*\cos(2*b*\log(x^n) + 2*a) + 2*(2*(\cos(2*b*\log(c))*\sin(4*b*\log(c)) - \cos(4*b*\log(c))*\sin(2*b*\log(c)))*\cos(2*b*\log(x^n) + 2*a) - 2*(\cos(4*b*\log(c))*\cos(2*b*\log(c)) + \sin(4*b*\log(c))*\sin(2*b*\log(c)))*\sin(2*b*\log(x^n) + 2*a) - \sin(4*b*\log(c)))*\sin(4*b*\log(x^n) + 4*a) + 4*\sin(2*b*\log(c))*\sin(2*b*\log(x^n) + 2*a) + 1)$

**Fricas [A]** time = 0.473887, size = 154, normalized size = 3.67

$$\frac{bnx \cos(bn \log(x) + b \log(c) + a) + x \sin(bn \log(x) + b \log(c) + a)}{\cos(bn \log(x) + b \log(c) + a)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b^2\*n^2+1)\*csc(a+b\*log(c\*x^n))+2\*b^2\*n^2\*csc(a+b\*log(c\*x^n))^3, x, algorithm="fricas")

[Out] (b\*n\*x\*cos(b\*n\*log(x) + b\*log(c) + a) + x\*sin(b\*n\*log(x) + b\*log(c) + a))/(cos(b\*n\*log(x) + b\*log(c) + a)^2 - 1)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b\*\*2\*n\*\*2+1)\*csc(a+b\*ln(c\*x\*\*n))+2\*b\*\*2\*n\*\*2\*csc(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int 2b^2n^2 \csc(b \log(cx^n) + a)^3 - (b^2n^2 + 1) \csc(b \log(cx^n) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(b^2\*n^2+1)\*csc(a+b\*log(c\*x^n))+2\*b^2\*n^2\*csc(a+b\*log(c\*x^n))^3, x, algorithm="giac")

[Out] integrate(2\*b^2\*n^2\*csc(b\*log(c\*x^n) + a)^3 - (b^2\*n^2 + 1)\*csc(b\*log(c\*x^n) + a), x)

$$3.302 \quad \int x^m \csc^3 \left( a + 2 \log \left( cx^{\frac{1}{2} \sqrt{-(1+m)^2}} \right) \right) dx$$

**Optimal.** Leaf size=110

$$\frac{x^{m+1} \csc \left( a + 2 \log \left( cx^{\frac{1}{2} \sqrt{-(m+1)^2}} \right) \right)}{2(m+1)} - \frac{x^{m+1} \cot \left( a + 2 \log \left( cx^{\frac{1}{2} \sqrt{-(m+1)^2}} \right) \right) \csc \left( a + 2 \log \left( cx^{\frac{1}{2} \sqrt{-(m+1)^2}} \right) \right)}{2\sqrt{-(m+1)^2}}$$

[Out]  $(x^{(1+m)} \text{Csc}[a + 2 \text{Log}[c x^{\sqrt{-(1+m)^2}/2}]])/(2*(1+m)) - (x^{(1+m)} \text{Cot}[a + 2 \text{Log}[c x^{\sqrt{-(1+m)^2}/2}]] * \text{Csc}[a + 2 \text{Log}[c x^{\sqrt{-(1+m)^2}/2}]])/(2 \sqrt{-(1+m)^2})$

**Rubi [C]** time = 0.183675, antiderivative size = 142, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {4510, 4506, 364}

$$\frac{8e^{3ia} x^{m+1} \left( cx^{\frac{1}{2} \sqrt{-(m+1)^2}} \right)^{6i} {}_2F_1 \left( 3, \frac{1}{2} \left( 3 - \frac{i(m+1)}{\sqrt{-(m+1)^2}} \right); \frac{1}{2} \left( 5 - \frac{i(m+1)}{\sqrt{-(m+1)^2}} \right); e^{2ia} \left( cx^{\frac{1}{2} \sqrt{-(m+1)^2}} \right)^{4i} \right)}{im - 3\sqrt{-(m+1)^2} + i}$$

Warning: Unable to verify antiderivative.

[In] Int[x^m\*Csc[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]

[Out]  $(-8 * E^{((3*I)*a)} * x^{(1+m)} * (c * x^{\sqrt{-(1+m)^2}/2})^{(6*I)} * \text{Hypergeometric2F1}[3, (3 - (I*(1+m))/\sqrt{-(1+m)^2})/2, (5 - (I*(1+m))/\sqrt{-(1+m)^2})/2, E^{((2*I)*a)} * (c * x^{\sqrt{-(1+m)^2}/2})^{(4*I)}]) / (I + I*m - 3*\sqrt{-(1+m)^2})$

#### Rule 4510

Int[Csc[((a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m+1)/(e\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[x^((m+1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[(-2\*I)^p \* E^(I\*a\*d\*p), Int[((e\*x)^m \* x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p \* (c\*x)^(m+1) \* Hypergeometric2F1[-p, (m+1)/n, (m+1)/n + 1, -(b\*x^n)/a]) / (c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \csc^3\left(a + 2 \log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right) dx = \frac{\left(2x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{-\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right) \text{Subst}\left(\int x^{-1+\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\csc^3(a + 2 \log(x)) dx, x, \right)}{\sqrt{-(1+m)^2}}$$

$$= \frac{\left(16ie^{3ia}x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{-\frac{2(1+m)}{\sqrt{-(1+m)^2}}}\right) \text{Subst}\left(\int \frac{x^{\frac{(-1+6i)+\frac{2(1+m)}{\sqrt{-(1+m)^2}}}{\sqrt{-(1+m)^2}}}}{(1-e^{2ia}x^{4i})^3} dx, x, cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)}{\sqrt{-(1+m)^2}}$$

$$= -\frac{8e^{3ia}x^{1+m}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{6i} {}_2F_1\left(3, \frac{1}{2}\left(3 - \frac{i(1+m)}{\sqrt{-(1+m)^2}}\right); \frac{1}{2}\left(5 - \frac{i(1+m)}{\sqrt{-(1+m)^2}}\right); e^{2ia}\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{i + im - 3\sqrt{-(1+m)^2}}$$

**Mathematica [A]** time = 2.0796, size = 79, normalized size = 0.72

$$\frac{x^{m+1}\left(\sqrt{-(m+1)^2} \cot\left(a + 2 \log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right) + m + 1\right) \csc\left(a + 2 \log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right)}{2(m+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Csc[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]

[Out] (x^(1 + m)\*(1 + m + Sqrt[-(1 + m)^2]\*Cot[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2) ]])\*Csc[a + 2\*Log[c\*x^(Sqrt[-(1 + m)^2]/2) ]]/(2\*(1 + m)^2)

**Maple [F]** time = 0.181, size = 0, normalized size = 0.

$$\int x^m \left( \csc\left(a + 2 \ln\left(cx^{1/2\sqrt{-(1+m)^2}}\right)\right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*csc(a+2\*ln(c\*x^(1/2\*(-(1+m)^2)^(1/2))))^3,x)

[Out] int(x^m\*csc(a+2\*ln(c\*x^(1/2\*(-(1+m)^2)^(1/2))))^3,x)

**Maxima [B]** time = 1.44556, size = 1315, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*csc(a+2\*log(c\*x^(1/2\*(-(1+m)^2)^(1/2))))^3,x, algorithm="maxima")

[Out] 2\*((cos(2\*log(c))\*sin(a) + cos(a)\*sin(2\*log(c)))\*x\*e^(m\*log(x) + 14\*arctan2(sin(1/2\*m\*log(x)), cos(1/2\*m\*log(x))) + 14\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x)))) + 2\*(((cos(a)\*sin(2\*a) - cos(2\*a)\*sin(a))\*cos(2\*log(c)) - (cos(2\*a)\*cos(a) + sin(2\*a)\*sin(a))\*sin(2\*log(c)))\*cos(4\*log(c)) + ((cos(2\*a)\*co

$$\begin{aligned} & s(a) + \sin(2a)\sin(a)\cos(2\log(c)) + (\cos(a)\sin(2a) - \cos(2a)\sin(a)) \\ & * \sin(2\log(c)) * \sin(4\log(c)) * x * e^{(m\log(x) + 10\arctan2(\sin(1/2*m\log(x)), \\ & \cos(1/2*m\log(x))) + 10\arctan2(\sin(1/2*\log(x)), \cos(1/2*\log(x))))} - (((\cos(a)\sin(4a) - \cos(4a)\sin(a))\cos(2\log(c)) - (\cos(4a)\cos(a) + \sin(4a) \\ & a)\sin(a)\sin(2\log(c)))\cos(8\log(c)) + ((\cos(4a)\cos(a) + \sin(4a)\sin(a))\cos(2\log(c)) + (\cos(a)\sin(4a) - \cos(4a)\sin(a))\sin(2\log(c)))\sin( \\ & 8\log(c))) * x * e^{(m\log(x) + 6\arctan2(\sin(1/2*m\log(x)), \cos(1/2*m\log(x))) \\ & + 6\arctan2(\sin(1/2*\log(x)), \cos(1/2*\log(x))))} / ((\cos(4a)^2 + \sin(4a)^2) * \\ & \cos(8\log(c))^2 + (\cos(4a)^2 + \sin(4a)^2)\sin(8\log(c))^2 + ((\cos(4a)^2 \\ & + \sin(4a)^2)\cos(8\log(c))^2 + (\cos(4a)^2 + \sin(4a)^2)\sin(8\log(c))^2) * \\ & m + (m + 1) * e^{(16\arctan2(\sin(1/2*m\log(x)), \cos(1/2*m\log(x))) + 16\arctan \\ & 2(\sin(1/2*\log(x)), \cos(1/2*\log(x))))} - 4 * ((\cos(2a)\cos(4\log(c)) - \sin(2a) \\ & )\sin(4\log(c))) * m + \cos(2a)\cos(4\log(c)) - \sin(2a)\sin(4\log(c)) * e^{(12 \\ & * \arctan2(\sin(1/2*m\log(x)), \cos(1/2*m\log(x))) + 12\arctan2(\sin(1/2*\log(x)) \\ & , \cos(1/2*\log(x))))} + 2 * (2 * (\cos(2a)^2 + \sin(2a)^2)\cos(4\log(c))^2 + 2 * (\cos(2a)^2 + \sin(2a)^2)\sin(4\log(c))^2 + (2 * (\cos(2a)^2 + \sin(2a)^2)\cos( \\ & 4\log(c))^2 + 2 * (\cos(2a)^2 + \sin(2a)^2)\sin(4\log(c))^2 + \cos(4a)\cos(8\log(c)) - \sin(4a)\sin(8\log(c))) * m + \cos(4a)\cos(8\log(c)) - \sin(4a)\sin( \\ & 8\log(c)) * e^{(8\arctan2(\sin(1/2*m\log(x)), \cos(1/2*m\log(x))) + 8\arctan2( \\ & \sin(1/2*\log(x)), \cos(1/2*\log(x))))} - 4 * (((\cos(4a)\cos(2a) + \sin(4a)\sin \\ & (2a))\cos(4\log(c)) + (\cos(2a)\sin(4a) - \cos(4a)\sin(2a))\sin(4\log(c) \\ & ))\cos(8\log(c)) - ((\cos(2a)\sin(4a) - \cos(4a)\sin(2a))\cos(4\log(c)) - \\ & (\cos(4a)\cos(2a) + \sin(4a)\sin(2a))\sin(4\log(c)))\sin(8\log(c))) * m + \\ & (((\cos(4a)\cos(2a) + \sin(4a)\sin(2a))\cos(4\log(c)) + (\cos(2a)\sin(4a) \\ & - \cos(4a)\sin(2a))\sin(4\log(c)))\cos(8\log(c)) - ((\cos(2a)\sin(4a) - \\ & \cos(4a)\sin(2a))\cos(4\log(c)) - (\cos(4a)\cos(2a) + \sin(4a)\sin(2a))\sin(4\log(c)))\sin(8\log(c))) * e^{(4\arctan2(\sin(1/2*m\log(x)), \cos(1/2*m\log \\ & (x))) + 4\arctan2(\sin(1/2*\log(x)), \cos(1/2*\log(x))))} \end{aligned}$$

**Fricas [C]** time = 0.489028, size = 236, normalized size = 2.15

$$\frac{-4i x^2 x^{2m} e^{(3ia+6i \log(c))} + 2i e^{(5ia+10i \log(c))}}{(m+1)x^4 x^{4m} - 2(m+1)x^2 x^{2m} e^{(2ia+4i \log(c))} + (m+1)e^{(4ia+8i \log(c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*csc(a+2\*log(c\*x^(1/2\*(-(1+m)^2)^(1/2))))^3,x, algorithm="fricas")

[Out] (-4\*I\*x^2\*x^(2\*m)\*e^(3\*I\*a + 6\*I\*log(c)) + 2\*I\*e^(5\*I\*a + 10\*I\*log(c)))/((m + 1)\*x^4\*x^(4\*m) - 2\*(m + 1)\*x^2\*x^(2\*m)\*e^(2\*I\*a + 4\*I\*log(c)) + (m + 1)\*e^(4\*I\*a + 8\*I\*log(c)))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*csc(a+2\*ln(c\*x\*\*(1/2\*(-(1+m)\*\*2)\*\*(1/2))))\*\*3,x)

[Out] Timed out

**Giac [C]** time = 14.2014, size = 1133, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="giac")
```

```
[Out] I*c^(6*I)*m*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - I*c^(6*I)*x*x^m*x^abs(m + 1)*abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) + I*c^(6*I)*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - I*c^(2*I)*m*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - I*c^(2*I)*x*x^m*x^(3*abs(m + 1))*abs(m + 1)*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - I*c^(2*I)*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - I*c^(2*I)*x*x^m*x^(3*abs(m + 1))*abs(m + 1)*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - I*c^(2*I)*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) - I*c^(2*I)*x*x^m*x^(3*abs(m + 1))*abs(m + 1)*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1)))
```

### 3.303 $\int x \csc^3(a + 2 \log(cx^i)) dx$

**Optimal.** Leaf size=49

$$-\frac{ie^{ia}x^2(cx^i)^{2i}}{(1 - e^{2ia}(cx^i)^{4i})^2}$$

[Out]  $((-I)*E^{(I*a)}*(c*x^I)^{(2*I)}*x^2)/(1 - E^{((2*I)*a)}*(c*x^I)^{(4*I)})^2$

**Rubi [A]** time = 0.0422573, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4510, 4506, 261}

$$-\frac{ie^{ia}x^2(cx^i)^{2i}}{(1 - e^{2ia}(cx^i)^{4i})^2}$$

Antiderivative was successfully verified.

[In] Int[x\*Csc[a + 2\*Log[c\*x^I]]^3,x]

[Out]  $((-I)*E^{(I*a)}*(c*x^I)^{(2*I)}*x^2)/(1 - E^{((2*I)*a)}*(c*x^I)^{(4*I)})^2$

#### Rule 4510

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(-2\*I)^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int x \csc^3(a + 2 \log(cx^i)) dx &= -\left( (i(cx^i)^{2i} x^2) \text{Subst} \left( \int x^{-1-2i} \csc^3(a + 2 \log(x)) dx, x, cx^i \right) \right) \\ &= \left( 8e^{3ia} (cx^i)^{2i} x^2 \right) \text{Subst} \left( \int \frac{x^{-1+4i}}{(1 - e^{2ia}x^{4i})^3} dx, x, cx^i \right) \\ &= -\frac{ie^{ia} (cx^i)^{2i} x^2}{(1 - e^{2ia} (cx^i)^{4i})^2} \end{aligned}$$

**Mathematica [B]** time = 0.178873, size = 127, normalized size = 2.59

$$\frac{\csc^2(a + 2 \log(cx^i)) \left( (2x^4 + 1) \sin(a + 2 \log(cx^i) - 2i \log(x)) + i(2x^4 - 1) \cos(a + 2 \log(cx^i) - 2i \log(x)) \right) \left( i \sin(2(a + 2 \log(cx^i) - 2i \log(x))) \right)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Csc[a + 2\*Log[c\*x^I]]^3,x]

[Out] (Csc[a + 2\*Log[c\*x^I]]^2\*(I\*(-1 + 2\*x^4)\*Cos[a + 2\*Log[c\*x^I] - (2\*I)\*Log[x]] + (1 + 2\*x^4)\*Sin[a + 2\*Log[c\*x^I] - (2\*I)\*Log[x]])\*(Cos[2\*(a + 2\*Log[c\*x^I] - (2\*I)\*Log[x])] + I\*Sin[2\*(a + 2\*Log[c\*x^I] - (2\*I)\*Log[x])])/(4\*x^4)

**Maple [C]** time = 0.126, size = 215, normalized size = 4.4

$$\frac{-ix^2 e^{i(\pi (\operatorname{csgn}(icx^i))^2 \operatorname{csgn}(ix^i) - i\pi \operatorname{csgn}(icx^i) \operatorname{csgn}(ic) \operatorname{csgn}(ix^i) - i\pi (\operatorname{csgn}(icx^i))^3 + i\pi (\operatorname{csgn}(icx^i))^2 \operatorname{csgn}(ic) + 2 \ln(c) + 2 \ln(x^i) + a)}}{\left( e^{-2i(i\pi (\operatorname{csgn}(icx^i))^3 - i\pi (\operatorname{csgn}(icx^i))^2 \operatorname{csgn}(ic) - i\pi (\operatorname{csgn}(icx^i))^2 \operatorname{csgn}(ix^i) + i\pi \operatorname{csgn}(icx^i) \operatorname{csgn}(ic) \operatorname{csgn}(ix^i) - 2 \ln(c) - 2 \ln(x^i) - a)} - 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*csc(a+2\*ln(c\*x^I))^3,x)

[Out] -I\*x^2/(exp(-2\*I\*(I\*Pi\*csgn(I\*c\*x^I)^3-I\*Pi\*csgn(I\*c\*x^I)^2\*csgn(I\*c)-I\*Pi\*csgn(I\*c\*x^I)^2\*csgn(I\*x^I)+I\*Pi\*csgn(I\*c\*x^I)\*csgn(I\*c)\*csgn(I\*x^I)-2\*ln(c)-2\*ln(x^I)-a))-1)^2\*exp(I\*(I\*Pi\*csgn(I\*c\*x^I)^2\*csgn(I\*x^I)-I\*Pi\*csgn(I\*c\*x^I)\*csgn(I\*c)\*csgn(I\*x^I)-I\*Pi\*csgn(I\*c\*x^I)^3+I\*Pi\*csgn(I\*c\*x^I)^2\*csgn(I\*c)+2\*ln(c)+2\*ln(x^I)+a))

**Maxima [B]** time = 1.12438, size = 192, normalized size = 3.92

$$\frac{((-i \cos(a) + \sin(a)) \cos(2 \log(c)) + (\cos(a) + i \sin(a)) \sin(2 \log(c))) \sin(2 \arctan_2(\sin(\log(x)), \cos(\log(x))))}{(\cos(4a) + i \sin(4a)) \cos(8 \log(c)) - ((2 \cos(2a) + 2i \sin(2a)) \cos(4 \log(c)) + 2(i \cos(2a) - \sin(2a)) \sin(4 \log(c)))} + \frac{(\cos(a) + i \sin(a)) \sin(2 \log(c))}{(\cos(4a) + i \sin(4a)) \cos(8 \log(c)) - ((2 \cos(2a) + 2i \sin(2a)) \cos(4 \log(c)) + 2(i \cos(2a) - \sin(2a)) \sin(4 \log(c)))} + \frac{(-i \cos(a) + \sin(a)) \cos(2 \log(c))}{(\cos(4a) + i \sin(4a)) \cos(8 \log(c)) - ((2 \cos(2a) + 2i \sin(2a)) \cos(4 \log(c)) + 2(i \cos(2a) - \sin(2a)) \sin(4 \log(c)))} + \frac{(\cos(a) + i \sin(a)) \sin(2 \log(c))}{(\cos(4a) + i \sin(4a)) \cos(8 \log(c)) - ((2 \cos(2a) + 2i \sin(2a)) \cos(4 \log(c)) + 2(i \cos(2a) - \sin(2a)) \sin(4 \log(c)))} + \frac{e^{(4 \arctan_2(\sin(\log(x)), \cos(\log(x))))}}{(\cos(4a) + i \sin(4a)) \cos(8 \log(c)) - ((2 \cos(2a) + 2i \sin(2a)) \cos(4 \log(c)) + 2(i \cos(2a) - \sin(2a)) \sin(4 \log(c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*csc(a+2\*log(c\*x^I))^3,x, algorithm="maxima")

[Out] ((-I\*cos(a) + sin(a))\*cos(2\*log(c)) + (cos(a) + I\*sin(a))\*sin(2\*log(c)))\*x^2\*e^(6\*arctan2(sin(log(x)), cos(log(x))))/((cos(4\*a) + I\*sin(4\*a))\*cos(8\*log(c)) - ((2\*cos(2\*a) + 2\*I\*sin(2\*a))\*cos(4\*log(c)) + 2\*(I\*cos(2\*a) - sin(2\*a))\*sin(4\*log(c))))\*e^(4\*arctan2(sin(log(x)), cos(log(x)))) + (I\*cos(4\*a) - sin(4\*a))\*sin(8\*log(c)) + e^(8\*arctan2(sin(log(x)), cos(log(x))))

**Fricas [A]** time = 0.442479, size = 131, normalized size = 2.67

$$\frac{ix^2 e^{i(a+2i \log(cx^i))}}{e^{(4ia+8i \log(cx^i))} - 2e^{(2ia+4i \log(cx^i))} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x*csc(a+2*log(c*x^I))^3,x, algorithm="fricas")
```

```
[Out] -I*x^2*e^(I*a + 2*I*log(c*x^I))/(e^(4*I*a + 8*I*log(c*x^I)) - 2*e^(2*I*a + 4*I*log(c*x^I)) + 1)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \csc^3(a + 2 \log(cx^i)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+2*ln(c*x**I))**3,x)
```

```
[Out] Integral(x*csc(a + 2*log(c*x**I))**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \csc(a + 2 \log(cx^i))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+2*log(c*x^I))^3,x, algorithm="giac")
```

```
[Out] integrate(x*csc(a + 2*log(c*x^I))^3, x)
```

### 3.304 $\int \csc^3 \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) dx$

**Optimal.** Leaf size=58

$$\frac{1}{2}x \csc \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) + \frac{1}{2}ix \cot \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) \csc \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right)$$

[Out] (x\*Csc[a + 2\*Log[c\*x^(I/2)])/2 + (I/2)\*x\*Cot[a + 2\*Log[c\*x^(I/2)]]\*Csc[a + 2\*Log[c\*x^(I/2)]]

**Rubi [A]** time = 0.0352578, antiderivative size = 51, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4504, 4506, 261}

$$\frac{2ie^{ia}x \left( cx^{\frac{i}{2}} \right)^{2i}}{\left( 1 - e^{2ia} \left( cx^{\frac{i}{2}} \right)^{4i} \right)^2}$$

Warning: Unable to verify antiderivative.

[In] Int[Csc[a + 2\*Log[c\*x^(I/2)]]^3,x]

[Out] ((-2\*I)\*E^(I\*a)\*(c\*x^(I/2))^(2\*I)\*x)/(1 - E^((2\*I)\*a)\*(c\*x^(I/2))^(4\*I))^2

#### Rule 4504

Int[Csc[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(-2\*I)^p\*E^(I\*a\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \csc^3 \left( a + 2 \log \left( cx^{\frac{i}{2}} \right) \right) dx &= - \left( \left( 2i \left( cx^{\frac{i}{2}} \right)^{2i} x \right) \text{Subst} \left( \int x^{-1-2i} \csc^3(a + 2 \log(x)) dx, x, cx^{\frac{i}{2}} \right) \right) \\ &= \left( 16e^{3ia} \left( cx^{\frac{i}{2}} \right)^{2i} x \right) \text{Subst} \left( \int \frac{x^{-1+4i}}{\left( 1 - e^{2ia}x^{4i} \right)^3} dx, x, cx^{\frac{i}{2}} \right) \\ &= - \frac{2ie^{ia} \left( cx^{\frac{i}{2}} \right)^{2i} x}{\left( 1 - e^{2ia} \left( cx^{\frac{i}{2}} \right)^{4i} \right)^2} \end{aligned}$$

**Mathematica [B]** time = 0.146669, size = 137, normalized size = 2.36

$$\frac{\csc^2\left(a + 2 \log\left(cx^{\frac{i}{2}}\right)\right)\left(\left(2x^2 + 1\right) \sin\left(a + 2 \log\left(cx^{\frac{i}{2}}\right) - i \log(x)\right) + i\left(2x^2 - 1\right) \cos\left(a + 2 \log\left(cx^{\frac{i}{2}}\right) - i \log(x)\right)\right)\left(i \sin\left(\frac{a}{2}\right) \cos\left(\frac{a}{2}\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + 2\*Log[c\*x^(I/2)]]^3,x]

[Out] (Csc[a + 2\*Log[c\*x^(I/2)]]^2\*(I\*(-1 + 2\*x^2)\*Cos[a + 2\*Log[c\*x^(I/2)]] - I\*Log[x] + (1 + 2\*x^2)\*Sin[a + 2\*Log[c\*x^(I/2)]] - I\*Log[x]))\*(Cos[2\*(a + 2\*Log[c\*x^(I/2)]] - I\*Log[x]]) + I\*Ssin[2\*(a + 2\*Log[c\*x^(I/2)]] - I\*Log[x]))/(2\*x^2)

**Maple [C]** time = 0.136, size = 215, normalized size = 3.7

$$\frac{-2ixe^{-i\left(i\pi\left(\operatorname{csgn}\left(icx^{\frac{i}{2}}\right)\right)^3 - i\pi\left(\operatorname{csgn}\left(icx^{\frac{i}{2}}\right)\right)^2 \operatorname{csgn}(ic) - i\pi\left(\operatorname{csgn}\left(icx^{\frac{i}{2}}\right)\right)^2 \operatorname{csgn}\left(ix^{\frac{i}{2}}\right) + i\pi \operatorname{csgn}\left(icx^{\frac{i}{2}}\right) \operatorname{csgn}(ic) \operatorname{csgn}\left(ix^{\frac{i}{2}}\right) - 2 \ln(c) - 2 \ln(x^{i/2}) - a\right)}}{\left(e^{-2i\left(i\pi\left(\operatorname{csgn}\left(icx^{\frac{i}{2}}\right)\right)^3 - i\pi\left(\operatorname{csgn}\left(icx^{\frac{i}{2}}\right)\right)^2 \operatorname{csgn}(ic) - i\pi\left(\operatorname{csgn}\left(icx^{\frac{i}{2}}\right)\right)^2 \operatorname{csgn}\left(ix^{\frac{i}{2}}\right) + i\pi \operatorname{csgn}\left(icx^{\frac{i}{2}}\right) \operatorname{csgn}(ic) \operatorname{csgn}\left(ix^{\frac{i}{2}}\right) - 2 \ln(c) - 2 \ln(x^{i/2}) - a}\right) - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+2\*ln(c\*x^(1/2\*I)))^3,x)

[Out] -2\*I\*x\*exp(-I\*(I\*Pi\*csgn(I\*c\*x^(1/2\*I)))^3 - I\*Pi\*csgn(I\*c\*x^(1/2\*I))^2\*csgn(I\*c) - I\*Pi\*csgn(I\*c\*x^(1/2\*I))^2\*csgn(I\*x^(1/2\*I)) + I\*Pi\*csgn(I\*c\*x^(1/2\*I))\*csgn(I\*c)\*csgn(I\*x^(1/2\*I)) - 2\*ln(c) - 2\*ln(x^(1/2\*I)) - a)/(exp(-2\*I\*(I\*Pi\*csgn(I\*c\*x^(1/2\*I)))^3 - I\*Pi\*csgn(I\*c\*x^(1/2\*I))^2\*csgn(I\*c) - I\*Pi\*csgn(I\*c\*x^(1/2\*I))^2\*csgn(I\*x^(1/2\*I)) + I\*Pi\*csgn(I\*c\*x^(1/2\*I))\*csgn(I\*c)\*csgn(I\*x^(1/2\*I)) - 2\*ln(c) - 2\*ln(x^(1/2\*I)) - a) - 1)^2

**Maxima [B]** time = 1.18178, size = 215, normalized size = 3.71

$$(2(i \cos(a) - \sin(a)) \cos(2 \log(c)) - (2 \cos(a) + 2i \sin(a)) \sin(2 \log(c)))$$

$$(\cos(4a) + i \sin(4a)) \cos(8 \log(c)) - ((2 \cos(2a) + 2i \sin(2a)) \cos(4 \log(c)) + 2(i \cos(2a) - \sin(2a)) \sin(4 \log(c)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+2\*log(c\*x^(1/2\*I)))^3,x, algorithm="maxima")

[Out] -(2\*(I\*cos(a) - sin(a))\*cos(2\*log(c)) - (2\*cos(a) + 2\*I\*sin(a))\*sin(2\*log(c)))\*x\*e^(6\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x))))/((cos(4\*a) + I\*sin(4\*a))\*cos(8\*log(c)) - ((2\*cos(2\*a) + 2\*I\*sin(2\*a))\*cos(4\*log(c)) + 2\*(I\*cos(2\*a) - sin(2\*a))\*sin(4\*log(c))))\*e^(4\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x)))) + (I\*cos(4\*a) - sin(4\*a))\*sin(8\*log(c)) + e^(8\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x))))

**Fricas [A]** time = 0.447306, size = 155, normalized size = 2.67

$$\frac{2i x e^{\left(i a+2i \log\left(c x^{\frac{1}{2} i}\right)\right)}}{e^{\left(4i a+8i \log\left(c x^{\frac{1}{2} i}\right)\right)}-2 e^{\left(2i a+4i \log\left(c x^{\frac{1}{2} i}\right)\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+2\*log(c\*x^(1/2\*I)))^3,x, algorithm="fricas")

[Out] -2\*I\*x\*e^(I\*a + 2\*I\*log(c\*x^(1/2\*I)))/(e^(4\*I\*a + 8\*I\*log(c\*x^(1/2\*I))) - 2\*e^(2\*I\*a + 4\*I\*log(c\*x^(1/2\*I))) + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^3\left(a+2 \log\left(c x^{\frac{i}{2}}\right)\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+2\*ln(c\*x\*\*(1/2\*I)))\*\*3,x)

[Out] Integral(csc(a + 2\*log(c\*x\*\*(I/2)))\*\*3, x)

**Giac [A]** time = 4.53702, size = 100, normalized size = 1.72

$$\frac{2i c^{10i} e^{(5i a)}}{c^{8i} e^{(4i a)} - 2 c^{4i} x^2 e^{(2i a)} + x^4} - \frac{4i c^{6i} x^2 e^{(3i a)}}{c^{8i} e^{(4i a)} - 2 c^{4i} x^2 e^{(2i a)} + x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+2\*log(c\*x^(1/2\*I)))^3,x, algorithm="giac")

[Out] 2\*I\*c^(10\*I)\*e^(5\*I\*a)/(c^(8\*I)\*e^(4\*I\*a) - 2\*c^(4\*I)\*x^2\*e^(2\*I\*a) + x^4) - 4\*I\*c^(6\*I)\*x^2\*e^(3\*I\*a)/(c^(8\*I)\*e^(4\*I\*a) - 2\*c^(4\*I)\*x^2\*e^(2\*I\*a) + x^4)

$$3.305 \quad \int \csc^3 \left( a + 2 \log \left( cx^{-\frac{i}{2}} \right) \right) dx$$

**Optimal.** Leaf size=51

$$\frac{2ie^{3ia}x \left( cx^{-\frac{i}{2}} \right)^{6i}}{\left( 1 - e^{2ia} \left( cx^{-\frac{i}{2}} \right)^{4i} \right)^2}$$

[Out]  $((2*I)*E^{((3*I)*a)*(c/x^{(I/2)})^{(6*I)*x}}/(1 - E^{((2*I)*a)*(c/x^{(I/2)})^{(4*I)})}^2$

**Rubi [A]** time = 0.0403293, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {4504, 4506, 264}

$$\frac{2ie^{3ia}x \left( cx^{-\frac{i}{2}} \right)^{6i}}{\left( 1 - e^{2ia} \left( cx^{-\frac{i}{2}} \right)^{4i} \right)^2}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + 2\*Log[c/x^(I/2)]]^3,x]

[Out]  $((2*I)*E^{((3*I)*a)*(c/x^{(I/2)})^{(6*I)*x}}/(1 - E^{((2*I)*a)*(c/x^{(I/2)})^{(4*I)})}^2$

#### Rule 4504

Int[Csc[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4506

Int[Csc[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(-2\*I)^p\*E^(I\*a\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]

#### Rule 264

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \csc^3\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right)\right) dx &= \left(2i\left(cx^{-\frac{i}{2}}\right)^{-2i} x\right) \text{Subst}\left(\int x^{-1+2i} \csc^3(a + 2 \log(x)) dx, x, cx^{-\frac{i}{2}}\right) \\ &= -\left(\left(16e^{3ia}\left(cx^{-\frac{i}{2}}\right)^{-2i} x\right) \text{Subst}\left(\int \frac{x^{-1+8i}}{(1 - e^{2ia}x^{4i})^3} dx, x, cx^{-\frac{i}{2}}\right)\right) \\ &= \frac{2ie^{3ia}\left(cx^{-\frac{i}{2}}\right)^{6i} x}{\left(1 - e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2} \end{aligned}$$

**Mathematica [B]** time = 0.147083, size = 137, normalized size = 2.69

$$\frac{\csc^2\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right)\right)\left(i\left(2x^2 + 1\right) \sin\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right) + i \log(x)\right) + \left(2x^2 - 1\right) \cos\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right) + i \log(x)\right)\right)\left(\sin\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right) + i \log(x)\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + 2\*Log[c/x^(I/2)]]^3, x]

[Out] -(Csc[a + 2\*Log[c/x^(I/2)]]^2\*((-1 + 2\*x^2)\*Cos[a + 2\*Log[c/x^(I/2)]] + I\*Log[x]) + I\*(1 + 2\*x^2)\*Sin[a + 2\*Log[c/x^(I/2)]] + I\*Log[x]))\*(I\*Cos[2\*(a + 2\*Log[c/x^(I/2)]] + I\*Log[x]]) + Sin[2\*(a + 2\*Log[c/x^(I/2)]] + I\*Log[x])))/(2\*x^2)

**Maple [C]** time = 0.144, size = 239, normalized size = 4.7

$$2ixe^{-3i\left(i\pi\left(\operatorname{csgn}\left(\frac{ic}{x^2}\right)\right)^3 - i\pi\left(\operatorname{csgn}\left(\frac{ic}{x^2}\right)\right)^2 \operatorname{csgn}(ic) - i\pi\left(\operatorname{csgn}\left(\frac{ic}{x^2}\right)\right) \operatorname{csgn}\left(\frac{i}{x^2}\right) + i\pi \operatorname{csgn}\left(\frac{ic}{x^2}\right) \operatorname{csgn}(ic) \operatorname{csgn}\left(\frac{i}{x^2}\right) - 2 \ln(c) + 2 \ln(x^{i/2}) - a\right)} \left( -2i\left(i\pi\left(\operatorname{csgn}\left(\frac{ic}{x^2}\right)\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+2\*ln(c/(x^(1/2\*I))))^3, x)

[Out] 2\*I\*x\*exp(-3\*I\*(I\*Pi\*csgn(I\*c/(x^(1/2\*I))))^3 - I\*Pi\*csgn(I\*c/(x^(1/2\*I))))^2\*csgn(I\*c) - I\*Pi\*csgn(I\*c/(x^(1/2\*I))))^2\*csgn(I/(x^(1/2\*I))) + I\*Pi\*csgn(I\*c/(x^(1/2\*I))) \* csgn(I\*c) \* csgn(I/(x^(1/2\*I))) - 2\*ln(c) + 2\*ln(x^(1/2\*I)) - a) / (exp(-2\*I\*(I\*Pi\*csgn(I\*c/(x^(1/2\*I))))^3 - I\*Pi\*csgn(I\*c/(x^(1/2\*I))))^2\*csgn(I\*c) - I\*Pi\*csgn(I\*c/(x^(1/2\*I))))^2\*csgn(I/(x^(1/2\*I))) + I\*Pi\*csgn(I\*c/(x^(1/2\*I))) \* csgn(I\*c) \* csgn(I/(x^(1/2\*I))) - 2\*ln(c) + 2\*ln(x^(1/2\*I)) - a) - 1)^2

**Maxima [B]** time = 1.21784, size = 224, normalized size = 4.39

$$\frac{(2(i \cos(3a) - \sin(3a)) \cos(6 \log(c)) - (2 \cos(3a) + 2i \sin(3a)) \sin(6 \log(c))) \cos(8 \log(c)) - ((\cos(4a) + i \sin(4a)) \cos(8 \log(c)) - (-i \cos(4a) + \sin(4a)) \sin(8 \log(c))) e^{(8 \arctan(\sin(\frac{1}{2} \log(x)), \cos(\frac{1}{2} \log(x))))} - ((2 \cos(4a) + 2i \sin(4a)) \cos(8 \log(c)) - (-2i \cos(4a) + 2 \sin(4a)) \sin(8 \log(c)))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+2\*log(c/(x^(1/2\*I))))^3,x, algorithm="maxima")

[Out] (2\*(I\*cos(3\*a) - sin(3\*a))\*cos(6\*log(c)) - (2\*cos(3\*a) + 2\*I\*sin(3\*a))\*sin(6\*log(c)))\*x\*e^(6\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x))))/(((cos(4\*a) + I\*sin(4\*a))\*cos(8\*log(c)) - (-I\*cos(4\*a) + sin(4\*a))\*sin(8\*log(c)))\*e^(8\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x)))) - ((2\*cos(2\*a) + 2\*I\*sin(2\*a))\*cos(4\*log(c)) - 2\*(-I\*cos(2\*a) + sin(2\*a))\*sin(4\*log(c)))\*e^(4\*arctan2(sin(1/2\*log(x)), cos(1/2\*log(x)))) + 1)

**Fricas [A]** time = 0.450296, size = 161, normalized size = 3.16

$$\frac{2i x e^{\left(3i a + 6i \log\left(cx^{-\frac{1}{2}i}\right)\right)}}{e^{\left(4i a + 8i \log\left(cx^{-\frac{1}{2}i}\right)\right)} - 2 e^{\left(2i a + 4i \log\left(cx^{-\frac{1}{2}i}\right)\right)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+2\*log(c/(x^(1/2\*I))))^3,x, algorithm="fricas")

[Out] 2\*I\*x\*e^(3\*I\*a + 6\*I\*log(c\*x^(-1/2\*I)))/(e^(4\*I\*a + 8\*I\*log(c\*x^(-1/2\*I))) - 2\*e^(2\*I\*a + 4\*I\*log(c\*x^(-1/2\*I))) + 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^3\left(a + 2 \log\left(cx^{-\frac{i}{2}}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+2\*ln(c/(x\*\*(1/2\*I))))\*\*3,x)

[Out] Integral(csc(a + 2\*log(c\*x\*\*(-I/2)))\*\*3, x)

**Giac [B]** time = 4.75083, size = 112, normalized size = 2.2

$$\frac{4i c^{4i} x^2 e^{(2i a)}}{c^{10i} x^4 e^{(5i a)} - 2 c^{6i} x^2 e^{(3i a)} + c^{2i} e^{(i a)}} - \frac{2i}{c^{10i} x^4 e^{(5i a)} - 2 c^{6i} x^2 e^{(3i a)} + c^{2i} e^{(i a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+2\*log(c/(x^(1/2\*I))))^3,x, algorithm="giac")

[Out] 4\*I\*c^(4\*I)\*x^2\*e^(2\*I\*a)/(c^(10\*I)\*x^4\*e^(5\*I\*a) - 2\*c^(6\*I)\*x^2\*e^(3\*I\*a) + c^(2\*I)\*e^(I\*a)) - 2\*I/(c^(10\*I)\*x^4\*e^(5\*I\*a) - 2\*c^(6\*I)\*x^2\*e^(3\*I\*a) + c^(2\*I)\*e^(I\*a))

$$3.306 \quad \int \csc^p \left( a + \frac{i \log(cx^n)}{n(-2+p)} \right) dx$$

**Optimal.** Leaf size=96

$$\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}} \left( 1 - e^{2ia}(cx^n)^{\frac{2}{n(2-p)}} \right) \csc^p \left( a - \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)}$$

[Out]  $-\left((2-p)*x*(1 - E^{\left((2*I)*a\right)*(c*x^n)^{\frac{2}{n(2-p)}})}\right)*\text{Csc}\left[a - \frac{(I*\text{Log}[c*x^n])}{n(2-p)}\right]^p / \left(2*E^{\left((2*I)*a\right)*(1-p)*(c*x^n)^{\frac{2}{n(2-p)}}}\right)$

**Rubi [A]** time = 0.0883863, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4504, 4508, 261}

$$\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}} \left( 1 - e^{2ia}(cx^n)^{\frac{2}{n(2-p)}} \right) \csc^p \left( a - \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Csc}[a + (I*\text{Log}[c*x^n])/n*(-2 + p)]]^p, x]$

[Out]  $-\left((2-p)*x*(1 - E^{\left((2*I)*a\right)*(c*x^n)^{\frac{2}{n(2-p)}}}\right)*\text{Csc}\left[a - \frac{(I*\text{Log}[c*x^n])}{n(2-p)}\right]^p / \left(2*E^{\left((2*I)*a\right)*(1-p)*(c*x^n)^{\frac{2}{n(2-p)}}}\right)$

#### Rule 4504

$\text{Int}[\text{Csc}[\left((a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]\right)*(b_.)*(d_.)]^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[x/(n*(c*x^n)^{(1/n)}), \text{Subst}[\text{Int}[x^{(1/n - 1)}*\text{Csc}[d*(a + b*\text{Log}[x])]^p, x], x, c*x^n], x] /;$  FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

$\text{Int}[\text{Csc}[\left((a_.) + \text{Log}[x]*(b_.)\right)*(d_.)]^{(p_.)*\left((e_.)*(x_.)\right)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[\left(\text{Csc}[d*(a + b*\text{Log}[x])]^p*(1 - E^{(2*I*a*d)*x^{(2*I*b*d)}})\right)^p/x^{(I*b*d*p)}, \text{Int}[\left((e*x)^m*x^{(I*b*d*p)}\right)/(1 - E^{(2*I*a*d)*x^{(2*I*b*d)}})^p, x], x] /;$  FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 261

$\text{Int}[(x_.)^{(m_.)*\left((a_.) + (b_.)*(x_.)^{(n_.)}\right)^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p+1)}/(b*n*(p+1)), x] /;$  FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rubi steps



$$\begin{aligned} \int \csc^p \left( a + \frac{i \log(cx^n)}{n(-2+p)} \right) dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst} \left( \int x^{-1+\frac{1}{n}} \csc^p \left( a + \frac{i \log(x)}{n(-2+p)} \right) dx, x, cx^n \right)}{n} \\ &= \frac{\left( x(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}} \left( 1 - e^{2ia} (cx^n)^{-\frac{2}{n(-2+p)}} \right)^p \csc^p \left( a + \frac{i \log(cx^n)}{n(-2+p)} \right) \right) \operatorname{Subst} \left( \int x^{-1+\frac{1}{n}-\frac{p}{n(-2+p)}} \left( 1 - e^{2ia} (cx^n)^{-\frac{2}{n(-2+p)}} \right)^p \csc^p \left( a + \frac{i \log(cx^n)}{n(-2+p)} \right) dx, x, cx^n \right)}{n} \\ &= -\frac{e^{-2ia} (2-p) x (cx^n)^{-\frac{2}{n(2-p)}} \left( 1 - e^{2ia} (cx^n)^{\frac{2}{n(2-p)}} \right) \csc^p \left( a - \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)} \end{aligned}$$

**Mathematica [A]** time = 2.06899, size = 155, normalized size = 1.61

$$\frac{2^{p-1} (p-2) x e^{-\frac{2iap}{p-2}} \left( e^{\frac{2iap}{p-2}} - e^{\frac{4ia}{p-2}} (cx^n)^{\frac{2}{n(p-2)}} \right) \left( -\frac{ie^{\frac{ia(p+2)}{p-2}} (cx^n)^{\frac{1}{n(p-2)}}}{e^{\frac{4ia}{p-2}} (cx^n)^{\frac{2}{n(p-2)}} - e^{\frac{2iap}{p-2}}} \right)^p}{p-1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + (I\*Log[c\*x^n])/(n\*(-2 + p))]^p, x]

[Out] (2^(-1 + p)\*(-2 + p)\*x\*(E^(((2\*I)\*a\*p)/(-2 + p)) - E^(((4\*I)\*a)/(-2 + p)))\*(c\*x^n)^(2/(n\*(-2 + p))))\*(((-I)\*E^(((I\*a\*(2 + p))/(-2 + p)))\*(c\*x^n)^(1/(n\*(-2 + p)))))/(-E^(((2\*I)\*a\*p)/(-2 + p)) + E^(((4\*I)\*a)/(-2 + p))\*(c\*x^n)^(2/(n\*(-2 + p))))^p)/(E^(((2\*I)\*a\*p)/(-2 + p))\*(-1 + p))

**Maple [F]** time = 0.278, size = 0, normalized size = 0.

$$\int \left( \csc \left( a + \frac{i \ln(cx^n)}{n(p-2)} \right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+I\*ln(c\*x^n)/n/(p-2))^p, x)

[Out] int(csc(a+I\*ln(c\*x^n)/n/(p-2))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc \left( a + \frac{i \log(cx^n)}{n(p-2)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+I\*log(c\*x^n)/n/(-2+p))^p, x, algorithm="maxima")

[Out] integrate(csc(a + I\*log(c\*x^n)/(n\*(p - 2)))^p, x)

**Fricas [A]** time = 0.510156, size = 343, normalized size = 3.57

$$\frac{\left( (p-2)x e^{\left( \frac{2(ianp-2ian-\log(cx^n))}{np-2n} \right)} - (p-2)x \right) \left( \frac{2i e^{\left( \frac{i anp-2i an-\log(cx^n)}{np-2n} \right)}}{e^{\left( \frac{2(ianp-2ian-\log(cx^n))}{np-2n} \right)} - 1} \right)^p e^{\left( -\frac{2(ianp-2ian-\log(cx^n))}{np-2n} \right)}}{2(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="fricas")

[Out] 1/2\*((p - 2)\*x\*e^(2\*(I\*a\*n\*p - 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n)) - (p - 2)\*x)\*(2\*I\*e^((I\*a\*n\*p - 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n))/(e^(2\*(I\*a\*n\*p - 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n)) - 1))^p\*e^(-2\*(I\*a\*n\*p - 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n))/(p - 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^p \left( a + \frac{i \log(cx^n)}{n(p-2)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+I\*ln(c\*x\*\*n)/n/(-2+p))\*\*p,x)

[Out] Integral(csc(a + I\*log(c\*x\*\*n)/(n\*(p - 2)))\*\*p, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc \left( a + \frac{i \log(cx^n)}{n(p-2)} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="giac")

[Out] integrate(csc(a + I\*log(c\*x^n)/(n\*(p - 2)))^p, x)

$$3.307 \quad \int \csc^p \left( a - \frac{i \log(cx^n)}{n(-2+p)} \right) dx$$

**Optimal.** Leaf size=71

$$\frac{(2-p)x \left( 1 - e^{2ia} (cx^n)^{-\frac{2}{n(2-p)}} \right) \csc^p \left( a + \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)}$$

[Out] ((2 - p)\*x\*(1 - E^((2\*I)\*a)/(c\*x^n)^(2/(n\*(2 - p))))\*Csc[a + (I\*Log[c\*x^n])/(n\*(2 - p))]^p)/(2\*(1 - p))

**Rubi [A]** time = 0.0759444, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$ , Rules used = {4504, 4508, 264}

$$\frac{(2-p)x \left( 1 - e^{2ia} (cx^n)^{-\frac{2}{n(2-p)}} \right) \csc^p \left( a + \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)}$$

Antiderivative was successfully verified.

[In] Int[Csc[a - (I\*Log[c\*x^n])/(n\*(-2 + p))]^p, x]

[Out] ((2 - p)\*x\*(1 - E^((2\*I)\*a)/(c\*x^n)^(2/(n\*(2 - p))))\*Csc[a + (I\*Log[c\*x^n])/(n\*(2 - p))]^p)/(2\*(1 - p))

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 264

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))]^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \csc^p \left( a - \frac{i \log(cx^n)}{n(-2+p)} \right) dx &= \frac{(x (cx^n)^{-1/n}) \operatorname{Subst} \left( \int x^{-1+\frac{1}{n}} \csc^p \left( a - \frac{i \log(x)}{n(-2+p)} \right) dx, x, cx^n \right)}{n} \\ &= \frac{\left( x (cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}} \left( 1 - e^{2ia} (cx^n)^{\frac{2}{n(-2+p)}} \right)^p \csc^p \left( a - \frac{i \log(cx^n)}{n(-2+p)} \right) \right) \operatorname{Subst} \left( \int x^{-1+\frac{1}{n}+\frac{p}{n(-2+p)}} \left( 1 - e^{2ia} (cx^n)^{\frac{2}{n(-2+p)}} \right)^p dx, x, cx^n \right)}{n} \\ &= \frac{(2-p)x \left( 1 - e^{2ia} (cx^n)^{-\frac{2}{n(2-p)}} \right) \csc^p \left( a + \frac{i \log(cx^n)}{n(2-p)} \right)}{2(1-p)} \end{aligned}$$

**Mathematica [A]** time = 3.00541, size = 128, normalized size = 1.8

$$\frac{2^{p-1}(p-2)x \left( \frac{ie^{ia}(cx^n)^{\frac{1}{n(p-2)}}}{-1+e^{2ia}(cx^n)^{\frac{2}{n(p-2)}}} \right)^p \left( 1 + e^{2ia} (cx^n)^{\frac{2}{n(p-2)}} \left( -1 + \left( 1 - e^{-2ia} (cx^n)^{-\frac{2}{n(p-2)}} \right)^p \right) \right)}{p-1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a - (I\*Log[c\*x^n])/(n\*(-2 + p))]^p,x]

[Out] (2^(-1 + p)\*(-2 + p)\*x\*((I\*E^(I\*a))\*(c\*x^n)^(1/(n\*(-2 + p))))/(-1 + E^((2\*I)\*a)\*(c\*x^n)^(2/(n\*(-2 + p)))))^p\*(1 + E^((2\*I)\*a)\*(c\*x^n)^(2/(n\*(-2 + p))))\*(-1 + (1 - 1/(E^((2\*I)\*a)\*(c\*x^n)^(2/(n\*(-2 + p))))))^p)/(-1 + p)

**Maple [F]** time = 0.278, size = 0, normalized size = 0.

$$\int \left( \csc \left( a - \frac{i \ln(cx^n)}{n(p-2)} \right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a-I\*ln(c\*x^n)/n/(p-2))^p,x)

[Out] int(csc(a-I\*ln(c\*x^n)/n/(p-2))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( -\csc \left( -a + \frac{i \log(cx^n)}{n(p-2)} \right) \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a-I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="maxima")

[Out] integrate((-csc(-a + I\*log(c\*x^n)/(n\*(p - 2))))^p, x)

**Fricas [B]** time = 0.499228, size = 350, normalized size = 4.93

$$\frac{\left( (p-2)xe^{\frac{2(-ianp+2ian-\log(cx^n))}{np-2n}} - (p-2)x \right) \left( -\frac{2ie^{\frac{-ianp+2ian-\log(cx^n)}{np-2n}}}{e^{\frac{2(-ianp+2ian-\log(cx^n))}{np-2n}} - 1} \right)^p e^{\frac{-2(-ianp+2ian-\log(cx^n))}{np-2n}}}{2(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a-I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="fricas")

[Out] 1/2\*((p - 2)\*x\*e^(2\*(-I\*a\*n\*p + 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n)) - (p - 2)\*x)\*(-2\*I\*e^((-I\*a\*n\*p + 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n))/(e^(2\*(-I\*a\*n\*p + 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n)) - 1))^p\*e^(-2\*(-I\*a\*n\*p + 2\*I\*a\*n - log(c\*x^n))/(n\*p - 2\*n))/(p - 1)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^p \left( a - \frac{i \log(cx^n)}{n(p-2)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a-I\*ln(c\*x\*\*n)/n/(-2+p))\*\*p,x)

[Out] Integral(csc(a - I\*log(c\*x\*\*n)/(n\*(p - 2)))\*\*p, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc \left( a - \frac{i \log(cx^n)}{n(p-2)} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a-I\*log(c\*x^n)/n/(-2+p))^p,x, algorithm="giac")

[Out] integrate(csc(a - I\*log(c\*x^n)/(n\*(p - 2)))^p, x)

### 3.308 $\int \sqrt{\csc(a + b \log(cx^n))} dx$

**Optimal.** Leaf size=109

$$\frac{2x\sqrt{1 - e^{2ia}(cx^n)^{2ib}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right), \frac{1}{4}\left(5 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sqrt{\csc(a + b \log(cx^n))}}{2 + ibn}$$

[Out] (2\*x\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*Hypergeometric2F1[1/2, (1 - (2\*I)/(b\*n))/4, (5 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/(2 + I\*b\*n)

**Rubi [A]** time = 0.0705614, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4504, 4508, 364}

$$\frac{2x\sqrt{1 - e^{2ia}(cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right); \frac{1}{4}\left(5 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \sqrt{\csc(a + b \log(cx^n))}}{2 + ibn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csc[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*Hypergeometric2F1[1/2, (1 - (2\*I)/(b\*n))/4, (5 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/(2 + I\*b\*n)

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d))\*x^(2\*I\*b\*d)]^p/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d))\*x^(2\*I\*b\*d)]^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))]^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \sqrt{\csc(a + b \log(cx^n))} dx &= \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \sqrt{\csc(a + b \log(x))} dx, x, cx^n\right)}{n} \\ &= \frac{\left(x(cx^n)^{-\frac{ib}{2}-\frac{1}{n}} \sqrt{1 - e^{2ia}(cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))}\right) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1}{n}}}{\sqrt{1 - e^{2ia}x^{2ib}}} dx, x, cx^n\right)}{n} \\ &= \frac{2x \sqrt{1 - e^{2ia}(cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))} {}_2F_1\left(\frac{1}{2}, \frac{1}{4}\left(1 - \frac{2i}{bn}\right); \frac{1}{4}\left(5 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{2 + ibn} \end{aligned}$$

**Mathematica [A]** time = 0.645026, size = 115, normalized size = 1.06

$$\frac{2ie^{-2ia}x(cx^n)^{-2ib}(-1 + e^{2i(a+b \log(cx^n))})\sqrt{\csc(a + b \log(cx^n))}\operatorname{Hypergeometric2F1}\left(1, \frac{3}{4} + \frac{i}{2bn}, \frac{5}{4} + \frac{i}{2bn}, e^{-2i(a+b \log(cx^n))}\right)}{bn + 2i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Csc[a + b\*Log[c\*x^n]]], x]

[Out] ((2\*I)\*(-1 + E^((2\*I)\*(a + b\*Log[c\*x^n])))\*x\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*Hypergeometric2F1[1, 3/4 + (I/2)/(b\*n), 5/4 + (I/2)/(b\*n), E^((-2\*I)\*(a + b\*Log[c\*x^n]))])/(E^((2\*I)\*a)\*(2\*I + b\*n)\*(c\*x^n)^((2\*I)\*b))

**Maple [F]** time = 0.364, size = 0, normalized size = 0.

$$\int \sqrt{\csc(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(csc(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\csc(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(csc(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\csc(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(a + b*log(c*x**n))), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```



$$3.309 \quad \int \frac{\sqrt{\csc(a+b \log(cx^n))}}{x} dx$$

**Optimal.** Leaf size=59

$$\frac{2\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}\text{EllipticF}\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right), 2\right)}{bn}$$

[Out] (2\*Sqrt[Csc[a + b\*Log[c\*x^n]])\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(b\*n)

**Rubi [A]** time = 0.0412643, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3771, 2641}

$$\frac{2\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}F\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle| 2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csc[a + b\*Log[c\*x^n]]]/x, x]

[Out] (2\*Sqrt[Csc[a + b\*Log[c\*x^n]])\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(b\*n)

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\csc(a+b \log(cx^n))}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{\csc(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\left(\sqrt{\csc(a+b \log(cx^n))}\sqrt{\sin(a+b \log(cx^n))}\right) \text{Subst}\left(\int \frac{1}{\sqrt{\sin(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2\sqrt{\csc(a+b \log(cx^n))}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b \log(cx^n)\right)\middle| 2\right)\sqrt{\sin(a+b \log(cx^n))}}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.116522, size = 58, normalized size = 0.98

$$\frac{2\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}\text{EllipticF}\left(\frac{1}{4}(-2a-2b \log(cx^n)+\pi), 2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Csc[a + b\*Log[c\*x^n]]]/x,x]

[Out] (-2\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*EllipticF[(-2\*a + Pi - 2\*b\*Log[c\*x^n])/4, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(b\*n)

**Maple [A]** time = 0.799, size = 102, normalized size = 1.7

$$\frac{1}{n \cos(a + b \ln(cx^n)) b} \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n)) + 2} \sqrt{-\sin(a + b \ln(cx^n))} \text{EllipticF}\left(\sqrt{\sin(a + b \ln(cx^n))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^(1/2)/x,x)

[Out] 1/n\*(sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticF((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))/cos(a+b\*ln(c\*x^n))/sin(a+b\*ln(c\*x^n))^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\csc(b \log(cx^n) + a)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(csc(b\*log(c\*x^n) + a))/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\csc(b \log(cx^n) + a)}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(csc(b\*log(c\*x^n) + a))/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\csc(a + b \log(cx^n))}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*(1/2)/x,x)

[Out]  $\text{Integral}(\sqrt{\csc(a + b \cdot \log(cx^n))}/x, x)$

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")`

[Out] Timed out

### 3.310 $\int \csc^{\frac{3}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=109

$$\frac{2x(1 - e^{2ia}(cx^n)^{2ib})^{3/2} \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), \frac{1}{4}\left(7 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \csc^{\frac{3}{2}}(a + b \log(cx^n))}{2 + 3ibn}$$

[Out] (2\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Csc[a + b\*Log[c\*x^n]]^(3/2)\*Hypergeometric2F1[3/2, (3 - (2\*I)/(b\*n))/4, (7 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/(2 + (3\*I)\*b\*n)

**Rubi [A]** time = 0.0701248, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4504, 4508, 364}

$$\frac{2x(1 - e^{2ia}(cx^n)^{2ib})^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right); \frac{1}{4}\left(7 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \csc^{\frac{3}{2}}(a + b \log(cx^n))}{2 + 3ibn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Csc[a + b\*Log[c\*x^n]]^(3/2)\*Hypergeometric2F1[3/2, (3 - (2\*I)/(b\*n))/4, (7 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/(2 + (3\*I)\*b\*n)

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \csc^{\frac{3}{2}}(a + b \log(cx^n)) dx = \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int x^{-1+\frac{1}{n}} \csc^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{3ib}{2}-\frac{1}{n}} (1 - e^{2ia}(cx^n)^{2ib})\right)^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n)) \text{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1}{n}}}{(1-e^{2ia}x^{2ib})^{3/2}} dx, x, cx^n\right)}{n}$$

$$= \frac{2x(1 - e^{2ia}(cx^n)^{2ib})^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n)) {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right); \frac{1}{4}\left(7 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{2 + 3ibn}$$

**Mathematica [B]** time = 6.05775, size = 411, normalized size = 3.77

$$\frac{x \left( (b^2 n^2 + 4) x^{ibn} \sqrt{2 - 2e^{2ia}(cx^n)^{2ib}} \sqrt{\frac{e^{ia}(cx^n)^{ib}}{-1 + e^{2ia}(cx^n)^{2ib}}} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4} - \frac{i}{2bn}, \frac{7}{4} - \frac{i}{2bn}, e^{2ia}(cx^n)^{2ib}\right) - (3bn - 2) \right)}{bn(3bn - 2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (x\*((4 + b^2\*n^2)\*x^(I\*b\*n)\*Sqrt[2 - 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[(I\*E^(I\*a)\*(c\*x^n)^(I\*b))/(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)] - ((-2\*I + 3\*b\*n)\*((2\*I - b\*n)\*Sqrt[2 - 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[(I\*E^(I\*a)\*(c\*x^n)^(I\*b))/(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Hypergeometric2F1[1/2, -(2\*I + b\*n)/(4\*b\*n), 3/4 - (I/2)/(b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)] + 2\*x^(I\*b\*n)\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*(b\*n\*Cos[b\*n\*Log[x]] - 2\*Sin[b\*n\*Log[x]])))/x^(I\*b\*n))/(b\*n\*(-2\*I + 3\*b\*n)\*(b\*n\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]] + 2\*Sin[a - b\*n\*Log[x] + b\*Log[c\*x^n]]))

**Maple [F]** time = 0.295, size = 0, normalized size = 0.

$$\int (\csc(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(csc(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a)^(3/2), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.311 \quad \int \frac{\csc^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=94

$$\frac{2 \cos(a + b \log(cx^n)) \sqrt{\csc(a + b \log(cx^n))}}{bn} - \frac{2 \sqrt{\sin(a + b \log(cx^n))} \sqrt{\csc(a + b \log(cx^n))} E\left(\frac{1}{2}(a + b \log(cx^n) - \frac{\pi}{2})\right)}{bn}$$

[Out] (-2\*Cos[a + b\*Log[c\*x^n]]\*Sqrt[Csc[a + b\*Log[c\*x^n]]])/(b\*n) - (2\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*EllipticE[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(b\*n)

**Rubi [A]** time = 0.0549318, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3768, 3771, 2639}

$$\frac{2 \cos(a + b \log(cx^n)) \sqrt{\csc(a + b \log(cx^n))}}{bn} - \frac{2 \sqrt{\sin(a + b \log(cx^n))} \sqrt{\csc(a + b \log(cx^n))} E\left(\frac{1}{2}(a + b \log(cx^n) - \frac{\pi}{2})\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^(3/2)/x, x]

[Out] (-2\*Cos[a + b\*Log[c\*x^n]]\*Sqrt[Csc[a + b\*Log[c\*x^n]]])/(b\*n) - (2\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*EllipticE[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(b\*n)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{\csc^{\frac{3}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \csc^{\frac{3}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2 \cos(a + b \log(cx^n)) \sqrt{\csc(a + b \log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\csc(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2 \cos(a + b \log(cx^n)) \sqrt{\csc(a + b \log(cx^n))}}{bn} - \frac{(\sqrt{\csc(a + b \log(cx^n))} \sqrt{\sin(a + b \log(cx^n))})}{n} \\
&= -\frac{2 \cos(a + b \log(cx^n)) \sqrt{\csc(a + b \log(cx^n))}}{bn} - \frac{2\sqrt{\csc(a + b \log(cx^n))} E\left(\frac{1}{2}\left(a - \frac{\pi}{2} + b \log(cx^n)\right)\right)}{bn}
\end{aligned}$$

**Mathematica [A]** time = 0.152216, size = 72, normalized size = 0.77

$$\frac{2\sqrt{\csc(a + b \log(cx^n))} \left( \cos(a + b \log(cx^n)) - \sqrt{\sin(a + b \log(cx^n))} E\left(\frac{1}{4}(-2a - 2b \log(cx^n) + \pi) \middle| 2\right) \right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^(3/2)/x,x]

[Out] (-2\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*(Cos[a + b\*Log[c\*x^n]] - EllipticE[(-2\*a + Pi - 2\*b\*Log[c\*x^n])/4, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]]))/(b\*n)

**Maple [A]** time = 1.276, size = 190, normalized size = 2.

$$\frac{1}{n \cos(a + b \ln(cx^n)) b} \left( 2 \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n)) + 2} \sqrt{-\sin(a + b \ln(cx^n))} \text{EllipticE}\left(\sqrt{\sin(a + b \ln(cx^n)) + 1}, \frac{1}{2}\right) - (\sin(a + b \ln(cx^n)) + 1)^{1/2} \text{EllipticE}\left(\frac{\sin(a + b \ln(cx^n)) + 1}{2}, \frac{1}{2}\right) - (\sin(a + b \ln(cx^n)) + 1)^{1/2} \text{EllipticF}\left(\frac{\sin(a + b \ln(cx^n)) + 1}{2}, \frac{1}{2}\right) - 2 \cos(a + b \ln(cx^n)) \sqrt{-\sin(a + b \ln(cx^n))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^(3/2)/x,x)

[Out] 1/n\*(2\*(sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticE((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))-sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticF((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))-2\*cos(a+b\*ln(c\*x^n))^(1/2)/cos(a+b\*ln(c\*x^n))/sin(a+b\*ln(c\*x^n))^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a)^(3/2)/x, x)



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(b \log(cx^n) + a)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="fricas")

[Out] integral(csc(b\*log(c\*x^n) + a)^(3/2)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*(3/2)/x,x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(3/2)/x,x, algorithm="giac")

[Out] Timed out

### 3.312 $\int \csc^{\frac{5}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=109

$$\frac{2x(1 - e^{2ia}(cx^n)^{2ib})^{5/2} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right), \frac{1}{4}\left(9 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \csc^{\frac{5}{2}}(a + b \log(cx^n))}{2 + 5ibn}$$

[Out] (2\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Csc[a + b\*Log[c\*x^n]]^(5/2)\*Hypergeometric2F1[5/2, (5 - (2\*I)/(b\*n))/4, (9 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/(2 + (5\*I)\*b\*n)

**Rubi [A]** time = 0.0717096, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4504, 4508, 364}

$$\frac{2x(1 - e^{2ia}(cx^n)^{2ib})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right); \frac{1}{4}\left(9 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right) \csc^{\frac{5}{2}}(a + b \log(cx^n))}{2 + 5ibn}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Csc[a + b\*Log[c\*x^n]]^(5/2)\*Hypergeometric2F1[5/2, (5 - (2\*I)/(b\*n))/4, (9 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/(2 + (5\*I)\*b\*n)

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \csc^{\frac{5}{2}}(a + b \log(cx^n)) dx = \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int x^{-1+\frac{1}{n}} \csc^{\frac{5}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{5ib}{2}-\frac{1}{n}}(1 - e^{2ia}(cx^n)^{2ib})\right)^{5/2} \csc^{\frac{5}{2}}(a + b \log(cx^n)) \text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1}{n}}}{(1 - e^{2ia}x^{2ib})^{5/2}} dx, x, cx^n\right)}{n}$$

$$= \frac{2x(1 - e^{2ia}(cx^n)^{2ib})^{5/2} \csc^{\frac{5}{2}}(a + b \log(cx^n)) {}_2F_1\left(\frac{5}{2}, \frac{1}{4}\left(5 - \frac{2i}{bn}\right); \frac{1}{4}\left(9 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{2 + 5ibn}$$

**Mathematica [A]** time = 1.82415, size = 174, normalized size = 1.6

$$\frac{2x^{1-2ibn} e^{-2i(a+b \log(cx^n)-bn \log(x))} \sqrt{\csc(a + b \log(cx^n))} \left( (2 + ibn) (-1 + e^{2ia}(cx^n)^{2ib}) \text{Hypergeometric2F1}\left(1, \frac{3}{4} + \frac{i}{2bn}, \frac{5}{4} + \frac{i}{2bn}, e^{2ia}(cx^n)^{2ib}\right) \right)}{3b^2 n^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x^(1 - (2\*I)\*b\*n)\*Sqrt[Csc[a + b\*Log[c\*x^n]])\*(-(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)\*(2 + b\*n\*Cot[a + b\*Log[c\*x^n]])) + (2 + I\*b\*n)\*(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, 3/4 + (I/2)/(b\*n), 5/4 + (I/2)/(b\*n), E^((-2\*I)\*(a + b\*Log[c\*x^n]))])/(3\*b^2\*E^((2\*I)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))\*n^2)

**Maple [F]** time = 0.296, size = 0, normalized size = 0.

$$\int (\csc(a + b \ln(cx^n)))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^(5/2), x)

[Out] int(csc(a+b\*ln(c\*x^n))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(5/2), x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a)^(5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.313 \quad \int \frac{\csc^2(a+b \log(cx^n))}{x} dx$$

**Optimal.** Leaf size=98

$$\frac{2\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}\text{EllipticF}\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right), 2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n)) \csc^{\frac{3}{2}}(a+b \log(cx^n))}{3bn}$$

[Out]  $(-2*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Csc}[a + b*\text{Log}[c*x^n]]^{(3/2)})/(3*b*n) + (2*\text{Sqrt}[\text{Cs}c[a + b*\text{Log}[c*x^n]]]*\text{EllipticF}[(a - \text{Pi}/2 + b*\text{Log}[c*x^n])/2, 2]*\text{Sqrt}[\text{Sin}[a + b*\text{Log}[c*x^n]]])/(3*b*n)$

**Rubi [A]** time = 0.0583563, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3768, 3771, 2641}

$$\frac{2\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}F\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n)) \csc^{\frac{3}{2}}(a+b \log(cx^n))}{3bn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Csc}[a + b*\text{Log}[c*x^n]]^{(5/2)}/x, x]$

[Out]  $(-2*\text{Cos}[a + b*\text{Log}[c*x^n]]*\text{Csc}[a + b*\text{Log}[c*x^n]]^{(3/2)})/(3*b*n) + (2*\text{Sqrt}[\text{Cs}c[a + b*\text{Log}[c*x^n]]]*\text{EllipticF}[(a - \text{Pi}/2 + b*\text{Log}[c*x^n])/2, 2]*\text{Sqrt}[\text{Sin}[a + b*\text{Log}[c*x^n]]])/(3*b*n)$

#### Rule 3768

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Csc}[c + d*x])^{(n-1)})/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

#### Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n * \text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$

#### Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x\_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

#### Rubi steps

$$\begin{aligned}
\int \frac{\csc^{\frac{5}{2}}(a + b \log(cx^n))}{x} dx &= \frac{\text{Subst}\left(\int \csc^{\frac{5}{2}}(a + bx) dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2 \cos(a + b \log(cx^n)) \csc^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\csc(a + bx)} dx, x, \log(cx^n)\right)}{3n} \\
&= -\frac{2 \cos(a + b \log(cx^n)) \csc^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{\left(\sqrt{\csc(a + b \log(cx^n))} \sqrt{\sin(a + b \log(cx^n))}\right)}{3n} \\
&= -\frac{2 \cos(a + b \log(cx^n)) \csc^{\frac{3}{2}}(a + b \log(cx^n))}{3bn} + \frac{2\sqrt{\csc(a + b \log(cx^n))} F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + b \log(cx^n)\right)\right)}{3bn}
\end{aligned}$$

**Mathematica [A]** time = 0.186015, size = 73, normalized size = 0.74

$$\frac{2 \csc^{\frac{3}{2}}(a + b \log(cx^n)) \left( \sin^{\frac{3}{2}}(a + b \log(cx^n)) \text{EllipticF}\left(\frac{1}{4}(-2a - 2b \log(cx^n) + \pi), 2\right) + \cos(a + b \log(cx^n)) \right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^(5/2)/x,x]

[Out] (-2\*Csc[a + b\*Log[c\*x^n]]^(3/2)\*(Cos[a + b\*Log[c\*x^n]] + EllipticF[(-2\*a + Pi - 2\*b\*Log[c\*x^n])/4, 2]\*Sin[a + b\*Log[c\*x^n]]^(3/2)))/(3\*b\*n)

**Maple [A]** time = 1.118, size = 131, normalized size = 1.3

$$\frac{1}{3n \cos(a + b \ln(cx^n)) b} \left( \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n)) + 2} \sqrt{-\sin(a + b \ln(cx^n))} \text{EllipticF}\left(\sqrt{\sin(a + b \ln(cx^n)) + 1}, \frac{1}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^(5/2)/x,x)

[Out] 1/3/n/sin(a+b\*ln(c\*x^n))^(3/2)\*((sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticF((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))\*sin(a+b\*ln(c\*x^n))-2\*cos(a+b\*ln(c\*x^n))^2)/cos(a+b\*ln(c\*x^n))/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(b \log(cx^n) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a)^(5/2)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\csc(b \log(cx^n) + a)^{\frac{5}{2}}}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="fricas")

[Out] integral(csc(b\*log(c\*x^n) + a)^(5/2)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*ln(c\*x\*\*n))\*\*(5/2)/x,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^(5/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.314 \quad \int \frac{1}{\sqrt{\csc(a+b \log(cx^n))}} dx$$

**Optimal.** Leaf size=110

$$\frac{{}_2F_1\left(-\frac{1}{2}, -\frac{bn+2i}{4bn}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b \log(cx^n))}}$$

[Out] (2\*x\*Hypergeometric2F1[-1/2, -(2\*I + b\*n)/(4\*b\*n), (3 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/((2 - I\*b\*n)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Csc[a + b\*Log[c\*x^n]]])

**Rubi [A]** time = 0.0687148, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4504, 4508, 364}

$$\frac{{}_2F_1\left(-\frac{1}{2}, -\frac{bn+2i}{4bn}, \frac{1}{4}\left(3 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Csc[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x\*Hypergeometric2F1[-1/2, -(2\*I + b\*n)/(4\*b\*n), (3 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/((2 - I\*b\*n)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Csc[a + b\*Log[c\*x^n]]])

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.)]^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\int \frac{1}{\sqrt{\csc(a + b \log(cx^n))}} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sqrt{\csc(a+b \log(x))}} dx, x, cx^n\right)}{n}$$

$$= \frac{(x(cx^n)^{\frac{ib}{2}-\frac{1}{n}}) \operatorname{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1}{n}} \sqrt{1 - e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n\sqrt{1 - e^{2ia}(cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))}}$$

$$= \frac{2x {}_2F_1\left(-\frac{1}{2}, -\frac{2i+bn}{4bn}; \frac{1}{4}\left(3 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{(2 - ibn)\sqrt{1 - e^{2ia}(cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))}}$$

**Mathematica [B]** time = 4.03723, size = 377, normalized size = 3.43

$$\frac{2x \sin(a + b \log(cx^n) - bn \log(x))}{\sqrt{\csc(a + b \log(cx^n))} (2 \sin(a + b \log(cx^n) - bn \log(x)) + bn \cos(a + b \log(cx^n) - bn \log(x)))} - \frac{2e^{ia}bnx (cx^n)^{ib} \sqrt{2 - 2E^{((2I)a)(cx^n)^{(2I)b}}}}{\dots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[Csc[a + b*Log[c*x^n]]], x]

[Out] (-2*b*E^(I*a)*n*x*(c*x^n)^(I*b)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*((2*I + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)] + (-2*I + 3*b*n)*Hypergeometric2F1[1/2, -(2*I + b*n)/(4*b*n), 3/4 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2*I + b*n)*(-2*I + 3*b*n)*((2*I + b*n)*x^((2*I)*b*n) + E^((2*I)*a)*(-2*I + b*n)*(c*x^n)^((2*I)*b))) + (2*x*Sin[a - b*n*Log[x] + b*Log[c*x^n]])/(Sqrt[Csc[a + b*Log[c*x^n]]]*(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + 2*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

**Maple [F]** time = 0.319, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\csc(a + b \ln(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/csc(a+b*ln(c*x^n))^(1/2), x)

[Out] int(1/csc(a+b*ln(c*x^n))^(1/2), x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\csc(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/csc(a+b*log(c*x^n))^(1/2), x, algorithm="maxima")
```

[Out] integrate(1/sqrt(csc(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csc(a+b\*log(c\*x^n))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\csc(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csc(a+b\*ln(c\*x\*\*n))\*\*(1/2),x)

[Out] Integral(1/sqrt(csc(a + b\*log(c\*x\*\*n))), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csc(a+b\*log(c\*x^n))^(1/2),x, algorithm="giac")

[Out] Timed out

$$3.315 \quad \int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}} dx$$

**Optimal.** Leaf size=59

$$\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

[Out] (2\*Sqrt[Csc[a + b\*Log[c\*x^n]])\*EllipticE[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(b\*n)

**Rubi [A]** time = 0.0404361, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {3771, 2639}

$$\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[Csc[a + b\*Log[c\*x^n]]]), x]

[Out] (2\*Sqrt[Csc[a + b\*Log[c\*x^n]])\*EllipticE[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(b\*n)

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^n, x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] :> Simp[(2\*EllipticE[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\csc(a+bx)}} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\left(\sqrt{\csc(a+b\log(cx^n))}\sqrt{\sin(a+b\log(cx^n))}\right) \text{Subst}\left(\int \sqrt{\sin(a+bx)} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{2\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+b\log(cx^n)\right)\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.108274, size = 58, normalized size = 0.98

$$\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{4}(-2a-2b\log(cx^n)+\pi)\middle|2\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[Csc[a + b\*Log[c\*x^n]]]),x]

[Out]  $(-2\sqrt{\text{Csc}[a + b\text{Log}[c*x^n]]}\text{EllipticE}[(-2*a + \text{Pi} - 2*b*\text{Log}[c*x^n])/4, 2]*\sqrt{\text{Sin}[a + b*\text{Log}[c*x^n]]})/(b*n)$

**Maple [A]** time = 1.305, size = 129, normalized size = 2.2

$$-\frac{1}{n \cos(a + b \ln(cx^n))b} \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n)) + 2} \sqrt{-\sin(a + b \ln(cx^n))} \left( 2 \text{EllipticE} \left( \sqrt{\sin(a + b \ln(cx^n)) + 1}, \frac{1}{2} \right) - \text{EllipticF} \left( \sqrt{\sin(a + b \ln(cx^n)) + 1}, \frac{1}{2} \right) \right) / \cos(a + b \ln(cx^n)) / \sin(a + b \ln(cx^n))^{1/2} / b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/csc(a+b\*ln(c\*x^n))^(1/2),x)

[Out]  $-1/n*(\sin(a+b*\ln(c*x^n))+1)^{(1/2)}*(-2*\sin(a+b*\ln(c*x^n))+2)^{(1/2)}*(-\sin(a+b*\ln(c*x^n)))^{(1/2)}*(2*\text{EllipticE}((\sin(a+b*\ln(c*x^n))+1)^{(1/2)},1/2*2^{(1/2)})-\text{EllipticF}((\sin(a+b*\ln(c*x^n))+1)^{(1/2)},1/2*2^{(1/2)}))/\cos(a+b*\ln(c*x^n))/\sin(a+b*\ln(c*x^n))^{(1/2)}/b$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\text{csc}(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*log(c\*x^n))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(x\*sqrt(csc(b\*log(c\*x^n) + a))), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{x\sqrt{\text{csc}(b \log(cx^n) + a)}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*log(c\*x^n))^(1/2),x, algorithm="fricas")

[Out] integral(1/(x\*sqrt(csc(b\*log(c\*x^n) + a))), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\text{csc}(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*ln(c\*x\*\*n))\*\*(1/2),x)

[Out]  $\text{Integral}(1/(x\sqrt{\csc(a + b\log(cx^n))}), x)$

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")`

[Out] Timed out

$$3.316 \quad \int \frac{1}{\csc^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=109

$$\frac{{}_2x\text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right), \frac{1}{4}\left(1 - \frac{2i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1 - e^{2ia}(cx^n)^{2ib}\right)^{3/2} \csc^{\frac{3}{2}}(a+b \log(cx^n))}$$

[Out] (2\*x\*Hypergeometric2F1[-3/2, (-3 - (2\*I)/(b\*n))/4, (1 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 - (3\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Csc[a + b\*Log[c\*x^n]]^(3/2)

**Rubi [A]** time = 0.0707769, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4504, 4508, 364}

$$\frac{{}_2x_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right); \frac{1}{4}\left(1 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1 - e^{2ia}(cx^n)^{2ib}\right)^{3/2} \csc^{\frac{3}{2}}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^(-3/2), x]

[Out] (2\*x\*Hypergeometric2F1[-3/2, (-3 - (2\*I)/(b\*n))/4, (1 - (2\*I)/(b\*n))/4, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 - (3\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Csc[a + b\*Log[c\*x^n]]^(3/2)

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d))\*x^(2\*I\*b\*d)]^p/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d))\*x^(2\*I\*b\*d)]^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.)]^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{\csc^{\frac{3}{2}}(a + b \log(cx^n))} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\csc^{\frac{3}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{(x(cx^n)^{\frac{3ib}{2}-\frac{1}{n}}) \operatorname{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1}{n}} (1 - e^{2ia}x^{2ib})^{3/2} dx, x, cx^n\right)}{n(1 - e^{2ia}(cx^n)^{2ib})^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n))}$$

$$= \frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i}{bn}\right); \frac{1}{4}\left(1 - \frac{2i}{bn}\right); e^{2ia}(cx^n)^{2ib}\right)}{(2 - 3ibn)(1 - e^{2ia}(cx^n)^{2ib})^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n))}$$

**Mathematica [A]** time = 2.31086, size = 186, normalized size = 1.71

$$\frac{2ix \left( (2 - ibn) (3bn \cot(a + b \log(cx^n)) - 2) - 3e^{-2ia}b^2n^2 (cx^n)^{-2ib} (-1 + e^{2ia}(cx^n)^{2ib}) \csc^2(a + b \log(cx^n)) \right) \operatorname{Hypergeometric2F1}\left[1, \frac{3}{4} + \frac{i}{2}(bn), \frac{5}{4} + \frac{i}{2}(bn), E^{(-2i)(a + b \log(cx^n))}\right]}{(-3bn + 2i)(bn + 2i)(3bn + 2i) \csc^{\frac{3}{2}}(a + b \log(cx^n))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^(-3/2), x]

[Out]  $((2I)*x*((2 - I*b*n)*(-2 + 3*b*n*\cot[a + b*\log[c*x^n]])) - (3*b^2*n^2*(-1 + E^{((2I)*a)*(c*x^n)^{((2I)*b)}})*\csc[a + b*\log[c*x^n]]^2*\operatorname{Hypergeometric2F1}[1, 3/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n), E^{((-2I)*(a + b*\log[c*x^n])}]])/ (E^{((2I)*a)*(c*x^n)^{((2I)*b)}})) / ((2I - 3*b*n)*(2I + b*n)*(2I + 3*b*n)*\csc[a + b*\log[c*x^n]]^{(3/2)})$

**Maple [F]** time = 0.299, size = 0, normalized size = 0.

$$\int (\csc(a + b \ln(cx^n)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/csc(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(1/csc(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\csc(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csc(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a)^(-3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csc(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\csc^{\frac{3}{2}}(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csc(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Integral(csc(a + b\*log(c\*x\*\*n))\*\*(-3/2), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/csc(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] Timed out



$$3.317 \quad \int \frac{1}{x \csc^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=98

$$\frac{2\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}\text{EllipticF}\left(\frac{1}{2}(a+b \log(cx^n)-\frac{\pi}{2}), 2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n))}{3bn\sqrt{\csc(a+b \log(cx^n))}}$$

[Out] (-2\*Cos[a + b\*Log[c\*x^n]])/(3\*b\*n\*Sqrt[Csc[a + b\*Log[c\*x^n]]]) + (2\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(3\*b\*n)

**Rubi [A]** time = 0.0589235, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}F\left(\frac{1}{2}(a+b \log(cx^n)-\frac{\pi}{2})\middle|2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n))}{3bn\sqrt{\csc(a+b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Csc[a + b\*Log[c\*x^n]]^(3/2)), x]

[Out] (-2\*Cos[a + b\*Log[c\*x^n]])/(3\*b\*n\*Sqrt[Csc[a + b\*Log[c\*x^n]]]) + (2\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*EllipticF[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(3\*b\*n)

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] :> Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] :> Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] :> Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x \csc^{\frac{3}{2}}(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2 \cos(a + b \log(cx^n))}{3bn\sqrt{\csc(a + b \log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\csc(a + bx)} dx, x, \log(cx^n)\right)}{3n} \\
&= -\frac{2 \cos(a + b \log(cx^n))}{3bn\sqrt{\csc(a + b \log(cx^n))}} + \frac{(\sqrt{\csc(a + b \log(cx^n))}\sqrt{\sin(a + b \log(cx^n))}) \text{Subst}\left(\int \frac{1}{\sqrt{s}}\right)}{3n} \\
&= -\frac{2 \cos(a + b \log(cx^n))}{3bn\sqrt{\csc(a + b \log(cx^n))}} + \frac{2\sqrt{\csc(a + b \log(cx^n))}F\left(\frac{1}{2}\left(a - \frac{\pi}{2} + b \log(cx^n)\right)\middle| 2\right)\sqrt{\sin(a + b \log(cx^n))}}{3bn}
\end{aligned}$$

**Mathematica [A]** time = 0.165752, size = 76, normalized size = 0.78

$$\frac{\sqrt{\csc(a + b \log(cx^n))} \left( 2\sqrt{\sin(a + b \log(cx^n))} \text{EllipticF}\left(\frac{1}{4}(-2a - 2b \log(cx^n) + \pi), 2\right) + \sin(2(a + b \log(cx^n))) \right)}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Csc[a + b\*Log[c\*x^n]]^(3/2)),x]

[Out] -(Sqrt[Csc[a + b\*Log[c\*x^n]]]\*(2\*EllipticF[(-2\*a + Pi - 2\*b\*Log[c\*x^n])/4, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]] + Sin[2\*(a + b\*Log[c\*x^n])]))/(3\*b\*n)

**Maple [A]** time = 1.423, size = 131, normalized size = 1.3

$$\frac{1}{n \cos(a + b \ln(cx^n)) b} \left( \frac{1}{3} \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n)) + 2} \sqrt{-\sin(a + b \ln(cx^n))} \text{EllipticF}\left(\sqrt{\sin(a + b \ln(cx^n)) + 1}, \frac{1}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/csc(a+b\*ln(c\*x^n))^(3/2),x)

[Out] 1/n\*(1/3\*(sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticF((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))-2/3\*cos(a+b\*ln(c\*x^n))^2\*sin(a+b\*ln(c\*x^n)))/cos(a+b\*ln(c\*x^n))/sin(a+b\*ln(c\*x^n))^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \csc(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*log(c\*x^n))^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x\*csc(b\*log(c\*x^n) + a)^(3/2)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \csc(b \log(cx^n) + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*log(c\*x^n))^(3/2),x, algorithm="fricas")

[Out] integral(1/(x\*csc(b\*log(c\*x^n) + a)^(3/2)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*ln(c\*x\*\*n))\*\*(3/2),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*log(c\*x^n))^(3/2),x, algorithm="giac")

[Out] Timed out

$$3.318 \quad \int \frac{1}{\csc^{\frac{5}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=110

$$\frac{{}_2x\text{Hypergeometric2F1}\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right), -\frac{bn+2i}{4bn}, e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1 - e^{2ia}(cx^n)^{2ib}\right)^{5/2} \csc^{\frac{5}{2}}(a+b \log(cx^n))}$$

[Out] (2\*x\*Hypergeometric2F1[-5/2, (-5 - (2\*I)/(b\*n))/4, -(2\*I + b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 - (5\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Csc[a + b\*Log[c\*x^n]]^(5/2))

**Rubi [A]** time = 0.0731525, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4504, 4508, 364}

$$\frac{{}_2x_2F_1\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right); -\frac{bn+2i}{4bn}; e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1 - e^{2ia}(cx^n)^{2ib}\right)^{5/2} \csc^{\frac{5}{2}}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^(-5/2), x]

[Out] (2\*x\*Hypergeometric2F1[-5/2, (-5 - (2\*I)/(b\*n))/4, -(2\*I + b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 - (5\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Csc[a + b\*Log[c\*x^n]]^(5/2))

#### Rule 4504

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d))\*x^(2\*I\*b\*d)]^p/x^(I\*b\*d\*p), Int[((e\*x)^(m\*x^(I\*b\*d\*p)))/(1 - E^(2\*I\*a\*d))\*x^(2\*I\*b\*d)]^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.)]^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{\csc^{\frac{5}{2}}(a + b \log(cx^n))} dx = \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\csc^{\frac{5}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{\frac{5ib}{2}-\frac{1}{n}}\right) \operatorname{Subst}\left(\int x^{-1-\frac{5ib}{2}+\frac{1}{n}} (1 - e^{2ia}x^{2ib})^{5/2} dx, x, cx^n\right)}{n(1 - e^{2ia}(cx^n)^{2ib})^{5/2} \csc^{\frac{5}{2}}(a + b \log(cx^n))}$$

$$= \frac{{}_2F_1\left(-\frac{5}{2}, \frac{1}{4}\left(-5 - \frac{2i}{bn}\right); -\frac{2i+bn}{4bn}; e^{2ia}(cx^n)^{2ib}\right)}{(2 - 5ibn)(1 - e^{2ia}(cx^n)^{2ib})^{5/2} \csc^{\frac{5}{2}}(a + b \log(cx^n))}$$

**Mathematica [B]** time = 8.75875, size = 876, normalized size = 7.96

$$\sqrt{\csc(a + bn \log(x) + b(\log(cx^n) - n \log(x)))} \left( -\frac{x \cos(bn \log(x)) (-55b^2n^2 + 65b^2 \cos(2(a + b(\log(cx^n) - n \log(x))))}{4(5bn - 2i)(5bn + 2i)(bn \cos(a + b \log(x)))} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^(-5/2), x]

[Out] (-30\*b^3\*E^(I\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* n^3 \* x^(1 - I\*b\*n) \* Sqrt[2 - 2\*E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n)] \* Sqrt[(I\*E^(I\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^(I\*b\*n))/(-1 + E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n))] \* ((2\*I + b\*n) \* x^((2\*I)\*b\*n) \* Hypergeometric2F1[1/2, 3/4 - (I/2)/(b\*n), 7/4 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n)] + (-2\*I + 3\*b\*n) \* Hypergeometric2F1[1/2, -(2\*I + b\*n)/(4\*b\*n), 3/4 - (I/2)/(b\*n), E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* x^((2\*I)\*b\*n))]/((2\*I + b\*n) \* (-2\*I + 3\*b\*n) \* (-2\*I + 5\*b\*n) \* (2\*I + 5\*b\*n) \* (2\*I + b\*n + E^((2\*I)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))) \* (-2\*I + b\*n))) + Sqrt[Csc[a + b\*n\*Log[x] + b\*(-(n\*Log[x]) + Log[c\*x^n])] \* (-x \* Cos[b\*n\*Log[x]] \* (-12 - 55\*b^2\*n^2 + 12\*Cos[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 65\*b^2\*n^2 \* Cos[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 4\*b\*n\*Sin[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))])/(4\*(-2\*I + 5\*b\*n) \* (2\*I + 5\*b\*n) \* (b\*n \* Cos[a + b\*(-(n\*Log[x]) + Log[c\*x^n])] + 2\*Sin[a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + (x\*Sin[b\*n\*Log[x]] \* (16\*b\*n - 4\*b\*n\*Cos[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 12\*Sin[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 65\*b^2\*n^2 \* Sin[2\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))])/(4\*(-2\*I + 5\*b\*n) \* (2\*I + 5\*b\*n) \* (b\*n \* Cos[a + b\*(-(n\*Log[x]) + Log[c\*x^n])] + 2\*Sin[a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + (x\*Cos[3\*b\*n\*Log[x]] \* (5\*b\*n\*Cos[3\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] - 2\*Sin[3\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))])/(2\*(-2\*I + 5\*b\*n) \* (2\*I + 5\*b\*n)) - (x\*Sin[3\*b\*n\*Log[x]] \* (2\*Cos[3\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))] + 5\*b\*n\*Sin[3\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n]))])/(2\*(-2\*I + 5\*b\*n) \* (2\*I + 5\*b\*n)))

**Maple [F]** time = 0.3, size = 0, normalized size = 0.

$$\int (\csc(a + b \ln(cx^n)))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/csc(a+b\*ln(c\*x^n))^(5/2), x)

[Out] `int(1/csc(a+b*ln(c*x^n))^(5/2),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\csc(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")`

[Out] `integrate(csc(b*log(c*x^n) + a)^(-5/2), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/csc(a+b*ln(c*x**n))**(5/2),x)`

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")`

[Out] Timed out

$$3.319 \quad \int \frac{1}{x \csc^{\frac{5}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=98

$$\frac{6\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}E\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn} - \frac{2 \cos(a+b \log(cx^n))}{5bn \csc^{\frac{3}{2}}(a+b \log(cx^n))}$$

[Out] (-2\*Cos[a + b\*Log[c\*x^n]])/(5\*b\*n\*Csc[a + b\*Log[c\*x^n]]^(3/2)) + (6\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*EllipticE[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(5\*b\*n)

**Rubi [A]** time = 0.0583067, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3769, 3771, 2639}

$$\frac{6\sqrt{\sin(a+b \log(cx^n))}\sqrt{\csc(a+b \log(cx^n))}E\left(\frac{1}{2}\left(a+b \log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn} - \frac{2 \cos(a+b \log(cx^n))}{5bn \csc^{\frac{3}{2}}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Csc[a + b\*Log[c\*x^n]]^(5/2)),x]

[Out] (-2\*Cos[a + b\*Log[c\*x^n]])/(5\*b\*n\*Csc[a + b\*Log[c\*x^n]]^(3/2)) + (6\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*EllipticE[(a - Pi/2 + b\*Log[c\*x^n])/2, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]]])/(5\*b\*n)

#### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(n\_), x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

#### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_.)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x \csc^2(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{\csc^2(a+bx)} dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2 \cos(a + b \log(cx^n))}{5bn \csc^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{3 \text{Subst}\left(\int \frac{1}{\sqrt{\csc(a+bx)}} dx, x, \log(cx^n)\right)}{5n} \\
&= -\frac{2 \cos(a + b \log(cx^n))}{5bn \csc^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{(3\sqrt{\csc(a + b \log(cx^n))}\sqrt{\sin(a + b \log(cx^n))}) \text{Subst}\left(\int \sqrt{\sin} \right)}{5n} \\
&= -\frac{2 \cos(a + b \log(cx^n))}{5bn \csc^{\frac{3}{2}}(a + b \log(cx^n))} + \frac{6\sqrt{\csc(a + b \log(cx^n))}E\left(\frac{1}{2}\left(a - \frac{\pi}{2} + b \log(cx^n)\right)\middle| 2\right)\sqrt{\sin}}{5bn}
\end{aligned}$$

**Mathematica [A]** time = 0.220296, size = 88, normalized size = 0.9

$$\frac{2\sqrt{\csc(a + b \log(cx^n))} \left( \sin^2(a + b \log(cx^n)) \cos(a + b \log(cx^n)) + 3\sqrt{\sin(a + b \log(cx^n))} E\left(\frac{1}{4}(-2a - 2b \log(cx^n) + \pi)\right) \right)}{5bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Csc[a + b\*Log[c\*x^n]]^(5/2)),x]

[Out] (-2\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*(3\*EllipticE[(-2\*a + Pi - 2\*b\*Log[c\*x^n])/4, 2]\*Sqrt[Sin[a + b\*Log[c\*x^n]] + Cos[a + b\*Log[c\*x^n]]\*Sin[a + b\*Log[c\*x^n]]^2))/(5\*b\*n)

**Maple [A]** time = 1.183, size = 205, normalized size = 2.1

$$\frac{1}{n \cos(a + b \ln(cx^n)) b} \left( \frac{2 (\sin(a + b \ln(cx^n)))^4}{5} - \frac{2 (\sin(a + b \ln(cx^n)))^2}{5} - \frac{6}{5} \sqrt{\sin(a + b \ln(cx^n)) + 1} \sqrt{-2 \sin(a + b \ln(cx^n))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/csc(a+b\*ln(c\*x^n))^(5/2),x)

[Out] 1/n\*(2/5\*sin(a+b\*ln(c\*x^n))^4-2/5\*sin(a+b\*ln(c\*x^n))^2-6/5\*(sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticE((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2))+3/5\*(sin(a+b\*ln(c\*x^n))+1)^(1/2)\*(-2\*sin(a+b\*ln(c\*x^n))+2)^(1/2)\*(-sin(a+b\*ln(c\*x^n)))^(1/2)\*EllipticF((sin(a+b\*ln(c\*x^n))+1)^(1/2),1/2\*2^(1/2)))/cos(a+b\*ln(c\*x^n))/sin(a+b\*ln(c\*x^n))^(1/2)/b

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \csc(b \log(cx^n) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/x/csc(a+b\*log(c\*x^n))^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x\*csc(b\*log(c\*x^n) + a)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x \csc(b \log(cx^n) + a)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*log(c\*x^n))^(5/2),x, algorithm="fricas")

[Out] integral(1/(x\*csc(b\*log(c\*x^n) + a)^(5/2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*ln(c\*x\*\*n))\*\*(5/2),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/csc(a+b\*log(c\*x^n))^(5/2),x, algorithm="giac")

[Out] Timed out

### 3.320 $\int (ex)^m \csc^3(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=122

$$\frac{8e^{3iad}(ex)^{m+1}(cx^n)^{3ibd} \operatorname{Hypergeometric2F1}\left(3, -\frac{-3bdn+i(m+1)}{2bdn}, -\frac{-5bdn+i(m+1)}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right)}{e(-3bdn+i(m+1))}$$

[Out]  $(-8E^{((3I)*a*d)}*(e*x)^{(1+m)}*(c*x^n)^{((3I)*b*d)}*\operatorname{Hypergeometric2F1}[3, -(I*(1+m) - 3*b*d*n)/(2*b*d*n), -(I*(1+m) - 5*b*d*n)/(2*b*d*n), E^{((2I)*a*d)}*(c*x^n)^{((2I)*b*d)}]/(e*(I*(1+m) - 3*b*d*n))$

**Rubi [A]** time = 0.109617, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4510, 4506, 364}

$$\frac{8e^{3iad}(ex)^{m+1}(cx^n)^{3ibd} {}_2F_1\left(3, -\frac{i(m+1)-3bdn}{2bdn}; -\frac{i(m+1)-5bdn}{2bdn}; e^{2iad}(cx^n)^{2ibd}\right)}{e(-3bdn+i(m+1))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^m*\operatorname{Csc}[d*(a + b*\operatorname{Log}[c*x^n])]^3, x]$

[Out]  $(-8E^{((3I)*a*d)}*(e*x)^{(1+m)}*(c*x^n)^{((3I)*b*d)}*\operatorname{Hypergeometric2F1}[3, -(I*(1+m) - 3*b*d*n)/(2*b*d*n), -(I*(1+m) - 5*b*d*n)/(2*b*d*n), E^{((2I)*a*d)}*(c*x^n)^{((2I)*b*d)}]/(e*(I*(1+m) - 3*b*d*n))$

#### Rule 4510

$\operatorname{Int}[\operatorname{Csc}[(a_.) + \operatorname{Log}[c_.*(x_.)^{n_.}]]*(b_.)*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[x^{((m+1)/n-1)}*\operatorname{Csc}[d*(a + b*\operatorname{Log}[x])]^p, x], x, c*x^n], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x\} \&\& (\operatorname{NeQ}[c, 1] \mid\mid \operatorname{NeQ}[n, 1])$

#### Rule 4506

$\operatorname{Int}[\operatorname{Csc}[(a_.) + \operatorname{Log}[x_*]]*(b_.)*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(-2*I)^p * E^{(I*a*d*p)}, \operatorname{Int}[(e*x)^m * x^{(I*b*d*p)}]/(1 - E^{(2*I*a*d)*x^{(2*I*b*d)}})^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, d, e, m\}, x\} \&\& \operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c_.*(x_.))^{(m_.)}*(a_. + (b_.)*(x_.)^{n_.})^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a^p * (c*x)^{(m+1)} * \operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -(b*x^n)/a])]/(c*(m+1)), x] /;$   $\operatorname{FreeQ}\{a, b, c, m, n, p\}, x\} \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \mid\mid \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\int (ex)^m \csc^3(d(a + b \log(cx^n))) dx = \frac{\left( (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{n}} \csc^3(d(a + b \log(x))) dx, x, cx^n \right)}{en}$$

$$= \frac{\left( 8ie^{3iad} (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int \frac{x^{-1+3ibd+\frac{1+m}{n}}}{(1-e^{2iad}x^{2ibd})^3} dx, x, cx^n \right)}{en}$$

$$= -\frac{8e^{3iad} (ex)^{1+m} (cx^n)^{3ibd} {}_2F_1 \left( 3, -\frac{i(1+m)-3bdn}{2bdn}; -\frac{i(1+m)-5bdn}{2bdn}; e^{2iad} (cx^n)^{2ibd} \right)}{i(e + em) - 3bden}$$

**Mathematica [B]** time = 2.28485, size = 367, normalized size = 3.01

$$x(ex)^m \left( 8(-ibdn + m + 1)x^{ibdn} (\sin(d(a + b \log(cx^n) - bn \log(x))) - i \cos(d(a + b \log(cx^n) - bn \log(x)))) \text{Hypergeometric} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (x*(e*x)^m*(-(b*d*n*Csc[(d*(a + b*Log[c*x^n]))/2]^2) - 4*(1 + m)*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])] + b*d*n*Sec[(d*(a + b*Log[c*x^n]))/2]^2 + 2*(1 + m)*Csc[(d*(a + b*Log[c*x^n]))/2]*Csc[(d*(a - b*n*Log[x] + b*Log[c*x^n]))/2]*Sin[(b*d*n*Log[x])/2] - 2*(1 + m)*Sec[(d*(a + b*Log[c*x^n]))/2]*Sec[(d*(a - b*n*Log[x] + b*Log[c*x^n]))/2]*Sin[(b*d*n*Log[x])/2] + 8*(1 + m - I*b*d*n)*x^(I*b*d*n)*Hypergeometric2F1[1, (-I - I*m + b*d*n)/(2*b*d*n), ((-I/2)*(1 + m + (3*I)*b*d*n))/(b*d*n), x^((2*I)*b*d*n)*(Cos[2*d*(a - b*n*Log[x] + b*Log[c*x^n])) + I*Sin[2*d*(a - b*n*Log[x] + b*Log[c*x^n]])])*((-I)*Cos[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Sin[d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(8*b^2*d^2*n^2)
```

**Maple [F]** time = 5.298, size = 0, normalized size = 0.

$$\int (ex)^m (\csc(d(a + b \ln(cx^n))))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^3,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] -((b*d*e^m*n*cos(b*d*log(c)) - e^m*m*sin(b*d*log(c)) - e^m*sin(b*d*log(c)))*x*x^m*cos(b*d*log(x^n) + a*d) - (b*d*e^m*n*sin(b*d*log(c)) + e^m*m*cos(b*d*log(c))
```



$$\begin{aligned}
& \hat{m} * n^4 * \cos(2 * b * d * \log(x^n) + 2 * a * d) - 2 * ((b^6 * d^6 * \cos(2 * b * d * \log(c)) * \sin(4 * \\
& b * d * \log(c)) - b^6 * d^6 * \cos(4 * b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m} * n^6} + ((b^4 * \\
& d^4 * \cos(2 * b * d * \log(c)) * \sin(4 * b * d * \log(c)) - b^4 * d^4 * \cos(4 * b * d * \log(c)) * \sin(2 * b \\
& * d * \log(c))) * e^{\hat{m} * m^2} + 2 * (b^4 * d^4 * \cos(2 * b * d * \log(c)) * \sin(4 * b * d * \log(c)) - b^4 * \\
& d^4 * \cos(4 * b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m} * m} + (b^4 * d^4 * \cos(2 * b * d * \log(c)) \\
& * \sin(4 * b * d * \log(c)) - b^4 * d^4 * \cos(4 * b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m}} * n^4 \\
& * \sin(2 * b * d * \log(x^n) + 2 * a * d)) * \cos(4 * b * d * \log(x^n) + 4 * a * d) - 4 * (b^6 * d^6 * e^{\hat{m} * \\
& n^6 * \cos(2 * b * d * \log(c)) + (b^4 * d^4 * e^{\hat{m} * m^2 * \cos(2 * b * d * \log(c)) + 2 * b^4 * d^4 * e^{\hat{m} * \\
& m * \cos(2 * b * d * \log(c)) + b^4 * d^4 * e^{\hat{m} * \cos(2 * b * d * \log(c))}) * n^4) * \cos(2 * b * d * \log(x^n \\
& ) + 2 * a * d) - 2 * (b^6 * d^6 * e^{\hat{m} * n^6 * \sin(4 * b * d * \log(c)) + (b^4 * d^4 * e^{\hat{m} * m^2 * \sin(4 * \\
& b * d * \log(c)) + 2 * b^4 * d^4 * e^{\hat{m} * m * \sin(4 * b * d * \log(c)) + b^4 * d^4 * e^{\hat{m} * \sin(4 * b * d * \log \\
& (c))}) * n^4 - 2 * ((b^6 * d^6 * \cos(2 * b * d * \log(c)) * \sin(4 * b * d * \log(c)) - b^6 * d^6 * \cos(4 \\
& * b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m} * n^6} + ((b^4 * d^4 * \cos(2 * b * d * \log(c)) * \sin(4 \\
& * b * d * \log(c)) - b^4 * d^4 * \cos(4 * b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m} * m^2} + 2 * (b^4 \\
& * d^4 * \cos(2 * b * d * \log(c)) * \sin(4 * b * d * \log(c)) - b^4 * d^4 * \cos(4 * b * d * \log(c)) * \sin(2 \\
& * b * d * \log(c))) * e^{\hat{m} * m} + (b^4 * d^4 * \cos(2 * b * d * \log(c)) * \sin(4 * b * d * \log(c)) - b^4 * d^4 * \\
& \cos(4 * b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m}} * n^4) * \cos(2 * b * d * \log(x^n) + 2 * a * d) \\
& ) + 2 * ((b^6 * d^6 * \cos(4 * b * d * \log(c)) * \cos(2 * b * d * \log(c)) + b^6 * d^6 * \sin(4 * b * d * \log \\
& (c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m} * n^6} + ((b^4 * d^4 * \cos(4 * b * d * \log(c)) * \cos(2 * b * d * \log \\
& (c)) + b^4 * d^4 * \sin(4 * b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m} * m^2} + 2 * (b^4 * d^4 * \cos \\
& (4 * b * d * \log(c)) * \cos(2 * b * d * \log(c)) + b^4 * d^4 * \sin(4 * b * d * \log(c)) * \sin(2 * b * d * \log \\
& (c))) * e^{\hat{m} * m} + (b^4 * d^4 * \cos(4 * b * d * \log(c)) * \cos(2 * b * d * \log(c)) + b^4 * d^4 * \sin(4 * \\
& b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m}} * n^4) * \sin(2 * b * d * \log(x^n) + 2 * a * d)) * \sin(4 \\
& * b * d * \log(x^n) + 4 * a * d) + 4 * (b^6 * d^6 * e^{\hat{m} * n^6 * \sin(2 * b * d * \log(c)) + (b^4 * d^4 * e^{\hat{m} * \\
& m^2 * \sin(2 * b * d * \log(c)) + 2 * b^4 * d^4 * e^{\hat{m} * m * \sin(2 * b * d * \log(c)) + b^4 * d^4 * e^{\hat{m} * \sin \\
& (2 * b * d * \log(c))}) * n^4) * \sin(2 * b * d * \log(x^n) + 2 * a * d)) * \int (1 / 4 * (x^{\hat{m}} * \cos(b * \\
& d * \log(x^n) + a * d) * \sin(b * d * \log(c)) + x^{\hat{m}} * \cos(b * d * \log(c)) * \sin(b * d * \log(x^n) \\
& + a * d)) / (2 * b^4 * d^4 * n^4 * \cos(b * d * \log(c)) * \cos(b * d * \log(x^n) + a * d) - 2 * b^4 * d^4 * \\
& n^4 * \sin(b * d * \log(c)) * \sin(b * d * \log(x^n) + a * d) + b^4 * d^4 * n^4 + (b^4 * d^4 * \cos(b * \\
& d * \log(c))^2 + b^4 * d^4 * \sin(b * d * \log(c))^2) * n^4 * \cos(b * d * \log(x^n) + a * d)^2 + (b \\
& ^4 * d^4 * \cos(b * d * \log(c))^2 + b^4 * d^4 * \sin(b * d * \log(c))^2) * n^4 * \sin(b * d * \log(x^n) \\
& + a * d)^2), x) + 2 * (b^6 * d^6 * e^{\hat{m} * n^6} + (b^4 * d^4 * e^{\hat{m} * m^2} + 2 * b^4 * d^4 * e^{\hat{m} * m} + b \\
& ^4 * d^4 * e^{\hat{m}}) * n^4 + ((b^6 * d^6 * \cos(4 * b * d * \log(c))^2 + b^6 * d^6 * \sin(4 * b * d * \log(c)) \\
& ^2) * e^{\hat{m} * n^6} + ((b^4 * d^4 * \cos(4 * b * d * \log(c))^2 + b^4 * d^4 * \sin(4 * b * d * \log(c))^2) * \\
& e^{\hat{m} * m^2} + 2 * (b^4 * d^4 * \cos(4 * b * d * \log(c))^2 + b^4 * d^4 * \sin(4 * b * d * \log(c))^2) * e^{\hat{m} * \\
& m} + (b^4 * d^4 * \cos(4 * b * d * \log(c))^2 + b^4 * d^4 * \sin(4 * b * d * \log(c))^2) * e^{\hat{m}}) * n^4) * \\
& \cos(4 * b * d * \log(x^n) + 4 * a * d)^2 + 4 * ((b^6 * d^6 * \cos(2 * b * d * \log(c))^2 + b^6 * d^6 * \sin \\
& (2 * b * d * \log(c))^2) * e^{\hat{m} * n^6} + ((b^4 * d^4 * \cos(2 * b * d * \log(c))^2 + b^4 * d^4 * \sin(2 \\
& * b * d * \log(c))^2) * e^{\hat{m} * m^2} + 2 * (b^4 * d^4 * \cos(2 * b * d * \log(c))^2 + b^4 * d^4 * \sin(2 * b * \\
& d * \log(c))^2) * e^{\hat{m} * m} + (b^4 * d^4 * \cos(2 * b * d * \log(c))^2 + b^4 * d^4 * \sin(2 * b * d * \log(c) \\
& ))^2) * e^{\hat{m}}) * n^4) * \cos(2 * b * d * \log(x^n) + 2 * a * d)^2 + ((b^6 * d^6 * \cos(4 * b * d * \log(c)) \\
& ^2 + b^6 * d^6 * \sin(4 * b * d * \log(c))^2) * e^{\hat{m} * n^6} + ((b^4 * d^4 * \cos(4 * b * d * \log(c))^2 + b^4 \\
& * d^4 * \sin(4 * b * d * \log(c))^2) * e^{\hat{m} * m^2} + 2 * (b^4 * d^4 * \cos(4 * b * d * \log(c))^2 + b^4 \\
& * d^4 * \sin(4 * b * d * \log(c))^2) * e^{\hat{m} * m} + (b^4 * d^4 * \cos(4 * b * d * \log(c))^2 + b^4 * d^4 * \sin \\
& (4 * b * d * \log(c))^2) * e^{\hat{m}}) * n^4) * \sin(4 * b * d * \log(x^n) + 4 * a * d)^2 + 4 * ((b^6 * d^6 * \cos \\
& (2 * b * d * \log(c))^2 + b^6 * d^6 * \sin(2 * b * d * \log(c))^2) * e^{\hat{m} * n^6} + ((b^4 * d^4 * \cos(2 \\
& * b * d * \log(c))^2 + b^4 * d^4 * \sin(2 * b * d * \log(c))^2) * e^{\hat{m} * m^2} + 2 * (b^4 * d^4 * \cos(2 * b * \\
& d * \log(c))^2 + b^4 * d^4 * \sin(2 * b * d * \log(c))^2) * e^{\hat{m} * m} + (b^4 * d^4 * \cos(2 * b * d * \log(c) \\
& ))^2 + b^4 * d^4 * \sin(2 * b * d * \log(c))^2) * e^{\hat{m}}) * n^4) * \sin(2 * b * d * \log(x^n) + 2 * a * d)^2 \\
& + 2 * (b^6 * d^6 * e^{\hat{m} * n^6 * \cos(4 * b * d * \log(c)) + (b^4 * d^4 * e^{\hat{m} * m^2 * \cos(4 * b * d * \log(c)) \\
& ) + 2 * b^4 * d^4 * e^{\hat{m} * m * \cos(4 * b * d * \log(c)) + b^4 * d^4 * e^{\hat{m} * \cos(4 * b * d * \log(c))}) * n^4 \\
& - 2 * ((b^6 * d^6 * \cos(4 * b * d * \log(c)) * \cos(2 * b * d * \log(c)) + b^6 * d^6 * \sin(4 * b * d * \log(c) \\
& )) * \sin(2 * b * d * \log(c))) * e^{\hat{m} * n^6} + ((b^4 * d^4 * \cos(4 * b * d * \log(c)) * \cos(2 * b * d * \log(c) \\
& )) + b^4 * d^4 * \sin(4 * b * d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m} * m^2} + 2 * (b^4 * d^4 * \cos \\
& (4 * b * d * \log(c)) * \cos(2 * b * d * \log(c)) + b^4 * d^4 * \sin(4 * b * d * \log(c)) * \sin(2 * b * d * \log(c) \\
& )) * e^{\hat{m} * m} + (b^4 * d^4 * \cos(4 * b * d * \log(c)) * \cos(2 * b * d * \log(c)) + b^4 * d^4 * \sin(4 * b * \\
& d * \log(c)) * \sin(2 * b * d * \log(c))) * e^{\hat{m}}) * n^4) * \cos(2 * b * d * \log(x^n) + 2 * a * d) - 2 * ((b^6 \\
& * d^6 * \cos(2 * b * d * \log(c)) * \sin(4 * b * d * \log(c)) - b^6 * d^6 * \cos(4 * b * d * \log(c)) * \sin(2 \\
& * b * d * \log(c))) * e^{\hat{m} * n^6} + ((b^4 * d^4 * \cos(2 * b * d * \log(c)) * \sin(4 * b * d * \log(c)) - b^4
\end{aligned}$$



```

og(c)) + b*d*sin(2*b*d*log(c))*sin(b*d*log(c))) * e^m*n + (cos(b*d*log(c))*sin(
n(2*b*d*log(c)) - cos(2*b*d*log(c))*sin(b*d*log(c))) * e^m * x*x^m*sin(b*d*log
(x^n) + a*d)) * sin(2*b*d*log(x^n) + 2*a*d)) / (4*b^2*d^2*n^2*cos(2*b*d*log(c))
*cos(2*b*d*log(x^n) + 2*a*d) - 4*b^2*d^2*n^2*sin(2*b*d*log(c))*sin(2*b*d*lo
g(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(4*b*d*log(c))^2 + b^2*d^2*sin(
4*b*d*log(c))^2) * n^2*cos(4*b*d*log(x^n) + 4*a*d)^2 - 4*(b^2*d^2*cos(2*b*d*l
og(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2) * n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 -
(b^2*d^2*cos(4*b*d*log(c))^2 + b^2*d^2*sin(4*b*d*log(c))^2) * n^2*sin(4*b*d*
log(x^n) + 4*a*d)^2 - 4*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*lo
g(c))^2) * n^2*sin(2*b*d*log(x^n) + 2*a*d)^2 - 2*(b^2*d^2*n^2*cos(4*b*d*log(c)
)) - 2*(b^2*d^2*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^2*d^2*sin(4*b*d*log
(c))*sin(2*b*d*log(c))) * n^2*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b^2*d^2*cos(2*
b*d*log(c))*sin(4*b*d*log(c)) - b^2*d^2*cos(4*b*d*log(c))*sin(2*b*d*log(c))
) * n^2*sin(2*b*d*log(x^n) + 2*a*d)) * cos(4*b*d*log(x^n) + 4*a*d) + 2*(b^2*d^2
*n^2*sin(4*b*d*log(c)) - 2*(b^2*d^2*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b
^2*d^2*cos(4*b*d*log(c))*sin(2*b*d*log(c))) * n^2*cos(2*b*d*log(x^n) + 2*a*d)
+ 2*(b^2*d^2*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b^2*d^2*sin(4*b*d*log(c)
))*sin(2*b*d*log(c))) * n^2*sin(2*b*d*log(x^n) + 2*a*d)) * sin(4*b*d*log(x^n) +
4*a*d))

```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((ex)^m \csc(bd \log(cx^n) + ad)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*csc(b*d*log(c*x^n) + a*d)^3, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*csc(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \csc((b \log(cx^n) + a)d)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^3, x)
```

### 3.321 $\int (ex)^m \csc^2(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=119

$$\frac{4e^{2iad}(ex)^{m+1}(cx^n)^{2ibd} \operatorname{Hypergeometric2F1}\left(2, -\frac{-2bdn+i(m+1)}{2bdn}, -\frac{-4bdn+i(m+1)}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right)}{e(2ibd + m + 1)}$$

[Out]  $(-4 * E^{((2 * I) * a * d)} * (e * x)^{(1 + m)} * (c * x^n)^{((2 * I) * b * d)} * \operatorname{Hypergeometric2F1}[2, -(I * (1 + m) - 2 * b * d * n) / (2 * b * d * n), -(I * (1 + m) - 4 * b * d * n) / (2 * b * d * n), E^{((2 * I) * a * d)} * (c * x^n)^{((2 * I) * b * d)}]) / (e * (1 + m + (2 * I) * b * d * n))$

**Rubi [A]** time = 0.0966439, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4510, 4506, 364}

$$\frac{4e^{2iad}(ex)^{m+1}(cx^n)^{2ibd} {}_2F_1\left(2, -\frac{i(m+1)-2bdn}{2bdn}; -\frac{i(m+1)-4bdn}{2bdn}; e^{2iad}(cx^n)^{2ibd}\right)}{e(2ibd + m + 1)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e * x)^m * \operatorname{Csc}[d * (a + b * \operatorname{Log}[c * x^n])]^2, x]$

[Out]  $(-4 * E^{((2 * I) * a * d)} * (e * x)^{(1 + m)} * (c * x^n)^{((2 * I) * b * d)} * \operatorname{Hypergeometric2F1}[2, -(I * (1 + m) - 2 * b * d * n) / (2 * b * d * n), -(I * (1 + m) - 4 * b * d * n) / (2 * b * d * n), E^{((2 * I) * a * d)} * (c * x^n)^{((2 * I) * b * d)}]) / (e * (1 + m + (2 * I) * b * d * n))$

#### Rule 4510

$\operatorname{Int}[\operatorname{Csc}[(a_.) + \operatorname{Log}[c_. * (x_.)^{n_.}] * (b_.)] * (d_.)]^{(p_.)} * ((e_.) * (x_.))^{(m_.)}, x\_Symbol] := \operatorname{Dist}[(e * x)^{(m + 1)} / (e * n * (c * x^n)^{((m + 1) / n)}), \operatorname{Subst}[\operatorname{Int}[x^{((m + 1) / n - 1)} * \operatorname{Csc}[d * (a + b * \operatorname{Log}[x])]^p, x], x, c * x^n], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\operatorname{NeQ}[c, 1] \mid \mid \operatorname{NeQ}[n, 1])$

#### Rule 4506

$\operatorname{Int}[\operatorname{Csc}[(a_.) + \operatorname{Log}[x_] * (b_.)] * (d_.)]^{(p_.)} * ((e_.) * (x_.))^{(m_.)}, x\_Symbol] := \operatorname{Dist}[(-2 * I)^p * E^{(I * a * d * p)}, \operatorname{Int}[(e * x)^m * x^{(I * b * d * p)} / (1 - E^{(2 * I * a * d)} * x^{(2 * I * b * d)})^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, d, e, m\}, x] \&\& \operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c_.) * (x_.)]^{(m_.)} * ((a_.) + (b_.) * (x_.)^{n_.})^{(p_.)}, x\_Symbol] := \operatorname{Simp}[(a^p * (c * x)^{(m + 1)} * \operatorname{Hypergeometric2F1}[-p, (m + 1) / n, (m + 1) / n + 1, -(b * x^n) / a]) / (c * (m + 1)), x] /;$   $\operatorname{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \mid \mid \operatorname{GtQ}[a, 0])$

#### Rubi steps



$$\int (ex)^m \csc^2(d(a + b \log(cx^n))) dx = \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \csc^2(d(a + b \log(x))) dx, x, cx^n\right)}{en}$$

$$= \frac{\left(4e^{2iad} (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+2ibd+\frac{1+m}{n}}}{(1-e^{2iad}x^{2ibd})^2} dx, x, cx^n\right)}{en}$$

$$= \frac{4e^{2iad} (ex)^{1+m} (cx^n)^{2ibd} {}_2F_1\left(2, -\frac{i(1+m)-2bdn}{2bdn}; -\frac{i(1+m)-4bdn}{2bdn}; e^{2iad} (cx^n)^{2ibd}\right)}{e(1 + m + 2ibdn)}$$

**Mathematica [B]** time = 6.54271, size = 534, normalized size = 4.49

$$\frac{x(ex)^m \sin(bdn \log(x)) \csc(d(a + b(\log(cx^n) - n \log(x)))) \csc(d(a + b(\log(cx^n) - n \log(x)) + bdn \log(x)))}{bdn}$$

(m + 1)

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csc[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]])/(b*d*n) - ((1 + m)*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(x^(1 + m)*Csc[d*(a + b*Log[c*x^n]])*Sin[b*d*n*Log[x]])/(1 + m) - (I*(I*E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m + (2*I)*b*d*n)*Cot[d*(a + b*Log[c*x^n])] - E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-I/2)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] - E^((a*(1 + 2*m + (2*I)*b*d*n))/(b*n) + (1 + m + (2*I)*b*d*n)*Log[x] + ((1 + 2*m + (2*I)*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, ((-I/2)*(1 + m + (2*I)*b*d*n))/(b*d*n), ((-I/2)*(1 + m + (4*I)*b*d*n))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))]*Sin[d*(a + b*(-(n*Log[x]) + Log[c*x^n])))]/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + (2*I)*b*d*n)))/(b*d*n*x^m)
```

**Maple [F]** time = 2.25, size = 0, normalized size = 0.

$$\int (ex)^m (\csc(d(a + b \ln(cx^n))))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^2,x)
```

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*csc(d\*(a+b\*log(c\*x^n)))^2,x, algorithm="maxima")

[Out] Timed out

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \csc(bd \log(cx^n) + ad)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*csc(d\*(a+b\*log(c\*x^n)))^2,x, algorithm="fricas")

[Out] integral((e\*x)^m\*csc(b\*d\*log(c\*x^n) + a\*d)^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*m\*csc(d\*(a+b\*ln(c\*x\*\*n)))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \csc((b \log(cx^n) + a)d)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*csc(d\*(a+b\*log(c\*x^n)))^2,x, algorithm="giac")

[Out] integrate((e\*x)^m\*csc((b\*log(c\*x^n) + a)\*d)^2, x)

### 3.322 $\int (ex)^m \csc(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=123

$$\frac{2e^{iad}(ex)^{m+1}(cx^n)^{ibd} \operatorname{Hypergeometric2F1}\left(1, -\frac{-bdn+im+i}{2bdn}, -\frac{-3bdn+i(m+1)}{2bdn}, e^{2iad}(cx^n)^{2ibd}\right)}{e(-bdn+i(m+1))}$$

[Out]  $(2E^{(I*a*d)}*(e*x)^{(1+m)}*(c*x^n)^{(I*b*d)}*\operatorname{Hypergeometric2F1}[1, -(I+I*m-b*d*n)/(2*b*d*n), -(I*(1+m)-3*b*d*n)/(2*b*d*n), E^{((2*I)*a*d)*(c*x^n)^{(2*I)*b*d}}]/(e*(I*(1+m)-b*d*n))$

**Rubi [A]** time = 0.0770617, antiderivative size = 118, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4510, 4506, 364}

$$\frac{2e^{iad}(ex)^{m+1}(cx^n)^{ibd} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i(m+1)}{bdn}\right); -\frac{i(m+1)-3bdn}{2bdn}; e^{2iad}(cx^n)^{2ibd}\right)}{e(-bdn+i(m+1))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^m*\operatorname{Csc}[d*(a + b*\operatorname{Log}[c*x^n])], x]$

[Out]  $(2E^{(I*a*d)}*(e*x)^{(1+m)}*(c*x^n)^{(I*b*d)}*\operatorname{Hypergeometric2F1}[1, (1-(I*(1+m))/(b*d*n))/2, -(I*(1+m)-3*b*d*n)/(2*b*d*n), E^{((2*I)*a*d)*(c*x^n)^{(2*I)*b*d}}]/(e*(I*(1+m)-b*d*n))$

#### Rule 4510

$\operatorname{Int}[\operatorname{Csc}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[x^{((m+1)/n-1)}*\operatorname{Csc}[d*(a + b*\operatorname{Log}[x])]^p, x], x, c*x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& (\operatorname{NeQ}[c, 1] \parallel \operatorname{NeQ}[n, 1])$

#### Rule 4506

$\operatorname{Int}[\operatorname{Csc}[(a_.) + \operatorname{Log}[x_]*(b_.)]*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(-2*I)^p E^{(I*a*d*p)}, \operatorname{Int}[(e*x)^m*x^{(I*b*d*p)}]/(1 - E^{(2*I*a*d)*x^{(2*I*b*d)}})^p, x], x] /; \operatorname{FreeQ}\{a, b, d, e, m\}, x] \&\& \operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a^p*(c*x)^{(m+1)}*\operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/ (c*(m+1)), x] /; \operatorname{FreeQ}\{a, b, c, m, n, p\}, x] \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \parallel \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\int (ex)^m \csc(d(a + b \log(cx^n))) dx = \frac{\left((ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \csc(d(a + b \log(x))) dx, x, cx^n\right)}{en}$$

$$= -\frac{\left(2ie^{iad}(ex)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+ibd+\frac{1+m}{n}}}{1-e^{2iad}x^{2ibd}} dx, x, cx^n\right)}{en}$$

$$= \frac{2e^{iad}(ex)^{1+m} (cx^n)^{ibd} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{i(1+m)}{bdn}\right); -\frac{i(1+m)-3bdn}{2bdn}; e^{2iad} (cx^n)^{2ibd}\right)}{i(e + em) - bden}$$

**Mathematica [A]** time = 0.425706, size = 181, normalized size = 1.47

$$\frac{2(ex)^m x^{1+ibdn} (\sin(d(a + b(\log(cx^n) - n \log(x)))) - i \cos(d(a + b(\log(cx^n) - n \log(x)))))) \text{Hypergeometric2F1}\left(1, \frac{bdn-1}{2b}, ibdn + m + 1\right)}{ibdn + m + 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m\*Csc[d\*(a + b\*Log[c\*x^n])],x]

[Out] (2\*x^(1 + I\*b\*d\*n)\*(e\*x)^m\*Hypergeometric2F1[1, (-I - I\*m + b\*d\*n)/(2\*b\*d\*n), ((-I/2)\*(1 + m + (3\*I)\*b\*d\*n))/(b\*d\*n), x^((2\*I)\*b\*d\*n)\*(Cos[2\*d\*(a + b\*(-n\*Log[x]) + Log[c\*x^n])] + I\*Sin[2\*d\*(a + b\*(-n\*Log[x]) + Log[c\*x^n]])]\*((-I)\*Cos[d\*(a + b\*(-n\*Log[x]) + Log[c\*x^n])] + Sin[d\*(a + b\*(-n\*Log[x]) + Log[c\*x^n])])]/(1 + m + I\*b\*d\*n)

**Maple [F]** time = 0.94, size = 0, normalized size = 0.

$$\int (ex)^m \csc(d(a + b \ln(cx^n))) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*csc(d\*(a+b\*ln(c\*x^n))),x)

[Out] int((e\*x)^m\*csc(d\*(a+b\*ln(c\*x^n))),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \csc((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*csc(d\*(a+b\*log(c\*x^n))),x, algorithm="maxima")

[Out] integrate((e\*x)^m\*csc((b\*log(c\*x^n) + a)\*d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex)^m \csc(bd \log(cx^n) + ad), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*csc(b*d*log(c*x^n) + a*d), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \csc(ad + bd \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*csc(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*csc(a*d + b*d*log(c*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \csc((b \log(cx^n) + a)d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*csc((b*log(c*x^n) + a)*d), x)
```

### 3.323 $\int x^m \csc^{\frac{5}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} (1 - e^{2ia} (cx^n)^{2ib})^{5/2} \csc^{\frac{5}{2}}(a + b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, -\frac{-5bn+2im+2i}{4bn}, -\frac{-9bn+2im+2i}{4bn}, e^{2ia} (cx^n)^{2ib}\right)}{5ibn + 2m + 2}$$

[Out] (2\*x^(1 + m)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Csc[a + b\*Log[c\*x^n]]^(5/2)\*Hypergeometric2F1[5/2, -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 9\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(2 + 2\*m + (5\*I)\*b\*n)

**Rubi [A]** time = 0.0961908, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4510, 4508, 364}

$$\frac{2x^{m+1} (1 - e^{2ia} (cx^n)^{2ib})^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{1}{4} \left(5 - \frac{2i(m+1)}{bn}\right); -\frac{2im-9bn+2i}{4bn}; e^{2ia} (cx^n)^{2ib}\right) \csc^{\frac{5}{2}}(a + b \log(cx^n))}{5ibn + 2m + 2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Csc[a + b\*Log[c\*x^n]]^(5/2),x]

[Out] (2\*x^(1 + m)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(5/2)\*Csc[a + b\*Log[c\*x^n]]^(5/2)\*Hypergeometric2F1[5/2, (5 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 9\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(2 + 2\*m + (5\*I)\*b\*n)

#### Rule 4510

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \csc^{\frac{5}{2}}(a + b \log(cx^n)) dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \csc^{\frac{5}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{5ib}{2}-\frac{1+m}{n}} (1 - e^{2ia} (cx^n)^{2ib})^{5/2} \csc^{\frac{5}{2}}(a + b \log(cx^n))\right) \text{Subst}\left(\int \frac{x^{-1+\frac{5ib}{2}+\frac{1+m}{n}}}{(1 - e^{2ia} x^{2ib})^{5/2}}\right)}{n}$$

$$= \frac{2x^{1+m} (1 - e^{2ia} (cx^n)^{2ib})^{5/2} \csc^{\frac{5}{2}}(a + b \log(cx^n)) {}_2F_1\left(\frac{5}{2}, \frac{1}{4} \left(5 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-9bn}{4bn}\right)}{2 + 2m + 5ibn}$$

**Mathematica [A]** time = 2.95451, size = 165, normalized size = 1.27

$$\frac{2x^{m+1} \sqrt{\csc(a + b \log(cx^n))} \left(e^{-2ia} (ibn + 2m + 2) (cx^n)^{-2ib} (-1 + e^{2ia} (cx^n)^{2ib}) \text{Hypergeometric2F1}\left(1, \frac{3bn+2im+2i}{4bn}, \frac{5bn}{4bn}\right)\right)}{3b^2 n^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Csc[a + b\*Log[c\*x^n]]^(5/2), x]

[Out] (2\*x^(1 + m)\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*(-2 - 2\*m - b\*n\*Cot[a + b\*Log[c\*x^n]] + ((2 + 2\*m + I\*b\*n)\*(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Hypergeometric2F1[1, (2\*I + (2\*I)\*m + 3\*b\*n)/(4\*b\*n), (2\*I + (2\*I)\*m + 5\*b\*n)/(4\*b\*n), E^((-2\*I)\*(a + b\*Log[c\*x^n]))])/(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))/(3\*b^2\*n^2)

**Maple [F]** time = 0.389, size = 0, normalized size = 0.

$$\int x^m (\csc(a + b \ln(cx^n)))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*csc(a+b\*ln(c\*x^n))^(5/2), x)

[Out] int(x^m\*csc(a+b\*ln(c\*x^n))^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \csc(b \log(cx^n) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*csc(a+b\*log(c\*x^n))^(5/2), x, algorithm="maxima")

[Out] integrate(x^m\*csc(b\*log(c\*x^n) + a)^(5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



### 3.324 $\int x^m \operatorname{csc}^{\frac{3}{2}}(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{3/2} \operatorname{csc}^{\frac{3}{2}}(a + b \log(cx^n)) \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, -\frac{-3bn+2im+2i}{4bn}, -\frac{-7bn+2im+2i}{4bn}, e^{2ia} (cx^n)^{2ib}\right)}{3ibn + 2m + 2}$$

[Out] (2\*x^(1 + m)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Csc[a + b\*Log[c\*x^n]]^(3/2)\*Hypergeometric2F1[3/2, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]]/(2 + 2\*m + (3\*I)\*b\*n)

**Rubi [A]** time = 0.09169, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4510, 4508, 364}

$$\frac{2x^{m+1} \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i(m+1)}{bn}\right); -\frac{2im-7bn+2i}{4bn}; e^{2ia} (cx^n)^{2ib}\right) \operatorname{csc}^{\frac{3}{2}}(a + b \log(cx^n))}{3ibn + 2m + 2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Csc[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^(1 + m)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Csc[a + b\*Log[c\*x^n]]^(3/2)\*Hypergeometric2F1[3/2, (3 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]]/(2 + 2\*m + (3\*I)\*b\*n)

#### Rule 4510

Int[Csc[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)]\*(d\_)^(p\_)\*((e\_)\*(x\_))^(m\_), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[((a\_) + Log[x\_]\*(b\_)]\*(d\_)^(p\_)\*((e\_)\*(x\_))^(m\_), x\_Symbol] :> Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \csc^{\frac{3}{2}}(a + b \log(cx^n)) dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \csc^{\frac{3}{2}}(a + b \log(x)) dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{3ib}{2}-\frac{1+m}{n}} (1 - e^{2ia} (cx^n)^{2ib})^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n))\right) \text{Subst}\left(\int \frac{x^{-1+\frac{3ib}{2}+\frac{1+m}{n}}}{(1-e^{2ia}x^{2ib})^{3/2}} dx\right)}{n}$$

$$= \frac{2x^{1+m} (1 - e^{2ia} (cx^n)^{2ib})^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n)) {}_2F_1\left(\frac{3}{2}, \frac{1}{4}\left(3 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-7bn}{4bn}; e^{2ia}\right)}{2 + 2m + 3ibn}$$

**Mathematica [B]** time = 9.53125, size = 466, normalized size = 3.58

$$x^{-ibn+m+1} \left( (b^2n^2 + 4m^2 + 8m + 4) x^{2ibn} \sqrt{2 - 2e^{2ia} (cx^n)^{2ib}} \sqrt{\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}} \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{i\left(\frac{3ibn}{2}+m+1\right)}{2bn}, -\frac{-7bn+2i}{4bn}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Csc[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (x^(1 + m - I\*b\*n))\*((4 + 8\*m + 4\*m^2 + b^2\*n^2)\*x^((2\*I)\*b\*n)\*Sqrt[2 - 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[(I\*E^(I\*a)\*(c\*x^n)^(I\*b))/(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Hypergeometric2F1[1/2, ((-I/2)\*(1 + m + ((3\*I)/2)\*b\*n))/(b\*n), -(2\*I + (2\*I)\*m - 7\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)] + (-2\*I - (2\*I)\*m + 3\*b\*n)\*((-2\*I - (2\*I)\*m + b\*n)\*Sqrt[2 - 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[(I\*E^(I\*a)\*(c\*x^n)^(I\*b))/(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m + b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)] - 2\*x^(I\*b\*n)\*Sqrt[Csc[a + b\*Log[c\*x^n]]\*(b\*n\*Cos[b\*n\*Log[x]] - 2\*(1 + m)\*Sin[b\*n\*Log[x]])])/(b\*n\*(-2\*I - (2\*I)\*m + 3\*b\*n)\*(b\*n\*Cos[a - b\*n\*Log[x] + b\*Log[c\*x^n]] + 2\*(1 + m)\*Sin[a - b\*n\*Log[x] + b\*Log[c\*x^n]]))

**Maple [F]** time = 0.298, size = 0, normalized size = 0.

$$\int x^m (\csc(a + b \ln(cx^n)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*csc(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(x^m\*csc(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \csc(b \log(cx^n) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*csc(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] `integrate(x^m*csc(b*log(c*x^n) + a)^(3/2), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*csc(a+b*ln(c*x**n))**(3/2),x)`

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")`

[Out] Timed out

### 3.325 $\int x^m \sqrt{\csc(a + b \log(cx^n))} dx$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \sqrt{1 - e^{2ia} (cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{-bn+2im+2i}{4bn}, -\frac{-5bn+2im+2i}{4bn}, e^{2ia} (cx^n)^{2ib}\right)}{ibn + 2m + 2}$$

[Out] (2\*x^(1 + m)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Csc[a + b\*Log[c\*x^n]])\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]]/(2 + 2\*m + I\*b\*n)

**Rubi [A]** time = 0.0924365, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4510, 4508, 364}

$$\frac{2x^{m+1} \sqrt{1 - e^{2ia} (cx^n)^{2ib}} {}_2F_1\left(\frac{1}{2}, -\frac{2im-bn+2i}{4bn}; -\frac{2im-5bn+2i}{4bn}; e^{2ia} (cx^n)^{2ib}\right) \sqrt{\csc(a + b \log(cx^n))}}{ibn + 2m + 2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sqrt[Csc[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x^(1 + m)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Csc[a + b\*Log[c\*x^n]])\*Hypergeometric2F1[1/2, -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 5\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]]/(2 + 2\*m + I\*b\*n)

#### Rule 4510

Int[Csc[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^p, x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int x^m \sqrt{\csc(a + b \log(cx^n))} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1+\frac{1+m}{n}} \sqrt{\csc(a + b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{-\frac{ib}{2}-\frac{1+m}{n}} \sqrt{1 - e^{2ia} (cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{ib}{2}+\frac{1+m}{n}}}{\sqrt{1 - e^{2ia} x^{2ib}}} dx, x, cx^n\right)}{n}$$

$$= \frac{2x^{1+m} \sqrt{1 - e^{2ia} (cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))} {}_2F_1\left(\frac{1}{2}, -\frac{2i+2im-bn}{4bn}; -\frac{2i+2im-5bn}{4bn}; e^{2ia} (cx^n)^{2ib}\right)}{2 + 2m + ibn}$$

**Mathematica [A]** time = 0.969139, size = 138, normalized size = 1.06

$$\frac{2e^{-2ia} x^{m+1} (cx^n)^{-2ib} (-1 + e^{2ia} (cx^n)^{2ib}) \sqrt{\csc(a + b \log(cx^n))} \text{Hypergeometric2F1}\left(1, \frac{3bn+2im+2i}{4bn}, \frac{5bn+2im+2i}{4bn}, e^{-2i(a+b \log(cx^n))}\right)}{-ibn + 2m + 2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m\*Sqrt[Csc[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x^(1 + m)\*(-1 + E^((2\*I)\*a))\*(c\*x^n)^((2\*I)\*b))\*Sqrt[Csc[a + b\*Log[c\*x^n]]]\*Hypergeometric2F1[1, (2\*I + (2\*I)\*m + 3\*b\*n)/(4\*b\*n), (2\*I + (2\*I)\*m + 5\*b\*n)/(4\*b\*n), E^((-2\*I)\*(a + b\*Log[c\*x^n]))]/(E^((2\*I)\*a)\*(2 + 2\*m - I\*b\*n)\*(c\*x^n)^((2\*I)\*b))

**Maple [F]** time = 0.289, size = 0, normalized size = 0.

$$\int x^m \sqrt{\csc(a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*csc(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(x^m\*csc(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\csc(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*csc(a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(x^m\*sqrt(csc(b\*log(c\*x^n) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\csc(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(csc(a + b*log(c*x**n))), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.326 \quad \int \frac{x^m}{\sqrt{\csc(a+b \log(cx^n))}} dx$$

**Optimal.** Leaf size=129

$$\frac{2x^{m+1} \text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{bn+2im+2i}{4bn}, -\frac{-3bn+2im+2i}{4bn}, e^{2ia} (cx^n)^{2ib}\right)}{(-ibn + 2m + 2)\sqrt{1 - e^{2ia} (cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))}}$$

[Out] (2\*x^(1 + m)\*Hypergeometric2F1[-1/2, -(2\*I + (2\*I)\*m + b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 + 2\*m - I\*b\*n)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Csc[a + b\*Log[c\*x^n]]])

**Rubi [A]** time = 0.0883437, antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4510, 4508, 364}

$$\frac{2x^{m+1} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-\frac{2i(m+1)}{bn} - 1\right); -\frac{2im-3bn+2i}{4bn}; e^{2ia} (cx^n)^{2ib}\right)}{(-ibn + 2m + 2)\sqrt{1 - e^{2ia} (cx^n)^{2ib}} \sqrt{\csc(a + b \log(cx^n))}}$$

Antiderivative was successfully verified.

[In] Int[x^m/Sqrt[Csc[a + b\*Log[c\*x^n]]], x]

[Out] (2\*x^(1 + m)\*Hypergeometric2F1[-1/2, (-1 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - 3\*b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 + 2\*m - I\*b\*n)\*Sqrt[1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]\*Sqrt[Csc[a + b\*Log[c\*x^n]]])

#### Rule 4510

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x\_]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))]^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{x^m}{\sqrt{\csc(a+b \log(cx^n))}} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\sqrt{\csc(a+b \log(x))}} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{\frac{ib}{2}-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1-\frac{ib}{2}+\frac{1+m}{n}} \sqrt{1-e^{2ia}x^{2ib}} dx, x, cx^n\right)}{n\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b \log(cx^n))}}$$

$$= \frac{2x^{1+m} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left(-1-\frac{2i(1+m)}{bn}\right); -\frac{2i+2im-3bn}{4bn}; e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b \log(cx^n))}}$$

**Mathematica [B]** time = 7.34828, size = 441, normalized size = 3.42

$$\frac{2e^{ia}bnx^{m+1} (cx^n)^{m+1} \sin(a+b \log(cx^n) - bn \log(x))}{\sqrt{\csc(a+b \log(cx^n))} (2(m+1) \sin(a+b \log(cx^n) - bn \log(x)) + bn \cos(a+b \log(cx^n) - bn \log(x)))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m/Sqrt[Csc[a + b\*Log[c\*x^n]]], x]

[Out]  $(-2*b*E^{(I*a)*n}*x^{(1+m)}*(c*x^n)^{(I*b)}*\text{Sqrt}[2-2*E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}}]*\text{Sqrt}[(I*E^{(I*a)*(c*x^n)^{(I*b)}})/(-1+E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}})]*((2*I+(2*I)*m+b*n)*x^{((2*I)*b*n)}*\text{Hypergeometric2F1}[1/2, ((-I/2)*(1+m+(3*I)/2)*b*n)/(b*n), -(2*I+(2*I)*m-7*b*n)/(4*b*n), E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}}] + (-2*I-(2*I)*m+3*b*n)*\text{Hypergeometric2F1}[1/2, -(2*I+(2*I)*m+b*n)/(4*b*n), -(2*I+(2*I)*m-3*b*n)/(4*b*n), E^{((2*I)*a)*(c*x^n)^{((2*I)*b)}}])/(2+2*m-I*b*n)*(2+2*m+(3*I)*b*n)*((2*I+(2*I)*m+b*n)*x^{((2*I)*b*n)} + E^{((2*I)*a)*(-2*I-(2*I)*m+b*n)*(c*x^n)^{((2*I)*b)}}) + (2*x^{(1+m)}*\text{Sin}[a-b*n*\text{Log}[x]+b*\text{Log}[c*x^n]])/(\text{Sqrt}[Csc[a+b*\text{Log}[c*x^n]]])*(b*n*\text{Cos}[a-b*n*\text{Log}[x]+b*\text{Log}[c*x^n]] + 2*(1+m)*\text{Sin}[a-b*n*\text{Log}[x]+b*\text{Log}[c*x^n]]))$

**Maple [F]** time = 0.279, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{\csc(a+b \ln(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/csc(a+b\*ln(c\*x^n))^(1/2), x)

[Out] int(x^m/csc(a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\csc(b \log(cx^n) + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x<sup>m</sup>/csc(a+b\*log(c\*x<sup>n</sup>))<sup>(1/2)</sup>,x, algorithm="maxima")

[Out] integrate(x<sup>m</sup>/sqrt(csc(b\*log(c\*x<sup>n</sup>) + a)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/csc(a+b\*log(c\*x<sup>n</sup>))<sup>(1/2)</sup>,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\csc(a + b \log(cx^n))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/csc(a+b\*ln(c\*x\*\*n))\*\*(1/2),x)

[Out] Integral(x\*\*m/sqrt(csc(a + b\*log(c\*x\*\*n))), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/csc(a+b\*log(c\*x<sup>n</sup>))<sup>(1/2)</sup>,x, algorithm="giac")

[Out] Timed out

$$3.327 \quad \int \frac{x^m}{\csc^{\frac{3}{2}}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=130

$$\frac{2x^{m+1} \text{Hypergeometric2F1}\left(-\frac{3}{2}, -\frac{3bn+2im+2i}{4bn}, -\frac{-bn+2im+2i}{4bn}, e^{2ia} (cx^n)^{2ib}\right)}{(-3ibn + 2m + 2) \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n))}$$

[Out] (2\*x^(1 + m)\*Hypergeometric2F1[-3/2, -(2\*I + (2\*I)\*m + 3\*b\*n)/(4\*b\*n), -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 + 2\*m - (3\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Csc[a + b\*Log[c\*x^n]]^(3/2))

**Rubi [A]** time = 0.0943862, antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {4510, 4508, 364}

$$\frac{2x^{m+1} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-\frac{2i(m+1)}{bn} - 3\right); -\frac{2im-bn+2i}{4bn}; e^{2ia} (cx^n)^{2ib}\right)}{(-3ibn + 2m + 2) \left(1 - e^{2ia} (cx^n)^{2ib}\right)^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x^m/Csc[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^(1 + m)\*Hypergeometric2F1[-3/2, (-3 - ((2\*I)\*(1 + m))/(b\*n))/4, -(2\*I + (2\*I)\*m - b\*n)/(4\*b\*n), E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)])/((2 + 2\*m - (3\*I)\*b\*n)\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^(3/2)\*Csc[a + b\*Log[c\*x^n]]^(3/2))

#### Rule 4510

Int[Csc[(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[(a\_.) + Log[x]\*(b\_.)]\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))]^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a + b \log(cx^n))} dx = \frac{\left(x^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{\csc^{\frac{3}{2}}(a+b \log(x))} dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x^{1+m} (cx^n)^{\frac{3ib}{2}-\frac{1+m}{n}}\right) \text{Subst}\left(\int x^{-1-\frac{3ib}{2}+\frac{1+m}{n}} (1 - e^{2ia} x^{2ib})^{3/2} dx, x, cx^n\right)}{n(1 - e^{2ia} (cx^n)^{2ib})^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n))}$$

$$= \frac{2x^{1+m} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{2i(1+m)}{bn}\right); -\frac{2i+2im-bn}{4bn}; e^{2ia} (cx^n)^{2ib}\right)}{(2 + 2m - 3ibn)(1 - e^{2ia} (cx^n)^{2ib})^{3/2} \csc^{\frac{3}{2}}(a + b \log(cx^n))}$$

**Mathematica [A]** time = 2.40706, size = 218, normalized size = 1.68

$$\frac{2x^{m+1} \left(3e^{-2ia} b^2 n^2 (cx^n)^{-2ib} (-1 + e^{2ia} (cx^n)^{2ib}) \csc^2(a + b \log(cx^n)) \text{Hypergeometric2F1}\left(1, \frac{3bn+2im+2i}{4bn}, \frac{5bn+2im+2i}{4bn}, e^{-2ia} (cx^n)^{2ib}\right)\right)}{(-ibn + 2m + 2)(-3ibn + 2m + 2)(3ibn + 2m + 2) \csc^{\frac{3}{2}}(a + b \log(cx^n))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^m/Csc[a + b\*Log[c\*x^n]]^(3/2), x]

[Out] (2\*x^(1 + m)\*((2 + 2\*m - I\*b\*n)\*(2 + 2\*m - 3\*b\*n\*Cot[a + b\*Log[c\*x^n]])) + (3\*b^2\*n^2\*(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))\*Csc[a + b\*Log[c\*x^n]]^2\*Hypergeometric2F1[1, (2\*I + (2\*I)\*m + 3\*b\*n)/(4\*b\*n), (2\*I + (2\*I)\*m + 5\*b\*n)/(4\*b\*n), E^((-2\*I)\*(a + b\*Log[c\*x^n]))])/(E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))/((2 + 2\*m - I\*b\*n)\*(2 + 2\*m - (3\*I)\*b\*n)\*(2 + 2\*m + (3\*I)\*b\*n)\*Csc[a + b\*Log[c\*x^n]]^(3/2))

**Maple [F]** time = 0.307, size = 0, normalized size = 0.

$$\int x^m (\csc(a + b \ln(cx^n)))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/csc(a+b\*ln(c\*x^n))^(3/2), x)

[Out] int(x^m/csc(a+b\*ln(c\*x^n))^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\csc(b \log(cx^n) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/csc(a+b\*log(c\*x^n))^(3/2), x, algorithm="maxima")

[Out] integrate(x^m/csc(b\*log(c\*x^n) + a)^(3/2), x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/csc(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.328 $\int (ex)^m \csc^p(d(a + b \log(cx^n))) dx$

**Optimal.** Leaf size=139

$$\frac{(ex)^{m+1} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^p \csc^p(d(a + b \log(cx^n))) \operatorname{Hypergeometric2F1}\left(p, -\frac{bdnp+im+i}{2bdn}, \frac{1}{2}\left(-\frac{i(m+1)}{bdn} + p + 2\right), e^{2iad}\right)}{e(ibdn + m + 1)}$$

[Out]  $((e*x)^{(1+m)}*(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p * Csc[d*(a + b*Log[c*x^n])]^p * \operatorname{Hypergeometric2F1}[p, -(I + I*m - b*d*n*p)/(2*b*d*n), (2 - (I*(1 + m))/(b*d*n) + p)/2, E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}]/(e*(1 + m + I*b*d*n*p))$

**Rubi [A]** time = 0.113112, antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4510, 4508, 364}

$$\frac{(ex)^{m+1} \left(1 - e^{2iad} (cx^n)^{2ibd}\right)^p {}_2F_1\left(p, \frac{1}{2}\left(p - \frac{i(m+1)}{bdn}\right); \frac{1}{2}\left(-\frac{i(m+1)}{bdn} + p + 2\right); e^{2iad} (cx^n)^{2ibd}\right) \csc^p(d(a + b \log(cx^n)))}{e(ibdn + m + 1)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^m * Csc[d*(a + b*Log[c*x^n])]^p, x]$

[Out]  $((e*x)^{(1+m)}*(1 - E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)})^p * Csc[d*(a + b*Log[c*x^n])]^p * \operatorname{Hypergeometric2F1}[p, (((-I)*(1 + m))/(b*d*n) + p)/2, (2 - (I*(1 + m))/(b*d*n) + p)/2, E^{((2*I)*a*d)}*(c*x^n)^{((2*I)*b*d)}]/(e*(1 + m + I*b*d*n*p))$

#### Rule 4510

$\operatorname{Int}[Csc[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] :> \operatorname{Dist}[(e*x)^{(m+1)}/(e*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[x^{((m+1)/n-1)}*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, m, n, p\}, x\} \&\& (\operatorname{NeQ}[c, 1] \parallel \operatorname{NeQ}[n, 1])$

#### Rule 4508

$\operatorname{Int}[Csc[(a_.) + \operatorname{Log}[x_]* (b_.)]*(d_.)]^{(p_.)}*((e_.)*(x_.))^{(m_.)}, x\_Symbol] :> \operatorname{Dist}[(Csc[d*(a + b*Log[x])]^p * (1 - E^{(2*I*a*d)}*x^{(2*I*b*d)})^p)/x^{(I*b*d*p)}, \operatorname{Int}[(e*x)^m * x^{(I*b*d*p)}]/(1 - E^{(2*I*a*d)}*x^{(2*I*b*d)})^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, d, e, m, p\}, x\} \&\& !\operatorname{IntegerQ}[p]$

#### Rule 364

$\operatorname{Int}[(c_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] :> \operatorname{Simp}[(a^p * (c*x)^{(m+1)} * \operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -(b*x^n)/a])/ (c*(m+1)), x] /;$   $\operatorname{FreeQ}\{a, b, c, m, n, p\}, x\} \&\& !\operatorname{IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \parallel \operatorname{GtQ}[a, 0])$

#### Rubi steps

$$\begin{aligned} \int (ex)^m \csc^p(d(a + b \log(cx^n))) dx &= \frac{\left( (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{n}} \csc^p(d(a + b \log(x))) dx, x, cx^n \right)}{en} \\ &= \frac{\left( (ex)^{1+m} (cx^n)^{-\frac{1+m}{n}-ibdp} \left( 1 - e^{2iad} (cx^n)^{2ibd} \right)^p \csc^p(d(a + b \log(cx^n))) \right) \text{Subst} \left( \int x^{-1+\frac{1+m}{n}-ibdp} \csc^p(d(a + b \log(x))) dx, x, cx^n \right)}{en} \\ &= \frac{(ex)^{1+m} \left( 1 - e^{2iad} (cx^n)^{2ibd} \right)^p \csc^p(d(a + b \log(cx^n))) {}_2F_1 \left( p, \frac{1}{2} \left( -\frac{i(1+m)}{bdn} + p \right); \frac{1}{2} \left( 2 - \frac{i(1+m)}{bdn} + p \right); e^{2iad} (cx^n)^{2ibd} \right)}{e(1 + m + ibdnp)} \end{aligned}$$

**Mathematica [A]** time = 1.69334, size = 169, normalized size = 1.22

$$\frac{x(ex)^m \left( 2 - 2e^{2iad} (cx^n)^{2ibd} \right)^p \left( \frac{ie^{iad} (cx^n)^{ibd}}{-1 + e^{2iad} (cx^n)^{2ibd}} \right)^p \text{Hypergeometric2F1} \left( p, -\frac{i(bdnp+m+1)}{2bdn}, \frac{1}{2} \left( -\frac{i(m+1)}{bdn} + p + 2 \right), e^{2iad} (cx^n)^{2ibd} \right)}{ibdnp + m + 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e\*x)^m\*Csc[d\*(a + b\*Log[c\*x^n])]^p,x]

[Out] (x\*(e\*x)^m\*(2 - 2\*E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d))^p\*((I\*E^(I\*a\*d)\*(c\*x^n)^((I\*b\*d)))/(-1 + E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)))^p\*Hypergeometric2F1[p, ((-I/2)\*(1 + m + I\*b\*d\*n\*p))/(b\*d\*n), (2 - (I\*(1 + m))/(b\*d\*n) + p)/2, E^((2\*I)\*a\*d)\*(c\*x^n)^((2\*I)\*b\*d)]/(1 + m + I\*b\*d\*n\*p)

**Maple [F]** time = 0.283, size = 0, normalized size = 0.

$$\int (ex)^m (\csc(d(a + b \ln(cx^n))))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^m\*csc(d\*(a+b\*ln(c\*x^n)))^p,x)

[Out] int((e\*x)^m\*csc(d\*(a+b\*ln(c\*x^n)))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \csc((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^m\*csc(d\*(a+b\*log(c\*x^n)))^p,x, algorithm="maxima")

[Out] integrate((e\*x)^m\*csc((b\*log(c\*x^n) + a)\*d)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (ex)^m \csc(bd \log(cx^n) + ad)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*csc(b*d*log(c*x^n) + a*d)^p, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*csc(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \csc((b \log(cx^n) + a)d)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^p, x)
```

### 3.329 $\int x \csc^p(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=106

$$\frac{x^2 (1 - e^{2ia} (cx^n)^{2ib})^p \operatorname{Hypergeometric2F1}\left(p, \frac{1}{2}\left(p - \frac{2i}{bn}\right), \frac{1}{2}\left(-\frac{2i}{bn} + p + 2\right), e^{2ia} (cx^n)^{2ib}\right) \csc^p(a + b \log(cx^n))}{2 + ibnp}$$

[Out] (x^2\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*Csc[a + b\*Log[c\*x^n]]^p\*Hypergeometric2F1[p, ((-2\*I)/(b\*n) + p)/2, (2 - (2\*I)/(b\*n) + p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(2 + I\*b\*n\*p)

**Rubi [A]** time = 0.0755096, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {4510, 4508, 364}

$$\frac{x^2 (1 - e^{2ia} (cx^n)^{2ib})^p {}_2F_1\left(p, \frac{1}{2}\left(p - \frac{2i}{bn}\right); \frac{1}{2}\left(p - \frac{2i}{bn} + 2\right); e^{2ia} (cx^n)^{2ib}\right) \csc^p(a + b \log(cx^n))}{2 + ibnp}$$

Antiderivative was successfully verified.

[In] Int[x\*Csc[a + b\*Log[c\*x^n]]^p, x]

[Out] (x^2\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*Csc[a + b\*Log[c\*x^n]]^p\*Hypergeometric2F1[p, ((-2\*I)/(b\*n) + p)/2, (2 - (2\*I)/(b\*n) + p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(2 + I\*b\*n\*p)

#### Rule 4510

Int[Csc[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(e\*x)^(m + 1)/(e\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[x^((m + 1)/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[((a\_.) + Log[x]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\begin{aligned} \int x \csc^p(a + b \log(cx^n)) dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int x^{-1+\frac{2}{n}} \csc^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(x^2 (cx^n)^{-\frac{2}{n}-ibp} (1 - e^{2ia} (cx^n)^{2ib})^p \csc^p(a + b \log(cx^n))) \text{Subst}\left(\int x^{-1+\frac{2}{n}+ibp} (1 - e^{2ia} x^{\frac{2}{n}})^p dx, x, cx^n\right)}{n} \\ &= \frac{x^2 (1 - e^{2ia} (cx^n)^{2ib})^p \csc^p(a + b \log(cx^n)) {}_2F_1\left(p, \frac{1}{2}\left(-\frac{2i}{bn} + p\right); \frac{1}{2}\left(2 - \frac{2i}{bn} + p\right); e^{2ia} (cx^n)^{2ib}\right)}{2 + ibnp} \end{aligned}$$

**Mathematica [A]** time = 1.11793, size = 142, normalized size = 1.34

$$\frac{ix^2 (2 - 2e^{2ia} (cx^n)^{2ib})^p \left(\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}\right)^p \text{Hypergeometric2F1}\left(\frac{p}{2} - \frac{i}{bn}, p, -\frac{i}{bn} + \frac{p}{2} + 1, e^{2ia} (cx^n)^{2ib}\right)}{bnp - 2i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*Csc[a + b\*Log[c\*x^n]]^p,x]

[Out] ((-I)\*x^2\*(2 - 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*((I\*E^(I\*a)\*(c\*x^n)^(I\*b))/(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*Hypergeometric2F1[(-I)/(b\*n) + p/2, p, 1 - I/(b\*n) + p/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/(-2\*I + b\*n\*p)

**Maple [F]** time = 0.232, size = 0, normalized size = 0.

$$\int x (\csc(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*csc(a+b\*ln(c\*x^n))^p,x)

[Out] int(x\*csc(a+b\*ln(c\*x^n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \csc(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*csc(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] integrate(x\*csc(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x \csc(b \log(cx^n) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(x*csc(b*log(c*x^n) + a)^p, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \csc^p(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(x*csc(a + b*log(c*x**n))**p, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \csc(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(x*csc(b*log(c*x^n) + a)^p, x)
```

### 3.330 $\int \csc^p(a + b \log(cx^n)) dx$

**Optimal.** Leaf size=107

$$\frac{x(1 - e^{2ia}(cx^n)^{2ib})^p \operatorname{Hypergeometric2F1}\left(p, -\frac{-bnp+i}{2bn}, \frac{1}{2}\left(-\frac{i}{bn} + p + 2\right), e^{2ia}(cx^n)^{2ib}\right) \csc^p(a + b \log(cx^n))}{1 + ibnp}$$

[Out] (x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*Csc[a + b\*Log[c\*x^n]]^p\*Hypergeometric2F1[p, -(I - b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) + p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(1 + I\*b\*n\*p)

**Rubi [A]** time = 0.0664191, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {4504, 4508, 364}

$$\frac{x(1 - e^{2ia}(cx^n)^{2ib})^p {}_2F_1\left(p, -\frac{i-bnp}{2bn}; \frac{1}{2}\left(p - \frac{i}{bn} + 2\right); e^{2ia}(cx^n)^{2ib}\right) \csc^p(a + b \log(cx^n))}{1 + ibnp}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b\*Log[c\*x^n]]^p, x]

[Out] (x\*(1 - E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*Csc[a + b\*Log[c\*x^n]]^p\*Hypergeometric2F1[p, -(I - b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) + p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b))]/(1 + I\*b\*n\*p)

#### Rule 4504

Int[Csc[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(d\_.)]^(p\_.), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[x^(1/n - 1)\*Csc[d\*(a + b\*Log[x])]^p, x], x, c\*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])

#### Rule 4508

Int[Csc[((a\_.) + Log[x\_]\*(b\_.))\*(d\_.)]^(p\_.)\*((e\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(Csc[d\*(a + b\*Log[x])]^p\*(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p]/x^(I\*b\*d\*p), Int[((e\*x)^m\*x^(I\*b\*d\*p))/(1 - E^(2\*I\*a\*d)\*x^(2\*I\*b\*d))^p, x], x] /; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \csc^p(a + b \log(cx^n)) dx &= \frac{(x (cx^n)^{-1/n}) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}} \csc^p(a + b \log(x)) dx, x, cx^n\right)}{n} \\ &= \frac{(x (cx^n)^{-\frac{1}{n}-ibp} (1 - e^{2ia} (cx^n)^{2ib})^p \csc^p(a + b \log(cx^n))) \operatorname{Subst}\left(\int x^{-1+\frac{1}{n}+ibp} (1 - e^{2ia} x^{2ib})^{-p} dx, x, cx^n\right)}{n} \\ &= \frac{x (1 - e^{2ia} (cx^n)^{2ib})^p \csc^p(a + b \log(cx^n)) {}_2F_1\left(p, -\frac{i-bnp}{2bn}; \frac{1}{2}\left(2 - \frac{i}{bn} + p\right); e^{2ia} (cx^n)^{2ib}\right)}{1 + ibnp} \end{aligned}$$

**Mathematica [A]** time = 0.869853, size = 142, normalized size = 1.33

$$\frac{ix (2 - 2e^{2ia} (cx^n)^{2ib})^p \left(\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}\right)^p \operatorname{Hypergeometric2F1}\left(p, \frac{bnp-i}{2bn}, \frac{1}{2}\left(-\frac{i}{bn} + p + 2\right), e^{2ia} (cx^n)^{2ib}\right)}{bnp - i}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[a + b\*Log[c\*x^n]]^p, x]

[Out] ((-I)\*x\*(2 - 2\*E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*((I\*E^(I\*a)\*(c\*x^n)^(I\*b))/(-1 + E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)))^p\*Hypergeometric2F1[p, (-I + b\*n\*p)/(2\*b\*n), (2 - I/(b\*n) + p)/2, E^((2\*I)\*a)\*(c\*x^n)^((2\*I)\*b)]/(-I + b\*n\*p)

**Maple [F]** time = 0.208, size = 0, normalized size = 0.

$$\int (\csc(a + b \ln(cx^n)))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(a+b\*ln(c\*x^n))^p, x)

[Out] int(csc(a+b\*ln(c\*x^n))^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(a+b\*log(c\*x^n))^p, x, algorithm="maxima")

[Out] integrate(csc(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\csc(b \log(cx^n) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral(csc(b*log(c*x^n) + a)^p, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc^p(a + b \log(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*ln(c*x**n))**p,x)
```

```
[Out] Integral(csc(a + b*log(c*x**n))**p, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \csc(b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(csc(b*log(c*x^n) + a)^p, x)
```



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58             If[Head[expn]===Plus || Head[expn]===Times,
59               Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60             If[ElementaryFunctionQ[Head[expn]],
61               Max[3,ExpnType[expn[[1]]],
62             If[SpecialFunctionQ[Head[expn]],
63               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64             If[HypergeometricFunctionQ[Head[expn]],
65               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66             If[AppellFunctionQ[Head[expn]],
67               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68             If[Head[expn]===RootSum,
69               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70             If[Head[expn]===Integrate || Head[expn]===Int,
71               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72             9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82   },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+`') or type(expn,'*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #     Port of original Maple grading function by
3 #     Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #     added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124     else:
125         return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34         #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```



```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```